



The University of Jordan  
School of Engineering  
Department of Mechatronics



## Mechanics of Machines (0944331)

### Course project

### Radial engine

Submitted to: Dr. Ibrahim Abu-alshaikh

Date: 14-1-2024

Day: Sunday

Submitted by:

Name	ID
Rashed Al Ashqar	0195746
Anas Abdeen	0193740

## Contents

List Of figures .....	2
Introduction .....	3
History .....	3
Radial engines golden age. ....	5
Calculations .....	6
Position Analysis .....	6
Velocity Analysis .....	7
Acceleration Analysis.....	7
Jerk Analysis.....	8
Force Analysis .....	9
Using MATLAB .....	13
Using Analytical method.....	16
References.....	18

## List Of figures

Figure 1 The radial engines used in ww2 aircrafts .....	3
Figure 2 Leon levavasseur .....	4
Figure 3 George Mead .....	4
Figure 4 Charles Lawrence.....	4
Figure 5 parallel motion.....	4
Figure 6 Radial engine Design used to 3d print it .....	5

# Introduction

## History

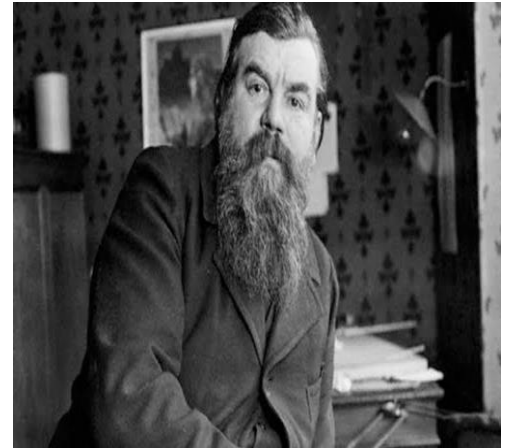
The concept of the radial engine dates to the late 19th century when inventors and engineers began experimenting with different engine configurations. One of the earliest successful radial engines was built by French engineer Leon Levavasseur in 1908.

The 1920s and 1930s saw the golden age of aviation, and radial engines became the dominant powerplants for many aircraft, the engine powered a wide range of aircrafts, from small biplanes to large bombers.



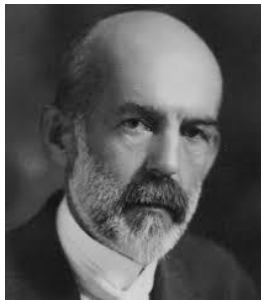
*Figure 1 radial engine used in WW2 for aircraft FW-190*

The development of the first practical radial engine is credited to the French engineer Léon Levavasseur. In 1908, Levavasseur designed and built the Antoinette 8V, a pioneering aircraft engine that featured a radial arrangement of cylinders. The Antoinette 8V was a water-cooled engine with eight cylinders arranged in a circular fashion around the crankshaft. This design provided a compact and efficient powerplant for early aircraft.



*Figure 2 Leon Levavasseur*

The engine was improved by George Mead and Charles Lawrence. These engineers and their respective companies made significant advancements in radial engine technology, enhancing performance, reliability, and efficiency.

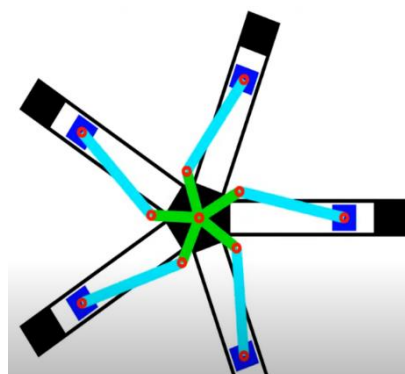


*Figure 4 George Mead*



*Figure 3 Charles Lawrence*

Their work contributed to the widespread adoption of radial engines during the golden age of aviation, and aircraft industry in WW2.



