9.3 Dimensionality Reduction

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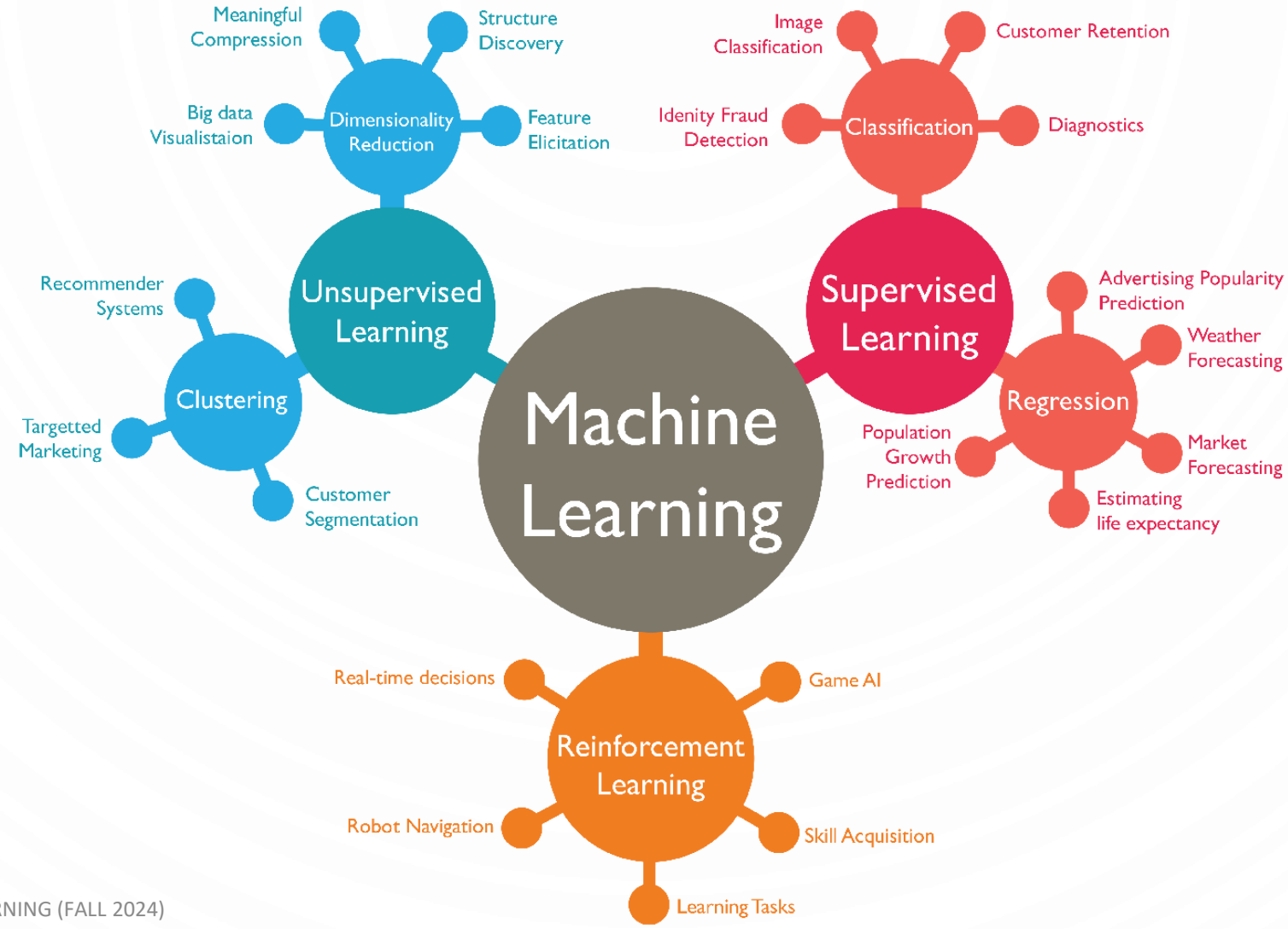
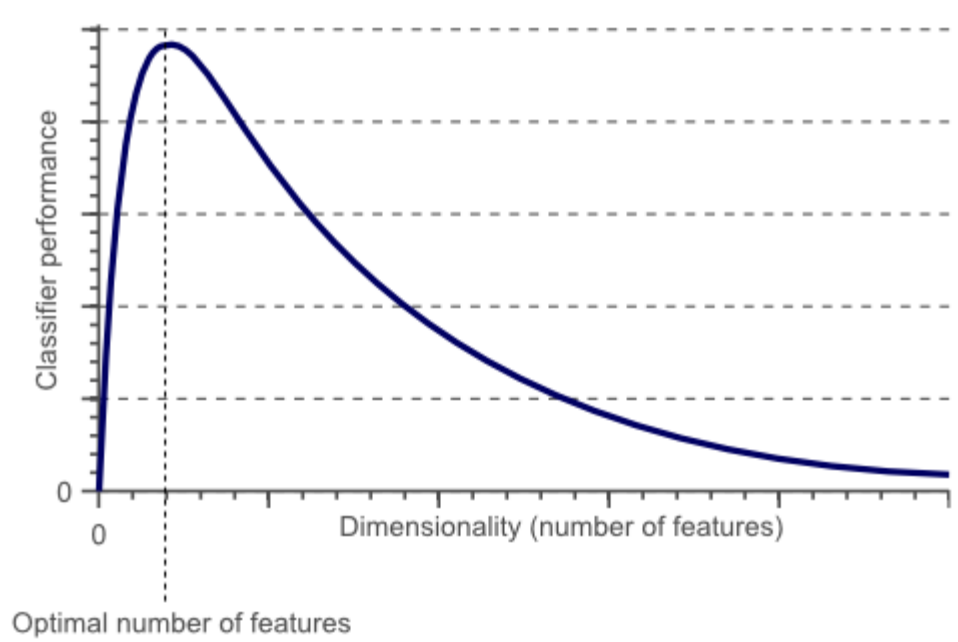
Machine Learning Approaches



