Modern and Digital Control Systems

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Modern and Digital Control Systems

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2. Write the transfer functions in polynomial format.
3. Write the transfer functions in factored format.
4. Write the transfer functions in the partial fraction format.
5. Draw the state diagram of the initial state space format.
6. Write the state space model in equation format (A, B, C, D).
7. Write the state space model in matrix format (A, B, C, D).
8. Evidence and a check on the Observability and Controllability of the Transfer functions (newly added) .</td></tr>
<tr><td>Task 1</td><td>Chapter 3: Decomposition of Transfer Functions</td><td>Decomposition to Controllability Canonical Form.</td><td>A) Draw the state diagram.
B) Derive the state space model in equations format.
C) Write the state space model in matrix format (A, B, C, D).
D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.
E) Compare the converted transfer function with the original transfer function that you started with.
F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.
G) Find the characteristic equation $|sI - A|$.
H) Find the Eigenvalues of matrix A.</td></tr>
<tr><td>Task 2</td><td>Chapter 3: Decomposition of Transfer Functions</td><td>Decomposition to Observability Canonical Form.</td><td>A) Draw the state diagram.
B) Derive the state space model in equations format.
C) Write the state space model in matrix format (A, B, C, D).
D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.
E) Compare the converted transfer function with the original transfer function that you started with.
F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.
G) Find the characteristic equation $|sI - A|$.</td></tr>
</table>

			H) Find the Eigenvalues of matrix A.
Task 3	**Chapter 3: Decomposition of Transfer Functions**	**Cascaded Decomposition to Upper Triangular A Matrix.**	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$. D) Convert the resulting state space model to transfer function format using the equation $\boldsymbol{C}(\boldsymbol{sI}-\boldsymbol{A})^{-1}\,\boldsymbol{B}+\boldsymbol{D}$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert \boldsymbol{sI}-\boldsymbol{A}\rvert$. H) Find the Eigenvalues of matrix A.
Task 4	**Chapter 3: Decomposition of Transfer Functions**	**Cascaded Decomposition to lower triangular A matrix.**	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$. D) Convert the resulting state space model to transfer function format using the equation $\boldsymbol{C}(\boldsymbol{sI}-\boldsymbol{A})^{-1}\,\boldsymbol{B}+\boldsymbol{D}$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert \boldsymbol{sI}-\boldsymbol{A}\rvert$. H) Find the Eigenvalues of matrix A.
Task 5	**Chapter 3: Decomposition of Transfer Functions**	**Diagonal Decomposition Based on Controllability Canonical Form (If Applicable)**	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$. D) Convert the resulting state space model to transfer function format using the equation $\boldsymbol{C}(\boldsymbol{sI}-\boldsymbol{A})^{-1}\,\boldsymbol{B}+\boldsymbol{D}$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert \boldsymbol{sI}-\boldsymbol{A}\rvert$. H) Find the Eigenvalues of matrix A.

Task 6	**Chapter 3: Decomposition of Transfer Functions**	**Diagonal Decomposition Based on Observability Canonical Form (If Applicable).**	**A) Draw the state diagram.** **B) Derive the state space model in equations format.** **C) Write the state space model in matrix format (A, B, C, D).** **D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\,B + D$ in details.** **E) Compare the converted transfer function with the original transfer function that you started with.** **F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **G) Find the characteristic equation $\|sI - A\|$.** **H) Find the Eigenvalues of matrix A.**
Task 9	**Chapter 4: Similarity Transformations for State Space Models**	**Similarity Transformation to Controllability Canonical Form.**	**A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.** **B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).** **C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.** **D) Compare the converted transfer function with the original transfer function that you started with.** **E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **F) Find the characteristic equation $\|sI - A\|$.** **A) Find the Eigenvalues values of matrix (A).**
Task 10	**Chapter 4: Similarity Transformations for State Space Models**	**Similarity Transformation to Observability Canonical Form.**	**A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.** **B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).** **C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.** **D) Compare the converted transfer function with the original transfer function that you started with.** **E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **F) Find the characteristic equation $\|sI - A\|$.** **Find the Eigenvalues values of matrix (A).**

Task 11	**Chapter 4: Similarity Transformations for State Space Models**	**Similarity Transformation to Diagonal Canonical Form (If Applicable)**	**A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.** **B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).** **C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.** **D) Compare the converted transfer function with the original transfer function that you started with.** **E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **F) Find the characteristic equation $\|sI - A\|$.** **G) Find the Eigenvalues values of matrix (A).**
Task 13	**Chapter 6: Similarity Transformations for State Space Models**	**Controllability Tests and Observability Tests based on decomposed DCF**	**A) Perform the controllability test based on the state diagram of decomposed MIMO control system.** **B) Perform the Observability test based on the state diagram of decomposed MIMO control system.** **C) Perform the controllability test using Gilbert's Test for decomposed (A, B, C,D).** **D) Perform the observability test using Gilbert's Test for decomposed (A, B, C,D).** **E) Perform the controllability test using Kalman's Test for decomposed (A, B, C,D).** **F) Perform the observability test using Kalman's Test for decomposed (A, B, C,D).** **G) Perform the controllability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt)** **H) Perform the observability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt)** **I) Perform the controllability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt).** **J) Perform the observability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt).**

1. General Requirements for all tasks

Table 2 General Requirements Description

Task Number	Chapter	Task Name	Task Description
Task 0	---	General Requirements for All Tasks	1. Generate your own random transfer functions. 2. Write the transfer functions in polynomial format. 3. Write the transfer functions in factored format. 4. Write the transfer functions in the partial fraction format. 5. Draw the state diagram of the initial state space format. 6. Write the state space model in equation format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$. 7. Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$.

1.1 Transfer Functions

Therefore, the generated Transfer Functions based on previously requirements:

1)$M(s) = \frac{Y(s)}{U(s)} = \frac{(s+2)(s+4)(s+9)}{(s+5)(s+7)(s+3)(s+1)}$ (controllable and observable)

2) $M(s) = \frac{Y(s)}{U(s)} = \frac{(s+14)(s+1)(s+3)}{(s+5)(s+7)(s+3)(s+1)}$ (uncontrollable and unobservable)

3) $M(s) = \frac{Y(s)}{U(s)} = \frac{(s+7)(s+12)(s+4)}{(s+5)(s+7)(s+3)(s+1)}$ (uncontrollable and observable)

4) $M(s) = \frac{Y(s)}{U(s)} = \frac{(s+16)(s+6)(s+3)}{(s+5)(s+3)(s+7)(s+1)}$ (controllable and unobservable)

1.2 Transfer Functions in Polynomial Format

Consider for this case the order of the numerator polynomial is less than the order of the denominator polynomial by at least one degree as below:

$$\boldsymbol{\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + b_{n-3}s^{n-3} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} + \cdots + a_1 s + a_0}} \quad (\mathbf{1.2.1})$$

So, we rewrite the transfer function as follows:

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

1.3 Transfer Function in Factored Format

1.The general form of factored format for the numerator and denominator of any transfer function is as follows:

$$\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \boldsymbol{K}\frac{(s+z_1)(s+z_2)(s+z_3)\dots(s+z_{n-2})(s+z_{n-1})}{(s+p_1)^k(s+p_{k+1})\dots\ (s+p_{n-2})(s+p_{n-1})(s+p_n)} \qquad (\mathbf{1.3.1})$$

2. Decompose the factored transfer function into n terms of the following form

$$\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \boldsymbol{M_1} \times \boldsymbol{M_2} \times \dots \times \boldsymbol{M_n} = \boldsymbol{K}\frac{s+z_1}{s+p_1} + \frac{s+z_2}{s+p_1} + \cdots + \frac{1}{s+p_n} \qquad (\mathbf{1.3.2})$$

So, we rewrite the transfer functions in factored format as follows:

$$1)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+2}{s+5}\right)\left(\frac{s+4}{s+7}\right)\left(\frac{s+9}{s+3}\right)\left(\frac{1}{s+1}\right)$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+14}{s+5}\right)\left(\frac{1}{s+7}\right)\left(\frac{s+3}{s+3}\right)\left(\frac{s+1}{s+1}\right)$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+7}{s+5}\right)\left(\frac{s+12}{s+7}\right)\left(\frac{s+4}{s+3}\right)\left(\frac{1}{s+1}\right)$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+16}{s+5}\right)\left(\frac{s+6}{s+3}\right)\left(\frac{s+3}{s+7}\right)\left(\frac{1}{s+1}\right)$$

1.4 Transfer Function in the Partial Fraction Format

Consider this case some of roots of the characteristic equation are real repeated (multiple-order eigenvalues). If we consider that ***the first root is repeated k times***, then the transfer function is written as:

$$\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + b_{n-3}s^{n-3} + \cdots + b_1 s + b_0}{(s+p_1)^k(s+p_2)\dots\ (s+p_{n-1})(s+p_n)}$$

$$= \frac{B_1}{s+p_1} + \frac{B_2}{s+p_2} + \cdots + \frac{B_k}{s+p_k} + \sum_{i=k+1}^{n}\frac{r_i}{s+p_i} \qquad (\mathbf{1.4.1})$$

So, we rewrite the transfer functions in partial fraction format as follows:

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{0.75}{(s+5)} + \frac{-0.625}{(s+7)} + \frac{0.375}{(s+3)} + \frac{0.5}{(s+1)}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{4.5}{(s+5)} + \frac{-3.5}{(s+7)} + \frac{0}{(s+3)} + \frac{0}{(s+1)}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{-0.875}{(s+5)}+\frac{0}{(s+7)}+\frac{-2.25}{(s+3)}+\frac{4.125}{(s+1)}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{-1.375}{(s+5)}+\frac{-0.75}{(s+7)}+\frac{0}{(s+3)}+\frac{3.125}{(s+1)}$$

1.5 Draw the state diagram of the initial State Space Format

According to general requirements to using ***Cascaded Lower Triangular Matrix Form of initial state space format***. We draw the state space diagram in OCF format using the transfer function in factored format:

Its to be noted that:

- ❖ **The first transfer function is the one with the yellow color.**
- ❖ **The second transfer function is the one with the blue color.**
- ❖ **The third transfer function is the one with the red color.**
- ❖ **The fourth transfer function is the one with the green color.**
- ❖ **The Parts with the Black color are the ones common with all Transfer functions.**

- ❖ **And therefore, the State diagram of the Fourth order MIMO system with initial form: Cascaded lower triangle is Drawn here on a landscape:**

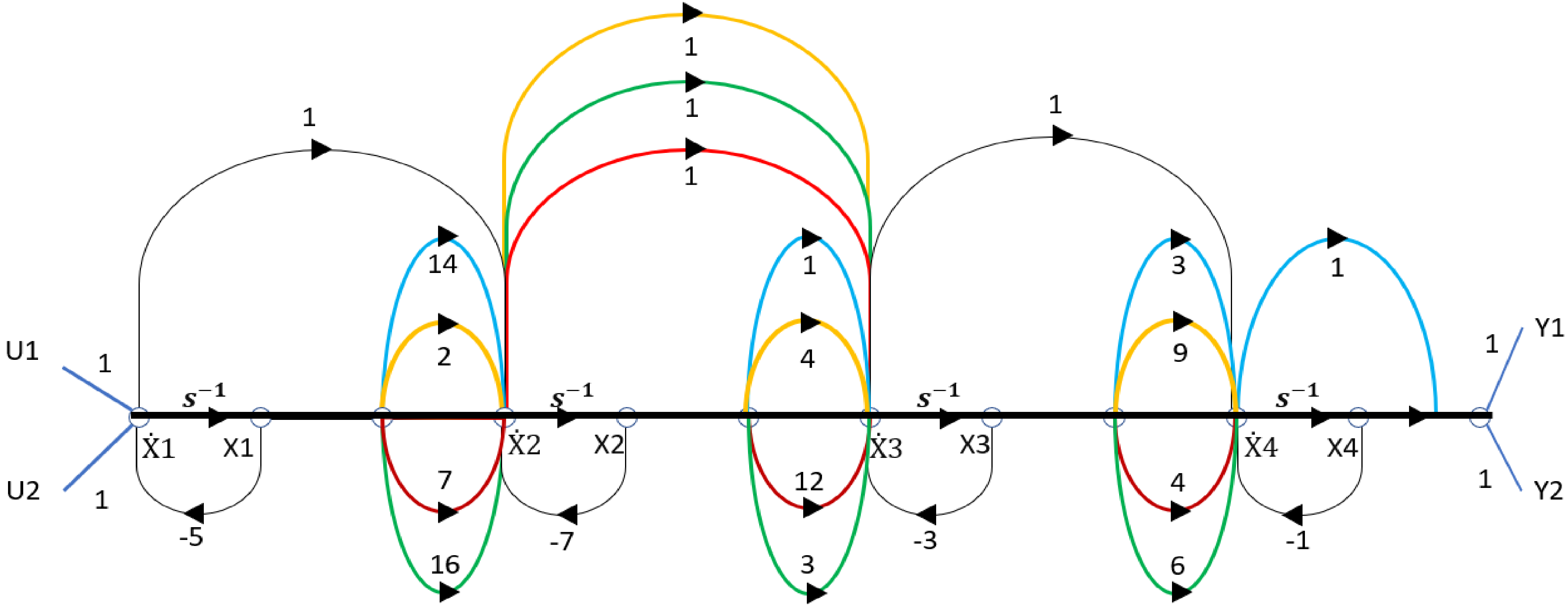
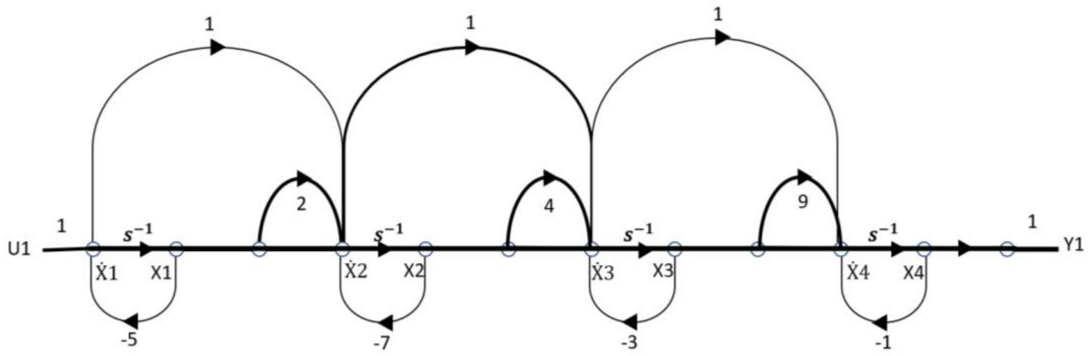


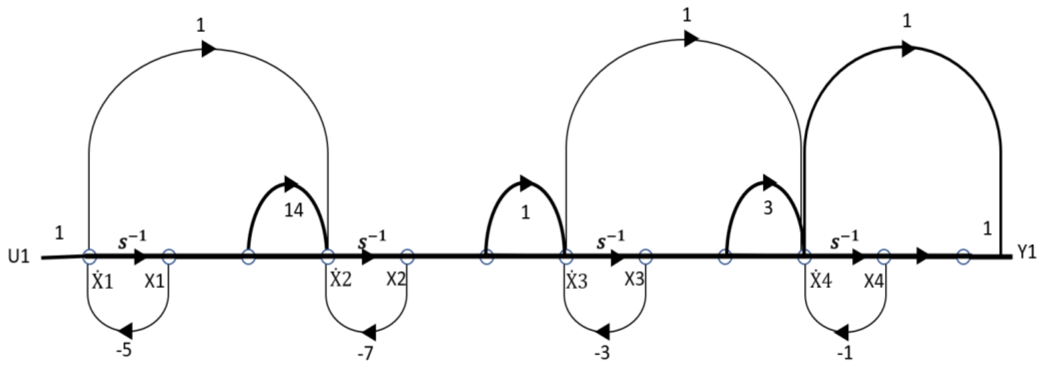
Figure 1 MIMO State-Space Diagram for General Transfer Function

For the SISO Transfer functions:

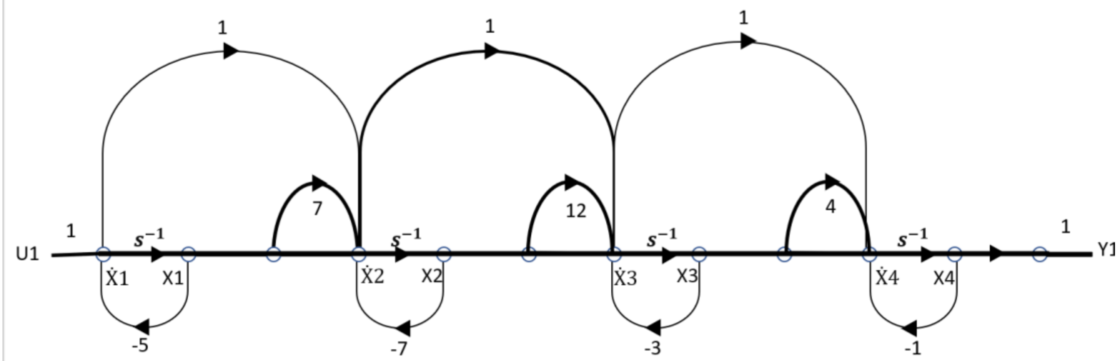
$$1)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+2}{s+5}\right)\left(\frac{s+4}{s+7}\right)\left(\frac{s+9}{s+3}\right)\left(\frac{1}{s+1}\right)$$



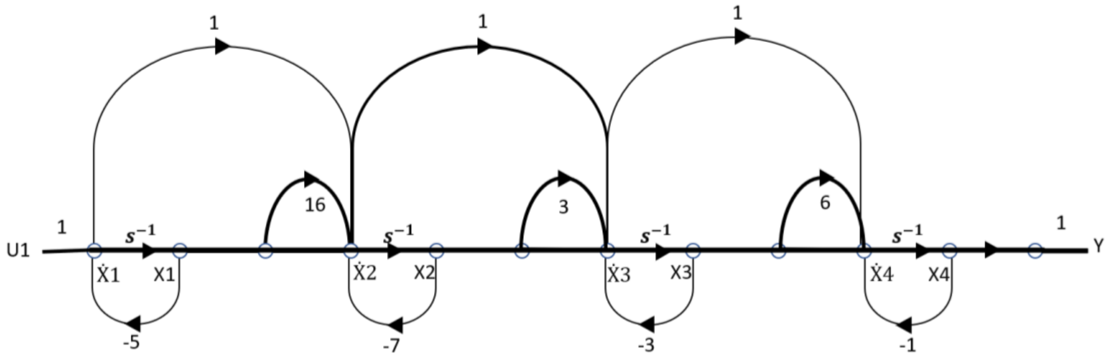
$$2)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+14}{s+5}\right)\left(\frac{1}{s+7}\right)\left(\frac{s+3}{s+3}\right)\left(\frac{s+1}{s+1}\right)$$



$$3)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+7}{s+5}\right)\left(\frac{s+12}{s+7}\right)\left(\frac{s+4}{s+3}\right)\left(\frac{1}{s+1}\right)$$



$$4) M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+16}{s+5}\right)\left(\frac{s+6}{s+3}\right)\left(\frac{s+3}{s+7}\right)\left(\frac{1}{s+1}\right)$$



1.6 Write the State Space model in Equation Format (A, B, C, D).

Throughout the previous section 1.5, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_{11}\,x_1 + a_{12}\,x_2 + \cdots + a_{1n}\,x_n + b_1\,u \\
\dot{x}_2 &= a_{21}\,x_1 + a_{22}\,x_2 + \cdots + a_{2n}\,x_n + b_2\,u \\
&\;\vdots \\
\dot{x}_n &= a_{n1}\,x_1 + a_{n2}\,x_2 + \cdots + a_{nn}\,x_n + b_n\,u \\
y &= a_1\,x_1 + a_2\,x_2 + \cdots + a_n\,x_n + b\,u
\end{aligned} \qquad (1.6.1)$$

The derived state space equations from for the **Error! Reference source not found.** as the following:

For MIMO:

$$\begin{aligned}
\dot{x}_1 &= -5x_1 + u \\
\dot{x}_2 &= 34x_1 - 7x_2 + u \\
\dot{x}_3 &= 102x_1 - x_2 - 3x_3 + 3u \\
\dot{x}_4 &= 102x_1 - x_2 - 19x_3 - x_4 + 3u \\
y &= 102x_1 - x_2 + 19x_3
\end{aligned}$$

For SISO:

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -5x_1 + u \\
\dot{x}_2 &= -3x_1 - 7x_2 + u \\
\dot{x}_3 &= -3x_1 - 3x_2 - 3x_3 + u \\
\dot{x}_4 &= -3x_1 - 3x_2 + 6x_3 - x_4 + u \\
y &= x_4
\end{aligned}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 9\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = \boldsymbol{x}_2 - 3\boldsymbol{x}_3$
$\dot{\boldsymbol{x}}_4 = \boldsymbol{x}_2 - \boldsymbol{x}_4$
$\boldsymbol{y} = \boldsymbol{x}_2$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 144s + 252}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 2\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = 2\boldsymbol{x}_1 + 5\boldsymbol{x}_2 - 3\boldsymbol{x}_3 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_4 = 2\boldsymbol{x}_1 + 5\boldsymbol{x}_2 + \boldsymbol{x}_3 - \boldsymbol{x}_4 + \boldsymbol{u}$
$\boldsymbol{y} = \boldsymbol{x}_4$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 11\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = 11\boldsymbol{x}_1 - 4\boldsymbol{x}_2 - 3\boldsymbol{x}_3 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_4 = 11\boldsymbol{x}_1 - 4\boldsymbol{x}_2 + 3\boldsymbol{x}_3 - \boldsymbol{x}_4 + \boldsymbol{u}$
$\boldsymbol{y} = \boldsymbol{x}_4$

1.7 Write the State Space model in Matrix Format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned}\dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A}_{n\times n}\,\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_{n\times 1}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C}_{1\times n}\,\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}_{1\times 1}\boldsymbol{u}(\boldsymbol{t})\end{aligned} \qquad (\mathbf{1.7.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$A = \begin{array}{c} \\ \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{nn} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \end{array} \qquad B = \begin{array}{c} \\ \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{array} \begin{array}{c} u \\ \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \end{array} \qquad (1.7.2)$$

$$C = y \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ [c_1 \quad c_2 \quad \cdots \quad c_n] \end{array} \qquad D = y \begin{array}{c} u \\ [d] \end{array}$$

Consider the derived state space, the state space diagram's matrix format as:

For MIMO:

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 34 & -7 & 0 & 0 \\ 102 & -1 & -3 & 0 \\ 102 & -1 & 19 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 3 & 3 \\ 3 & 3 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 102 & -1 & 19 & 0 \\ 102 & -1 & 19 & 0 \end{bmatrix} x(t) + [0]u(t).$$

For SISO:

$$\mathbf{1)}\boldsymbol{M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ -3 & -7 & 0 & 0 \\ -3 & -3 & -3 & 0 \\ -3 & -3 & 6 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0 \quad 0 \quad 0 \quad 1]\, x(t) + [0]u(t).$$

$$\mathbf{2)}\boldsymbol{M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 9 & -7 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [\,0 \quad 1 \quad 0 \quad 0]\, x(t) + [0]u(t).$$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) \ = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 2 & -7 & 0 & 0 \\ 2 & 5 & -3 & 0 \\ 2 & 5 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 0 \quad 0 \quad 0 \quad 1]\, x(t) + [0]u(t).$$

$$\mathbf{4)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) \ = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 11 & -7 & 0 & 0 \\ 11 & -4 & -3 & 0 \\ 11 & -4 & 3 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 0 \quad 0 \quad 0 \quad 1]\, x(t) + [0]u(t).$$

1.8 Using MATLAB to check the viability of the transfer functions:

- The first T.f can be visually checked based on the rule:

If there is no pole-zero cancellation in the closed loop transfer function of the control system, then the corresponding state space model is always completely state controllable and completely state observable regardless of the construction of the state diagram.

- The 2nd T.f is automatically uncontrollable and unobservable because we have 2 zero-pole cancellations and 0 repeated roots.
- The Third T.f :

```
>> A=[-5 0 0 0;2 -7 0 0;2 5 -3 0; 2 5 1 -1];
B=[1;1;1;1];
C=[0 0 0 1];
D=0;
>> Qc=ctrb(A,B);
>> rank(Qc)

ans =

     3

>> Q0=obsv(A,C);
>> rank(Q0)

ans =

     4

>>
```

- The 4Th T.f:

```
>> A=[-5 0 0 0 ;11 -3 0 0 ; 11 3 -7 0; 11 3 -4 -1];
>> B=[1;1;1;1];
>> C=[0 0 0 1];
>> D=0;
>> Qc=ctrb(A,B);
>> rank(Qc)

ans =

     4

>> Qo=obsv(A,C);
>> rank(Qo)

ans =

     3

>>
```

Figure 2 MATLAB Checks the viability of The Transfer functions.

2.1 Decomposition to controllability canonical Form.

Table 3 Task 1 Description

Task Number	Chapter	Task Name	Task Description
Task 1	**Chapter 3:** Decomposition of Transfer Functions	Decomposition to Controllability Canonical Form.	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1} B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert sI - A \rvert$. H) Find the Eigenvalues of matrix A.

2.1.1 Draw the state diagram.

To draw the state diagram, multiply the transfer function by s^{-5} for the numerator and denominator, yielding the following form:

$$\boldsymbol{\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 + a_{n-1}s^{-1} + a_{n-2}s^{-2} + a_{n-3}s^{-3} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}}} \qquad (\mathbf{2.1.1.1})$$

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 144s^{-3} + 288s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

Then, using (-1) as a common factor from the denominator, we get the following form:

$$\boldsymbol{\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 - [-a_{n-1}s^{-1} - a_{n-2}s^{-2} - a_{n-3}s^{-3} - \cdots - a_1 s^{-n+1} - a_0 s^{-n}]}} \qquad (\mathbf{2.1.1.2})$$

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+18s^{-2}+59s^{-3}+42s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+23s^{-2}+160s^{-3}+336s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+162s^{-3}+288s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

The CCF State-Space Diagram is shown in **Error! Reference source not found.**.