*Figure 5 parallel motion*

## Calculations

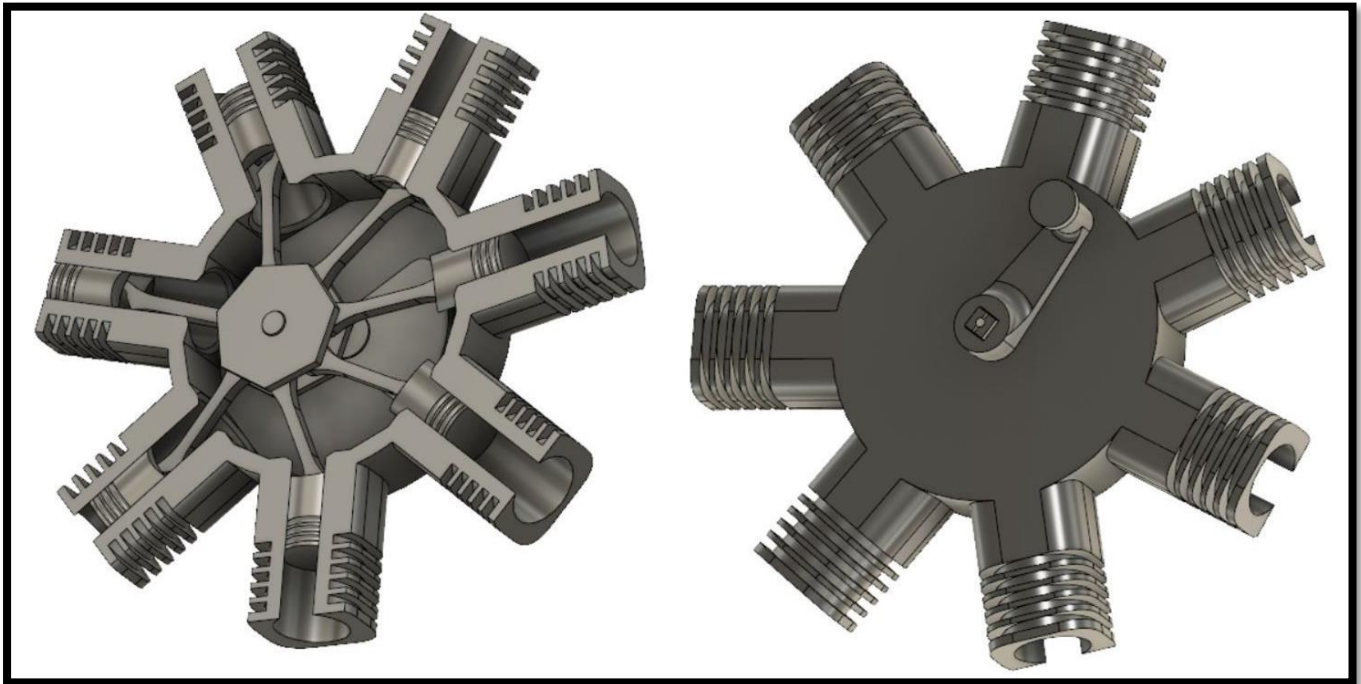


Figure 8: Radial engine design

### Position Analysis

From the loop

$$\vec{R}_2 - \vec{R}_3 - \vec{R}_4 - \vec{R}_1 = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$C = 0$  (there is no offset)

So, we have:

$$a \cos \theta_2 - b \cos \theta_3 - d \cos \theta_1 = 0 \quad \text{----- (1)}$$

$$a \sin \theta_2 - b \sin \theta_3 - d \sin \theta_1 = 0 \quad \text{----- (2)}$$

## Velocity Analysis

$$ae^{j\theta_2} - be^{j\theta_3} - de^{j\theta_1} = 0$$

Derive the equation.

$$aj\omega_2 e^{j\theta_2} + bj\omega_3 e^{j\theta_3} - cj\omega_4 e^{j\theta_4} - dj - fj\omega_1 e^{j\theta_1} = 0$$

So, we have:

$$a\omega_2 \cos \theta_2 - b\omega_3 \cos \theta_3 - d' = 0 \text{ ---(3)}$$

$$-a\omega_2 \sin \theta_2 + b\omega_3 \sin \theta_3 - d' = 0 \text{ ---(4)}$$

## Acceleration Analysis

$$aj\omega_2 e^{j\theta_2} - bj\omega_3 e^{j\theta_3} - d'' = 0$$

Derive the equation.