Its to be noted that:

- **The first transfer function is the one with the yellow color.**
- **The second transfer function is the one with the blue color.**
- **The third transfer function is the one with the red color.**
- **The fourth transfer function is the one with the green color.**
- **The Parts with the Black color are the ones common with all Transfer functions.**

- **And therefore, the State diagram of the Fourth order MIMO system with initial form: Cascaded lower triangle is Drawn here on a landscape:**

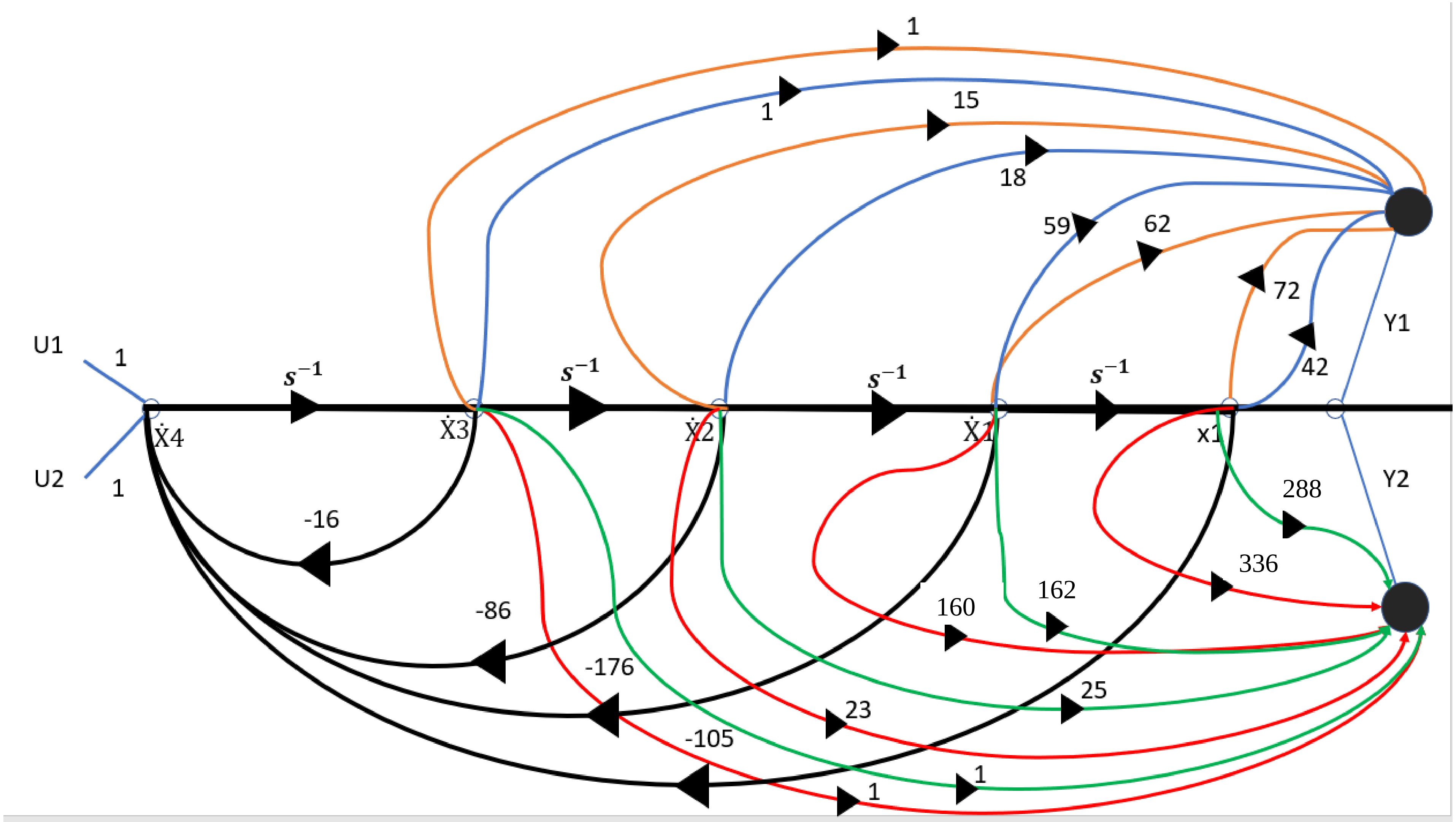


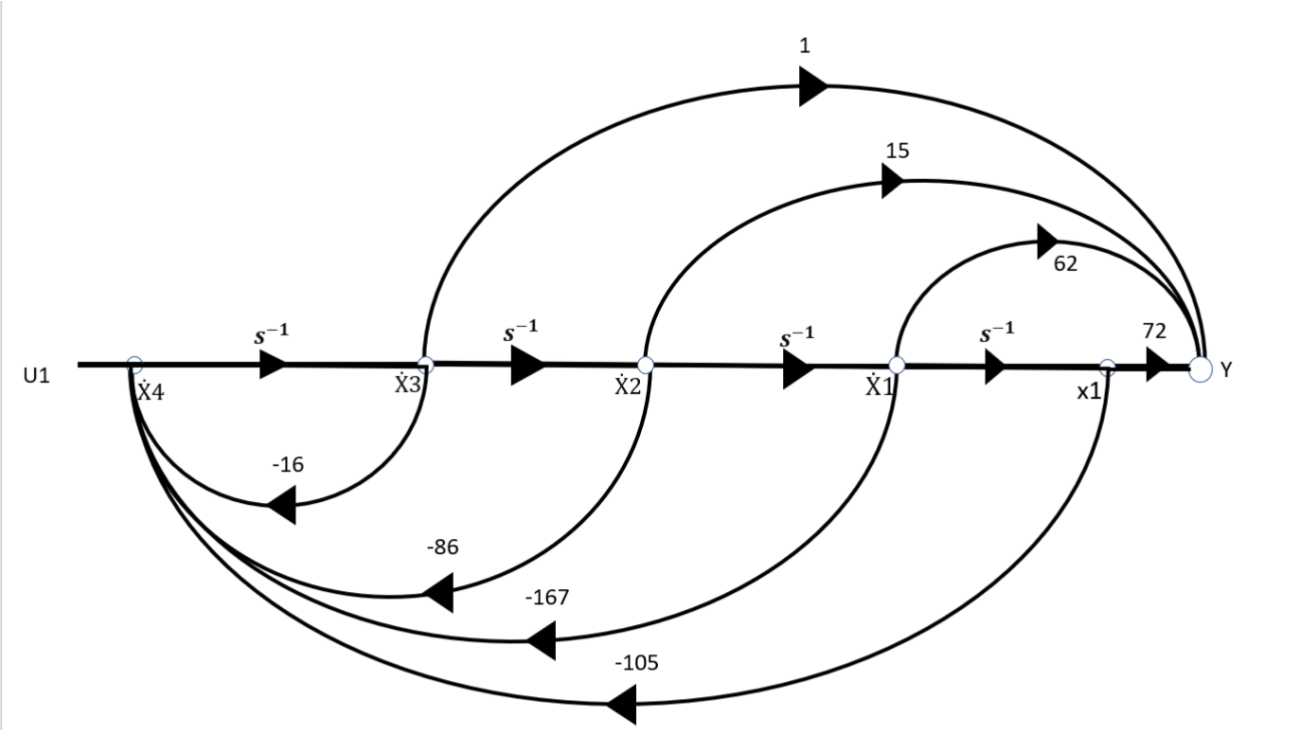
Figure 3 State Diagram of Controllability Canonical Form

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

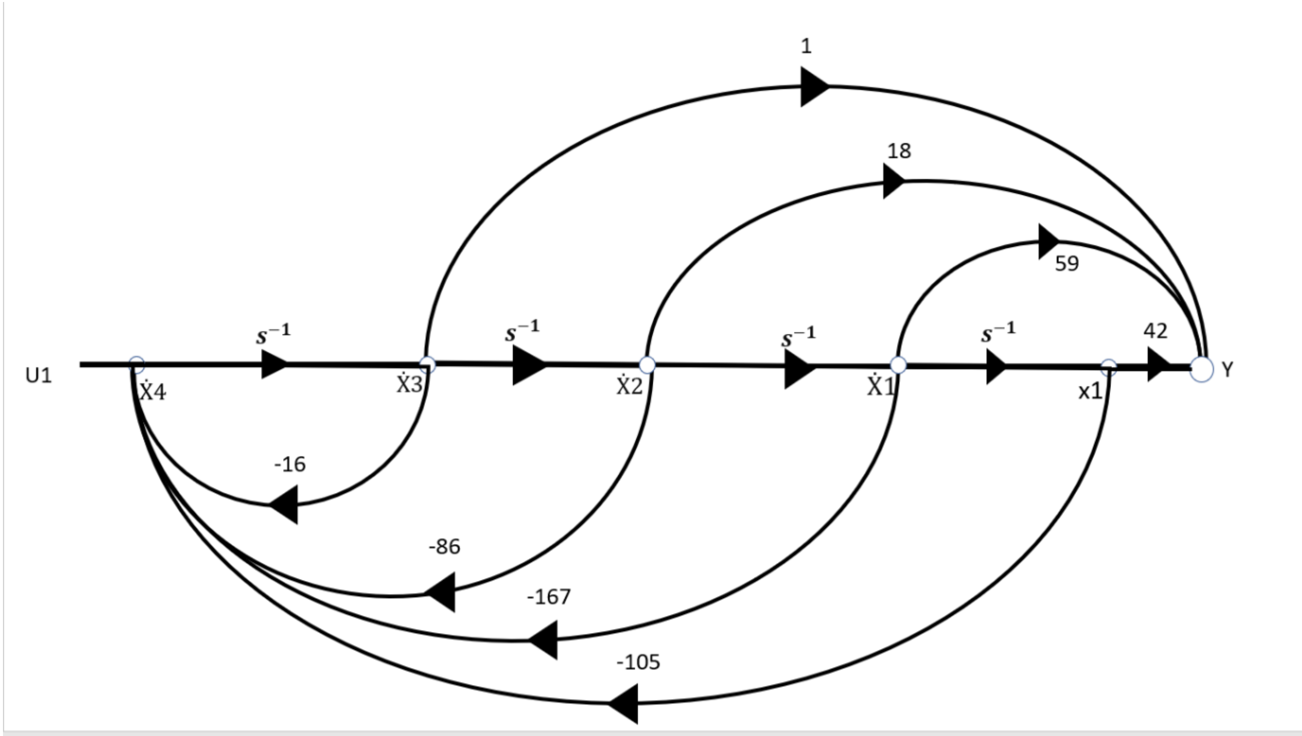
Evidence:

```
From input 1 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

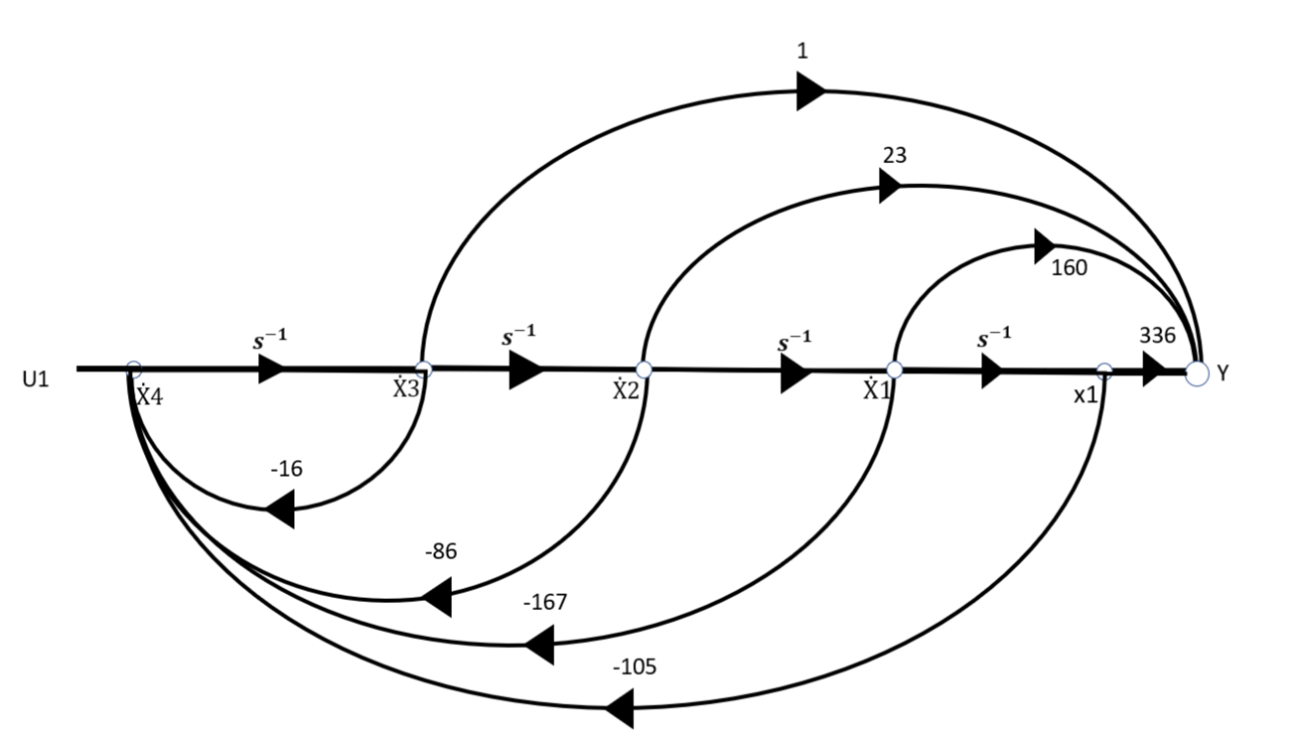
$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$



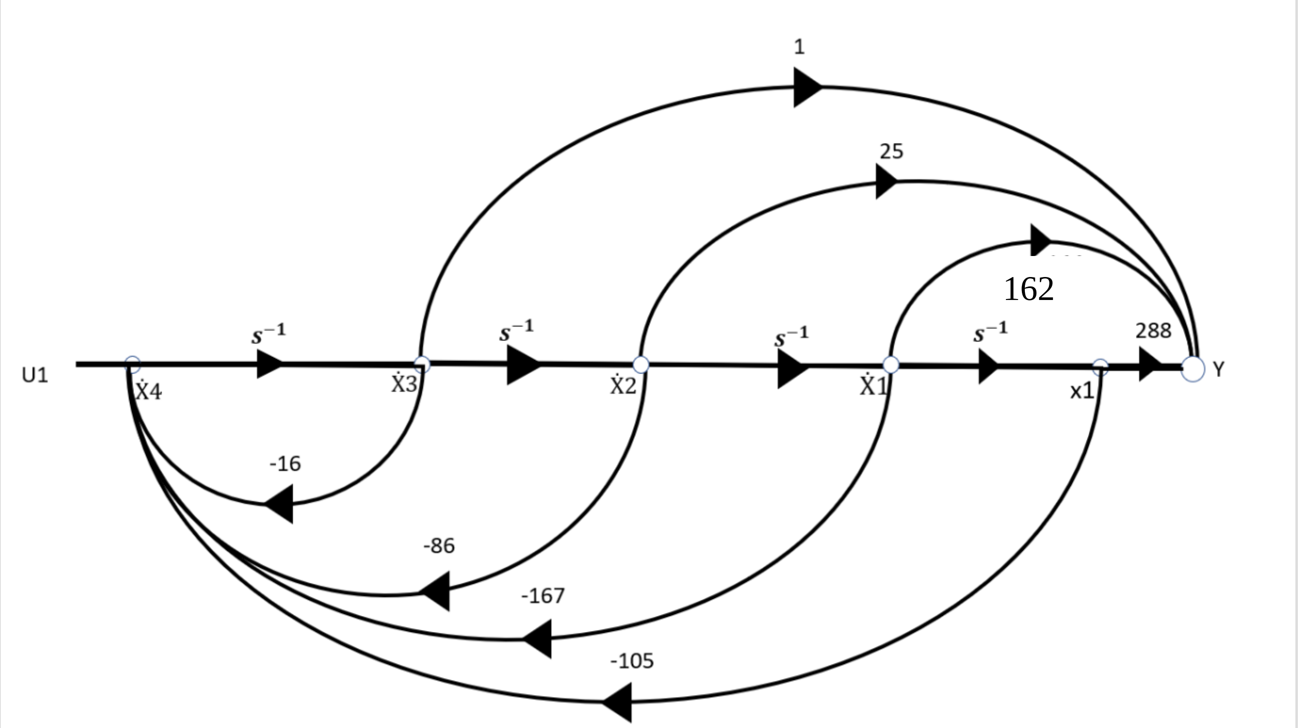
$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+18s^{-2}+59s^{-3}+42s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$



$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+23s^{-2}+144s^{-3}+252s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$



$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+162+288s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$



Derive the state space model in equations format.

Throughout the previous section 2.1.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\vdots\\ \dot{x}_n &= -a_0x_1-a_1x_2-a_2x_3-\cdots-a_{n-1}x_n+u\\ y &= b_0x_1+b_1x_2+b_2x_3-\cdots-b_{n-1}x_n\end{aligned} \qquad (2.1.2.1)$$

The derived state space equation from for the **Error! Reference source not found.** as the following:

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$\dot{x}_1=x_2$$
$$\dot{x}_2=x_3$$
$$\dot{x}_3=x_4$$

$$\dot{x}_4=-105x_1-176x_2-86\,x_3-16x_4+u$$

$$y_1=72\,x_1+62x_2+15x_3+x_4$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -105x_1 - 176x_2 - 86\,x_3 - 16x_4 + u$$

$$y_1 = 42\,x_1 + 59x_2 + 18 + x_4$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -105x_1 - 176x_2 - 86\,x_3 - 16x_4 + u$$

$$y_1 = 336\,x_1 + 160x_2 + 23x_3 + x_4$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -105x_1 - 176x_2 - 86\,x_3 - 16x_4 + u$$

$$y_1 = 288\,x_1 + 162x_2 + 25x_3 + x_4$$

2.1.2 Write the state space model in matrix format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\boxed{\begin{aligned} \dot{x}(t) &= A_{CFF}\,x(t) + B_{CCF}u(t) \\ y(t) &= C_{CFF}\,x(t) + D_{CCF}u(t) \end{aligned} \qquad (2.1.3.1)}$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$A_{CCF} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \qquad B_{CCF} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad (2.1.3.2)$$

$$C_{CCF} = [b_0 \quad b_1 \quad \cdots \quad b_{n-2} \quad b_{n-1}] \qquad D_{CCF} = [0]$$

Consider the derived state space, the state space diagram's matrix format as:

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 72 \quad 62 \quad 15 \quad 1]x(t) + [0]u(t)$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 42 \quad 59 \quad 18 \quad 1]x(t) + [0]u(t)$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 162s + 252}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 336 \quad 160 \quad 23 \quad 1]x(t) + [0]u(t)$$

$$4)\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 288 \quad 162 \quad 25 \quad 1]x(t) + [0]u(t)$$

2.1.3 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\ B + D$ in details.

$$\boldsymbol{C(sI - A)^{-1}B + D} \qquad (\mathbf{2.1.4.1})$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI - A}] = \begin{bmatrix} \boldsymbol{s} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s} & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{s} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a_0} & -\boldsymbol{a_1} & -\boldsymbol{a_2} & \dots & -\boldsymbol{a_{n-1}} \end{bmatrix} \qquad (\mathbf{2.1.4.2})$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 105 & 167 & 86 & s + 16 \end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[\boldsymbol{sI - A}]^{-1} = \frac{\boldsymbol{adj(sI - A)}}{\boldsymbol{det(sI - A)}} \quad \dots \quad \frac{\mathbf{1}}{\boldsymbol{det(sI - A)}} * [\boldsymbol{cof(sI - A)}]^{\boldsymbol{T}} \qquad (\mathbf{2.1.4.3})$$

- **[P]=** $\frac{\mathbf{1}}{\boldsymbol{det(sI-A)}} * [\mathbf{cof(sI - A)}]^{\mathbf{T}}$

$$1)\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+167s+105}\times[p]$$

$$p=\begin{bmatrix} s^3+16s^2+86s+176 & s^2+16s+86 & s+16 & 1 \\ -105 & s(s^2+16s+86) & s(s+16) & s \\ -105s & -176s-105 & s^2(s+16) & s^2 \\ -105s^2 & -s(176s+105) & -86s^2-176s-105 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+167s+105}\times[72 \quad 62 \quad 15 \quad 1]\times[p]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$=\frac{[(72s^3+1047s^2+4617s+5514) \quad (62s^3+897s^2+3874s+4617) \quad (15s^3+216s^2+897s+1047) \quad (s+2)(s+4)(s+9)]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+2)(s+4)(s+9)}{s^4+16s^3+86s^2+176s+105}$$

$$\mathbf{2)}\boldsymbol{M(s)}=\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}=\frac{\boldsymbol{s^3+18s^2+59s+42}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} s^3+16s^2+86s+176 & s^2+16s+86 & s+16 & 1 \\ -105 & s(s^2+16s+86) & s(s+16) & s \\ -105s & -176s-105 & s^2(s+16) & s^2 \\ -105s^2 & -s(176s+105) & -86s^2-176s-105 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+167s+105}\times[42 \quad 59 \quad 18 \quad 1]\times[p]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$=\frac{[(42s^3+567s^2+1722s+819) \quad (59s^3+819s^2+2635s+1722) \quad (18s^3+261s^2+819s+567) \quad (s+1)(s+3)(s+14)]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{s^4+16s^3+86s^2+167s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+1)(s+3)(s+14)}{s^4+16s^3+86s^2+167s+105}$$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} s^3 + 16s^2 + 86s + 176 & s^2 + 16s + 86 & s + 16 & 1 \\ -105 & s(s^2 + 16s + 86) & s(s + 16) & s \\ -105s & -176s - 105 & s^2(s + 16) & s^2 \\ -105s^2 & -s(176s + 105) & -86s^2 - 176s - 105 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [336 \quad 160 \quad 23 \quad 1] \times [p] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{[(252s^3 + 3927s^2 + 19257s + 26964) \quad (144s^3 + 2389s^2 + 12470s + 19257) \quad (23s^3 + 426s^2 + 2389s + 3927) \quad (s + 4)(s + 7)(s + 12)] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 167s + 105}$$

$$C(sI - A)^{-1}\ B + D = \frac{(s + 4)(s + 7)(s + 12)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\mathbf{4)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} s^3 + 16s^2 + 86s + 176 & s^2 + 16s + 86 & s + 16 & 1 \\ -105 & s(s^2 + 16s + 86) & s(s + 16) & s \\ -105s & -176s - 105 & s^2(s + 16) & s^2 \\ -105s^2 & -s(176s + 105) & -86s^2 - 176s - 105 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [288 \quad 162 \quad 25 \quad 1] \times [p] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{[(288s^3 + 4503s^2 + 22143s + 32976) \quad (144s^3 + 2425s^2 + 12712s + 22143) \quad (25s^3 + 458s^2 + 2424s + 4503) \quad (s + 16)(s + 6)(s + 3)] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 167s + 105}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$(sI - A)^{-1}\ B + D = \frac{(s + 16)(s + 6)(s + 3)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2.1.4 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Functions (Polynomial):

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Functions(Polynomial):

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

2.1.5 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

1. First Transfer function:

```
Command Window
>> At1=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -167 -86 -16];
Bt1=[0;0;0;1];
Ct1=[72 62 15 1];
Dt1=0;
>> [num_t1,den_t1]=ss2tf(At1,Bt1,Ct1,Dt1);
>> Mt1=tf(num_t1,den_t1)

Mt1 =

        s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 167 s + 105

Continuous-time transfer function.
```

```
>> Ai=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

        s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

2. Second Transfer function:

Command Window

```
>>
>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
Ct=[42 59 18 1];
Dt=0;
[num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
M_t=tf(num_t,den_t)

M_t =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> Ai=[-5 0 0 0;9 -7 0 0;0 1 -3 0;0 1 0 -1];
Bi=[1;1;0;0];
Ci=[0 1 0 0];
Di=0;
[num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
M_i=tf(num_i,den_i)

M_i =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

3rd Transfer function:

Command Window

```
>> clear
>> Ai=[-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=[0];
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
>> Ct=[336 160 23 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> M_t=tf(num_t,den_t)

M_t =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

4th Transfer function:

```
Command Window
>> clear
>> Ai=[-5 0 0 0 ;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
>> Ct=[288 144 25 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> M_t=tf(num_t,den_t)

M_t =

      s^3 + 25 s^2 + 144 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 4 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to CCF state-space

Table 3 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**		The **coefficients of Denominator**
The Original TF	1)	[1 15 62 72]	[1 16 66 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	
The Converted TF	1)	[1 15 62 72]	[1 16 66 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	

2.1.6 Find the characteristic equation $|sI - A|$.

We Will do it for the 4 SISO systems Initially, we apply the equation $|sI - A|$, which returns:

$$|s\boldsymbol{I}-\boldsymbol{A}| = \begin{vmatrix} s & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & s & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & s \end{vmatrix} - \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a}_0 & -\boldsymbol{a}_1 & -\boldsymbol{a}_2 & \dots & -\boldsymbol{a}_{n-1} \end{vmatrix} \qquad (\mathbf{2.1.7.1})$$

$$|sI-A| = 0$$

$$\begin{vmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 105 & 167 & 86 & s+16 \end{vmatrix}$$

$$|sI-A| = \begin{vmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 105 & 167 & 86 & s+16 \end{vmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.1.7.2})$$

2.1.7 Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda\boldsymbol{I}-\boldsymbol{A}| = \begin{vmatrix} \lambda & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \lambda \end{vmatrix} - \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a}_0 & -\boldsymbol{a}_1 & -\boldsymbol{a}_2 & \dots & -\boldsymbol{a}_{n-1} \end{vmatrix} \qquad (\mathbf{2.1.8.1})$$

$$|\lambda I-A| = 0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda+16 \end{vmatrix}$$

$$|\lambda I-A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda+16 \end{vmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.1.8.2})$$

Then we use the **MATLAB** command **<u>"roots"</u>** and get the following eigenvalues:

$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -1,$

2.2 Decomposition to observability canonical Form.

Table 4 Task 2 Description

Task Number	Chapter	Task Name	Task Description
Task 2	**Chapter 3:** Decomposition of Transfer Functions	Decomposition to Observability Canonical Form.	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\|sI - A\|$. H) Find the Eigenvalues of matrix A.

Draw the state diagram.

To draw the state diagram, multiply the transfer function by s^{-5} for the numerator and denominator, yielding the following form:

$$\boldsymbol{\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 + a_{n-1}s^{-1} + a_{n-2}s^{-2} + a_{n-3}s^{-3} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}}} \qquad \mathbf{(2.1.1.1)}$$

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 162s^{-3} + 288s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

Then, using (-1) as a common factor from the denominator, we get the following form:

$$\boldsymbol{\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 - [-a_{n-1}s^{-1} - a_{n-2}s^{-2} - a_{n-3}s^{-3} - \cdots - a_1 s^{-n+1} - a_0 s^{-n}]}} \qquad \mathbf{(2.1.1.2)}$$

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 162s^{-3} + 288s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

The CCF State-Space Diagram is shown in **Error! Reference source not found.**.

It's to be noted that:

- **The first transfer function is the one with the yellow color.**
- **The second transfer function is the one with the blue color.**
- **The third transfer function is the one with the red color.**
- **The fourth transfer function is the one with the green color.**
- **The Parts with the Black color are the ones common with all Transfer functions.**

- **And therefore, the State diagram of the Fourth order MIMO system with initial form: Cascaded lower triangle is Drawn here on a landscape:**

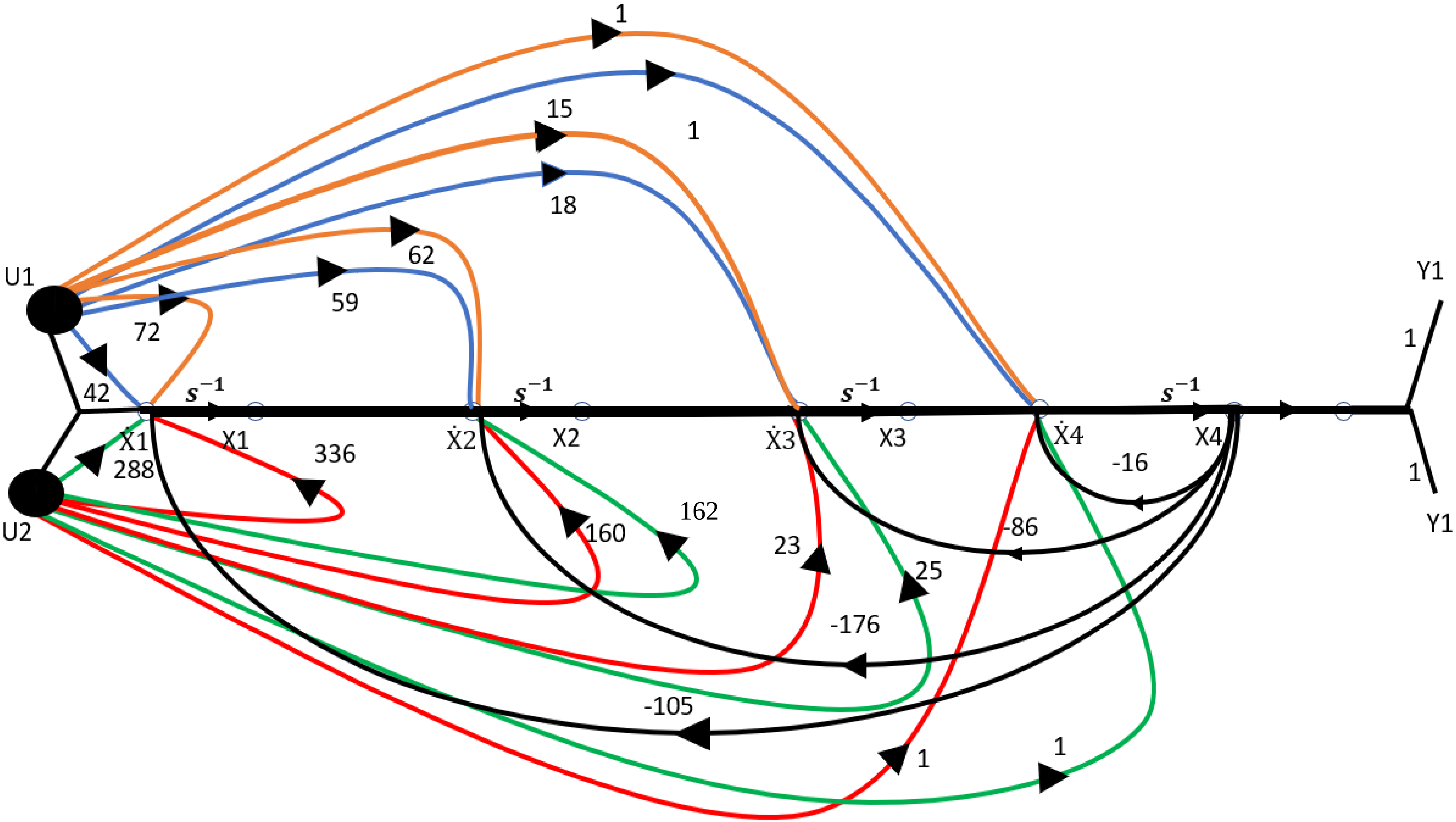


Figure 5 State Diagram of Observability Canonical Form

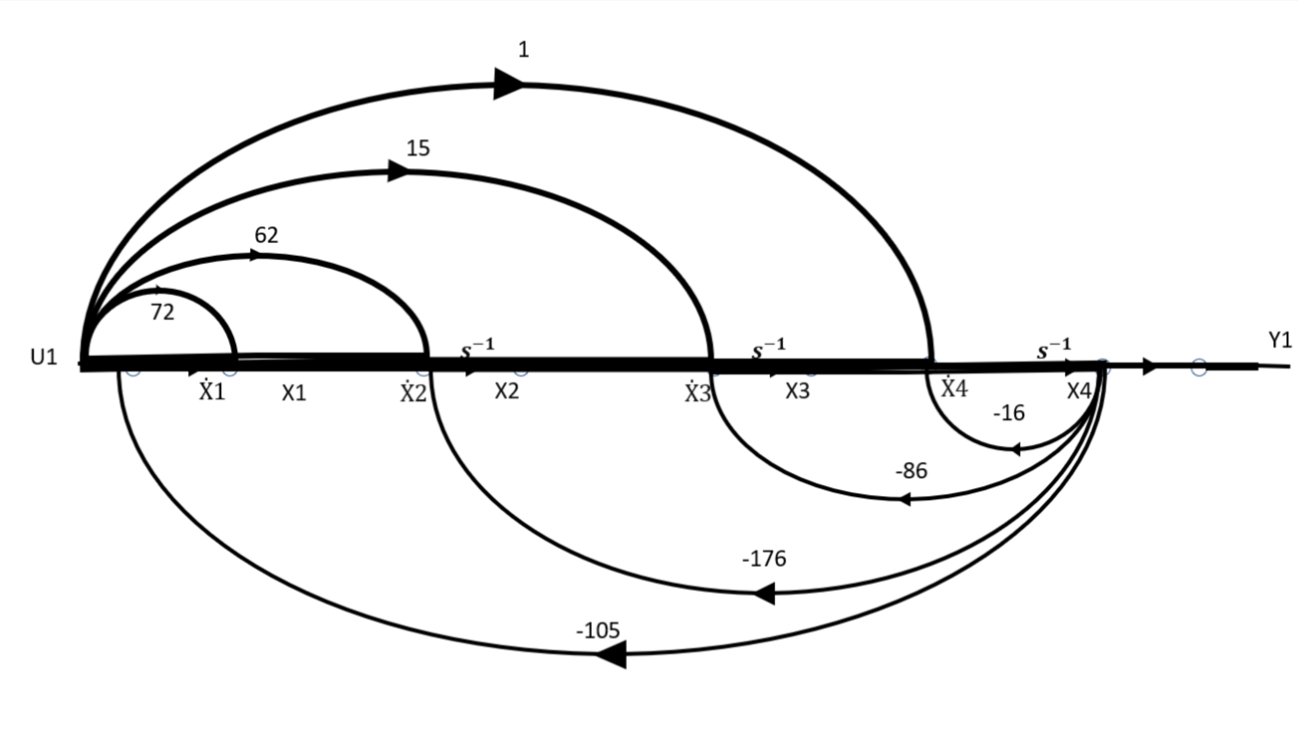
Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

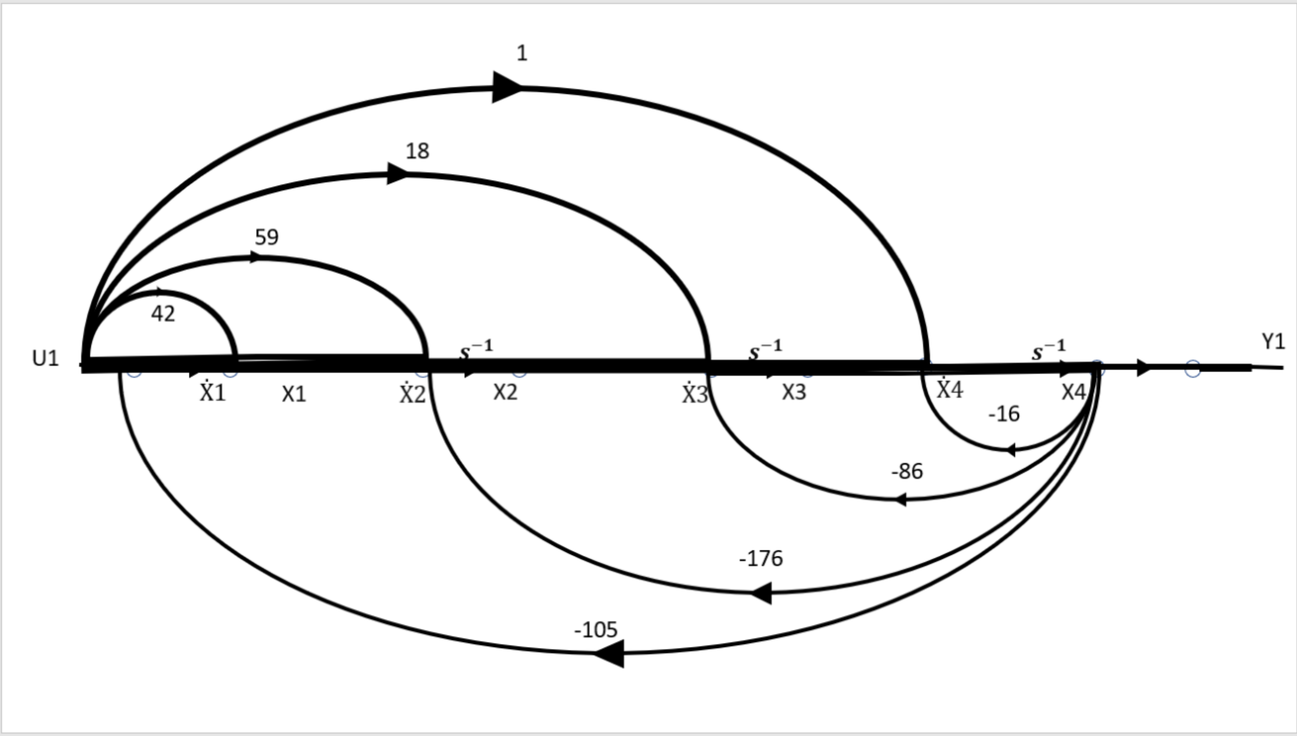
```
From input 1 to output...
         2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