$$(a\omega_2^2 e^{j\theta_2} + aj\alpha_2 e^{j\theta_2}) - (b\omega_3^2 e^{j\theta_3} + bj\alpha_3 e^{j\theta_3}) - (c\omega_4^2 e^{j\theta_4} + cj\alpha_4 e^{j\theta_4}) - (d'') = 0$$

So, we have:

$$-a\alpha_2 \sin \theta_2 - a\omega_2^2 \cos \theta_2 + b\alpha_3 \sin \theta_3 + b\omega_3^2 \cos \theta_3 - d'' = 0$$

----- (5)

$$a\alpha_2 \cos \theta_2 - a\omega_2^2 \sin \theta_2 - b\alpha_3 \cos \theta_3 + b\omega_3^2 \sin \theta_3 - d'' = 0$$

----- (6)

## Jerk Analysis

$$(a\omega_2^2 e^{j\theta_2} + aj\alpha_2 e^{j\theta_2}) + (b\omega_3^2 e^{j\theta_3} + bj\alpha_3 e^{j\theta_3}) - (d\omega_1^2 e^{j\theta_1} + dj\alpha_1 e^{j\theta_1}) = 0$$

So, we have:

$$\begin{aligned} & a\omega_2^3 \sin \theta_2 - 3a\alpha_2 \omega_2 \cos \theta_2 - a\varphi_2 \sin \theta_2 + b\omega_3^3 \sin \theta_3 - \\ & 3b\alpha_3 \omega_3 \cos \theta_3 - b\varphi_3 \sin \theta_3 - \ddot{d} = 0 \end{aligned}$$

------(7)

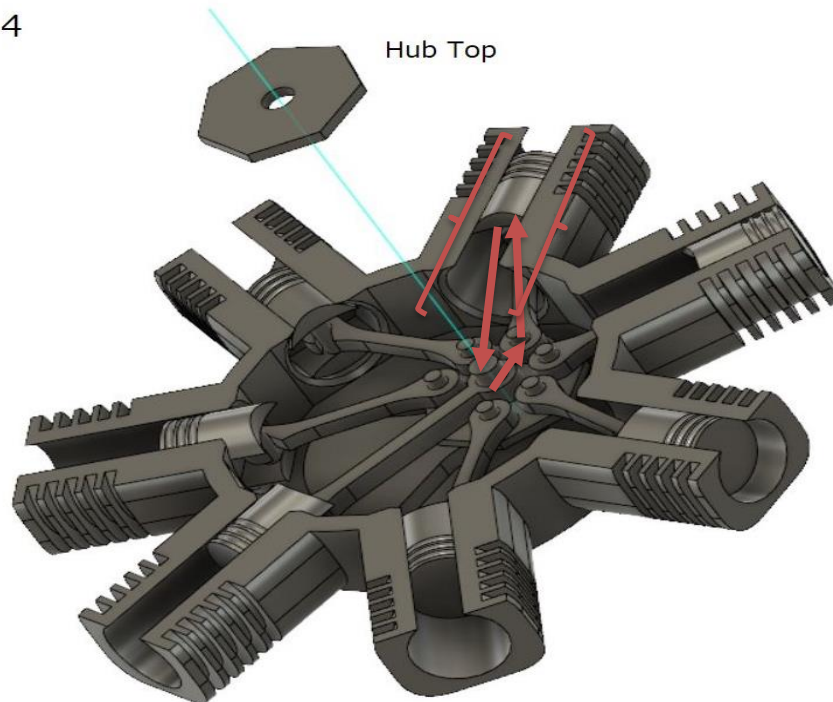
$$\begin{aligned} & -a\omega_2^3 \cos \theta_2 - 3a\alpha_2 \omega_2 \sin \theta_2 + a\varphi_2 \cos \theta_2 - b\omega_3^3 \cos \theta_3 - \\ & 3b\alpha_3 \omega_3 \sin \theta_3 + b\varphi_3 \cos \theta_3 - \ddot{d} = 0 \end{aligned}$$

------(8)