         2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
         2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

         2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

SISO Diagram:

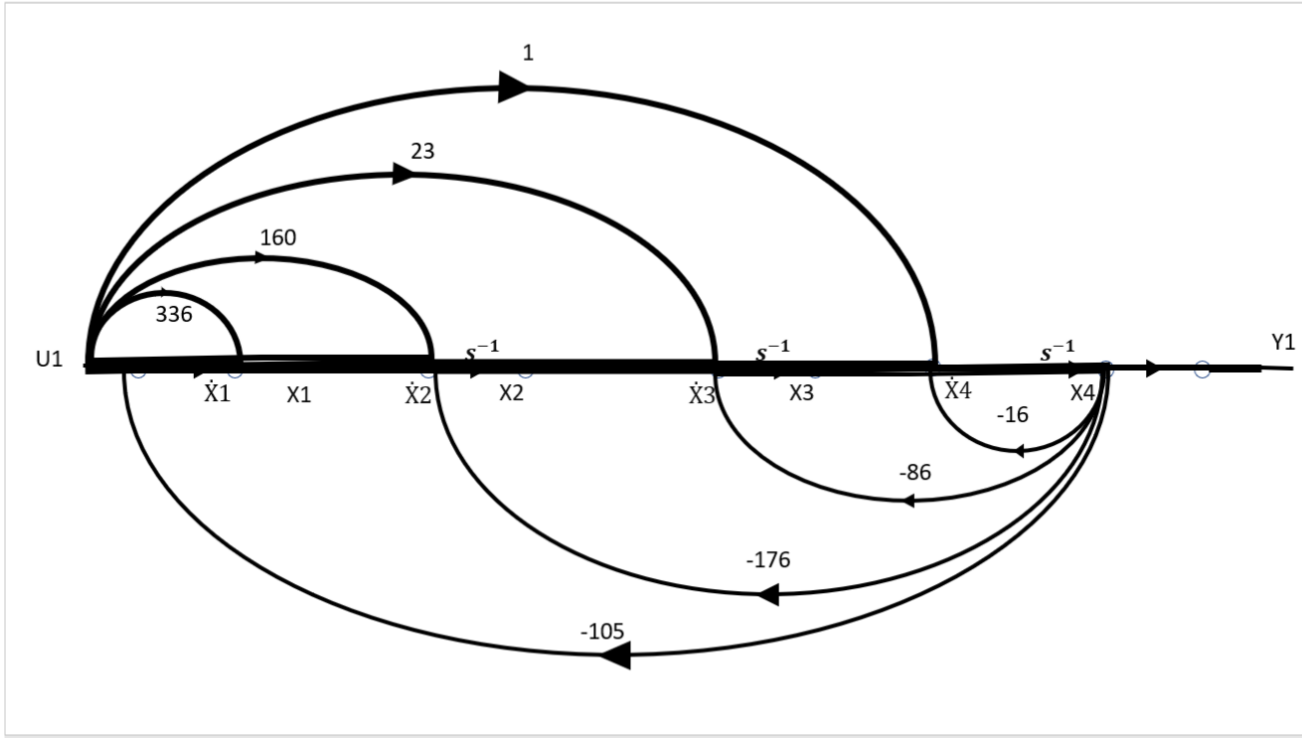
$$\mathbf{1)}\boldsymbol{M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+15s^{-2}+62s^{-3}+72s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}}$$



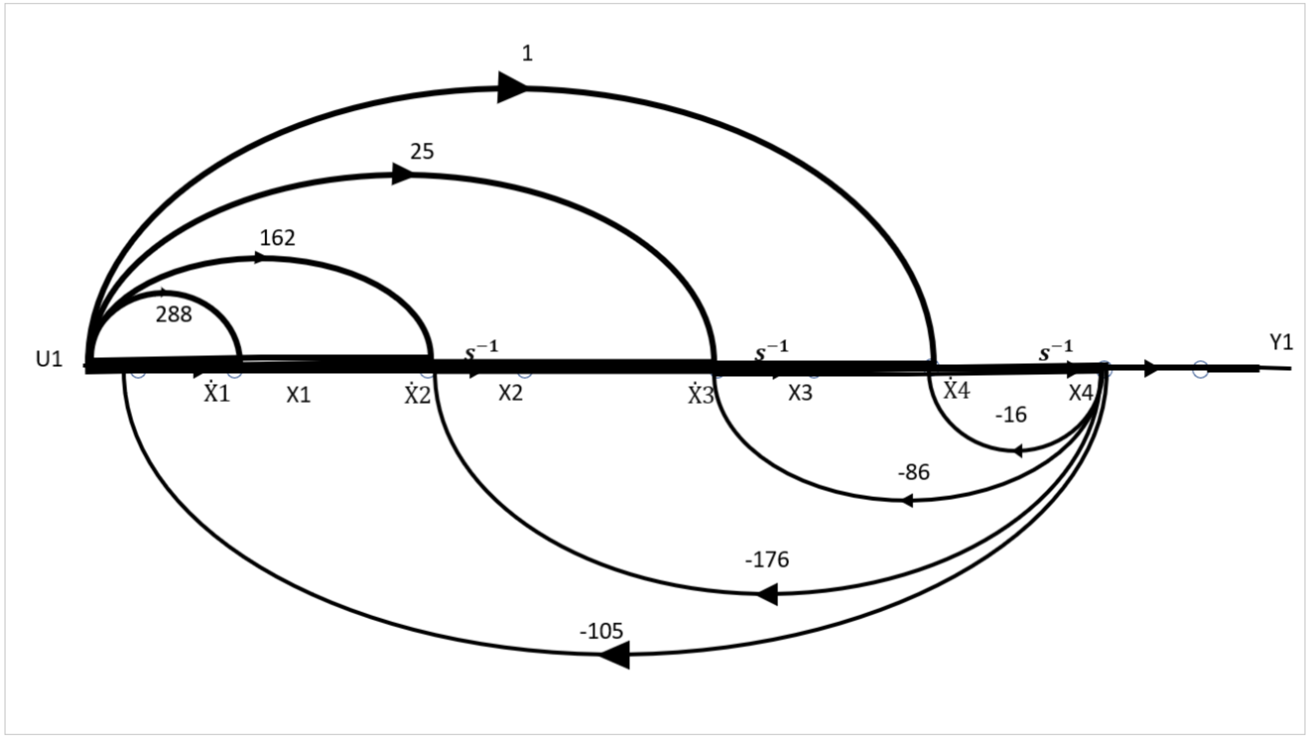
$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$



$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$



$$4)\ M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+162s^{-3}+288s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$



Derive the state space model in equations format.

Throughout the previous section 2.2.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned} \dot{x}_1 &= -a_0 x_n + b_0 u \\ \dot{x}_2 &= x_1 - a_1 x_n + b_1 u \\ &\vdots \\ \dot{x}_n &= x_{n-1} - a_{n-1} x_n + b_{n-1} u \\ y &= x_n \end{aligned} \qquad (2.2.2.1)$$

The derived state space equation from for the **Error! Reference source not found.** as the following:

$$\mathbf{1)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$\dot{x}_1 = -105\, x_4 + 72\, u$
$\dot{x}_2 = x_1 - 176\, x_4 + 62\, u$
$\dot{x}_3 = x_2 - 86x_4 + 15\, u$
$\dot{x}_4 = x_3 - 16x_4\ + u$
$y = x_4$

$$\mathbf{2)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 18s^2 + 59s + 42}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$\dot{x}_1 = -105\, x_4 + 42\, u$
$\dot{x}_2 = x_1 - 176\, x_4 + 59\, u$
$\dot{x}_3 = x_2 - 86x_4 + 18\, u$
$\dot{x}_4 = x_3 - 16x_4\ + u$
$y = x_4$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$\dot{x}_1 = -105\, x_4 + 336\, u$
$\dot{x}_2 = x_1 - 176\, x_4 + 160\, u$
$\dot{x}_3 = x_2 - 86x_4 + 23\, u$
$\dot{x}_4 = x_3 - 16x_4\ + u$
$y = x_4$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$\dot{x}_1=-105\,x_4+288\,u$$
$$\dot{x}_2=x_1-176\,x_4+\ 162u$$
$$\dot{x}_3=x_2-86x_4+25\,u$$
$$\dot{x}_4=x_3-16x_4\ +u$$
$$y=x_4$$

Write the state space model in matrix format (A,B,C,D).

We can write the derived state equation format in matrix format as the following general format:

$$\dot{x}(t)=A_{OFF}\,x(t)+B_{OCF}u(t)$$
$$y(t)=C_{OFF}\,x(t)\ +\ D_{OCF}u(t) \qquad (2.1.3.1)$$

Where the matrices $A,B,C,and\ D$ as the following general format:

$$A_{OCF}=\begin{bmatrix}0&0&\cdots&0&-a_0\\1&0&1&0&-a_1\\0&1&\vdots&0&\vdots\\\vdots&\vdots&\ddots&\vdots&-a_2\\0&0&\cdots&1&-a_{n-1}\end{bmatrix}\qquad B_{OCF}=\begin{bmatrix}B_0\\b_1\\b_2\\\vdots\\b_n\end{bmatrix}\qquad (2.2.3.2)$$

$$C_{OCF}=[0\quad 0\quad \dots\quad 1]\qquad D_{OCF}=[0]$$

Consider the derived state space, the state space diagram's matrix format as:

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 72 \\ 62 \\ 15 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t) + [0]u(t)$$

$$\mathbf{2)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 18s^2 + 59s + 42}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 42 \\ 59 \\ 18 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t) + [0]u(t)$$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 336 \\ 160 \\ 23 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t) + [0]u(t)$$

$$4)\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 288 \\ 162 \\ 25 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0 \quad 0 \quad 0 \quad 1\,]\, x(t) + [0]u(t)$$

Convert the resulting state space model to transfer function format using the equation $\boldsymbol{C(sI - A)^{-1}\, B + D}$ in details.

$$\boldsymbol{C(sI - A)^{-1}B + D} \qquad (\mathbf{2.2.4.1})$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI - A}] = \begin{bmatrix} \boldsymbol{s} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s} & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{s} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{a_0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & -\boldsymbol{a_1} \\ \mathbf{0} & \mathbf{1} & \vdots & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & -\boldsymbol{a_2} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{1} & -\boldsymbol{a_{n-1}} \end{bmatrix} \qquad (\mathbf{2.2.4.1})$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI-A]=\begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}=0$$

Then we find the inverse of the resulting matrix:

$$\boldsymbol{[sI-A]^{-1}=\frac{adj(sI-A)}{det(sI-A)} \quad \dots \quad \frac{1}{det(sI-A)} * [cof(sI-A)]^{T}} \qquad \mathbf{(2.2.4.3)}$$

$[p]$=$\boldsymbol{[cof(sI-A)]^{T}}$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} (s+8)(s^2+8s+22) & -105 & -105s & -105s^2 \\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105) \\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105 \\ 1 & s & s^2 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\times[p]\times\begin{bmatrix}72\\62\\15\\1\end{bmatrix}$$

$$=\frac{\begin{bmatrix}1 & s & s^2 & s^3\end{bmatrix}\times\begin{bmatrix}72\\62\\15\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+2)(s+4)(s+9)}{s^4+16s^3+86s^2+176s+105}$$

$$\mathbf{2)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 18s^2 + 59s + 42}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+8)(s^2+8s+22) & -105 & -105s & -105s^2 \\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105) \\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105 \\ 1 & s & s^2 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [0 \quad 0 \quad 0 \quad 1] \times [p] \times \begin{bmatrix} 42 \\ 59 \\ 18 \\ 1 \end{bmatrix}$$

$$= \frac{[1 \quad s \quad s^2 \quad s^3] \times \begin{bmatrix} 42 \\ 59 \\ 18 \\ 1 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI - A)^{-1}\ B + D = \frac{(s+1)(s+3)(s+14)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+8)(s^2+8s+22) & -105 & -105s & -105s^2 \\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105) \\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105 \\ 1 & s & s^2 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\times[p]\times\begin{bmatrix}336\\160\\23\\1\end{bmatrix}$$

$$=\frac{\begin{bmatrix}1 & s & s^2 & s^3\end{bmatrix}\times\begin{bmatrix}336\\160\\23\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+4)(s+7)(s+12))}{s^4+16s^3+86s^2+176s+105}$$

$$\mathbf{4)}\boldsymbol{M(s)}=\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}=\frac{\boldsymbol{s^3+25s^2+162s+288}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+8)(s^2+8s+22) & -105 & -105s & -105s^2\\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105)\\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105\\ 1 & s & s^2 & s^3\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\times[p]\times\begin{bmatrix}288\\162\\25\\1\end{bmatrix}$$

$$=\frac{\begin{bmatrix}1 & s & s^2 & s^3\end{bmatrix}\times\begin{bmatrix}288\\162\\25\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+3)(s+6)(s+16)}{s^4+16s^3+86s^2+176s+105}$$

Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Functions (Polynomial):

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Functions (Polynomial):

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

Command Window

```
>> Ai=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

       s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[72;62;15;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

       s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

Command Window

```
>> Ai=[-5 0 0 0;9 -7 0 0;0 1 -3 0;0 1 0 -1];
>> Bi=[1;1;0;0];
>> Ci=[0 1 0 0];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[42;59;18;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

```
Command Window
>> clear
>> Ai=[-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[336;160;23;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

```
Command Window
>> Ai=[-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[288;162;25;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 6 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to OCF state-space

Table 5 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	1) [1 15 62 72]	[1 16 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	
The Converted TF	1) [1 15 62 72]	[1 16 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	

Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|sI-A| = \begin{vmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & 1 & 0 & -a_1 \\ 0 & 1 & \vdots & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & -a_2 \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{vmatrix} \qquad (2.2.7.1)$$

$|sI - A| = 0$

$$[sI-A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI-A] = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix} = 0$$

$$s^4 + 16s^3 + 86s^2 + 176s + 105 = 0 \qquad (2.2.7.2)$$

Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda I-A| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & 1 & 0 & -a_1 \\ 0 & 1 & \vdots & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & -a_2 \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{vmatrix} \qquad (2.2.8.1)$$

$|\lambda I - A| = 0$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} \mathrm{s} & 0 & 0 & 105 \\ -1 & \mathrm{s} & 0 & 176 \\ 0 & -1 & \mathrm{s} & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} \mathrm{s} & 0 & 0 & 105 \\ -1 & \mathrm{s} & 0 & 176 \\ 0 & -1 & \mathrm{s} & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.2.8.2})$$

Then we use the **MATLAB** command **<u>"roots"</u>** and get the following eigenvalues:

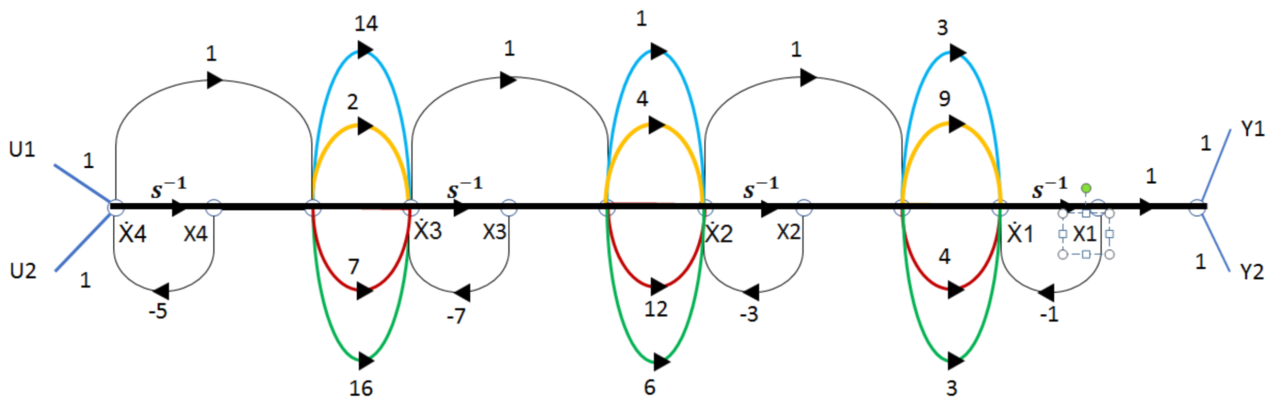
$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -3, \quad \lambda_5 = -1$

2.3 Cascaded Decomposition to Upper Triangular A Matrix.

Table 6 Task 3 Description

Task Number	Chapter	Task Name	Task Description
Task 3	**Chapter 3:** Decomposition of Transfer Functions	Cascaded Decomposition to Upper Triangular A Matrix.	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\|sI - A\|$. H) Find the Eigenvalues of matrix A.

This is the state diagram of Mimo system



Consider the derived state space, the state space diagram's matrix format as:

For MIMO:

$$\dot{x}(t) \ = \begin{bmatrix} -4 & -29 & -244 & -492 \\ 0 & -12 & -61 & -123 \\ 0 & 0 & = 38 & -41 \\ 0 & 0 & 0 & -20 \end{bmatrix} x(t) + \begin{bmatrix} 192 & 192 \\ 48 & 48 \\ 16 & 16 \\ 4 & 4 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & -29 & -244 & -492 \\ 0 & -29 & -244 & -492 \end{bmatrix} x(t) + [0]u(t).$$

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

```
From input 1 to output...
         2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

         2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
         2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105

         2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
      s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

2.3.1 Draw the state diagram.

To draw the state diagram, the factored form of the transfer function is as follows:

$$\frac{Y(s)}{U(s)} = K\frac{(s+z_1)(s+z_2)\ldots(s+z_{n-2})(s+z_{n-1})}{(s+p_1)(s+p_2)\ldots(s+p_{n-2})(s+p_{n-1})(s+p_n)} \qquad (2.3.1.1)$$

$$M(s) = \frac{Y(s)}{U(s)} = \frac{(s+2)(s+4)(s+9)}{(s+5)(s+7)(s+3)(s+1)}$$

It can be ***decomposed*** into the ***four*** following terms:

$$\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \boldsymbol{M_1(s) \times M_2(s) \times M_3(s) \times \ldots \times M_{n-1}(s) \times M_n(s)} \qquad (\mathbf{2.3.1.2})$$

$$M_1(s) = \frac{s+2}{s+5} \;, \qquad M_2(s) = \frac{s+4}{s+7}, \qquad M_3(s) = \frac{s+9}{s+3} \;,$$

$$M_4(s) = \frac{1}{s+1},$$

The state diagram of the system with **subsystems** connected in **Cascaded Form** from $\boldsymbol{M_1(s)}$ in the left to $\boldsymbol{M_4(s)}$ in the right based on **CCF** is shown in Figure 7**.**

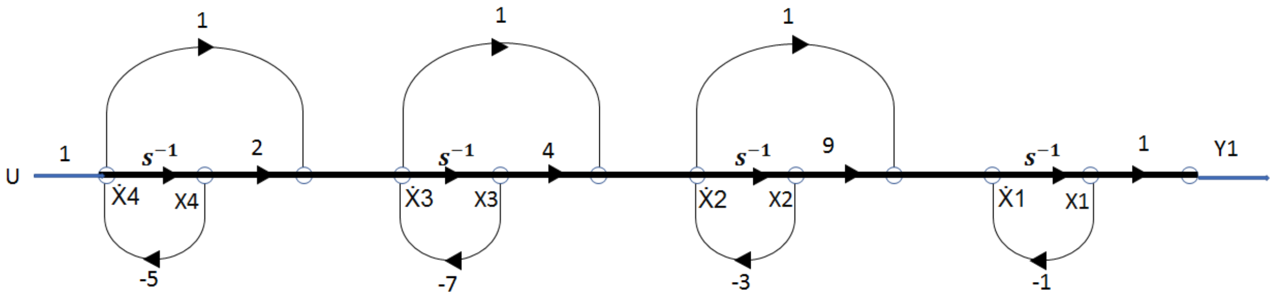


Figure 7 Cascaded Decomposition to upper triangular State-Space Diagram Based on CCF

2.3.2 Derive the state space model in equations format.

Throughout the previous section 2.3.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_1x_1 + a_2x_2 + \cdots + a_nx_m + b_1u \\
\dot{x}_2 &= a_2x_2 + a_3x_3 + \cdots + a_nx_n + b_2u \\
&\vdots \\
\dot{x}_{n-1} &= a_{n-1}x_{n-1} + a_nx_n + b_{n-1}u \\
\dot{x}_n &= a_nx_n + +b_nu \\
y &= x_1
\end{aligned} \qquad (\mathbf{2.3.2.1})$$

The derived state space equation from for the as the following:

$$\dot{x}_1 = -x_1 + 6x_2 - 3x_3 - 3x_4 + u$$
$$\dot{x}_2 = -3x_2 - 3x_3 - 3x_4 + u$$
$$\dot{x}_3 = -7x_3 - 3x_4 + u$$
$$\dot{x}_4 = -5x_4 + u$$
$$y = x_1$$

2.3.3 Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$.

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A}_{\boldsymbol{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_{\boldsymbol{CCF}} \boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C}_{\boldsymbol{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}_{\boldsymbol{CCF}} \boldsymbol{u}(\boldsymbol{t}) \end{aligned} \qquad (\mathbf{2.3.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A}_{\boldsymbol{CCF}} = \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \boldsymbol{a}_{13} & \cdots & \boldsymbol{a}_{1n} \\ \mathbf{0} & \boldsymbol{a}_{22} & \boldsymbol{a}_{23} & \cdots & \boldsymbol{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{a}_{(n-1)n} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{a}_{nn} \end{bmatrix} \qquad \boldsymbol{B}_{\boldsymbol{CCF}} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix} \qquad (\mathbf{2.3.3.2})$$

$$\boldsymbol{C}_{\boldsymbol{CCF}} = [\mathbf{1} \quad \mathbf{0} \quad \dots \quad \mathbf{0}] \qquad \boldsymbol{D}_{\boldsymbol{CCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\dot{x}(t) = \begin{bmatrix} -1 & 6 & -3 & -3 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & -7 & -3 \\ 0 & 0 & 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 1 \quad 0 \quad 0 \quad 0]$$

2.3.4 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1} B + D$ in details.

$$\boldsymbol{C(sI - A)^{-1}B + D} \qquad (2.3.4.1)$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[sI - A] = \begin{bmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & a_{(n-1)n} \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix} \qquad (2.3.4.2)$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 6 & -3 & -3 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & -7 & -3 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+1 & -6 & 3 & 3 \\ 0 & s+3 & 3 & 3 \\ 0 & 0 & s+7 & 3 \\ 0 & 0 & 0 & s+5 \end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[sI - A]^{-1} = \frac{adj(sI - A)}{det(sI - A)} \quad \cdots \quad \frac{1}{det(sI - A)} * [adj(sI - A)] \qquad (2.3.4.3)$$

$$\frac{1}{det(sI - A)} = \frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[\boldsymbol{adj(sI-A)}] = \begin{bmatrix} (s+3)(s+5)(s+7) & 6(s+5)(s+7) & -3(s+5)(s+9) & -3(s+4)(s+9) \\ 0 & (s+1)(s+5)(s+7) & -3(s+1)(s+5) & -3(s+1)(s+4) \\ 0 & 0 & (s+1)(s+5)(s+7) & -3(s+1)(s+3) \\ 0 & 0 & 0 & (s+1)(s+5)(s+7) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D = \frac{\mathbf{1}}{\boldsymbol{det(sI-A)}} \times [\boldsymbol{adj(sI-A)}]$$

$$C(sI-A)^{-1}\ B+D = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$M(s) = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

2.3.5 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s) = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s) = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

2.3.6 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

```
>> A=[-1 6 -3 -3;0 -3 -3 -3;0 0 -7 -3;0 0 0 -5];
>> B=[1;1;1;1];
>> C=[1 0 0 0];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> Mt1=tf(num,den)

Mt1 =

        s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 8 MATLAB Checks the similarity for the output transfer function and between initial state-space format and Cascaded Decomposition to upper triangular A matrix format Based on CCF

Note: **Since the transfer function is *identical for all canonical forms*, the converted transfer function is the same as the original transfer function.**

Table 7 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 15 62 72]	[1 16 86 176 105]
The Converted TF	[1 15 62 72]	[1 16 66 176 105]

2.3.7 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|\lambda I - A|$, which returns:

2.3.8 $$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{vmatrix} \qquad (2.1.8.1)$$

2.3.9 $|\lambda I - A| = 0$

2.3.10 $$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix}$$

2.3.11 $$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix} = 0$$

2.3.12 $$s^4 + 16s^3 + 86s^2 + 176s + 105 = 0 \qquad (2.1.8.2)$$

2.3.13 Then we use the MATLAB command "roots" and get the following eigenvalues:

$$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -1,$$

$$M_1(s) = \frac{s+14}{s+5}, \qquad M_2(s) = \frac{1}{s+7}, \qquad M_3(s) = \frac{s+3}{s+3},$$

$$M_4(s) = \frac{s+1}{s+1},$$

The state diagram of the system with **subsystems** connected in **Cascaded Form** from $M_1(s)$ in the left to $M_4(s)$ in the right based on **CCF** is shown in Figure 7**.**