---

## Force Analysis

Step 4

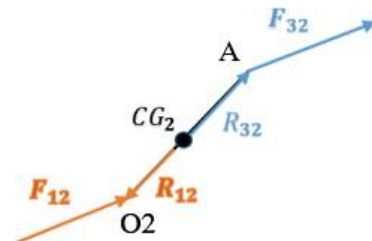


➤ For link 2:

$$\sum F_x = ma_x \rightarrow F_{12x} + F_{32x} = m_2 a_{G2x}$$

$$\sum F_y = ma_y \rightarrow F_{12y} + F_{32y} = m_2 a_{G2y}$$

$$\begin{aligned} \sum T = I_G \alpha \rightarrow T_{12} + (R_{12x}F_{12y} - R_{12y}F_{12x}) \\ + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{G2}\alpha_2 \end{aligned}$$

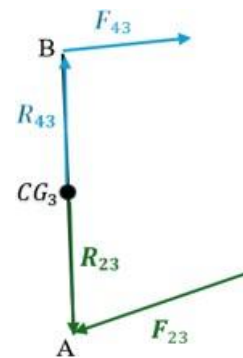


➤ For link 3:

$$\sum F_x = ma_x \rightarrow F_{43x} - F_{23x} = m_3 a_{G3x}$$

$$\sum F_y = ma_y \rightarrow F_{43y} - F_{23y} = m_3 a_{G3y}$$

$$\begin{aligned} \sum T = I_G \alpha \rightarrow (R_{43x}F_{43y} - R_{43y}F_{43x}) \\ + (R_{23x}F_{23y} - R_{23y}F_{23x}) = I_{G3}\alpha_3 \end{aligned}$$





➤ For link 4 :

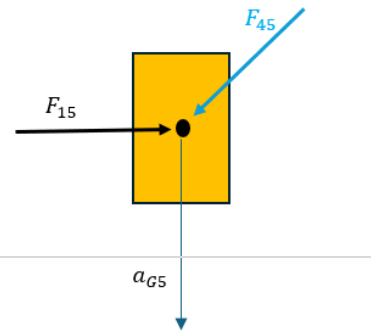
$$\sum F_x = ma_x \rightarrow F_{34x} + F_{14x} = m_4 a_{G4x}$$

$$\sum F_y = ma_y \rightarrow F_{34y} + F_{14y} = m_4 a_{G4y}$$

$$\sum T = I_G \alpha \rightarrow (R_{34x} F_{45y} - R_{34y} F_{34x})$$

---


$$+ (R_{14x} F_{14y} - R_{14y} F_{14x}) = I_{G5}$$

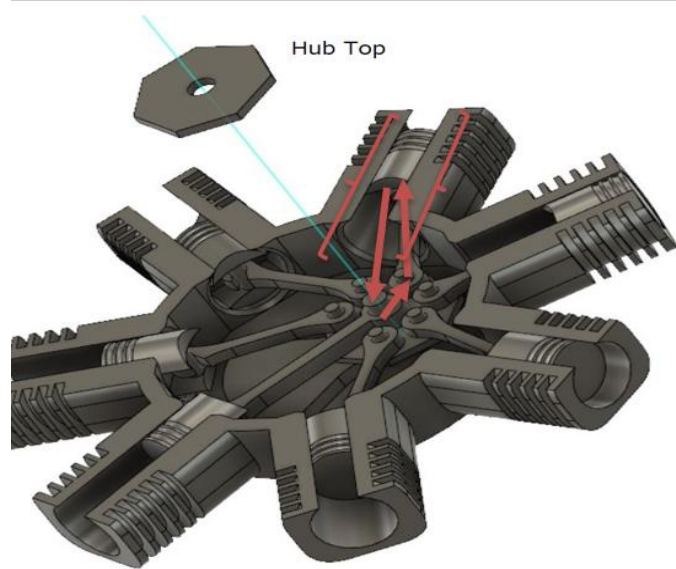


## Using MATLAB

→ Loop:

When:  $a=10\text{mm}$ ,  $b=36\text{mm}$ ,  $c=0\text{mm}$ ,  $d=\text{variable}$