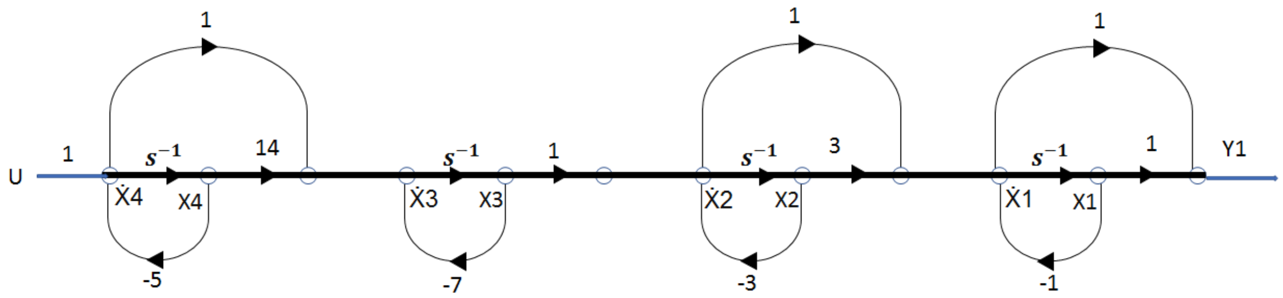


Figure 8 Cascaded Decomposition to upper triangular State-Space Diagram Based on CCF

2.3.14 Derive the state space model in equations format.

Throughout the previous section 2.3.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_1x_1 + a_2x_2 + \cdots + a_nx_m + b_1u \\
\dot{x}_2 &= a_2x_2 + a_3x_3 + \cdots + a_nx_n + b_2u \\
&\vdots \\
\dot{x}_{n-1} &= a_{n-1}x_{n-1} + a_nx_n + b_{n-1}u \\
\dot{x}_n &= a_nx_n + +b_nu \\
y &= x_1
\end{aligned} \qquad (2.3.2.1)$$

The derived state space equation from for the as the following:

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_3 \\
\dot{x}_2 &= -3x_2 + x_3 \\
\dot{x}_3 &= -7x_3 + 9x_4 + u \\
\dot{x}_4 &= -5x_4 + u \\
y &= x_3
\end{aligned}$$

2.3.15 Write the state space model in matrix format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned}\dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A_{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B_{CCF}}\boldsymbol{u}(\boldsymbol{t})\\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C_{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D_{CCF}}\boldsymbol{u}(\boldsymbol{t})\end{aligned} \qquad (\mathbf{2.3.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A_{CCF}} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} & \dots & \boldsymbol{a_{1n}} \\ \mathbf{0} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \dots & \boldsymbol{a_{2n}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{a_{(n-1)n}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{a_{nn}} \end{bmatrix} \qquad \boldsymbol{B_{CCF}} = \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \vdots \\ \boldsymbol{b_n} \end{bmatrix} \qquad (\mathbf{2.3.3.2})$$

$$\boldsymbol{C_{CCF}} = [\mathbf{1} \quad \mathbf{0} \quad \dots \quad \mathbf{0}] \qquad \boldsymbol{D_{CCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & -7 & 9 \\ 0 & 0 & 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 0 \quad 0 \quad 1 \quad 0]$$

2.3.16 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\, B + D$ in details.

$$\boldsymbol{C}(\boldsymbol{sI} - \boldsymbol{A})^{-\mathbf{1}}\boldsymbol{B} + \boldsymbol{D} \qquad (\mathbf{2.3.4.1})$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI}-\boldsymbol{A}]=\begin{bmatrix}\boldsymbol{s} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{s} & \ddots & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{s}\end{bmatrix}-\begin{bmatrix}\boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} & \dots & \boldsymbol{a_{1n}}\\ \boldsymbol{0} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \dots & \boldsymbol{a_{2n}}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{a_{(n-1)n}}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{a_{nn}}\end{bmatrix} \qquad (\mathbf{2.3.4.2})$$

$$[sI-A]=\begin{bmatrix}s & 0 & 0 & 0\\ 0 & s & 0 & 0\\ 0 & 0 & s & 0\\ 0 & 0 & 0 & s\end{bmatrix}-\begin{bmatrix}-1 & 0 & 1 & 0\\ 0 & -3 & 1 & 0\\ 0 & 0 & -7 & 9\\ 0 & 0 & 0 & -5\end{bmatrix}$$

$$[sI-A]=\begin{bmatrix}s+1 & 0 & -1 & 0\\ 0 & s+3 & -1 & 3\\ 0 & 0 & s+7 & -9\\ 0 & 0 & 0 & s+5\end{bmatrix}$$

$$\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}=\frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & (s+5)(s+3) & 9s+27\\ 0 & (s+1)(s+5)(s+7) & (s+1)(s+5) & -3(s+1)(s+4)\\ 0 & 0 & (s+1)(s+5)(s+3) & 9(s+1)(s+3)\\ 0 & 0 & 0 & (s+1)(s+3)(s+7)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}\times[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

2.3.17 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2.3.18 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

```
>> A=[-1 0 1 0;0 -3 1 0;0 0 -7 9;0 0 0 -5];
>> B=[0;0;1;1];
>> C=[0 0 1 0];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> Mt1=tf(num,den)

Mt1 =

        s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 9 MATLAB Checks the similarity for the output transfer function and between initial state-space format and Cascaded Decomposition to upper triangular A matrix format Based on CCF

Note: **Since the transfer function is *identical for all canonical forms*, the converted transfer function is the same as the original transfer function.**

Table 8 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 18 59 72]	[1 16 86 176 105]

The Converted TF	[1 18 59 72]	[1 16 66 176 105]

2.3.19 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{vmatrix} \qquad (2.1.8.1)$$

$$|\lambda I - A| = 0$$

2.3.20 $$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix}$$

2.3.21 $$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix} = 0$$

2.3.22 $$s^4 + 16s^3 + 86s^2 + 176s + 105 = 0$$

2.3.23 Then we use the MATLAB command "roots" and get the following eigenvalues:

2.3.24 $\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -1,$

$$M_1(s) = \frac{s+7}{s+5} \ , \qquad M_2(s) = \frac{s+12}{s+7}, \qquad M_3(s) = \frac{s+4}{s+3} \ ,$$
$$M_4(s) = \frac{1}{s+1},$$

The state diagram of the system with **subsystems** connected in **Cascaded Form** from $\boldsymbol{M_1(s)}$ in the left to $\boldsymbol{M_4(s)}$ in the right based on **CCF** is shown in Figure 7**.**

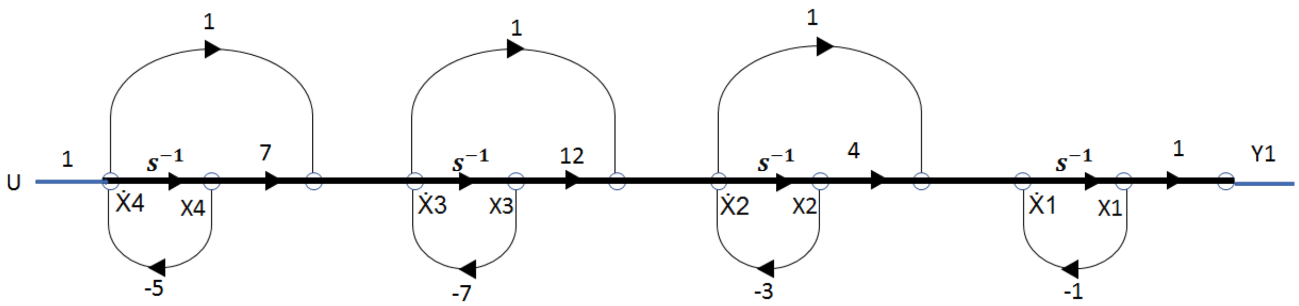


Figure 9 Cascaded Decomposition to upper triangular State-Space Diagram Based on CCF

2.3.25 Derive the state space model in equations format.

Throughout the previous section 2.3.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_1x_1 + a_2x_2 + \cdots + a_nx_m + b_1u \\
\dot{x}_2 &= a_2x_2 + a_3x_3 + \cdots + a_nx_n + b_2u \\
&\vdots \\
\dot{x}_{n-1} &= a_{n-1}x_{n-1} + a_nx_n + b_{n-1}u \\
\dot{x}_n &= a_nx_n + +b_nu \\
y &= x_1
\end{aligned} \qquad (2.3.2.1)$$

The derived state space equation from for the as the following:

$$\dot{x}_1 = -x_1 + x_2 + 5x_3 + 2x_4 + u$$

$$\dot{x}_2 = -3x_2 + 5x_3 + 2x_4 + u$$
$$\dot{x}_3 = -7x_3 + 2x_4 + u$$
$$\dot{x}_4 = -5x_4 + u$$
$$y = x_1$$

2.3.26 Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$.

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A}_{\boldsymbol{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_{\boldsymbol{CCF}} \boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C}_{\boldsymbol{CFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}_{\boldsymbol{CCF}} \boldsymbol{u}(\boldsymbol{t}) \end{aligned} \qquad (\mathbf{2.3.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A}_{\boldsymbol{CCF}} = \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \boldsymbol{a}_{13} & \cdots & \boldsymbol{a}_{1n} \\ \mathbf{0} & \boldsymbol{a}_{22} & \boldsymbol{a}_{23} & \cdots & \boldsymbol{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{a}_{(n-1)n} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{a}_{nn} \end{bmatrix} \qquad \boldsymbol{B}_{\boldsymbol{CCF}} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix} \qquad (\mathbf{2.3.3.2})$$

$$\boldsymbol{C}_{\boldsymbol{CCF}} = [\mathbf{1} \quad \mathbf{0} \quad \ldots \quad \mathbf{0}] \qquad \boldsymbol{D}_{\boldsymbol{CCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\dot{x}(t) = \begin{bmatrix} -1 & 1 & 5 & 2 \\ 0 & -3 & 5 & 2 \\ 0 & 0 & -7 & 2 \\ 0 & 0 & 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 1 \quad 0 \quad 0 \quad 0]$$

2.3.27 Convert the resulting state space model to transfer function format using the equation $\boldsymbol{C}(\boldsymbol{sI} - \boldsymbol{A})^{-1}\, \boldsymbol{B} + \boldsymbol{D}$ in details.

$$C(sI-A)^{-1}B+D \qquad (2.3.4.1)$$

Initially, we apply the equation $|sI-A|$, which returns:

$$[sI-A]=\begin{bmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & a_{(n-1)n} \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix} \qquad (2.3.4.2)$$

$$[sI-A]=\begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 & 5 & 2 \\ 0 & -3 & 5 & 2 \\ 0 & 0 & -7 & 2 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

$$[sI-A]=\begin{bmatrix} s+1 & -1 & -5 & -2 \\ 0 & s+3 & -5 & -2 \\ 0 & 0 & s+7 & -2 \\ 0 & 0 & 0 & s+5 \end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[sI-A]^{-1}=\frac{adj(sI-A)}{det(sI-A)} \quad \dots \quad \frac{1}{det(sI-A)} * [adj(sI-A)] \qquad (2.3.4.3)$$

$$\frac{1}{det(sI-A)}=\frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[adj(sI-A)] = \begin{bmatrix} (s+3)(s+5)(s+7) & (s+5)(s+7) & 5(s+5)(s+4) & 2(s+4)(s+12) \\ 0 & (s+1)(s+5)(s+7) & 5(s+1)(s+5) & 2(s+1)(s+12) \\ 0 & 0 & (s+1)(s+5)(s+3) & 2(s+1)(s+3) \\ 0 & 0 & 0 & (s+1)(s+3)(s+7) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}\times[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

2.3.28 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

2.3.29 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

```
>> A=[-1 1 5 2;0 -3 5 2;0 0 -7 2;0 0 0 -5];
>> B=[1;1;1;1];
>> C=[1 0 0 0];
>> [num,den]=ss2tf(A,B,C,D);
>> Mt1=tf(num,den)

Mt1 =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 10 MATLAB Checks the similarity for the output transfer function and between initial state-space format and Cascaded Decomposition to upper triangular A matrix format Based on CCF

Note: **Since the transfer function is *identical for all canonical forms*, the converted transfer function is the same as the original transfer function.**

Table 9 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	$[1 \quad 23 \quad 160 \quad 336]$	$[1 \quad 16 \quad 86 \quad 176 \quad 105]$
The Converted TF	$[1 \quad 23 \quad 160 \quad 336]$	$[1 \quad 16 \quad 66 \quad 176 \quad 105]$

2.3.30 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\boldsymbol{\lambda I - A}| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{vmatrix}$$

$$|\boldsymbol{\lambda I - A}| = \mathbf{0}$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix}$$

$$|\boldsymbol{\lambda I - A}| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix} = \mathbf{0}$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.1.8.2})$$

Then we use the MATLAB command “roots” and get the following eigenvalues:

$$\boldsymbol{\lambda_1 = -7}, \quad \boldsymbol{\lambda_2 = -5}, \quad \boldsymbol{\lambda_3 = -3}, \quad \boldsymbol{\lambda_4 = -1},$$

$$M_1(s) = \frac{s+16}{s+5} \;, \qquad M_2(s) = \frac{s+6}{s+7}, \qquad M_3(s) = \frac{s+3}{s+3} \;,$$

$$M_4(s) = \frac{1}{s+1},$$

The state diagram of the system with **subsystems** connected in **Cascaded Form** from $\boldsymbol{M_1(s)}$ in the left to $\boldsymbol{M_4(s)}$ in the right based on **CCF** is shown in Figure 7**.**

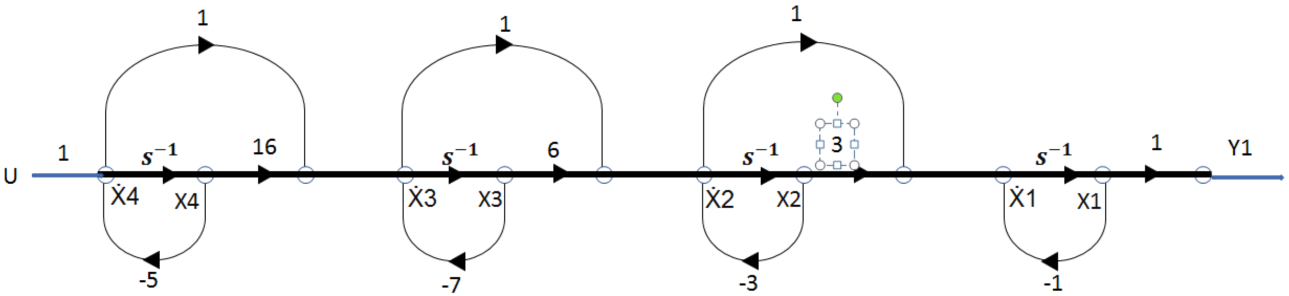


Figure 10 Cascaded Decomposition to upper triangular State-Space Diagram Based on CCF

2.3.31 Derive the state space model in equations format.

Throughout the previous section 2.3.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_1x_1 + a_2x_2 + \cdots + a_nx_m + b_1u \\
\dot{x}_2 &= a_2x_2 + a_3x_3 + \cdots + a_nx_n + b_2u \\
&\vdots \\
\dot{x}_{n-1} &= a_{n-1}x_{n-1} + a_nx_n + b_{n-1}u \\
\dot{x}_n &= a_nx_n + +b_nu \\
y &= x_1
\end{aligned} \qquad (2.3.2.1)$$

The derived state space equation from for the as the following:

$$\begin{aligned}
\dot{x}_1 &= -x_1 - 4x_2 + 3x_3 + 11x_4 + u \\
\dot{x}_2 &= -7x_2 + 3x_3 + 11x_4 + u \\
\dot{x}_3 &= -3x_3 + 11x_4 + u \\
\dot{x}_4 &= -5x_4 + u \\
y &= x_1
\end{aligned}$$

2.3.32 Write the state space model in matrix format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\dot{x}(t) = A_{CFF}\,x(t) + B_{CCF}u(t)$$
$$y(t) = C_{CFF}\,x(t) + D_{CCF}u(t) \qquad (2.3.3.1)$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$A_{CCF} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & a_{(n-1)n} \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix} \qquad B_{CCF} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \qquad (2.3.3.2)$$

$$C_{CCF} = [1 \quad 0 \quad \dots \quad 0] \qquad D_{CCF} = [0]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\dot{x}(t) = \begin{bmatrix} -1 & -4 & 3 & 11 \\ 0 & -3 & 3 & 11 \\ 0 & 0 & -3 & 11 \\ 0 & 0 & 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 1 \quad 0 \quad 0 \quad 0]$$

2.3.33 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\,B + D$ in details.

$$C(sI - A)^{-1}B + D \qquad (2.3.4.1)$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI}-\boldsymbol{A}]=\begin{bmatrix}\boldsymbol{s} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{s} & \ddots & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{s}\end{bmatrix}-\begin{bmatrix}\boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} & \dots & \boldsymbol{a_{1n}}\\ \boldsymbol{0} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \dots & \boldsymbol{a_{2n}}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{a_{(n-1)n}}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{a_{nn}}\end{bmatrix} \qquad (\mathbf{2.3.4.2})$$

$$[sI-A]=\begin{bmatrix}s & 0 & 0 & 0\\ 0 & s & 0 & 0\\ 0 & 0 & s & 0\\ 0 & 0 & 0 & s\end{bmatrix}-\begin{bmatrix}-1 & 0 & 3 & 11\\ 0 & -3 & -1 & 11\\ 0 & 0 & -7 & 11\\ 0 & 0 & 0 & -5\end{bmatrix}$$

$$[sI-A]=\begin{bmatrix}s+1 & 0 & -3 & -11\\ 0 & s+3 & 1 & -11\\ 0 & 0 & s+7 & -11\\ 0 & 0 & 0 & s+5\end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[\boldsymbol{sI}-\boldsymbol{A}]^{-1}=\frac{\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})} \quad \dots \quad \frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}*[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})] \qquad (\mathbf{2.3.4.3})$$

$$\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}=\frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 3(s+5)(s+3) & 11(s+3)(s+10)\\ 0 & (s+1)(s+5)(s+7) & -(s+1)(s+5) & 11(s+1)(s+6)\\ 0 & 0 & (s+1)(s+5)(s+3) & 11(s+1)(s+3)\\ 0 & 0 & 0 & (s+1)(s+3)(s+7)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})}\times[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2.3.34 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2.3.35 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

```
>> A=[-1 0 3 11;0 -3 -1 11;0 0 -7 11;0 0 0 -5];
>> B=[1;1;1;1];
>> C=[1 0 0 0];
>> [num,den]=ss2tf(A,B,C,D);
>> Mt1=tf(num,den)

Mt1 =

      s^3 + 29 s^2 + 238 s + 480
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 11 MATLAB Checks the similarity for the output transfer function and between initial state-space format and Cascaded Decomposition to upper triangular A matrix format Based on CCF

Note: **Since the transfer function is** ***identical for all canonical forms*****, the converted transfer function is the same as the original transfer function.**

Table 10 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 29 238 480]	[1 16 86 176 105]
The Converted TF	[1 29 238 480]	[1 16 66 176 105]

2.3.36 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{vmatrix}$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 176 & 86 & \lambda + 16 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 176 & 86 & \lambda + 16 \end{vmatrix} = 0$$

$$s^4 + 16s^3 + 86s^2 + 176s + 105 = 0$$

Then we use the MATLAB command “roots” and get the following eigenvalues:

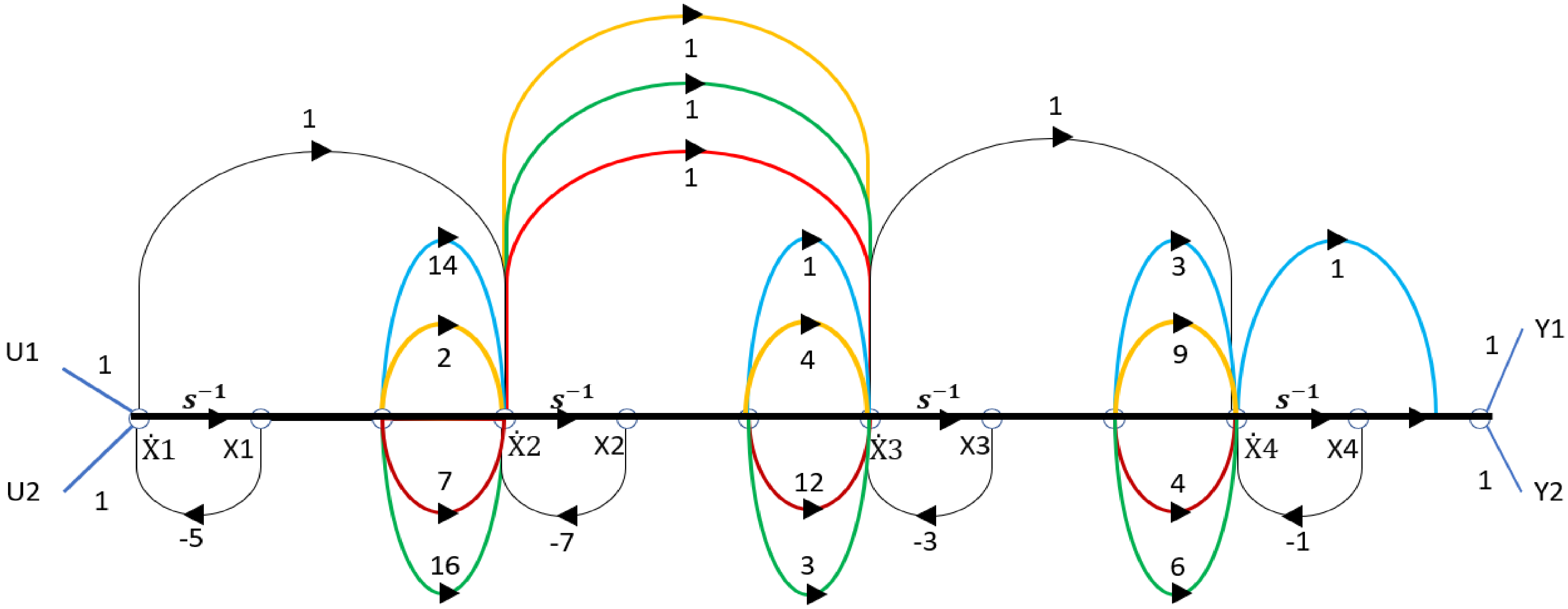
$\boldsymbol{\lambda_1 = -7}, \quad \boldsymbol{\lambda_2 = -5}, \quad \boldsymbol{\lambda_3 = -3}, \quad \boldsymbol{\lambda_4 = -1},$

2.4 Cascaded Decomposition to Lower Triangular A Matrix.

Table 11 Task 4 Descriptions

Task Number	Chapter	Task Name	Task Description
Task 4	**Chapter 3:** Decomposition of Transfer Functions	Cascaded Decomposition to lower triangular A matrix.	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert sI - A \rvert$. H) Find the Eigenvalues of matrix A.

This is the state diagram of Mimo equation



Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

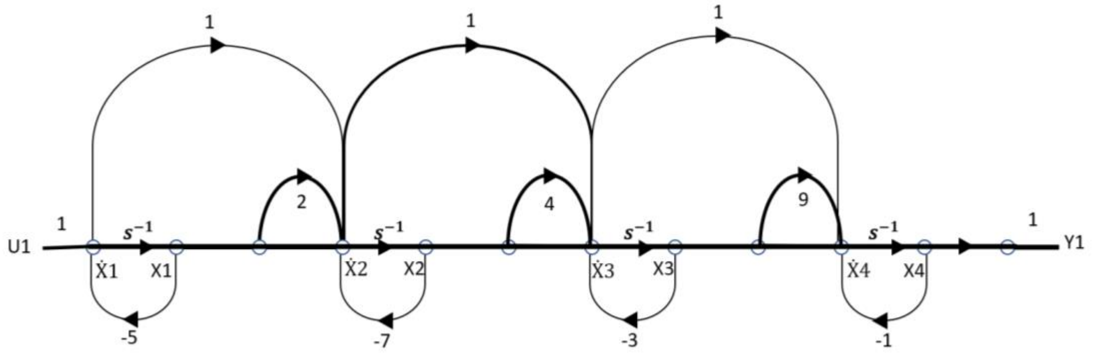
```
From input 1 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

Figure 12 MIMO State-Space Diagram for General Transfer Function

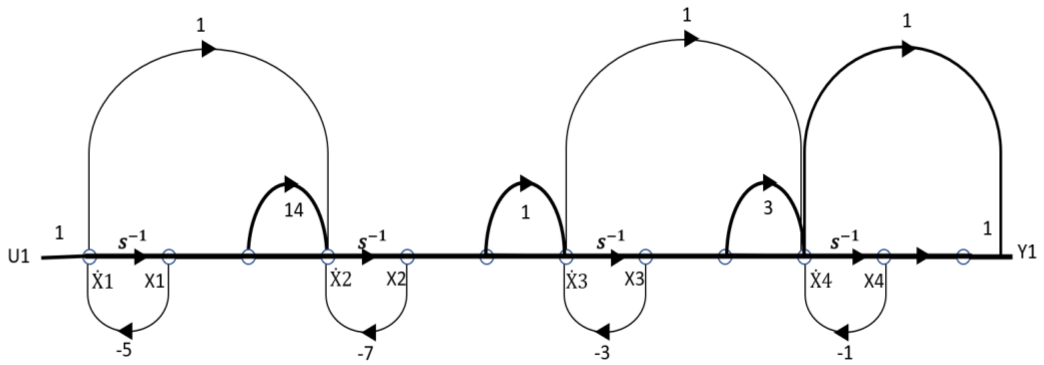
Modern and Digital Control System

For the SISO Transfer functions:

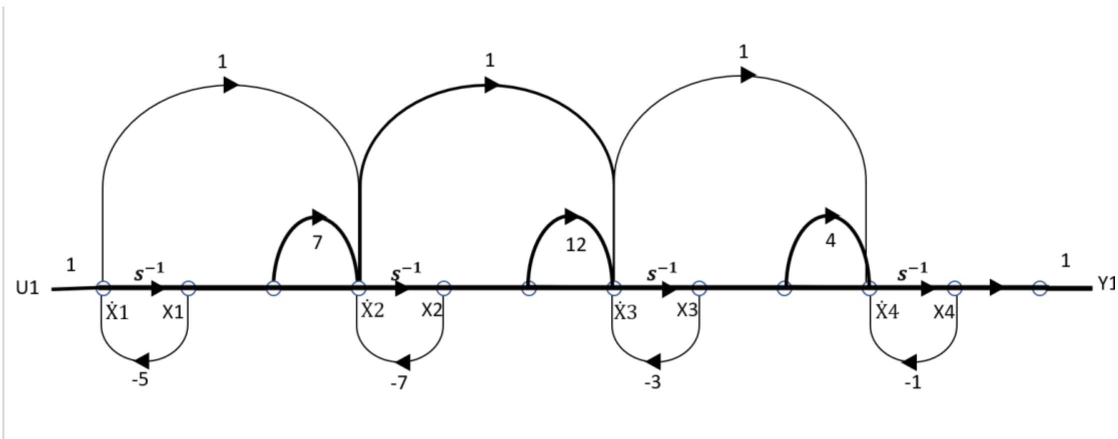
$$1)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+2}{s+5}\right)\left(\frac{s+4}{s+7}\right)\left(\frac{s+9}{s+3}\right)\left(\frac{1}{s+1}\right)$$



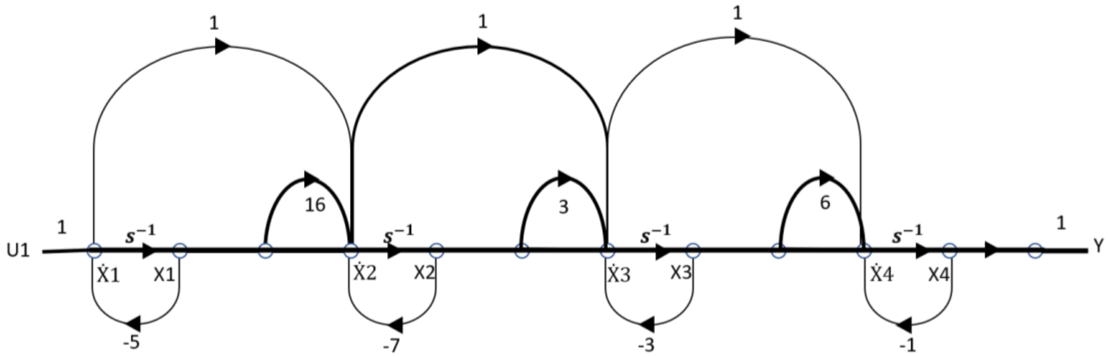
$$2)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+14}{s+5}\right)\left(\frac{1}{s+7}\right)\left(\frac{s+3}{s+3}\right)\left(\frac{s+1}{s+1}\right)$$



$$3)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+7}{s+5}\right)\left(\frac{s+12}{s+7}\right)\left(\frac{s+4}{s+3}\right)\left(\frac{1}{s+1}\right)$$



$$4)M(s) = \frac{Y(s)}{U(s)} = \left(\frac{s+16}{s+5}\right)\left(\frac{s+6}{s+3}\right)\left(\frac{s+3}{s+7}\right)\left(\frac{1}{s+1}\right)$$