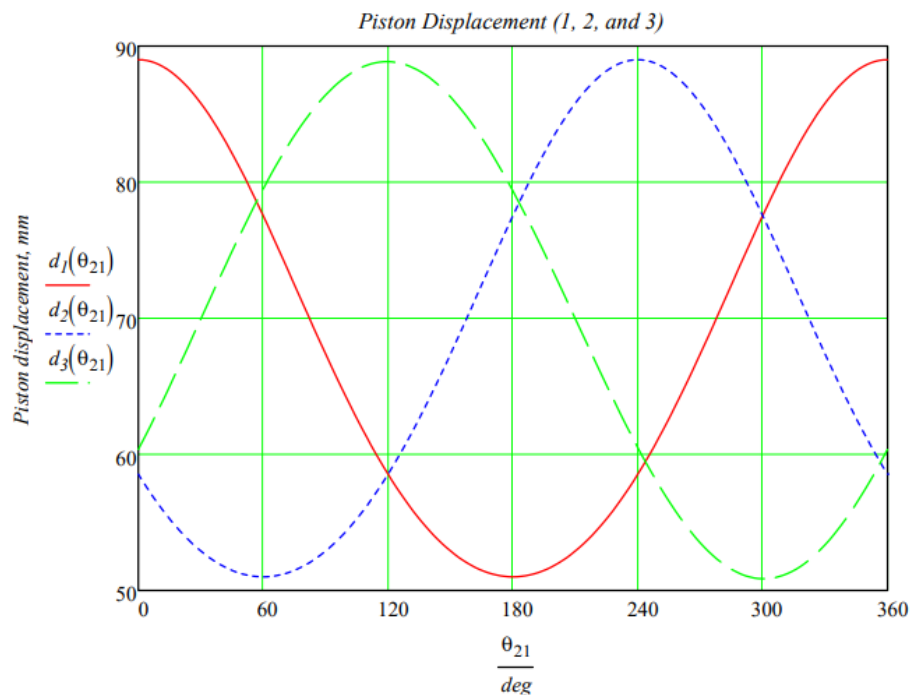
And  $\theta_1 = 90$ ,  $\theta_3 = 60$ ,  
 $\theta_2 = [0-360]$ ,



Position:

$$a \cos \theta_2 - b \cos \theta_3 - d \cos \theta_1 = 0 \quad \text{----- (1)}$$

$$a \sin \theta_2 - b \sin \theta_3 - d \sin \theta_1 = 0 \quad \text{----- (2)}$$



## MATLAB code

```
close all
clear all
clc

%% SET UP

r2 = 10;
r3 = 36;

% Set up for animation
figure
axis (gca, 'equal');
axis ([-400 600 -400 400])

zz(3,:) = [0, 0];

% Angles
th2 = deg2rad (0:20:720);

% Angular speed
om2 = 1;

%% LOOP

% i is the time to run the animation for
for i = 1:500

    th1 = om2*(i/10);
    aph = asin((r2*sin(th1))/(r3));

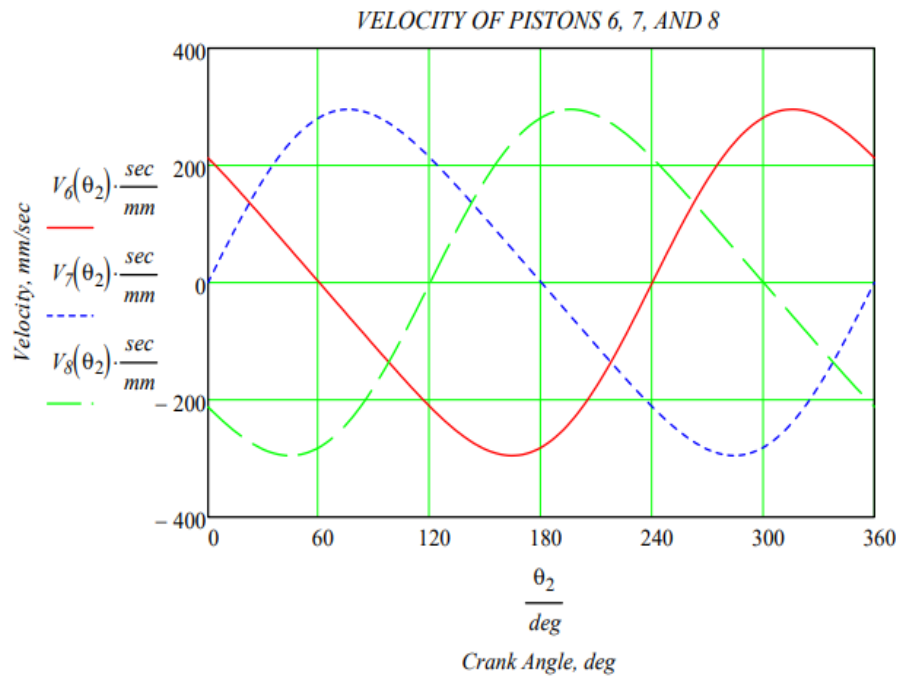
    % Distance between Slider and stationary point
    r1 = ((r2*cos(th1)) + (r3*cos(aph)));

    % Positions of links and joints
    zz(1,:) = [r1, 0];
    zz(2,:) = [r2*cos(th1), r2*sin(th1)];

    % Plot the results
    plot (zz(:,1), zz(:,2), 'o-')
    title ('Slider Crank Mechanism')
    axis ([-400 600 -400 400])
    pause (0.01)

end
```