1.9 Write the State Space model in Equation Format (A, B, C, D).

Throughout the previous section 1.5, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= a_{11}\,x_1 + a_{12}\,x_2 + \cdots + a_{1n}\,x_n + b_1\,u \\
\dot{x}_2 &= a_{21}\,x_1 + a_{22}\,x_2 + \cdots + a_{2n}\,x_n + b_2\,u \\
&\vdots \\
\dot{x}_n &= a_{n1}\,x_1 + a_{n2}\,x_2 + \cdots + a_{nn}\,x_n + b_n\,u \\
y &= a_1\,x_1 + a_2\,x_2 + \cdots + a_n\,x_n + b\,u
\end{aligned} \qquad (1.6.1)$$

The derived state space equations from for the **Error! Reference source not found.** as the following:

For MIMO:

$$\begin{aligned}
\dot{x}_1 &= -5x_1 + u \\
\dot{x}_2 &= 34x_1 - 7x_2 + u \\
\dot{x}_3 &= 102x_1 - x_2 - 3x_3 + 3u \\
\dot{x}_4 &= 102x_1 - x_2 - 19x_3 - x_4 + 3u \\
y &= 102x_1 - x_2 + 19x_3
\end{aligned}$$

For SISO:

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -5x_1 + u \\
\dot{x}_2 &= -3x_1 - 7x_2 + u \\
\dot{x}_3 &= -3x_1 - 3x_2 - 3x_3 + u \\
\dot{x}_4 &= -3x_1 - 3x_2 + 6x_3 - x_4 + u \\
y &= x_4
\end{aligned}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 9\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = \boldsymbol{x}_2 - 3\boldsymbol{x}_3$
$\dot{\boldsymbol{x}}_4 = \boldsymbol{x}_2 - \boldsymbol{x}_4$
$\boldsymbol{y} = \boldsymbol{x}_2$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 144s + 252}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 2\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = 2\boldsymbol{x}_1 + 5\boldsymbol{x}_2 - 3\boldsymbol{x}_3 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_4 = 2\boldsymbol{x}_1 + 5\boldsymbol{x}_2 + \boldsymbol{x}_3 - \boldsymbol{x}_4 + \boldsymbol{u}$
$\boldsymbol{y} = \boldsymbol{x}_4$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$\dot{\boldsymbol{x}}_1 = -5\boldsymbol{x}_1 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_2 = 11\boldsymbol{x}_1 - 7\boldsymbol{x}_2 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_3 = 11\boldsymbol{x}_1 - 4\boldsymbol{x}_2 - 3\boldsymbol{x}_3 + \boldsymbol{u}$
$\dot{\boldsymbol{x}}_4 = 11\boldsymbol{x}_1 - 4\boldsymbol{x}_2 + 3\boldsymbol{x}_3 - \boldsymbol{x}_4 + \boldsymbol{u}$
$\boldsymbol{y} = \boldsymbol{x}_4$

1.10 Write the State Space model in Matrix Format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A}_{n\times n}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_{n\times 1}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C}_{1\times n}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}_{1\times 1}\boldsymbol{u}(\boldsymbol{t}) \end{aligned} \qquad (\mathbf{1.7.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$A = \begin{array}{c c} & \begin{array}{cccc} \boldsymbol{x_1} & \boldsymbol{x_2} & \cdots & \boldsymbol{x_n} \end{array} \\ \begin{array}{c} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \vdots \\ \dot{\boldsymbol{x}}_n \end{array} & \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \cdots & \boldsymbol{a_{nn}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & \cdots & \boldsymbol{a_{2n}} \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{a_{n1}} & \boldsymbol{a_{n2}} & \cdots & \boldsymbol{a_{nn}} \end{bmatrix} \end{array} \qquad B = \begin{array}{c c} & \boldsymbol{u} \\ \begin{array}{c} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \vdots \\ \dot{\boldsymbol{x}}_n \end{array} & \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \vdots \\ \boldsymbol{b_n} \end{bmatrix} \end{array} \tag{1.7.2}$$

$$C = \begin{array}{c c} & \begin{array}{cccc} \boldsymbol{x_1} & \boldsymbol{x_2} & \cdots & \boldsymbol{x_n} \end{array} \\ \boldsymbol{y} & \begin{bmatrix} \boldsymbol{c_1} & \boldsymbol{c_2} & \cdots & \boldsymbol{c_n} \end{bmatrix} \end{array} \qquad D = \begin{array}{c c} & \boldsymbol{u} \\ \boldsymbol{y} & [\boldsymbol{d}] \end{array}$$

Consider the derived state space, the state space diagram's matrix format as:

For MIMO:

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 34 & -7 & 0 & 0 \\ 102 & -1 & -3 & 0 \\ 102 & -1 & 19 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 3 & 3 \\ 3 & 3 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 102 & -1 & 19 & 0 \\ 102 & -1 & 19 & 0 \end{bmatrix} x(t) + [0]u(t).$$

For SISO:

$$\mathbf{1)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ -3 & -7 & 0 & 0 \\ -3 & -3 & -3 & 0 \\ -3 & -3 & 6 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t) + [0]u(t).$$

$$\mathbf{2)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 18s^2 + 59s + 42}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 9 & -7 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x(t) + [0]u(t).$$

$$3) \boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 160s + 336}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 2 & -7 & 0 & 0 \\ 2 & 5 & -3 & 0 \\ 2 & 5 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 0 \quad 0 \quad 0 \quad 1]\ x(t) + [0]u(t).$$

$$4) \boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 11 & -7 & 0 & 0 \\ 11 & -4 & -3 & 0 \\ 11 & -4 & 3 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\ 0 \quad 0 \quad 0 \quad 1]\ x(t) + [0]u(t).$$

2.1.8 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\ B + D$ in details.

$$\boldsymbol{C(sI - A)^{-1}B + D} \qquad (\mathbf{2.1.4.1})$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI - A}] = \begin{bmatrix} \boldsymbol{s} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s} & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{s} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a_0} & -\boldsymbol{a_1} & -\boldsymbol{a_2} & \dots & -\boldsymbol{a_{n-1}} \end{bmatrix} \qquad (\mathbf{2.1.4.2})$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -5 & 1 & 0 & 0 \\ -3 & -7 & 0 & 0 \\ -3 & -3 & -3 & 0 \\ -3 & -3 & 6 & -1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+5 & 0 & 0 & 0 \\ 3 & s+7 & 0 & 0 \\ 3 & 3 & s+3 & 0 \\ 3 & 3 & -6 & s+1 \end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[sI - A]^{-1} = \frac{adj(sI - A)}{det(sI - A)} \quad \dots \quad \frac{1}{det(sI - A)} * [adj(sI - A)] \qquad (2.3.4.3)$$

$$\frac{1}{det(sI - A)} = \frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[adj(sI - A)] = \begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ -3(s+3)(s+1) & (s+1)(s+5)(s+3) & 0 & 0 \\ -3(s+4)(s+1) & -3(s+5)(s+9) & (s+1)(s+5)(s+7) & 0 \\ -3(s+4)(s+9) & -3(s+5)(s+1) & 6(s+5)(s+7) & (s+5)(s+3)(s+7) \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{det(sI - A)} \times [adj(sI - A)]$$

$$C(sI - A)^{-1}\ B + D = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\frac{1}{det(sI - A)} = \frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[adj(sI - A)] = \begin{bmatrix} (s+3)(s+1)(s+7) & 0 & 0 & 0 \\ 9(s+3)(s+1) & (s+1)(s+5)(s+3) & 0 & 0 \\ 9s+9 & (s+5)(s+1) & (s+1)(s+5)(s+7) & 0 \\ 9s+27 & (s+5)(s+3) & 0 & (s+5)(s+3)(s+7) \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{det(sI - A)} \times [adj(sI - A)]$$

$$C(sI - A)^{-1}\ B + D = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\frac{1}{det(sI - A)} = \frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})] = \begin{bmatrix} (s+3)(s+1)(s+7) & 0 & 0 & 0 \\ 2(s+3)(s+1) & (s+1)(s+5)(s+3) & 0 & 0 \\ 2(s+12)(s+1) & 5(s+5)(s+1) & (s+1)(s+5)(s+7) & 0 \\ (s+4)(s+12) & 5(s+5)(s+4) & (s+5)(s+7) & (s+5)(s+3)(s+7) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D = \frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})} \times [\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]$$

$$C(sI-A)^{-1}\ B+D = \frac{\boldsymbol{s^3+23s^2+160s+336}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$M(s) = \frac{\boldsymbol{s^3+23s^2+160s+336}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$\mathbf{4)}\boldsymbol{M}(\boldsymbol{s}) = \frac{\boldsymbol{Y}(\boldsymbol{s})}{\boldsymbol{U}(\boldsymbol{s})} = \frac{\boldsymbol{s^3+25s^2+162s+288}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$\frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})} = \frac{1}{(s+1)(s+3)(s+5)(s+7)}$$

$$[\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})] = \begin{bmatrix} (s+3)(s+1)(s+7) & 0 & 0 & 0 \\ 11(s+3)(s+1) & (s+1)(s+5)(s+3) & 0 & 0 \\ 11(s+3)(s+1) & -4(s+5)(s+1) & (s+1)(s+5)(s+7) & 0 \\ 11(s+3)(s+6) & -4(s+5)(s+6) & 3(s+5)(s+7) & (s+5)(s+3)(s+7) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D = \frac{\mathbf{1}}{\boldsymbol{det}(\boldsymbol{sI}-\boldsymbol{A})} \times [\boldsymbol{adj}(\boldsymbol{sI}-\boldsymbol{A})]$$

$$C(sI-A)^{-1}\ B+D = \frac{\boldsymbol{s^3+25s^2+162s+288}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$M(s) = \frac{\boldsymbol{s^3+25s^2+162s+288}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

2.1.9 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Functions (Polynomial):

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Functions(Polynomial):

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

2.2.1 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

```
>> A=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> B=[1;1;1;1];
>> C=[0 0 0 1];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> T=tf(num,den)

T
A=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
B=[1;1;1;1];
C=[0 0 0 1];

T =

       s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Table 12 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 15 62 72]	[1 16 86 176 105]
The Converted TF	[1 15 62 72]	[1 16 66 176 105]

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A=[-5 0 0 0;9 -7 0 0;0 1 -3 0;0 1 0 -1];
>> B=[1;1;0;0];
>> C=[0 1 0 0];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> T=tf(num,den)

T =

       s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Table 13 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 18 59 42]	[1 16 86 176 105]
The Converted TF	[1 18 59 42]	[1 16 66 176 105]

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A=[-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> B=[1;1;1;1];
>> C=[0 0 0 1];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> T=tf(num,den)

T =

       s^3 + 23 s^2 + 160 s + 336
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Table 14 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**

The Original TF	[1 23 160 336]	[1 16 86 176 105]
The Converted TF	[1 23 160 336]	[1 16 66 176 105]

$$4) M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

```
>> A=[-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> B=[1;1;1;1];
>> C=[0 0 0 1];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D);
>> T=tf(num,den)

T =

     s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Table 15 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	[1 29 238 480]	[1 16 86 176 105]
The Converted TF	[1 29 238 480]	[1 16 66 176 105]

2.3.37 Find the characteristic equation $|sI-A|$.

Initially, we apply the equation $|\lambda I-A|$, which returns:

$$|\lambda I-A|=\begin{vmatrix}\lambda & 0 & \cdots & \cdots & 0\\ 0 & \lambda & \ddots & \cdots & 0\\ \vdots & \ddots & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & \cdots & 0 & \lambda\end{vmatrix}-\begin{vmatrix}0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & 1\\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1}\end{vmatrix}$$

$$|\lambda I-A|=0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 176 & 86 & \lambda+16 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 176 & 86 & \lambda+16 \end{vmatrix} = 0$$

$$s^4 + 16s^3 + 86s^2 + 176s + 105 = 0$$

Then we use the MATLAB command “roots” and get the following eigenvalues:

$$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -1,$$

Note: All the equations have same eigenvalues

Task Number	Chapter	Task Name	Task Description
Task 5	**Chapter 3:** Decomposition of Transfer Functions	Diagonal Decomposition Based on Controllability Canonical Form (If Applicable)	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI-A)^{-1}B+D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\lvert sI-A\rvert$. H) Find the Eigenvalues of matrix A.

2.1.10 Draw the state diagram.

To draw the state diagram, multiply the transfer function by s^{-5} for the numerator and denominator, yielding the following form:

$$\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 + a_{n-1}s^{-1} + a_{n-2}s^{-2} + a_{n-3}s^{-3} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}} \quad (2.1.1.1)$$

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+23s^{-2}+160s^{-3}+336s^{-4}}{1+16s^{-1}+86s^{-2}+176s^{-3}+105s^{-4}}$$

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+144s^{-3}+288s^{-4}}{1+16s^{-1}+86s^{-2}+176s^{-3}+105s^{-4}}$$

Then, using (-1) as a common factor from the denominator, we get the following form:

$$\boldsymbol{\frac{Y(s)}{U(s)}=\frac{b_{n-1}s^{-1}+b_{n-2}s^{-2}+b_{n-3}s^{-3}+\cdots+b_1s^{-n+1}+b_0s^{-n}}{1-[-a_{n-1}s^{-1}-a_{n-2}s^{-2}-a_{n-3}s^{-3}-\cdots-a_1s^{-n+1}-a_0s^{-n}]}} \quad (\mathbf{2.1.1.2})$$

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+15s^{-2}+62s^{-3}+72s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+18s^{-2}+59s^{-3}+42s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+23s^{-2}+160s^{-3}+336s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+144s^{-3}+288s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

The CCF State-Space Diagram is shown in **Error! Reference source not found.**.

Its to be noted that:

- **The first transfer function is the one with the yellow color.**
- **The second transfer function is the one with the blue color.**
- **The third transfer function is the one with the red color.**
- **The fourth transfer function is the one with the green color.**
- **The Parts with the Black color are the ones common with all Transfer functions.**

- **And therefore, the State diagram of the Fourth order MIMO system with initial form: Cascaded lower triangle is Drawn here on a landscape:**

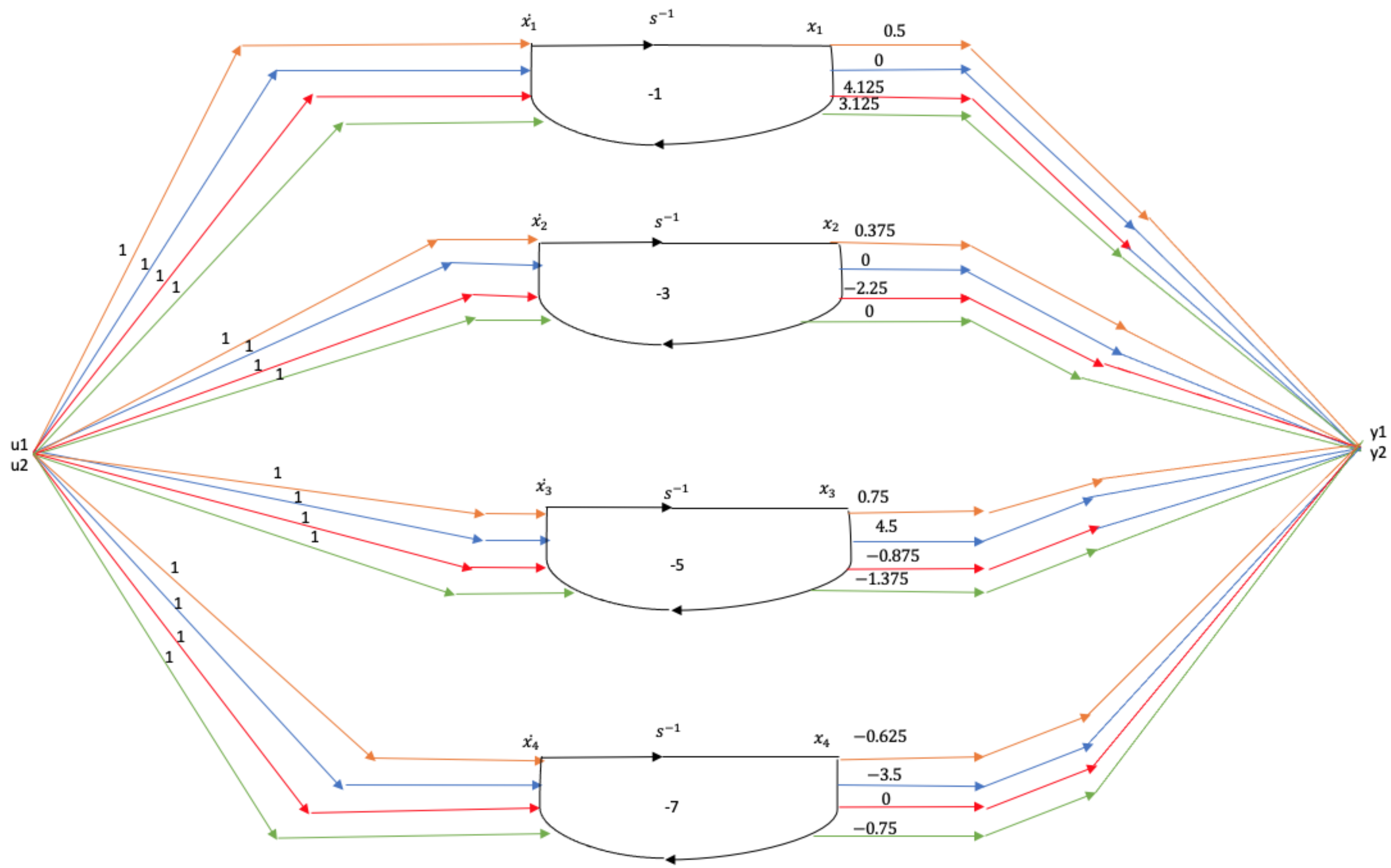


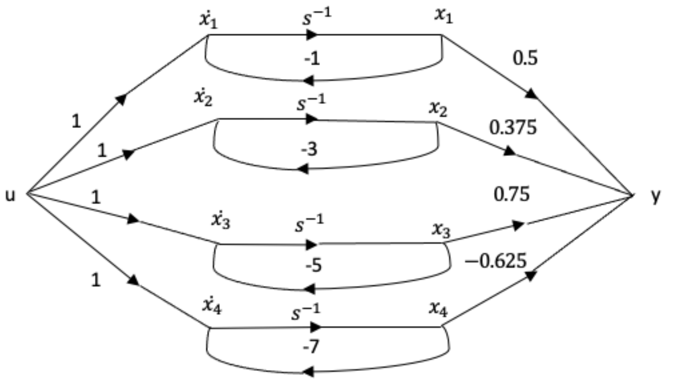
Figure 13 State Diagram of diagonal decomposition based on Controllability Canonical Form

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

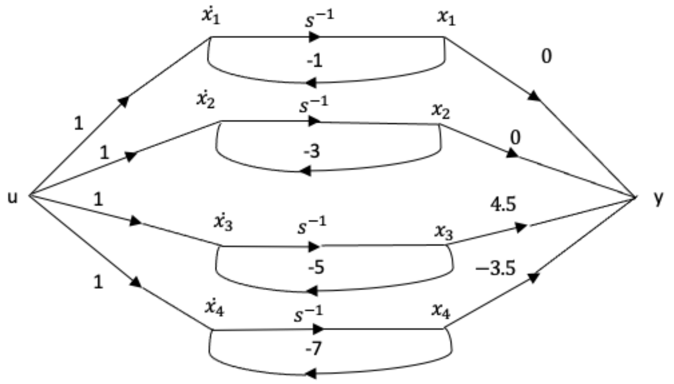
Evidence:

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1 1;1 1;1 1;1 1];
>> C = [0.5 0.375 5.25 -4.125;7.25 -2.25 -2.25 -0.75];
>> D = [0];
>> sys=ss(A,B,C,D);
>> sys1=tf(sys)

sys1 =

  From input 1 to output...
          2 s^3 + 33 s^2 + 121 s + 114
   1:  -----------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

          2 s^3 + 48 s^2 + 322 s + 624
   2:  -----------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

  From input 2 to output...
          2 s^3 + 33 s^2 + 121 s + 114
   1:  -----------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

          2 s^3 + 48 s^2 + 322 s + 624
   2:  -----------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

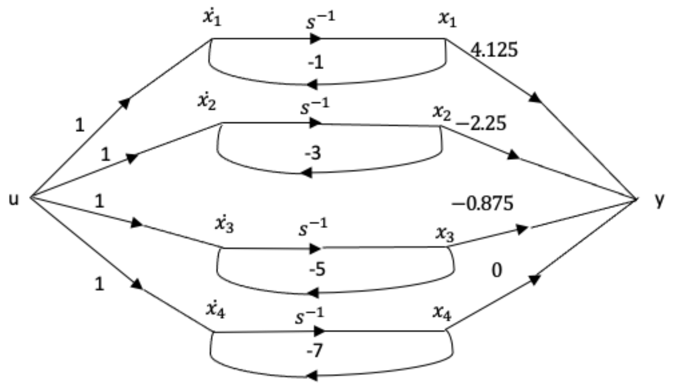
$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{0.75}{(s+5)} + \frac{-0.625}{(s+7)} + \frac{0.375}{(s+3)} + \frac{0.5}{(s+1)}$$



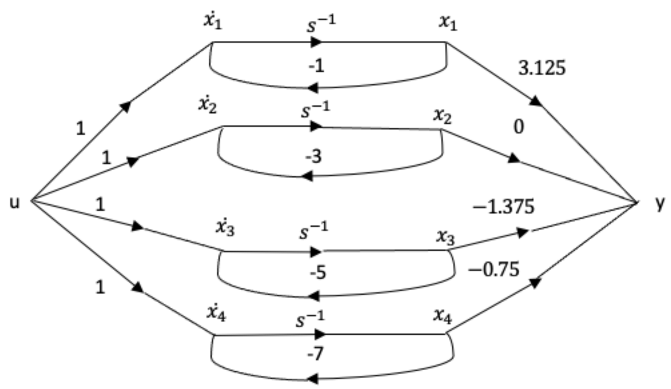
$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{4.5}{(s+5)} + \frac{-3.5}{(s+7)} + \frac{0}{(s+3)} + \frac{0}{(s+1)}$$



$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{-0.875}{(s+5)} + \frac{0}{(s+7)} + \frac{-2.25}{(s+3)} + \frac{4.125}{(s+1)}$$



$$4) M(s) = \frac{Y(s)}{U(s)} = \frac{-1.375}{(s+5)} + \frac{-0.75}{(s+7)} + \frac{0}{(s+3)} + \frac{3.125}{(s+1)}$$



Derive the state space model in equations format.

Throughout the previous section 2.2.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= -a_0 x_n + b_0 u\\
\dot{x}_2 &= x_1 - a_1 x_n + b_1 u\\
&\vdots\\
\dot{x}_n &= x_{n-1} - a_{n-1} x_n + b_{n-1} u\\
y &= x_n
\end{aligned} \qquad (2.2.2.1)$$

The derived state space equation from for the **Error! Reference source not found.** as the following:

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -1x_1 + u\\
\dot{x}_2 &= -3x_2 + u\\
\dot{x}_3 &= -5x_3 + u\\
\dot{x}_4 &= -7x_4 + u
\end{aligned}$$

$$y = 0.5x_1 + 0.375x_2 + 0.75x_3 - 0.625x_4$$

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -1x_1 + u\\
\dot{x}_2 &= -3x_2 + u\\
\dot{x}_3 &= -5x_3 + u
\end{aligned}$$

$$\dot{x}_4 = -7x_4 + u$$
$$y = 0\boldsymbol{x_1} + 0\boldsymbol{x_2} + 4.5\boldsymbol{x_3} - 3.5\boldsymbol{x_4}$$

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = -1\boldsymbol{x_1} + u$$
$$\dot{x}_2 = -3\boldsymbol{x_2} + u$$
$$\dot{x}_3 = -5x_3 + u$$
$$\dot{x}_4 = -7x_4 + u$$
$$y = 4.125\boldsymbol{x_1} - 2.25\boldsymbol{x_2} - 0.875\boldsymbol{x_3}$$

$$\frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = -1\boldsymbol{x_1} + u$$
$$\dot{x}_2 = -3\boldsymbol{x_2} + u$$
$$\dot{x}_3 = -5x_3 + u$$
$$\dot{x}_4 = -7x_4 + u$$
$$y = 3.125\boldsymbol{x_1} + 0\boldsymbol{x_2} - 1.375\boldsymbol{x_3} - 0.75\boldsymbol{x_4}$$

Write the state space model in matrix format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A_{OFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B_{OCF}}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C_{OFF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D_{OCF}}\boldsymbol{u}(\boldsymbol{t}) \end{aligned} \qquad (\mathbf{2.1.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A_{OCF}} = \begin{bmatrix} -\boldsymbol{p_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p_{n-1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p_n} \end{bmatrix} \qquad \boldsymbol{B_{OCF}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

$$\boldsymbol{C_{OCF}} = \begin{bmatrix} \boldsymbol{r_1} & \boldsymbol{r_2} & \dots & \boldsymbol{r_{n-1}} & \boldsymbol{r_n} \end{bmatrix} \qquad \boldsymbol{D_{OCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\mathbf{1)}\boldsymbol{M}(\boldsymbol{s}) = \frac{\boldsymbol{Y}(\boldsymbol{s})}{\boldsymbol{U}(\boldsymbol{s})} = \frac{\boldsymbol{s}^3 + \mathbf{15}\boldsymbol{s}^2 + \mathbf{62}\boldsymbol{s} + \mathbf{72}}{\boldsymbol{s}^4 + \mathbf{16}\boldsymbol{s}^3 + \mathbf{86}\boldsymbol{s}^2 + \mathbf{176}\boldsymbol{s} + \mathbf{105}}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\, x(t) + [0]u(t)$$

2) $)\boldsymbol{M}(s) = \frac{Y(s)}{U(s)} = \frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0 \;\; 0 \;\; 4.5 \;\; -3.5]\, x(t) + [0]u(t)$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,4.125 \quad -2.25 \;\; -0.875 \;\; 0]\, x(t) + [0]u(t)$$

4)

$$M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,3.125 \;\; 0 \;\; -1.375 \;\; -0.75]\, x(t) + [0]u(t)$$

Convert the resulting state space model to transfer function format using the equation $C(sI-A)^{-1}\,B+D$ in details.

$$C(sI-A)^{-1}B+D \qquad (2.2.4.1)$$

Initially, we apply the equation $|sI-A|$, which returns:

$$[sI-A]=\begin{bmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{bmatrix} - \begin{bmatrix} -p_1 & 0 & \cdots & 0 & 0 \\ 0 & -p_2 & 0 & 0 & 0 \\ 0 & 0 & -p_{n-1} & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \cdots & 0 & -p_n \end{bmatrix} \qquad (2.2.4.1)$$

$$[sI-A]=\begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}$$

$$[sI-A]=\begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}=0$$

Then we find the inverse of the resulting matrix:

$$[sI-A]^{-1}=\frac{adj(sI-A)}{det(sI-A)} \quad \cdots \quad \frac{1}{det(sI-A)}*[cof(sI-A)]^T \qquad (2.2.4.3)$$

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ 0 & (s+1)(s+5)(s+7) & 0 & 0 \\ 0 & 0 & (s+1)(s+3)(s+7) & 0 \\ 0 & 0 & 0 & (s+1)(s+3)(s+5) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$

$$=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[0.5\ \ 0.375\ \ 0.75\ \ -0.625]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[0.5(s+3)(s+5)(s+7)\ \ 0.375(s+1)(s+5)(s+7)\ \ 0.75(s+1)(s+3)(s+7)\ \ -0.625(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)\ \boldsymbol{M}(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[[\,0\ \ 0\ \ 4.5\ \ -3.5]\,]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[0(s+3)(s+5)(s+7)\ \ 0(s+1)(s+5)(s+7)\ \ 4.5(s+1)(s+3)(s+7)\ \ -3.5(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$

$$=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[\,4.125\quad -2.25\ -0.875\ \ 0]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$= \frac{[4.125(s+3)(s+5)(s+7)\ \ -2.25(s+1)(s+5)(s+7)\ \ -0.875(s+1)(s+3)(s+7)\ \ 0(s+1)(s+3)(s+5)] \times \begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D = \frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$M(s) = \frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1} = \frac{1}{s^4+16s^3+86s^2+176s+105} \times [p]$$

$$p = \begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ 0 & (s+1)(s+5)(s+7) & 0 & 0 \\ 0 & 0 & (s+1)(s+3)(s+7) & 0 \\ 0 & 0 & 0 & (s+1)(s+3)(s+5) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$
$$= \frac{1}{s^4+16s^3+86s^2+176s+105} \times [[\,3.125\ \ 0\ \ -1.375\ \ -0.75]] \times [p] \times \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$= \frac{[3.125(s+3)(s+5)(s+7)\ \ 0(s+1)(s+5)(s+7)\ \ -1.375(s+1)(s+3)(s+7)\ \ -0.75(s+1)(s+3)(s+5)] \times \begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D = \frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$M(s) = \frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s) = \frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

3)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

4)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [0.5 0.375 0.75 -0.625];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;9 -7 0 0;0 1 -3 0;0 0 0 -1];
>> B = [1;1;0;0];
>> C = [0 1 0 0];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

        s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [0 0 4.5 -3.5];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

        s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [4.125 -2.25 -0.875 0];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D=[0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [25/8 0 -11/8 -3/4];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

Figure 14 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to OCF state-space

Table 16 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**		The **coefficients of Denominator**
The Original TF	1)	[1 15 62 72]	[1 16 66 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	
The Converted TF	1)	[1 15 62 72]	[1 16 66 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	

Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|\boldsymbol{sI}-\boldsymbol{A}| = \begin{vmatrix} \boldsymbol{s} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s} & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{s} \end{vmatrix} - \begin{vmatrix} -\boldsymbol{p}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p}_{\boldsymbol{n}-\mathbf{1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p}_{\boldsymbol{n}} \end{vmatrix} \qquad (\mathbf{2.2.7.1})$$

$$|sI-A| = 0$$

$$|sI-A| = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}$$

$$|sI-A| = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix} = 0$$

$$\boldsymbol{s}^4 + \mathbf{16}\boldsymbol{s}^3 + \mathbf{86}\boldsymbol{s}^2 + \mathbf{176}\boldsymbol{s} + \mathbf{105} = \mathbf{0} \qquad (\mathbf{2.2.7.2})$$

Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda\boldsymbol{I}-\boldsymbol{A}| = \begin{vmatrix} \lambda & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \lambda \end{vmatrix} - \begin{vmatrix} -\boldsymbol{p}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p}_{\boldsymbol{n}-\mathbf{1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p}_{\boldsymbol{n}} \end{vmatrix} \qquad (\mathbf{2.2.8.1})$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix} = 0$$

$$\boldsymbol{\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0} \qquad (\mathbf{2.2.8.2})$$

Then we use the **MATLAB** command **<u>"roots"</u>** and get the following eigenvalues:

$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$

Table 17 Task 6 Description

Task Number	Chapter	Task Name	Task Description
Task 6	Chapter 3: Decomposition of Transfer Functions	Diagonal Decomposition Based on Observability Canonical Form (If Applicable).	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\|sI - A\|$. H) Find the Eigenvalues of matrix A.