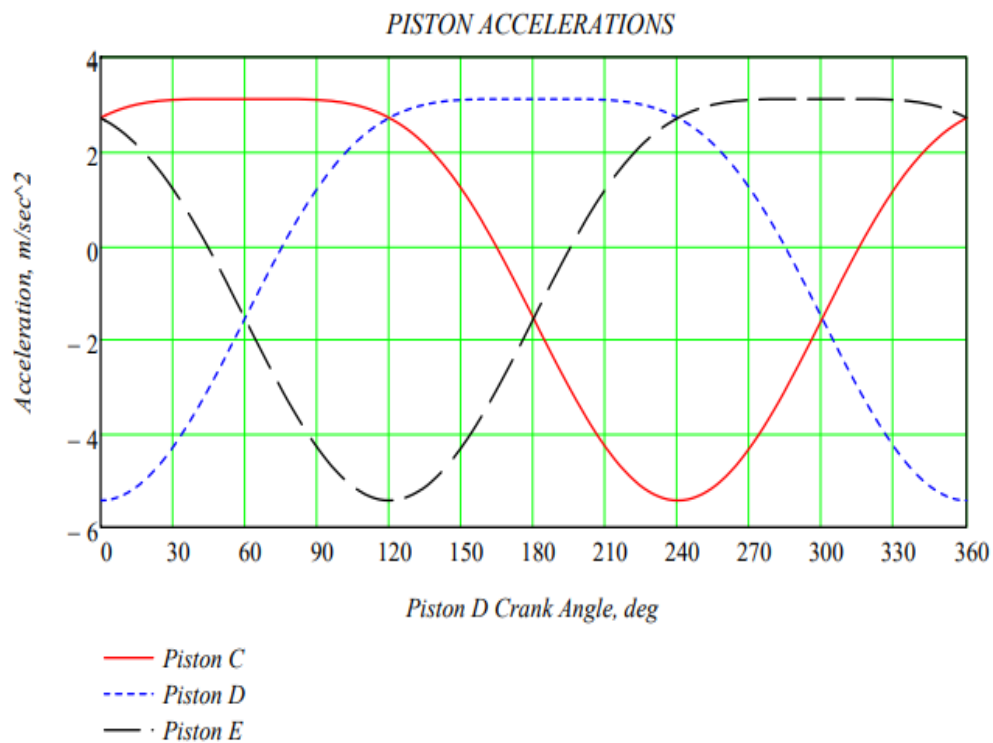
Velocity:



### Matlab Code

```
wab=6.28 r2=10 ;r3=36;  
theta=0:0.1:2*pi;  
phi = asin((r*sin(theta))/r3);  
wbc= (wab*r2*sin(theta))./(r3*sin(phi));  
v=wab*r2.*cos(theta)+wbc*r3.*cos(phi);  
plot(theta,v)
```

### Acceleration:



### **MATLAB code**

% Crank-Slider Mechanism Acceleration Calculation

% Input parameters

theta = linspace(0, 2\*pi, 100); % Crank angle in radians

r = 10; % Crank length (meters)

l = 36; % Connecting rod length (meters)

omega = 2\*pi; % Angular velocity of the crank (rad/s)