2.1.11 Draw the state diagram.

To draw the state diagram, multiply the transfer function by s^{-5} for the numerator and denominator, yielding the following form:

$$\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 + a_{n-1}s^{-1} + a_{n-2}s^{-2} + a_{n-3}s^{-3} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}} \quad (2.1.1.1)$$

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 144s^{-3} + 288s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

Then, using (-1) as a common factor from the denominator, we get the following form:

$$\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + b_{n-3}s^{-3} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}}{1 - [-a_{n-1}s^{-1} - a_{n-2}s^{-2} - a_{n-3}s^{-3} - \cdots - a_1 s^{-n+1} - a_0 s^{-n}]} \quad (2.1.1.2)$$

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 15s^{-2} + 62s^{-3} + 72s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$4) M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 144s^{-3} + 288s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

The CCF State-Space Diagram is shown in **Error! Reference source not found.**.

Its to be noted that:

- **The first transfer function is the one with the yellow color.**
- **The second transfer function is the one with the blue color.**
- **The third transfer function is the one with the red color.**
- **The fourth transfer function is the one with the green color.**
- **The Parts with the Black color are the ones common with all Transfer functions.**

- **And therefore, the State diagram of the Fourth order MIMO system with initial form: Cascaded lower triangle is Drawn here on a landscape:**

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

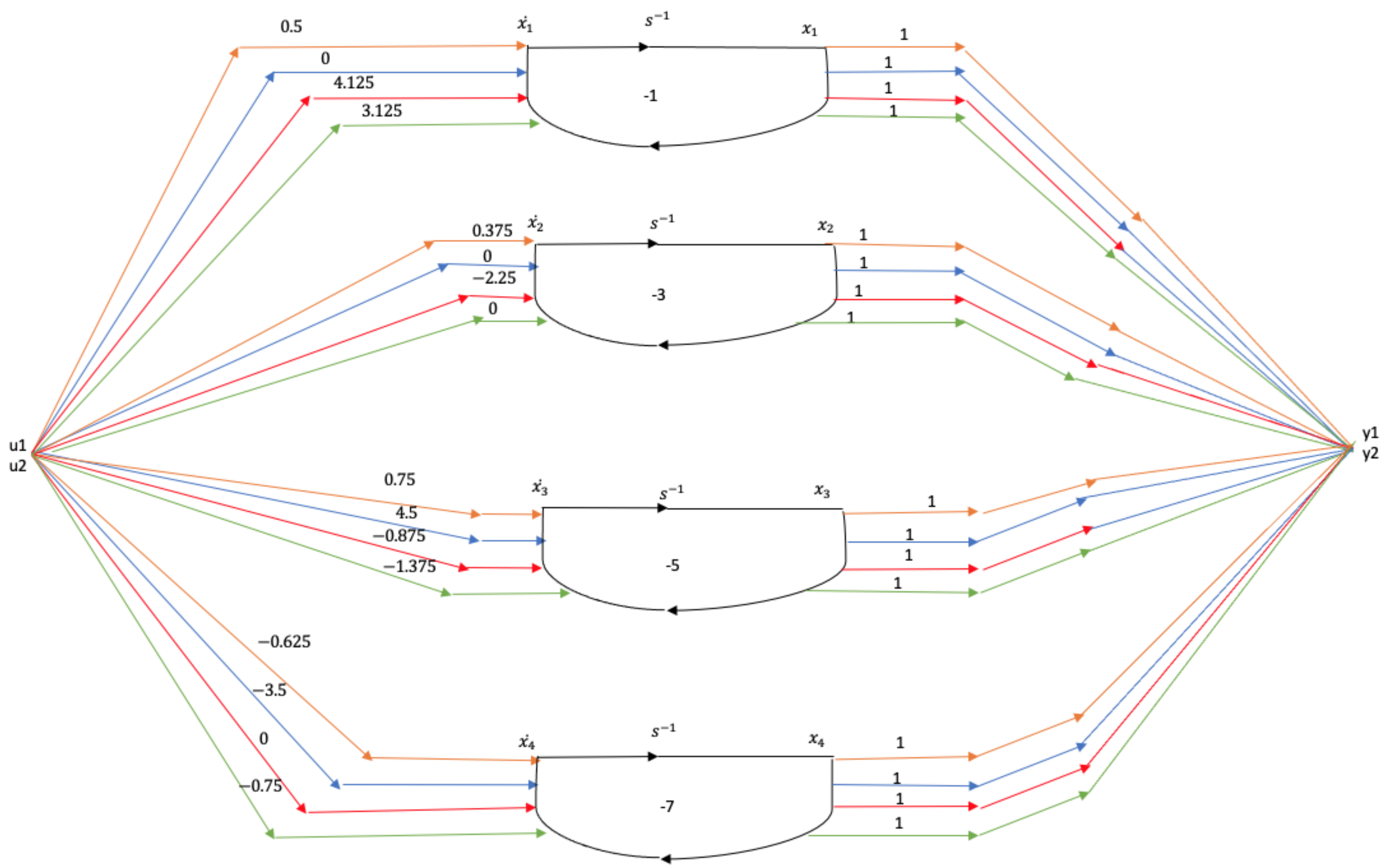
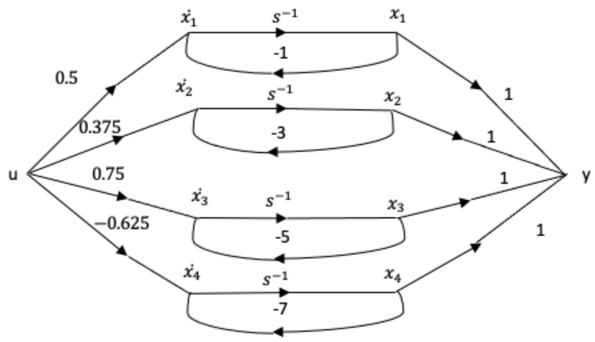


Figure 15 State Diagram of diagonal decomposition based on observability Canonical Form

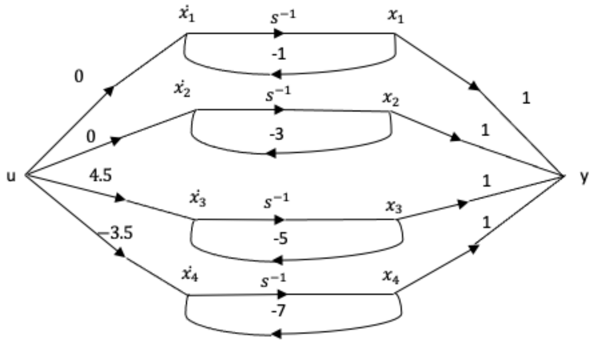
Evidence:

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [0.5 0.375;5.25 -4.125;7.25 -2.25;-2.25 -0.75];
>> C = [1 1 1 1;1 1 1 1];
>> D = [0];
>> sys=ss(A,B,C,D);
>> sys1=tf(sys)

sys1 =

  From input 1 to output...
       10.75 s^3 + 135.3 s^2 + 455.2 s + 354.7
   1:  ---------------------------------------
         s^4 + 16 s^3 + 86 s^2 + 176 s + 105

       10.75 s^3 + 135.3 s^2 + 455.2 s + 354.7
   2:  ---------------------------------------
         s^4 + 16 s^3 + 86 s^2 + 176 s + 105

  From input 2 to output...
       -6.75 s^3 - 79.5 s^2 - 254.3 s - 163.5
   1:  --------------------------------------
        s^4 + 16 s^3 + 86 s^2 + 176 s + 105

       -6.75 s^3 - 79.5 s^2 - 254.3 s - 163.5
   2:  --------------------------------------
        s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

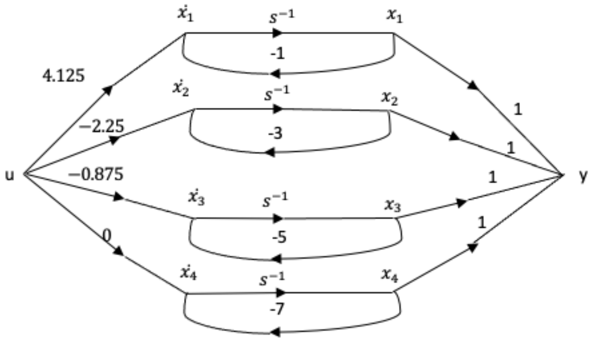
$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{0.75}{(s+5)}+\frac{-0.625}{(s+7)}+\frac{0.375}{(s+3)}+\frac{0.5}{(s+1)}$$



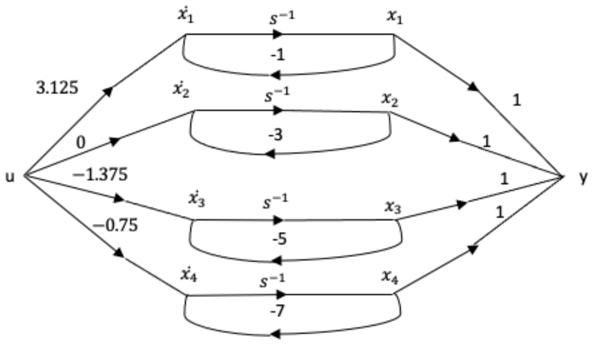
$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{4.5}{(s+5)}+\frac{-3.5}{(s+7)}+\frac{0}{(s+3)}+\frac{0}{(s+1)}$$



$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{-0.875}{(s+5)}+\frac{0}{(s+7)}+\frac{-2.25}{(s+3)}+\frac{4.125}{(s+1)}$$



$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{-1.375}{(s+5)} + \frac{-0.75}{(s+7)} + \frac{0}{(s+3)} + \frac{3.125}{(s+1)}$$



Derive the state space model in equations format.

Throughout the previous section 2.2.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}_{\mathbf{1}} &= -\boldsymbol{a}_{\mathbf{0}}\boldsymbol{x}_{\boldsymbol{n}} + \boldsymbol{b}_{\mathbf{0}}\boldsymbol{u} \\ \dot{\boldsymbol{x}}_{\mathbf{2}} &= \boldsymbol{x}_{\mathbf{1}} - \boldsymbol{a}_{\mathbf{1}}\boldsymbol{x}_{\boldsymbol{n}} + \boldsymbol{b}_{\mathbf{1}}\boldsymbol{u} \\ &\vdots \\ \dot{\boldsymbol{x}}_{\boldsymbol{n}} &= \boldsymbol{x}_{\boldsymbol{n-1}} - \boldsymbol{a}_{\boldsymbol{n-1}}\boldsymbol{x}_{\boldsymbol{n}} + \boldsymbol{b}_{\boldsymbol{n-1}}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{x}_{\boldsymbol{n}} \end{aligned} \qquad (\mathbf{2.2.2.1})$$

The derived state space equation from for the **Error! Reference source not found.** as the following:

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned} \dot{x}_1 &= -1\boldsymbol{x}_{\mathbf{1}} + 0.5\boldsymbol{x}_{\mathbf{1}} \\ \dot{x}_2 &= -3\boldsymbol{x}_{\mathbf{2}} + 0.375\mathrm{u} \\ \dot{x}_3 &= -5x_3 + 0.75u \\ \dot{x}_4 &= -7x_4 - 0.625\mathrm{u} \\ y &= \boldsymbol{x}_{\mathbf{1}} + \boldsymbol{x}_{\mathbf{2}} + \boldsymbol{x}_{\mathbf{3}} + \boldsymbol{x}_{\mathbf{4}} \end{aligned}$$

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned} \dot{x}_1 &= -1\boldsymbol{x}_{\mathbf{1}} \\ \dot{x}_2 &= -3\boldsymbol{x}_{\mathbf{2}} \\ \dot{x}_3 &= -5x_3 + 4.5u \\ \dot{x}_4 &= -7x_4 - 3.5 \\ y &= \boldsymbol{x}_{\mathbf{1}} + \boldsymbol{x}_{\mathbf{2}} + \boldsymbol{x}_{\mathbf{3}} + \boldsymbol{x}_{\mathbf{4}} \end{aligned}$$

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned} \dot{x}_1 &= -1\boldsymbol{x}_{\mathbf{1}} + 4.125u \\ \dot{x}_2 &= -3\boldsymbol{x}_{\mathbf{2}} - 2.25u \\ \dot{x}_3 &= -5x_3 - 0.875u \\ \dot{x}_4 &= -7x_4 \\ y &= \boldsymbol{x}_{\mathbf{1}} + \boldsymbol{x}_{\mathbf{2}} + \boldsymbol{x}_{\mathbf{3}} + \boldsymbol{x}_{\mathbf{4}} \end{aligned}$$

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned} \dot{x}_1 &= -1\boldsymbol{x}_{\mathbf{1}} + 3.125u \\ \dot{x}_2 &= -3\boldsymbol{x}_{\mathbf{2}} \\ \dot{x}_3 &= -5x_3 - 1.375 \\ \dot{x}_4 &= -7x_4 - 0.75 \end{aligned}$$

$y = \boldsymbol{x_1} + \boldsymbol{x_2} + \boldsymbol{x_3} + \boldsymbol{x_4}$

Write the state space model in matrix format (A, B, C, D).

We can write the derived state equation format in matrix format as the following general format:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A_{OFF}}\,\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B_{OCF}}\boldsymbol{u}(\boldsymbol{t})$$
$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C_{OFF}}\,\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D_{OCF}}\boldsymbol{u}(\boldsymbol{t}) \qquad (\mathbf{2.1.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A_{OCF}} = \begin{bmatrix} -\boldsymbol{p_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p_{n-1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p_n} \end{bmatrix} \qquad \boldsymbol{B_{OCF}} = \begin{bmatrix} \boldsymbol{r_1} \\ \boldsymbol{r_2} \\ \vdots \\ \boldsymbol{r_{n-1}} \\ \boldsymbol{r_n} \end{bmatrix}$$

$$\boldsymbol{C_{OCF}} = [\mathbf{1} \quad \mathbf{1} \quad \mathbf{1} \quad \mathbf{1}] \qquad \boldsymbol{D_{OCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

1)

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 0.5 \\ 0.375 \\ 0.75 \\ -0.625 \end{bmatrix} u(t)$$

$$y(t) = [1 \;\; 1 \;\; 1 \;\; 1]\, x(t) + [0]u(t)$$

2)

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) \;= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 4.5 \\ -3.5 \end{bmatrix} u(t)$$

$$y(t) = [\,1\;\;1\;\;1\;\;1]\, x(t) + [0]u(t)$$

3) $M(s) = \frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$

$$\dot{x}(t) \;= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 4.125 \\ -2.25 \\ 0.875 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [1\;\;1\;\;1\;\;1]\, x(t) + [0]u(t)$$

4)

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) \;= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 3.125 \\ 0 \\ -1.375 \\ -0.75 \end{bmatrix} u(t)$$

$$y(t) = [\,1\;\;1\;\;1\;\;1]\, x(t) + [0]u(t)$$

Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\, B + D$ in details.

$$\boldsymbol{C(sI - A)^{-1}B + D} \qquad (2.2.4.1)$$

Initially, we apply the equation $|sI - A|$, which returns:

$$[\boldsymbol{sI-A}] = \begin{bmatrix} \boldsymbol{s} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{s} & \ddots & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{s} \end{bmatrix} - \begin{bmatrix} -\boldsymbol{p_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{p_2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{p_{n-1}} & \boldsymbol{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & -\boldsymbol{p_n} \end{bmatrix} \qquad (\mathbf{2.2.4.1})$$

$$[sI-A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}$$

$$[sI-A] = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix} = 0$$

Then we find the inverse of the resulting matrix:

$$[\boldsymbol{sI-A}]^{-1} = \frac{\boldsymbol{adj(sI-A)}}{\boldsymbol{det(sI-A)}} \quad \cdots \quad \frac{\mathbf{1}}{\boldsymbol{det(sI-A)}} * [\boldsymbol{cof(sI-A)}]^{\boldsymbol{T}} \qquad (\mathbf{2.2.4.3})$$

1)

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$[sI-A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ 0 & (s+1)(s+5)(s+7) & 0 & 0 \\ 0 & 0 & (s+1)(s+3)(s+7) & 0 \\ 0 & 0 & 0 & (s+1)(s+3)(s+5) \end{bmatrix}$$

$$C(sI-A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [1\ \ 1\ \ 1\ \ 1] \times [p] \times \begin{bmatrix} 0.5 \\ 0.375 \\ 0.75 \\ -0.625 \end{bmatrix}$$

$$= \frac{[(s+3)(s+5)(s+7)(s+1)(s+5)(s+7)(s+1)(s+3)(s+7)(s+1)(s+3)(s+5)] \times \begin{bmatrix} 0.5 \\ 0.375 \\ 0.75 \\ -0.625 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI-A)^{-1}\ B + D = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2)

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ 0 & (s+1)(s+5)(s+7) & 0 & 0 \\ 0 & 0 & (s+1)(s+3)(s+7) & 0 \\ 0 & 0 & 0 & (s+1)(s+3)(s+5) \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [1\ \ 1\ \ 1\ \ 1\ ] \times [p] \times \begin{bmatrix} 0 \\ 0 \\ 4.5 \\ -3.5 \end{bmatrix}$$

$$= \frac{[(s+3)(s+5)(s+7)(s+1)(s+5)(s+7)(s+1)(s+3)(s+7)(s+1)(s+3)(s+5)] \times \begin{bmatrix} 0 \\ 0 \\ 4.5 \\ -3.5 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI - A)^{-1}\ B + D = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

3) $M(s) = \frac{s^3+23s^2+144s+252}{s^4+16s^3+86s^2+176s+105}$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+3)(s+5)(s+7) & 0 & 0 & 0 \\ 0 & (s+1)(s+5)(s+7) & 0 & 0 \\ 0 & 0 & (s+1)(s+3)(s+7) & 0 \\ 0 & 0 & 0 & (s+1)(s+3)(s+5) \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [1\ \ 1\ \ 1\ \ 1\ ] \times [p] \times \begin{bmatrix} 4.125 \\ -2.25 \\ 0.875 \\ 0 \end{bmatrix}$$

$$= \frac{[(s+3)(s+5)(s+7)(s+1)(s+5)(s+7)(s+1)(s+3)(s+7)(s+1)(s+3)(s+5)] \times \begin{bmatrix} 2.708 \\ 0 \\ -1.125 \\ -0.583 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

4)

$$M(s)=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[1\ \ 1\ \ 1\ \ 1]\times[p]\times\begin{bmatrix}3.125\\0\\-1.375\\-0.75\end{bmatrix}$$

$$=\frac{[(s+3)(s+5)(s+7)(s+1)(s+5)(s+7)(s+1)(s+3)(s+7)(s+1)(s+3)(s+5)]\times\begin{bmatrix}3.125\\0\\-1.375\\-0.75\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

2)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

3)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

4)

The converted Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Function:

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [0.5;0.375;0.75;-0.625];
>> C = [1 1 1 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;9 -7 0 0;0 1 -3 0;0 0 0 -1];
>> B = [1;1;0;0];
>> C = [0 1 0 0];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [0;0;4.5;-3.5];
>> C = [1 1 1 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 18 s^2 + 59 s + 42
  ------------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  ------------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [4.125;-2.25;-0.875;0];
>> C = [1 1 1 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  ------------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D=[0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [25/8 0 -11/8 -3/4];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

Figure 16 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition diagonal to OCF state-space

Table 18 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	
The Converted TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	

Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|\boldsymbol{sI - A}| = \begin{vmatrix} \boldsymbol{s} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s} & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \boldsymbol{s} \end{vmatrix} - \begin{vmatrix} -\boldsymbol{p_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p_{n-1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p_n} \end{vmatrix} \qquad (\mathbf{2.2.7.1})$$

$|sI - A| = 0$

$$|sI - A| = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}$$

$$|sI - A| = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.2.7.2})$$

Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\boldsymbol{\lambda I - A}| = \begin{vmatrix} \lambda & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \lambda \end{vmatrix} - \begin{vmatrix} -\boldsymbol{p_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p_{n-1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p_n} \end{vmatrix} \qquad (\mathbf{2.2.8.1})$$

$|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix} = 0$$

$$\boldsymbol{\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0} \qquad (\mathbf{2.2.8.2})$$

Then we use the **MATLAB** command **<u>"roots"</u>** and get the following eigenvalues:

$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$

Table 19 Task 7 Description

Task Number	Chapter	Task Name	Task Description
Task 7	**Chapter 3:** **Decomposition of Transfer Functions**	**Jordan Decomposition Based on Controllability Canonical Form (If Applicable)**	**A) Draw the state diagram.** **B) Derive the state space model in equations format.** **C) Write the state space model in matrix format (A, B, C, D).** **D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\,B + D$ in details.** **E) Compare the converted transfer function with the original transfer function that you started with.** **F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **G) Find the characteristic equation $\|sI - A\|$.** **H) Find the Eigenvalues of matrix A.**

Note: **Since this task has repeated roots we are not supposed to solve it**

Table 20 Task 8 Description

Task Number	Chapter	Task Name	Task Description
Task 8	Chapter 3: Decomposition of Transfer Functions	Jordan Decomposition Based on Observability Canonical Form (If Applicable)	A) Draw the state diagram. B) Derive the state space model in equations format. C) Write the state space model in matrix format (A, B, C, D). D) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}\,B + D$ in details. E) Compare the converted transfer function with the original transfer function that you started with. F) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. G) Find the characteristic equation $\|sI - A\|$. H) Find the Eigenvalues of matrix A.

Note: **Since this task has repeated roots we are not supposed to solve it**

Type equation here.

Table 21 Task 9 Description

Task Number	Chapter	Task Name	Task Description
Task 9	**Chapter 4:** **Similarity Transformations for State Space Models**	**Similarity Transformation to Controllability Canonical Form.**	**A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.** **B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).** **C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.** **D) Compare the converted transfer function with the original transfer function that you started with.** **E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **F) Find the characteristic equation $\|sI - A\|$.** **I) Find the Eigenvalues values of matrix (A).**

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

```
>> A = [0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> B = [0 0;0 0;0 0;1 1];
>> C = [114 121 33 2;624 322 48 2];
>> D = [0];
>> sys=ss(A,B,C,D);
>> sys1=tf(sys)

sys1 =

  From input 1 to output...
           2 s^3 + 33 s^2 + 121 s + 114
   1:  ---------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

           2 s^3 + 48 s^2 + 322 s + 624
   2:  ---------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

  From input 2 to output...
           2 s^3 + 33 s^2 + 121 s + 114
   1:  ---------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105

           2 s^3 + 48 s^2 + 322 s + 624
   2:  ---------------------------------
       s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

We will solve this task based on the values of the (A, B, C, D) matrices, and based on the general requirements:

$$\mathbf{1)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,72 \quad 62 \quad 15 \quad 1] x(t) + [0] u(t)$$

$$\mathbf{2)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 18s^2 + 59s + 42}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,42 \quad 59 \quad 18 \quad 1]x(t) + [0]u(t)$$

$$\mathbf{3)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 23s^2 + 144s + 252}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,336 \quad 160 \quad 23 \quad 1]x(t) + [0]u(t)$$

$$\mathbf{4)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,288 \quad 144 \quad 25 \quad 1]x(t) + [0]u(t)$$

3.1.1 Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.

We need to derive the **Transformation Matrix** ($\boldsymbol{T}$), to do so, we do as follow.

1. We first find the **controllability matrix ($\boldsymbol{Q_c}$):**

$$\boldsymbol{Q_c} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \quad \dots \quad \boldsymbol{A^{n-2}B} \quad \boldsymbol{A^{n-1}B}] \qquad \mathbf{(3.1.1.1)}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -16 \end{bmatrix}$$

$$A^2B = A \times AB = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -16 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -16 \\ 170 \end{bmatrix}$$

$$A^3B = A \times A^2B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -16 \\ 170 \end{bmatrix} = \begin{bmatrix} 1 \\ -16 \\ 170 \\ -1520 \end{bmatrix}$$

$$\boldsymbol{Q_c} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-16} \\ \mathbf{0} & \mathbf{1} & \mathbf{-16} & \mathbf{170} \\ \mathbf{1} & \mathbf{-16} & \mathbf{170} & \mathbf{-1520} \end{bmatrix}$$

2. Find the rank of matrix Q_c:

$$\boldsymbol{Rank\ Q_c = |Q_c|} \qquad (\mathbf{3.1.1.2})$$

$$|Q_C| = \begin{vmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-16} \\ \mathbf{0} & \mathbf{1} & \mathbf{-16} & \mathbf{170} \\ \mathbf{1} & \mathbf{-16} & \mathbf{170} & \mathbf{-1520} \end{vmatrix} = 4$$

$$\det(Q_C) = 4 \neq 0, Full\ Rank$$

Matrix determinant is nonzero; hence the system is full rank, which means it can be transformed into controllability canonical form.

3. Now, we find the characteristic equation $|sI-A|$, we do that as follows:

$$|s\boldsymbol{I} - \boldsymbol{A}| = \begin{vmatrix} s & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & s & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & s \end{vmatrix} - \begin{vmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} & \cdots & \boldsymbol{a_{1n}} \\ \mathbf{0} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \cdots & \boldsymbol{a_{2n}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{a_{(n-1)n}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{a_{nn}} \end{vmatrix} \qquad (\mathbf{3.1.1.3})$$

$$|sI - A| = 0$$

$$\begin{vmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} s & 1 & 0 & 0 \\ 0 & s & 1 & 0 \\ 0 & 0 & s & 1 \\ 105 & 176 & 86 & s+16 \end{vmatrix}$$

$$|sI - A| = \begin{vmatrix} s & 1 & 0 & 0 \\ 0 & s & 1 & 0 \\ 0 & 0 & s & 1 \\ 105 & 176 & 86 & s+16 \end{vmatrix} = 0$$

Therefore, the characteristic equation is:

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{3.1.1.4})$$

Comparing the above equation with the **fourth order standard equation**:

$$\boldsymbol{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \qquad (\mathbf{3.1.1.5})$$

$a_0 = 105 \quad a_1 = 176 \quad a_2 = 86 \quad a_3 = 16 \quad a_4 = 1$

4. Now, we **determine the Intermidiate Trasnformation Matrix ($\boldsymbol{W}$)**, we do that as follows:

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{a_1} & \boldsymbol{a_2} & & \boldsymbol{a_{n-1}} & \mathbf{1} \\ \boldsymbol{a_2} & \boldsymbol{a_3} & & \mathbf{1} & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \boldsymbol{a_{n-1}} & \mathbf{1} & & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & & \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad (\mathbf{3.1.1.6})$$

$$W = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & 1 \\ a_2 & a_3 & a_4 & 1 & 0 \\ a_3 & a_4 & 1 & 0 & 0 \\ a_4 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \left[\begin{bmatrix} 176 & 86 & 16 & 1 \\ 86 & 16 & 1 & 1 \\ 16 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}\right]$$

5. Determine the transformation matrix (T):

$$T = Q_c W = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-16} \\ \mathbf{0} & \mathbf{1} & \mathbf{-16} & \mathbf{170} \\ \mathbf{1} & \mathbf{-16} & \mathbf{170} & \mathbf{-1520} \end{bmatrix} \begin{bmatrix} 176 & 86 & 16 & 1 \\ 86 & 16 & 1 & 1 \\ 16 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$T = Q_c W = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0.073876710669735 & 0.012474264260627 & 0.000666101489645 \\ 0 & -0.274070485648541 & -0.018408622986557 & -0.000012110936175 \\ 0 & -3.111057284 & -0.2761293 & -0.0001816640 \\ 0 & -0.07387671 & -0.012474264 & -0.00066610 \end{bmatrix}$$

3.1.2 Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).

1. Find A_t matrix

$A_t = T^{-1}\, A\, T$

$$= \begin{bmatrix} 1 & 0.073876710669735 & 0.012474264260627 & 0.000666101489645 \\ 0 & -0.274070485648541 & -0.018408622986557 & -0.000012110936175 \\ 0 & -3.111057284 & -0.2761293 & -0.0001816640 \\ 0 & -0.07387671 & -0.012474264 & -0.00066610 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix} \rightarrow \ (It\ is\ in\ CCF\ format)$$

2. Find B_t matrix

$$B_t = T^{-1} B = \begin{bmatrix} 1 & 0.073876710669735 & 0.012474264260627 & 0.000666101489645 \\ 0 & -0.274070485648541 & -0.018408622986557 & -0.000012110936175 \\ 0 & -3.111057284 & -0.2761293 & -0.0001816640 \\ 0 & -0.07387671 & -0.012474264 & -0.00066610 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.000666 \\ -0.0000121 \\ -0.00018 \\ -0.000666154 \end{bmatrix}$$

$$\rightarrow \ (It\ is\ in\ CCF\ format)$$

3. Find C_t matrix

For the first T.F

$$C_t = CT = [72 \quad 62 \quad 15 \quad 1] \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$C_t = [73 \ \ 172 \ \ 7 \ \ 0] \rightarrow \ (It\ is\ in\ CCF\ format)$$

For the second T.F

$$C_t = CT = [42 \quad 59 \quad 18 \quad 1] \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$C_t = [42 \ \ 697 \ \ -41 \ \ 3] \rightarrow \ (It\ is\ in\ CCF\ format)$$

For the third T.F

$$C_t = CT = [336 \quad 160 \quad 23 \quad 1] \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$C_t = [336 \ \ 326 \ \ -15 \ \ 8] \rightarrow \ (It\ is\ in\ CCF\ format)$$

For the fourth T.F

$$C_t = CT = [288 \quad 144 \quad 25 \quad 1] \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -15 & 1 & 0 \\ 0 & 170 & -15 & 1 \\ 0 & -1520 & 170 & -15 \end{bmatrix}$$

$$C_t = [288 \ \ 858 \ \ -61 \ \ 10] \rightarrow \ (It\ is\ in\ CCF\ format)$$

4. Find D_t matrix

$$D_t = D = [0]$$

3.1.3 Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.

Initially, we apply the equation $|sI - A|$, which returns:

$$[sI - A] = \begin{bmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \quad (3.1.3.1)$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 1 & 0 & 0 \\ 0 & s & 1 & 0 \\ 0 & 0 & s & 1 \\ 105 & 176 & 86 & s + 16 \end{bmatrix}$$

Then we find the inverse of the resulting matrix:

$$[sI - A]^{-1} = \frac{adj(sI - A)}{det(sI - A)} \quad \dots \quad \frac{1}{det(sI - A)} * [cof(sI - A)]^T \quad (3.1.3.2)$$

- ❖ $[P] = \frac{1}{\det(sI - A)} * [\mathrm{cof}(sI - A)]^T$

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 167s + 105} \times [p]$$

$$p = \begin{bmatrix} s^3 + 16s^2 + 86s + 176 & s^2 + 16s + 86 & s + 16 & 1 \\ -105 & s(s^2 + 16s + 86) & s(s + 16) & s \\ -105s & -176s - 105 & s^2(s + 16) & s^2 \\ -105s^2 & -s(176s + 105) & -86s^2 - 176s - 105 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 167s + 105} \times [72 \quad 62 \quad 15 \quad 1] \times [p] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{[(72s^3 + 1047s^2 + 4617s + 5514) \quad (62s^3 + 897s^2 + 3874s + 4617) \quad (15s^3 + 216s^2 + 897s + 1047) \quad (s + 2)(s + 4)(s + 9)] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+2)(s+4)(s+9)}{s^4+16s^3+86s^2+176s+105}$$

$$\mathbf{2)}\boldsymbol{M(s)}=\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}=\frac{\boldsymbol{s^3+18s^2+59s+42}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} s^3+16s^2+86s+176 & s^2+16s+86 & s+16 & 1 \\ -105 & s(s^2+16s+86) & s(s+16) & s \\ -105s & -176s-105 & s^2(s+16) & s^2 \\ -105s^2 & -s(176s+105) & -86s^2-176s-105 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+167s+105}\times[42\quad 59\quad 18\quad 1]\times[p]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$=\frac{[(42s^3+567s^2+1722s+819)\quad(59s^3+819s^2+2635s+1722)\quad(18s^3+261s^2+819s+567)\quad(s+1)(s+3)(s+14)]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{s^4+16s^3+86s^2+167s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+1)(s+3)(s+14)}{s^4+16s^3+86s^2+167s+105}$$

$$\mathbf{3)}\boldsymbol{M(s)}=\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}=\frac{\boldsymbol{s^3+23s^2+160s+336}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} s^3+16s^2+86s+176 & s^2+16s+86 & s+16 & 1 \\ -105 & s(s^2+16s+86) & s(s+16) & s \\ -105s & -176s-105 & s^2(s+16) & s^2 \\ -105s^2 & -s(176s+105) & -86s^2-176s-105 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[336\quad 160\quad 23\quad 1]\times[p]\times\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$= \frac{[(252s^3 + 3927s^2 + 19257s + 26964) \quad (144s^3 + 2389s^2 + 12470s + 19257) \quad (23s^3 + 426s^2 + 2389s + 3927) \quad (s + 4)(s + 7)(s + 12)] \times}{s^4 + 16s^3 + 86s^2 + 167s + 105}$$

$$C(sI - A)^{-1} \; B + D = \frac{(s + 4)(s + 7)(s + 12)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\mathbf{4)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 25s^2 + 162s + 288}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} s^3 + 16s^2 + 86s + 176 & s^2 + 16s + 86 & s + 16 & 1 \\ -105 & s(s^2 + 16s + 86) & s(s + 16) & s \\ -105s & -176s - 105 & s^2(s + 16) & s^2 \\ -105s^2 & -s(176s + 105) & -86s^2 - 176s - 105 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1} \; B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [252 \quad 144 \quad 25 \quad 1] \times [p] \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{[(288s^3 + 4503s^2 + 22143s + 32976) \quad (144s^3 + 2425s^2 + 12712s + 22143) \quad (25s^3 + 458s^2 + 2424s + 4503) \quad (s + 16)(s + 6)(s + 3)] \times}{s^4 + 16s^3 + 86s^2 + 167s + 105}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$(sI - A)^{-1} \; B + D = \frac{(s + 16)(s + 6)(s + 3)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

3.1.4 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Functions (Polynomial):

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Functions(Polynomial):

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

3.1.5 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

1. First Transfer function:

```
Command Window
>> At1=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -167 -86 -16];
Bt1=[0;0;0;1];
Ct1=[72 62 15 1];
Dt1=0;
>> [num_t1,den_t1]=ss2tf(At1,Bt1,Ct1,Dt1);
>> Mt1=tf(num_t1,den_t1)

Mt1 =

       s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 167 s + 105

Continuous-time transfer function.
```

```
>> Ai=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

       s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

2. Second Transfer function:

Command Window

```
>>
>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
Ct=[42 59 18 1];
Dt=0;
[num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
M_t=tf(num_t,den_t)

M_t =

      s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> Ai=[-5 0 0 0;9 -7 0 0;0 1 -3 0;0 1 0 -1];
Bi=[1;1;0;0];
Ci=[0 1 0 0];
Di=0;
[num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
M_i=tf(num_i,den_i)

M_i =

      s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

3rd Transfer function:

Command Window

```
>> clear
>> Ai=[-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=[0];
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
>> Ct=[336 160 23 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> M_t=tf(num_t,den_t)

M_t =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

4th Transfer function:

```
Command Window
>> clear
>> Ai=[-5 0 0 0 ;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M_i=tf(num_i,den_i)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 1 0 0;0 0 1 0;0 0 0 1;-105 -176 -86 -16];
>> Bt=[0;0;0;1];
>> Ct=[288 144 25 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> M_t=tf(num_t,den_t)

M_t =

      s^3 + 25 s^2 + 144 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 17 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to CCF state-space

Note: **Since the transfer function is *identical for all canonical forms*, the converted transfer function is the same as the original transfer function.**

Table 22 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	

	4) [1 25 162 288]	
The Converted TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	

3.1.6 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|s\boldsymbol{I} - \boldsymbol{A}| = \begin{vmatrix} s & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & s & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & s \end{vmatrix} - \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a}_0 & -\boldsymbol{a}_1 & -\boldsymbol{a}_2 & \dots & -\boldsymbol{a}_{n-1} \end{vmatrix} \qquad (\mathbf{3.1.6.1})$$

$$|sI - A| = 0$$

$$\begin{vmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 105 & 167 & 86 & s+16 \end{vmatrix}$$

$$|sI - A| = \begin{vmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 105 & 167 & 86 & s+16 \end{vmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.1.7.2})$$

2.1.12 Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda\boldsymbol{I} - \boldsymbol{A}| = \begin{vmatrix} \lambda & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \lambda \end{vmatrix} - \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\boldsymbol{a}_0 & -\boldsymbol{a}_1 & -\boldsymbol{a}_2 & \dots & -\boldsymbol{a}_{n-1} \end{vmatrix} \qquad (\mathbf{2.1.8.1})$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -105 & -176 & -86 & -16 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda+16 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 105 & 167 & 86 & \lambda + 16 \end{vmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.1.8.2})$$

Then we use the **MATLAB** command **"roots"** and get the following eigenvalues:

$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -1,$

Table 23 Task 10 Description

Task Number	Chapter	Task Name	Task Description
Task 10	**Chapter 4: Similarity Transformations for State Space Models**	**Similarity Transformation to Observability Canonical Form.**	**A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts.** **B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t).** **C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details.** **D) Compare the converted transfer function with the original transfer function that you started with.** **E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.** **F) Find the characteristic equation $\|sI - A\|$.** **J) Find the Eigenvalues values of matrix (A).**

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

```
From input 1 to output...
      2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

      2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
      2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

      2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+15s^{-2}+62s^{-3}+72s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+18s^{-2}+59s^{-3}+42s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+23s^{-2}+160s^{-3}+336s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+25s^{-2}+144s^{-3}+288s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

Finding T

$$\boldsymbol{T=[M]=[M1\ \ M2 \ldots\ldots\ Mn]}$$

A= $\begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix}$, B= $\begin{bmatrix} \mathbf{72} \\ \mathbf{62} \\ \mathbf{15} \\ \mathbf{1} \end{bmatrix}$,

C= $[\mathbf{0}\quad \mathbf{0}\quad \mathbf{0}\quad \mathbf{1}]$, D= $[\mathbf{0}]$

Find the Eigenvalues of matrix A

|λI-A| = 0 $\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 & 105 \\ -1 & \lambda & 0 & 176 \\ 0 & -1 & \lambda & 86 \\ 0 & 0 & -1 & \lambda+16 \end{bmatrix}$

$$(\lambda+1)(\lambda+3)(\lambda+5)(\lambda+7) = 0$$

$$\lambda = -1, -3, -5, -7$$

When $\lambda = -1$

λI-A = $\begin{bmatrix} -1 & 0 & 0 & 105 \\ -1 & -1 & 0 & 176 \\ 0 & -1 & -1 & 86 \\ 0 & 0 & -1 & 15 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = , $\begin{bmatrix} 105 \\ 71 \\ 15 \\ 1 \end{bmatrix}$ = M1

When $\lambda = -3$

$\begin{bmatrix} -3 & 0 & 0 & 105 \\ -1 & -3 & 0 & 176 \\ 0 & -1 & -3 & 86 \\ 0 & 0 & -1 & 13 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 35 \\ 47 \\ 13 \\ 1 \end{bmatrix}$ = M2

When $\lambda = -5$

$\begin{bmatrix} -5 & 0 & 0 & 105 \\ -1 & -5 & 0 & 176 \\ 0 & -1 & -5 & 86 \\ 0 & 0 & -1 & 11 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 21 \\ 31 \\ 11 \\ 1 \end{bmatrix}$ = M3

When $\lambda = -7$

$\begin{bmatrix} -7 & 0 & 0 & 105 \\ -1 & -7 & 0 & 176 \\ 0 & -1 & -7 & 86 \\ 0 & 0 & -1 & 9 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 15 \\ 23 \\ 9 \\ 1 \end{bmatrix}$ = M4

T = $[M1 \quad M2 \quad M3 \quad M4]$

T = $\begin{bmatrix} 105 & 35 & 21 & 15 \\ 71 & 47 & 31 & 23 \\ 15 & 13 & 11 & 9 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Finding A_t , B_t , C_t , D_t

$A_t = T^{-1}AT$

$B_t = T^{-1}B$

$C_t = CT$

For M1(s)

$$T^{-1} = \begin{bmatrix} 105 & 75 & 15 & 1 \\ 35 & 47 & 13 & 1 \\ 21 & 31 & 11 & 1 \\ 15 & 23 & 9 & 1 \end{bmatrix}$$

$$A_t = \begin{bmatrix} -16292 & -21624 & -22860 & -23408 \\ -7208 & -10812 & -11680 & -12068 \\ -4572 & -7008 & -7620 & -7896 \\ -3344 & -5172 & -5640 & -5852 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 12188 \\ 5630 \\ 3600 \\ 2642 \end{bmatrix}$$

$C_t = [1 \quad 1 \quad 1 \quad 1]$

$D_t = 0$

For M2(s)

T is the same because the have the same dominator

$$T = \begin{bmatrix} 105 & 35 & 21 & 15 \\ 71 & 47 & 31 & 23 \\ 15 & 13 & 11 & 9 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 105 & 75 & 15 & 1 \\ 35 & 47 & 13 & 1 \\ 21 & 31 & 11 & 1 \\ 15 & 23 & 9 & 1 \end{bmatrix}$$

$$A_t = \begin{bmatrix} -16292 & -21624 & -22860 & -23408 \\ -7208 & -10812 & -11680 & -12068 \\ -4572 & -7008 & -7620 & -7896 \\ -3344 & -5172 & -5640 & -5852 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 8870 \\ 4478 \\ 2910 \\ 2150 \end{bmatrix}$$

$C_t = [1 \quad 1 \quad 1 \quad 1]$

$D_t = 0$

For M3(s)

$$T = \begin{bmatrix} 105 & 35 & 21 & 15 \\ 71 & 47 & 31 & 23 \\ 15 & 13 & 11 & 9 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 105 & 75 & 15 & 1 \\ 35 & 47 & 13 & 1 \\ 21 & 31 & 11 & 1 \\ 15 & 23 & 9 & 1 \end{bmatrix}$$

$$A_t = \begin{bmatrix} -16292 & -21624 & -22860 & -23408 \\ -7208 & -10812 & -11680 & -12068 \\ -4572 & -7008 & -7620 & -7896 \\ -3344 & -5172 & -5640 & -5852 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 46986 \\ 19580 \\ 12270 \\ 8928 \end{bmatrix}$$

$C_t = [1 \quad 1 \quad 1 \quad 1]$

$D_t = 0$

For M4(s)

$$T = \begin{bmatrix} 105 & 35 & 21 & 15 \\ 71 & 47 & 31 & 23 \\ 15 & 13 & 11 & 9 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 105 & 75 & 15 & 1 \\ 35 & 47 & 13 & 1 \\ 21 & 31 & 11 & 1 \\ 15 & 23 & 9 & 1 \end{bmatrix}$$

$$A_t = \begin{bmatrix} -16292 & -21624 & -22860 & -23408 \\ -7208 & -10812 & -11680 & -12068 \\ -4572 & -7008 & -7620 & -7896 \\ -3344 & -5172 & -5640 & -5852 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 42118 \\ 18020 \\ 11346 \\ 8272 \end{bmatrix}$$

$C_t = [1 \quad 1 \quad 1 \quad 1]$

$D_t = 0$

2.2.2 Convert the resulting state space model to transfer function format using the equation $C(sI-A)^{-1}\,B+D$ in details.

$$\boldsymbol{C(sI-A)^{-1}B+D} \qquad (2.2.4.1)$$

Initially, we apply the equation $|sI-A|$, which returns:

$$[sI-A]=\begin{bmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{bmatrix}-\begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & 1 & 0 & -a_1 \\ 0 & 1 & \vdots & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & -a_2 \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \qquad (2.2.4.1)$$

$$[sI-A]=\begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix}-\begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix}=\begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI-A]=\begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}=0$$

Then we find the inverse of the resulting matrix:

$$[sI-A]^{-1}=\frac{adj(sI-A)}{det(sI-A)} \quad \dots \quad \frac{1}{det(sI-A)}*[cof(sI-A)]^T \qquad (2.2.4.3)$$

$[p]$=$[cof(sI-A)]^T$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix} (s+8)(s^2+8s+22) & -105 & -105s & -105s^2 \\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105) \\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105 \\ 1 & s & s^2 & s^3 \end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}\times[p]\times\begin{bmatrix} 72 \\ 62 \\ 15 \\ 1 \end{bmatrix}$$

$$=\frac{\begin{bmatrix} 1 & s & s^2 & s^3 \end{bmatrix}\times\begin{bmatrix} 72 \\ 62 \\ 15 \\ 1 \end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+2)(s+4)(s+9)}{s^4+16s^3+86s^2+176s+105}$$

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+8)(s^2+8s+22) & -105 & -105s & -105s^2\\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105)\\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105\\ 1 & s & s^2 & s^3\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[0\quad 0\quad 0\quad 1]\times[p]\times\begin{bmatrix}42\\59\\18\\1\end{bmatrix}$$

$$=\frac{[1\quad s\quad s^2\quad s^3]\times\begin{bmatrix}42\\59\\18\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+1)(s+3)(s+14)}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+8)(s^2+8s+22) & -105 & -105s & -105s^2\\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105)\\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105\\ 1 & s & s^2 & s^3\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[0\quad 0\quad 0\quad 1]\times[p]\times\begin{bmatrix}336\\160\\23\\1\end{bmatrix}$$

$$=\frac{[1\quad s\quad s^2\quad s^3]\times\begin{bmatrix}336\\160\\23\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{(s+4)(s+7)(s+12))}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$[sI - A]^{-1} = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [p]$$

$$p = \begin{bmatrix} (s+8)(s^2+8s+22) & -105 & -105s & -105s^2 \\ s^2+16s+86 & s(s^2+16s+86) & -176s-105 & -s(176s+105) \\ s+16 & s(s+16) & s^2(s+16) & -86s^2-176s-105 \\ 1 & s & s^2 & s^3 \end{bmatrix}$$

$$C(sI - A)^{-1}\ B + D = \frac{1}{s^4 + 16s^3 + 86s^2 + 176s + 105} \times [0 \quad 0 \quad 0 \quad 1] \times [p] \times \begin{bmatrix} 288 \\ 162 \\ 25 \\ 1 \end{bmatrix}$$

$$= \frac{[1 \quad s \quad s^2 \quad s^3] \times \begin{bmatrix} 288 \\ 162 \\ 25 \\ 1 \end{bmatrix}}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$C(sI - A)^{-1}\ B + D = \frac{(s+3)(s+6)(s+16)}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

2.2.3 Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Functions (Polynomial):

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

The original Transfer Functions (Polynomial):

$$1) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$2) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

2.2.4 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

Command Window

```
>> Ai=[-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

       s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[72;62;15;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

       s^3 + 15 s^2 + 62 s + 72
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$2)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

```
Command Window
>> Ai=[-5 0 0 0;9 -7 0 0;0 1 -3 0;0 1 0 -1];
>> Bi=[1;1;0;0];
>> Ci=[0 1 0 0];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

       s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[42;59;18;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

       s^3 + 18 s^2 + 59 s + 42
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$3) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
Command Window
>> clear
>> Ai=[-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

       s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[336;160;23;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

       s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

$$4) M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
Command Window
>> Ai=[-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> Bi=[1;1;1;1];
>> Ci=[0 0 0 1];
>> Di=0;
>> [num_i,den_i]=ss2tf(Ai,Bi,Ci,Di);
>> M=tf(num_i,den_i)

M =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> At=[0 0 0 -105;1 0 0 -176;0 1 0 -86;0 0 1 -16];
>> Bt=[288;162;25;1];
>> Ct=[0 0 0 1];
>> Dt=0;
>> [num_t,den_t]=ss2tf(At,Bt,Ct,Dt);
>> Mt=tf(num_t,den_t)

Mt =

      s^3 + 25 s^2 + 162 s + 288
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

Figure 18 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to OCF state-space

Table 24 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**		The **coefficients of Denominator**
The Original TF	1)	[1 15 62 72]	[1 16 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	
The Converted TF	1)	[1 15 62 72]	[1 16 86 176 105]
	2)	[1 18 59 42]	
	3)	[1 23 160 336]	
	4)	[1 25 162 288]	

2.2.5 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|\boldsymbol{sI} - \boldsymbol{A}| = \begin{vmatrix} \boldsymbol{s} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{s} & \ddots & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{s} \end{vmatrix} - \begin{vmatrix} \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & -\boldsymbol{a_0} \\ \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} & -\boldsymbol{a_1} \\ \boldsymbol{0} & \boldsymbol{1} & \vdots & \boldsymbol{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & -\boldsymbol{a_2} \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{1} & -\boldsymbol{a_{n-1}} \end{vmatrix} \qquad (\mathbf{2.2.7.1})$$

$|sI - A| = 0$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.2.7.2})$$

2.2.6 Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda \boldsymbol{I} - \boldsymbol{A}| = \begin{vmatrix} \lambda & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda & \ddots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \lambda \end{vmatrix} - \begin{vmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{a_0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & -\boldsymbol{a_1} \\ \mathbf{0} & \mathbf{1} & \vdots & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & -\boldsymbol{a_2} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{1} & -\boldsymbol{a_{n-1}} \end{vmatrix} \qquad (\mathbf{2.2.8.1})$$

$$|\lambda I - A| = 0$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & -105 \\ 1 & 0 & 0 & -176 \\ 0 & 1 & 0 & -86 \\ 0 & 0 & 1 & -16 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 & 105 \\ -1 & s & 0 & 176 \\ 0 & -1 & s & 86 \\ 0 & 0 & -1 & s+16 \end{bmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (\mathbf{2.2.8.2})$$

Then we use the **MATLAB** command **<u>"roots"</u>** and get the following eigenvalues:

$\lambda_1 = -7, \quad \lambda_2 = -5, \quad \lambda_3 = -3, \quad \lambda_4 = -3, \quad \lambda_5 = -1$

Table 25 Task 11 Description

Task Number	Chapter	Task Name	Task Description
Task 11	**Chapter 4:** Similarity Transformations for State Space Models	Similarity Transformation to Diagonal Canonical Form (If Applicable)	A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts. B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t). C) Convert the resulting state space model to transfer function format using the equation $C(sI-A)^{-1}B+D$ in details. D) Compare the converted transfer function with the original transfer function that you started with. E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. F) Find the characteristic equation $\|sI-A\|$. K) Find the Eigenvalues values of matrix (A).

Unfortunately, The Transfer functions resulted from this MIMO Diagram are not logical and its all look like each other therefore we will proceed by converting it into A SiSO system.

Evidence:

```
From input 1 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

From input 2 to output...
        2 s^3 + 33 s^2 + 121 s + 114
 1:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105

        2 s^3 + 48 s^2 + 288 s + 504
 2:  -----------------------------------
     s^4 + 16 s^3 + 86 s^2 + 167 s + 105
```

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+15s^{-2}+62s^{-3}+72s^{-4}}{1-[-16s^{-1}-86s^{-2}-176s^{-3}-105s^{-4}]}$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 144s^{-3} + 288s^{-4}}{1 - [-16s^{-1} - 86s^{-2} - 176s^{-3} - 105s^{-4}]}$$

2.2.7 Derive the state space model in equations format.

Throughout the previous section 2.2.1, we are using the state space diagram to derive the state space equation as the following general format:

$$\begin{aligned}
\dot{x}_1 &= -a_0 x_n + b_0 u \\
\dot{x}_2 &= x_1 - a_1 x_n + b_1 u \\
&\vdots \\
\dot{x}_n &= x_{n-1} - a_{n-1} x_n + b_{n-1} u \\
y &= x_n
\end{aligned} \qquad (2.2.2.1)$$

The derived state space equation from for the **Error! Reference source not found.** as the following:

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -1x_1 + u \\
\dot{x}_2 &= -3x_2 + u \\
\dot{x}_3 &= -5x_3 + u \\
\dot{x}_4 &= -7x_4 + u
\end{aligned}$$

$$y = 0.5x_1 + 0.375x_2 + 0.75x_3 - 0.625x_4$$

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -1x_1 + u \\
\dot{x}_2 &= -3x_2 + u \\
\dot{x}_3 &= -5x_3 + u \\
\dot{x}_4 &= -7x_4 + u
\end{aligned}$$

$$y = 0x_1 + 0x_2 + 4.5x_3 - 3.5x_4$$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\begin{aligned}
\dot{x}_1 &= -1x_1 + u \\
\dot{x}_2 &= -3x_2 + u \\
\dot{x}_3 &= -5x_3 + u \\
\dot{x}_4 &= -7x_4 + u
\end{aligned}$$

$$y = 4.125x_1 - 2.25x_2 - 0.875x_3$$

$$\frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}_1 = -1x_1 + u$$

$$\dot{x}_2 = -3\boldsymbol{x_2} + u$$
$$\dot{x}_3 = -5x_3 + u$$
$$\dot{x}_4 = -7x_4 + u$$
$$y = 3.125\boldsymbol{x_1} + 0\boldsymbol{x_2} - 1.375\boldsymbol{x_3} - 0.75\boldsymbol{x_4}$$

2.2.8 Write the state space model in matrix format $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$.

We can write the derived state equation format in matrix format as the following general format:

$$\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A_{CCF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B_{CCF}}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C_{CCF}}\, \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D_{CCF}}\boldsymbol{u}(\boldsymbol{t}) \end{aligned} \qquad (\mathbf{2.1.3.1})$$

Where the matrices $A, B, C, and\ D$ as the following general format:

$$\boldsymbol{A_{CCF}} = \begin{bmatrix} -\boldsymbol{p_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{p_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{p_{n-1}} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{p_n} \end{bmatrix} \qquad \boldsymbol{B_{CCF}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

$$\boldsymbol{C_{CCF}} = \begin{bmatrix} \boldsymbol{r_1} & \boldsymbol{r_2} & \dots & \boldsymbol{r_{n-1}} & \boldsymbol{r_n} \end{bmatrix} \qquad \boldsymbol{D_{CCF}} = [\mathbf{0}]$$

Consider the derived state space, the state space diagram's matrix format as:

$$\mathbf{1})\boldsymbol{M}(\boldsymbol{s}) = \frac{\boldsymbol{Y}(\boldsymbol{s})}{\boldsymbol{U}(\boldsymbol{s})} = \frac{\boldsymbol{s}^3 + \mathbf{15}\boldsymbol{s}^2 + \mathbf{62}\boldsymbol{s} + \mathbf{72}}{\boldsymbol{s}^4 + \mathbf{16}\boldsymbol{s}^3 + \mathbf{86}\boldsymbol{s}^2 + \mathbf{176}\boldsymbol{s} + \mathbf{105}}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\, x(t) + [0]u(t)$$

$$2)\,)\boldsymbol{M}(s) = \frac{Y(s)}{U(s)} = \frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,0\ \ 0\ \ 4.5\ \ -3.5]\, x(t) + [0]u(t)$

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) \ = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,4.125 \quad -2.25\ \ -0.875\ \ 0]\, x(t) + [0]u(t)$

4)

$$M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) \ = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,3.125\ \ \ 0\ \ \ -1.375\ \ -0.75]\, x(t) + [0]u(t)$

Finding T

$\boldsymbol{T} = [\boldsymbol{M}] = [\boldsymbol{M1}\ \ \boldsymbol{M2} \ldots \ldots\ \boldsymbol{Mn}]$

A= $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$ B = $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ C= $[\,0.5\ \ 0.375\ \ 0.75\ \ -0.625]$ D=0

Find the Eigenvalues of matrix A

|λI-A| = 0 $\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} =$

$\begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$

$$(\lambda+1)(\lambda+3)(\lambda+5)(\lambda+7) = 0$$

$$\lambda = -1\ , -3, -5\ \ , -7$$

When $\lambda = -1$

λI-A = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = , $\begin{bmatrix} 48 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ = M1

When $\lambda = -3$

$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 0 \\ -20 \\ 0 \\ 0 \end{bmatrix}$ = M2

When $\lambda = -5$

$\begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 0 \\ 0 \\ 16 \\ 0 \end{bmatrix}$ = M3

When $\lambda = -7$

$\begin{bmatrix} -6 & 0 & 0 & 0 \\ -1 & -4 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}$ = $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -48 \end{bmatrix}$ = M4

T = $[M1 \quad M2 \quad M3 \quad M4]$

$$T = \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix}$$

Finding A_t , B_t , C_t , D_t

$A_t = T^{-1}AT$

$B_t = T^{-1}B$

$C_t = CT$

For M1(s)

$$T^{-1} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}$$

$$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$Bt = T^{-1}B = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.02 \\ -0.05 \\ 0.0625 \\ -0.02 \end{bmatrix}$$

$$C_t = CT = [\, 0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625 \,] \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [\, 24 \;\; -7.5 \;\; 12 \;\; 30 \,]$$

$D_t = 0$

For M2(s)

T is the same because the have the same dominator

$$T = \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}$$

$$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$Bt = T^{-1}B = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.02 \\ -0.05 \\ 0.0625 \\ -0.02 \end{bmatrix}$$

$$C_t = CT = [0 \ \ 0 \ \ 4.5 \ \ -3.5] \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [0 \ \ 0 \ \ 72 \ \ 168]$$