% Calculate position coordinates

x = r \* cos(theta) + sqrt(l^2 - r^2 \* sin(theta).^2);

% Calculate velocity components

v\_x = -r \* omega \* sin(theta);

v\_y = r \* omega \* cos(theta);

% Calculate acceleration components

a\_x = -r \* omega^2 \* cos(theta);

a\_y = -r \* omega^2 \* sin(theta);

```
% Total acceleration magnitude
a_total = sqrt(a_x.^2 + a_y.^2);

% Plotting
figure;

subplot(3, 1, 1);
plot(theta, x);
title('Position');

subplot(3, 1, 2);
plot(theta, v_x, theta, v_y);
legend('v_x', 'v_y');
title('Velocity');

subplot(3, 1, 3);
plot(theta, a_x, theta, a_y, theta, a_total);
legend('a_x', 'a_y', 'a_{total}');
title('Acceleration');

% Display maximum acceleration
max_acceleration = max(a_total);
fprintf('Maximum acceleration: %.2f m/s^2\n', max_acceleration);
```

## Using Analytical method:

$$d = 10 \text{ mm}$$

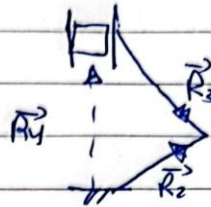
$$\omega_2 = 6.28 \text{ rad/s ccw}$$

$$b = 36 \text{ mm}$$

$$\theta_1 = 90^\circ$$

$$c = 0$$

$$\text{for } \theta_2 = 60^\circ$$



Solution:

Position  $\rightarrow$

$$\begin{aligned} -a \cos \theta_2 - b \cos \theta_3 - d \cos \theta_1 &= 0 \rightarrow \\ -a \sin \theta_2 - b \sin \theta_3 - d \sin \theta_1 &= 0 \end{aligned}$$

$$\begin{aligned} 10 \cos(60^\circ) - 36 \cos(\theta_3) - d \cos(90^\circ) &= 0 \rightarrow \\ 10 \sin(60^\circ) - 36 \sin(\theta_3) - d &= 0 \end{aligned}$$

$$\boxed{\theta_3 = -82^\circ}, \boxed{d = 44.3 \text{ mm}}$$

Velocity  $\rightarrow$

$$\begin{aligned} -a \omega_2 \sin \theta_2 + b \omega_3 \sin \theta_3 - \dot{d} &= 0 \\ a \omega_2 \cos \theta_2 - b \omega_3 \cos \theta_3 - \dot{d} &= 0 \end{aligned}$$

$$\begin{aligned} (-10)(6.28) \sin(60^\circ) + (36)(\omega_3) \sin(-82^\circ) - \dot{d} &= 0 \rightarrow \text{using calculator} \\ (10)(6.28) \cos(60^\circ) - (36)(\omega_3) \cos(-82^\circ) - \dot{d} &= 0 \end{aligned}$$

$$\boxed{\omega_3 = 2.799 \text{ rad/s}}$$

$$\boxed{\dot{d} = -45.427}$$

acceleration  $\rightarrow$  constant  $\omega_2$  so  $\alpha_2 = 0$

$$\begin{aligned} (-\alpha_2)(a) \sin \theta_2 - (a)(\omega_2)^2 \cos \theta_2 - b \alpha_3 \sin \theta_3 - b \omega_3^2 \cos \theta_3 - \ddot{d} &= 0 \\ (\alpha_2)(a) \cos \theta_2 - (a)(\omega_2)^2 \sin \theta_2 + b \alpha_3 \cos \theta_3 + b \omega_3^2 \sin \theta_3 + \ddot{d} &= 0 \end{aligned}$$

using calculator  $\rightarrow \boxed{\alpha_3 = 6.625 \text{ rad/s}^2} \quad \boxed{\ddot{d} = 29.1 \text{ mm/s}^2}$

## References

[https://en.wikipedia.org/wiki/Beam\\_engine](https://en.wikipedia.org/wiki/Beam_engine)

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[https://www.youtube.com/results?search\\_query=servo+motor+arduino](https://www.youtube.com/results?search_query=servo+motor+arduino)