$D_t = 0$

For M3(s)

$$T = \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}$$

$$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$Bt = T^{-1}B = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.02 \\ -0.05 \\ 0.0625 \\ -0.02 \end{bmatrix}$$

$$C_t = CT = [\,4.125 \quad -2.25 \quad -0.875 \quad 0\,] \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [198 \quad 45 \quad -14 \quad 0]$$

$$D_t = 0$$

For M4(s)

$$T = \begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}$$

$$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$Bt=T^{-1}B=\begin{bmatrix}0.02 & 0 & 0 & 0\\ 0 & -0.05 & 0 & 0\\ 0 & 0 & 0.0625 & 0\\ 0 & 0 & 0 & -0.02\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}0.02\\-0.05\\0.0625\\-0.02\end{bmatrix}$$

$$C_t = CT = [150 \quad 0 \quad -22 \quad 36]\begin{bmatrix}48 & 0 & 0 & 0\\ 0 & -20 & 0 & 0\\ 0 & 0 & 16 & 0\\ 0 & 0 & 0 & -48\end{bmatrix} = [\,0.01 \quad -0.01875 \quad 0.046875 \quad 0.0125]$$

$D_t = 0$

2.2.1 Convert the resulting state space model to transfer function format using the equation $C(sI-A)^{-1}\,B+D$ in details.

$$\boldsymbol{C(sI-A)^{-1}B+D} \qquad (2.2.4.1)$$

Initially, we apply the equation $|sI-A|$, which returns:

$$[\boldsymbol{sI-A}] = \begin{bmatrix}\boldsymbol{s} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{s} & \ddots & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{s}\end{bmatrix} - \begin{bmatrix}-\boldsymbol{p_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{p_2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{p_{n-1}} & \boldsymbol{0} & \vdots\\ \vdots & \vdots & \ddots & \vdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & -\boldsymbol{p_n}\end{bmatrix} \qquad (2.2.4.1)$$

$$[sI-A] = \begin{bmatrix}s & 0 & 0 & 0\\ 0 & s & 0 & 0\\ 0 & 0 & s & 0\\ 0 & 0 & 0 & s\end{bmatrix} - \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -3 & 0 & 0\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & -7\end{bmatrix} = \begin{bmatrix}s+1 & 0 & 0 & 0\\ 0 & s+3 & 0 & 0\\ 0 & 0 & s+5 & 0\\ 0 & 0 & 0 & s+7\end{bmatrix}$$

$$[sI-A] = \begin{bmatrix}s+1 & 0 & 0 & 0\\ 0 & s+3 & 0 & 0\\ 0 & 0 & s+5 & 0\\ 0 & 0 & 0 & s+7\end{bmatrix} = 0$$

Then we find the inverse of the resulting matrix:

$$[\boldsymbol{sI-A}]^{-1} = \frac{\boldsymbol{adj(sI-A)}}{\boldsymbol{det(sI-A)}} \quad \cdots \quad \frac{\boldsymbol{1}}{\boldsymbol{det(sI-A)}} * [\boldsymbol{cof(sI-A)}]^{\boldsymbol{T}} \qquad (2.2.4.3)$$

$$\mathbf{1)}\boldsymbol{M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$

$$=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[0.5\ \ 0.375\ \ 0.75\ \ -0.625]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[0.5(s+3)(s+5)(s+7)\ \ 0.375(s+1)(s+5)(s+7)\ \ 0.75(s+1)(s+3)(s+7)\ \ -0.625(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

$$2)\ \boldsymbol{M}(s)=\frac{Y(s)}{U(s)}=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[[\ 0\ \ 0\ \ 4.5\ \ -3.5]\ ]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[0(s+3)(s+5)(s+7)\ \ 0(s+1)(s+5)(s+7)\ \ 4.5(s+1)(s+3)(s+7)\ \ -3.5(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

$$3)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$

$$=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[\,4.125\quad -2.25\ -0.875\ \ 0]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[4.125(s+3)(s+5)(s+7)\ -2.25(s+1)(s+5)(s+7)\ \ -0.875(s+1)(s+3)(s+7)\ \ 0(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

$$4)M(s)=\frac{Y(s)}{U(s)}=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$[sI-A]^{-1}=\frac{1}{s^4+16s^3+86s^2+176s+105}\times[p]$$

$$p=\begin{bmatrix}(s+3)(s+5)(s+7) & 0 & 0 & 0\\ 0 & (s+1)(s+5)(s+7) & 0 & 0\\ 0 & 0 & (s+1)(s+3)(s+7) & 0\\ 0 & 0 & 0 & (s+1)(s+3)(s+5)\end{bmatrix}$$

$$C(sI-A)^{-1}\ B+D$$

$$=\frac{1}{s^4+16s^3+86s^2+176s+105}\times\big[[\,3.125\quad 0\quad -1.375\ -0.75]\big]\times[p]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\frac{[3.125(s+3)(s+5)(s+7)\ \ 0(s+1)(s+5)(s+7)\ \ -1.375(s+1)(s+3)(s+7)\ \ -0.75(s+1)(s+3)(s+5)]\times\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}{s^4+16s^3+86s^2+176s+105}$$

$$C(sI-A)^{-1}\ B+D=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

$$M(s)=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

Compare the converted transfer function with the original transfer function that you started with.

The converted Transfer Function:

$$M(s)=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s)=\frac{s^3+15s^2+62s+72}{s^4+16s^3+86s^2+176s+105}$$

2)

The converted Transfer Function:

$$M(s)=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s)=\frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$$

3)

The converted Transfer Function:

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s)=\frac{s^3+23s^2+160s+336}{s^4+16s^3+86s^2+176s+105}$$

4)

The converted Transfer Function:

$$M(s)=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

The original Transfer Function:

$$M(s)=\frac{s^3+25s^2+162s+288}{s^4+16s^3+86s^2+176s+105}$$

2.2.2 Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step.

$$M(s) = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;-3 -7 0 0;-3 -3 -3 0;-3 -3 6 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [0.5 0.375 0.75 -0.625];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 15 s^2 + 62 s + 72
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 18s^2 + 59s + 42}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;9 -7 0 0;0 1 -3 0;0 0 0 -1];
>> B = [1;1;0;0];
>> C = [0 1 0 0];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.
```

```
>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [0 0 4.5 -3.5];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

       s^3 + 18 s^2 + 59 s + 42
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;2 -7 0 0;2 5 -3 0;2 5 1 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [4.125 -2.25 -0.875 0];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 23 s^2 + 160 s + 336
  -----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

$$M(s) = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

```
>> A = [-5 0 0 0;11 -7 0 0;11 -4 -3 0;11 -4 3 -1];
>> B = [1;1;1;1];
>> C = [0 0 0 1];
>> D=[0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105

Continuous-time transfer function.

>> A = [-1 0 0 0;0 -3 0 0;0 0 -5 0;0 0 0 -7];
>> B = [1;1;1;1];
>> C = [25/8 0 -11/8 -3/4];
>> D = [0];
>> [num,den]= ss2tf (A,B,C,D);
>> M_i=tf(num,den)

M_i =

      s^3 + 25 s^2 + 162 s + 288
  ----------------------------------
  s^4 + 16 s^3 + 86 s^2 + 176 s + 105
```

Figure 19 MATLAB Checks similarity for the output transfer function between initial state-space format and Decomposition to OCF state-space

Table 26 The Coefficients of Numerator & Denominator for the original & converted TF

	The **coefficients of Numerator**	The **coefficients of Denominator**
The Original TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	
The Converted TF	1) [1 15 62 72]	[1 16 66 86 176 105]
	2) [1 18 59 42]	
	3) [1 23 160 336]	
	4) [1 25 162 288]	

2.2.3 Find the characteristic equation $|sI - A|$.

Initially, we apply the equation $|sI - A|$, which returns:

$$|\boldsymbol{sI}-\boldsymbol{A}| = \begin{vmatrix} s & 0 & \cdots & \cdots & 0 \\ 0 & s & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s \end{vmatrix} - \begin{vmatrix} -p_1 & 0 & \cdots & 0 & 0 \\ 0 & -p_2 & 0 & 0 & 0 \\ 0 & 0 & -p_{n-1} & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \cdots & 0 & -p_n \end{vmatrix} \qquad (2.2.7.1)$$

$$|sI-A| = 0$$

$$|sI-A| = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix}$$

$$|sI-A| = \begin{bmatrix} s+1 & 0 & 0 & 0 \\ 0 & s+3 & 0 & 0 \\ 0 & 0 & s+5 & 0 \\ 0 & 0 & 0 & s+7 \end{bmatrix} = 0$$

$$\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105 = 0} \qquad (2.2.7.2)$$

2.2.4 Find the Eigenvalues of matrix A.

Initially, we apply the equation $|\lambda I - A|$, which returns:

$$|\lambda\boldsymbol{I}-\boldsymbol{A}| = \begin{vmatrix} \lambda & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -p_1 & 0 & \cdots & 0 & 0 \\ 0 & -p_2 & 0 & 0 & 0 \\ 0 & 0 & -p_{n-1} & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \cdots & 0 & -p_n \end{vmatrix} \qquad (2.2.8.1)$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{vmatrix} = 0$$

$$\boldsymbol{\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0} \qquad (\mathbf{2.2.8.2})$$

Then we use the **MATLAB** command **"roots"** and get the following eigenvalues:

$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$

3.1 Similarity Transformation to Jordan Canonical Form (If Applicable).

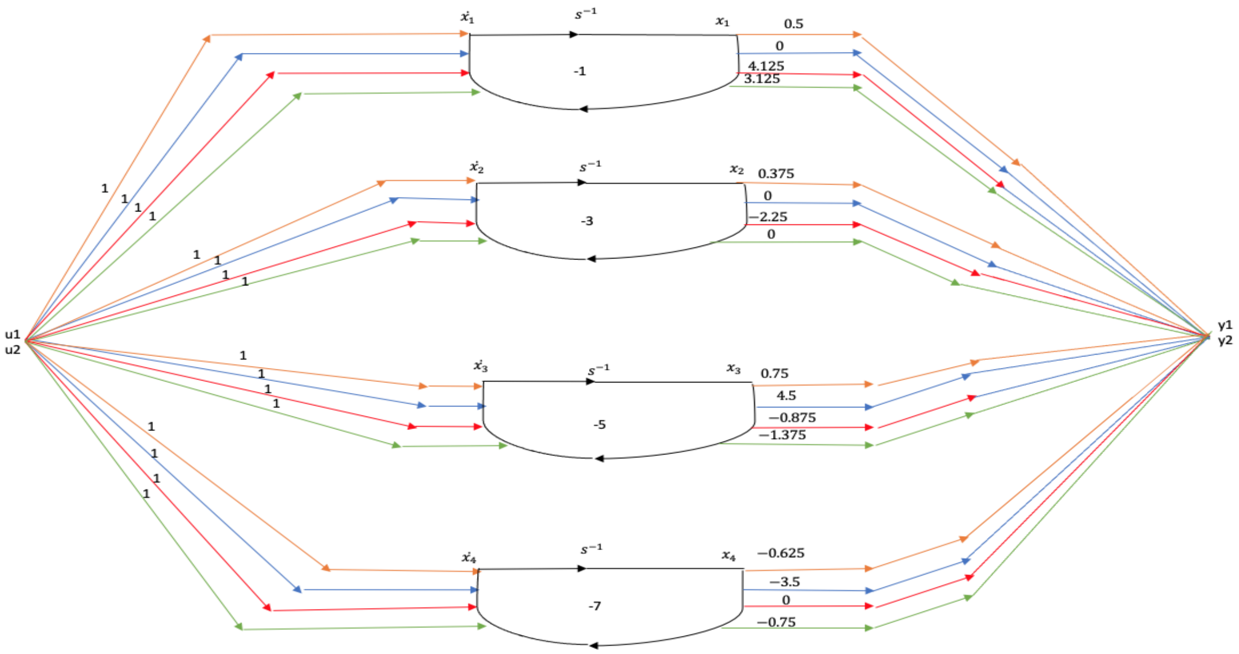
Table 27 Task 12 Description

Task Number	Chapter	Task Name	Task Description
Task 12	**Chapter 4:** Similarity Transformations for State Space Models	Similarity Transformation to Jordan Canonical Form (If Applicable).	A) Derive the transformation matrix (T) in extremely detailed steps according to the given algorithm in the course handouts. B) Find the transformed state space model in matrix format (A_t, B_t, C_t, D_t). C) Convert the resulting state space model to transfer function format using the equation $C(sI - A)^{-1}B + D$ in details. D) Compare the converted transfer function with the original transfer function that you started with. E) Using MATLAB, convert the initial state space format and the new state space format to transfer function format using the same command window, and add a print screen for the result in this step. F) Find the characteristic equation $\|sI - A\|$. G) Find the Eigenvalues values of matrix (A).

Note: **Since this task has repeated roots were not supposed to solve it**

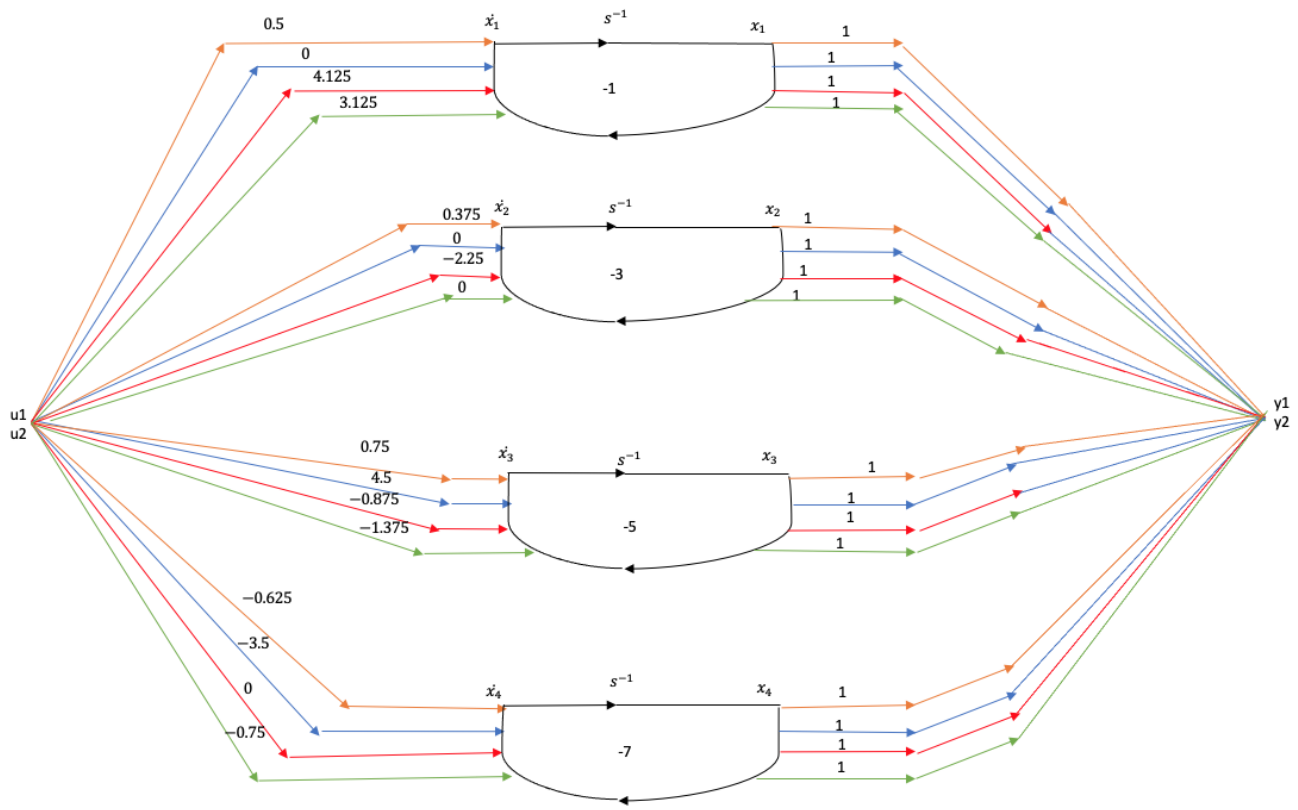
Task Number	Chapter	Task Name	Task Description
Task 13	Chapter 6: Controllability and Observability tests	Controllability Tests and Observability Tests based on decomposed DCF	A) Perform the controllability test based on the state diagram of decomposed MIMO control system. B) Perform the Observability test based on the state diagram of decomposed MIMO control system. C) Perform the controllability test using Gilbert's Test for decomposed (A, B, C, D). D) Perform the observability test using Gilbert's Test for decomposed (A, B, C, D). E) Perform the controllability test using Kalman's Test for decomposed (A, B, C, D). F) Perform the observability test using Kalman's Test for decomposed (A, B, C, D). G) Perform the controllability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt) H) Perform the observability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt) I) Perform the controllability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt). J) Perform the observability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt).

Perform the controllability test based on the state diagram of decomposed MIMO control system.



- ❖ **From the graph we can notice that the system is Completely state controllable and Not completely state observable Due to the Zero-pole cancellations in the outputs branches.**

Perform the Observability test based on the state diagram of decomposed MIMO control system.



- ❖ **From the Graph we notice that the system is Completely state Observable And not Completely state Controllable Due to the zero pole cancellations in the inputs Branches.**

Perform the controllability test using Gilbert's Test for decomposed (A, B, C, D).

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\, x(t) + [0]u(t)$$

$$|sI - A| = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$$

$$\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$$

For $\lambda = -1$

$$|\lambda I - A| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\text{M1}=\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}=\begin{bmatrix} 48 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -3$

$$|\lambda I - A| = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\text{M1}=\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}=\begin{bmatrix} 0 \\ -20 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -5$

$$|\lambda I - A| = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$M1=\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 16 \\ 0 \end{bmatrix}$$

For $\lambda = -7$

$$|\lambda I - A| = \begin{bmatrix} -6 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M1=\begin{bmatrix} C11 \\ C12 \\ C13 \\ C14 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ -48 \end{bmatrix}$$

$$T=M=\begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix}$$

$$T^{-1}=\begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}$$

$$Bt=T^{-1}B=\begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0.02 \\ -0.05 \\ 0.0625 \\ -0.02 \end{bmatrix}$$

As none of elements of Bt= $T^{-1}B$ are zero then system is completely state controllable based on gilbert's test.

Perform the Observability test using Gilbert's Test for decomposed (A, B, C, D).

$$1)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 15s^2 + 62s + 72}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\, x(t) + [0]u(t)$$

$$|sI - A| = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$$

$$\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$$

$$\text{T} = \text{V} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix}$$

$$\text{Ct} = \text{CT} = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625] \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix} =$$

$$CT = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]$$

There is no zero element in Ct therefore TF1 is completely state observable based on gilbert's test.

2))$\boldsymbol{M}(s) = \frac{Y(s)}{U(s)} = \frac{s^3+18s^2+59s+42}{s^4+16s^3+86s^2+176s+105}$

$$\dot{x}(t) \; = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,0 \;\; 0 \;\; 4.5 \;\; -3.5]\, x(t) + [0]u(t)$

$$|sI - A| = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$$

$\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0$

$\lambda_1 = -1, \quad \lambda_2 = \; -3, \qquad \lambda_3 = -5, \quad \lambda_4 = -7,$

$$\text{T = V =} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix}$$

$$\text{Ct = CT=} [\;0 \quad 0 \quad 4.5 \quad -3.5] \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix} =$$

$CT = [\,8 \;\; 135 \;\;\; 550 \;\; 1421]$

There is no zero element in Ct therefore TF2 is completely state observable based on gilbert's test.

3)$M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$

$$\dot{x}(t) \; = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,4.125 \quad -2.25\ -0.875\ \ 0]\,x(t) + [0]u(t)$

$$|sI - A| = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$$

$\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0$

$\lambda_1 = -1, \quad \lambda_2 = -3, \quad \lambda_3 = -5, \quad \lambda_4 = -7,$

$$\text{T = V =} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix}$$

$$\text{Ct = CT=} [\ 4.125 \quad -2.25\ -0.875\ \ 0] \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix} =$$

$CT = [\,7.25\ \ 18.75\ \ \ 27.25\ \ 62.75]$

There is no zero element in Ct therefore TF3 is completely state observable based on gilbert's test.

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,3.125\ \ \ 0\ \ -1.375\ -0.75]\,x(t) + [0]u(t)$

$$|sI-A| = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda+3 & 0 & 0 \\ 0 & 0 & \lambda+5 & 0 \\ 0 & 0 & 0 & \lambda+7 \end{bmatrix}$$

$\lambda^4 + 16\lambda^3 + 86\lambda^2 + 176\lambda + 105 = 0$

$\lambda_1 = -1,\quad \lambda_2 = -3,\quad \lambda_3 = -5,\quad \lambda_4 = -7,$

$$T = V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix}$$

$$Ct = CT = [\,3.125 \;\; 0 \;\; -1.375 \;\; -0.75\,] \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -3 & -5 & -7 \\ 1 & 9 & 25 & 49 \\ -1 & -27 & -125 & -343 \end{bmatrix} =$$

$CT = [\,2.5 \;\; 11 \;\; 62.5 \;\; 193\,]$

There is no zero element in Ct therefore TF4 is completely state observable based on gilbert's test.

Perform the Controllability test using Kalman's Test for decomposed (A, B, C, D).

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625\,]\, x(t) + [0]u(t)$

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Qc= [B AB A^2B A^3B]

$$Qc=\begin{bmatrix}1 & -1 & 1 & -1\\ 1 & -3 & 9 & -27\\ 1 & -5 & 25 & -125\\ 1 & -7 & 49 & -343\end{bmatrix}$$

Determinant of |Qc|=767

Which is not zero This means that the rank of the matrix Qc=4

The system is completely state controllable.

Perform the Observability test using Kalman's Test for decomposed (A, B, C, D).

$$\mathbf{1)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$\dot{x}(t) = \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -3 & 0 & 0\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & -7\end{bmatrix} x(t) + \begin{bmatrix}1\\1\\1\\1\end{bmatrix} u(t)$$

$$y(t) = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\, x(t) + [0]u(t)$$

$$Q0=\begin{bmatrix}C\\ CA\\ CA^2\\ CA^3\end{bmatrix}=\begin{bmatrix}0.5 & 0.375 & 0.75 & -0.625\\ -0.5 & -1.125 & -3.75 & 4.375\\ 0.5 & 3.375 & 18.75 & -30.625\\ -0.5 & -10.125 & -93.75 & 214.375\end{bmatrix}$$

- **|Q0|=-67.5 which is not zero so the rank is n=4 that means the system is completely state Observable.**

$$\mathbf{2)}\ \boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3+18s^2+59s+42}}{\boldsymbol{s^4+16s^3+86s^2+176s+105}}$$

$$\dot{x}(t) = \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -3 & 0 & 0\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & -7\end{bmatrix} x(t) + \begin{bmatrix}1\\1\\1\\1\end{bmatrix} u(t)$$

$$y(t) = [\,0 \;\; 0 \;\; 4.5 \;\; -3.5]\, x(t) + [0]u(t)$$

$$Q0=\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 4.5 & -3.5 \\ 0 & 0 & 22.5 & 24.5 \\ 0 & 0 & 112.5 & -171.5 \\ 0 & 0 & -526.5 & 1200.5 \end{bmatrix}$$

- ❖ **|Q0|= 0 so the rank is not 4 that means the system is not completely state Observable.**

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 23s^2 + 160s + 336}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,4.125 \quad -2.25 \; -0.875 \;\; 0]\, x(t) + [0]u(t)$$

$$Q0=\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}=\begin{bmatrix} 4.125 & -2.25 & -0.875 & 0 \\ -4.125 & 6.75 & 4.375 & 0 \\ 4.125 & -20.25 & -21.875 & 0 \\ -4.125 & 60.75 & 109.375 & 0 \end{bmatrix}$$

- ❖ **|Q0|= 0 so the rank is not 4 that means the system is not completely state Observable.**

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 25s^2 + 162s + 288}{s^4 + 16s^3 + 86s^2 + 176s + 105}$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,3.125 \;\; 0 \;\; -1.375 \; -0.75]\, x(t) + [0]u(t)$$

$$Q0=\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}=\begin{bmatrix} 3.125 & 0 & -1.375 & -0.75 \\ -3.125 & 0 & 6.875 & 5.25 \\ 3.125 & 0 & -34.375 & -36.75 \\ -3.125 & 0 & 171.87 & 257.25 \end{bmatrix}$$

|Q0|= 0 so the rank is not 4 that means the system is not completely state Observable.

Perform the controllability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt)

$$A_t = T^{-1}AT=\begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$Bt=T^{-1}B=\begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0.02 \\ -0.05 \\ 0.0625 \\ -0.02 \end{bmatrix}$$

System is completely state controllable since there is no zero element in Bt.

Perform the Observability test using Gilbert's Test for Transformed matrix format (At, Bt, Ct, Dt)

$$1)M(s)=\frac{Y(s)}{U(s)}=\frac{s^{-1}+15s^{-2}+62s^{-3}+72s^{-4}}{1+16s^{-1}+86s^{-2}+176s^{-3}+105s^{-4}}$$

$$C_t = CT = [\,0.5\ \ 0.375\ \ 0.75\ \ -0.625]\begin{bmatrix}48 & 0 & 0 & 0\\ 0 & -20 & 0 & 0\\ 0 & 0 & 16 & 0\\ 0 & 0 & 0 & -48\end{bmatrix} = [\,24 - 7.5\ \ 12\ \ 30]$$

Since there is no Zero element in the Ct the system is completely state observable.

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$C_t = CT = [\,0\ \ 0\ \ 4.5\ \ -3.5]\begin{bmatrix}48 & 0 & 0 & 0\\ 0 & -20 & 0 & 0\\ 0 & 0 & 16 & 0\\ 0 & 0 & 0 & -48\end{bmatrix} = [\,0\ \ \ 0\ \ 72\ \ 168]$$

Since there is zero element in the Ct the system is not completely state observable.

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$C_t = CT = [\,4.125\ \ \ -2.25\ \ -0.875\ \ 0]\begin{bmatrix}48 & 0 & 0 & 0\\ 0 & -20 & 0 & 0\\ 0 & 0 & 16 & 0\\ 0 & 0 & 0 & -48\end{bmatrix} = [198\ \ \ 45\ \ \ -14\ \ \ 0]$$

Since there is zero element in the Ct the system is not completely state observable.

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 162s^{-3} + 288s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$C_t = CT = [150\ \ \ 0\ \ \ -22\ \ \ 36]\begin{bmatrix}48 & 0 & 0 & 0\\ 0 & -20 & 0 & 0\\ 0 & 0 & 16 & 0\\ 0 & 0 & 0 & -48\end{bmatrix} = [\,0.01\ \ -0.01875\ \ 0.046875\ \ 0.0125]$$

Since there is no Zero element in the Ct the system is completely state observable.

Perform the Controllability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt)

$$At=\begin{bmatrix}-0.96 & 0 & 0 & 0\\ 0 & -3 & 0 & 0\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & -6.72\end{bmatrix} \qquad Bt=\begin{bmatrix}0.02\\ -0.05\\ 0.0625\\ -0.02\end{bmatrix}$$

Qc= [Bt AtBt At^2Bt At^3Bt]

$$Qc=\begin{bmatrix} 0.02 & -0.192 & 0.0184 & -0.177 \\ 1 & -3 & 9 & 1.25 \\ 0.0625 & -5 & 25 & -7.182 \\ -0.02 & 0.134 & -0.9032 & 6.092 \end{bmatrix}$$

Determinant of |Qc|=0.0075

Which is not zero This means that the rank of the matrix Qc=4

The system is completely state controllable.

Perform the Observability test using Kalman's Test for Transformed matrix format (At, Bt, Ct, Dt)

$$\mathbf{1)}\boldsymbol{M(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{s^3 + 15s^2 + 62s + 72}}{\boldsymbol{s^4 + 16s^3 + 86s^2 + 176s + 105}}$$

$$A_t = T^{-1}AT=\begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$$C_t = CT = [\,0.5 \;\; 0.375 \;\; 0.75 \;\; -0.625]\begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [\,24 - 7.5 \;\; 12 \;\; 30]$$

- **|Q0|=--393 which is not zero, so the rank is n=4 that means the system is completely state Observable.**

$$2)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 18s^{-2} + 59s^{-3} + 42s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$$A_t = T^{-1}AT=\begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$$

$C_t = CT = [\,0\ \ 0\ \ 4.5\ \ -3.5\,]\begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [\,0\ \ \ 0\ \ 72\ \ 168\,]$

- **|Q0|=0 which is zero, so the rank is n doesn't equal 4 that means the system is not completely state Observable.**

$$3)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 23s^{-2} + 160s^{-3} + 336s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$

$C_t = CT = [\,4.125\ \ \ -2.25\ \ -0.875\ \ 0\,]\begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [\,198\ \ \ 45\ \ \ -14\ \ \ 0\,]$

- **|Q0|=0 which is zero, so the rank is n doesn't equal 4 that means the system is not completely state Observable.**

$$4)M(s) = \frac{Y(s)}{U(s)} = \frac{s^{-1} + 25s^{-2} + 162s^{-3} + 288s^{-4}}{1 + 16s^{-1} + 86s^{-2} + 176s^{-3} + 105s^{-4}}$$

$A_t = T^{-1}AT = \begin{bmatrix} -0.96 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -6.72 \end{bmatrix}$

$C_t = CT = [\,150\ \ \ 0\ \ -22\ \ \ 36\,]\begin{bmatrix} 48 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -48 \end{bmatrix} = [\,0.01\ \ -0.01875\ \ 0.046875\ \ 0.0125\,]$

- **|Q0|=-6.39 which is not zero, so the rank is n=4 that means the system is completely state Observable.**