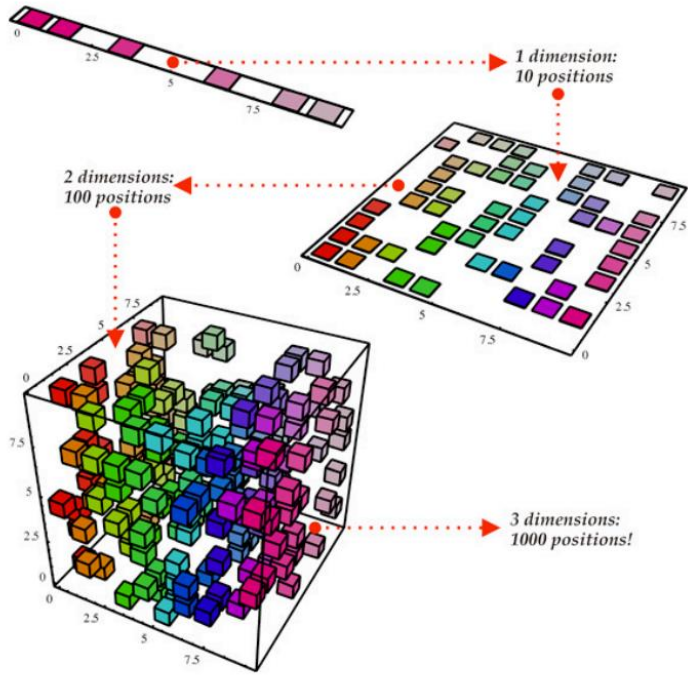
Hughes Phenomenon

- According to Hughes phenomenon, If the number of **training samples is fixed** and we keep on **increasing the number of dimensions** then the predictive power of our machine learning model first increases, but after a certain point it tends to decrease.
- Also known as the **curse of dimensionality**



Dimensionality Reduction

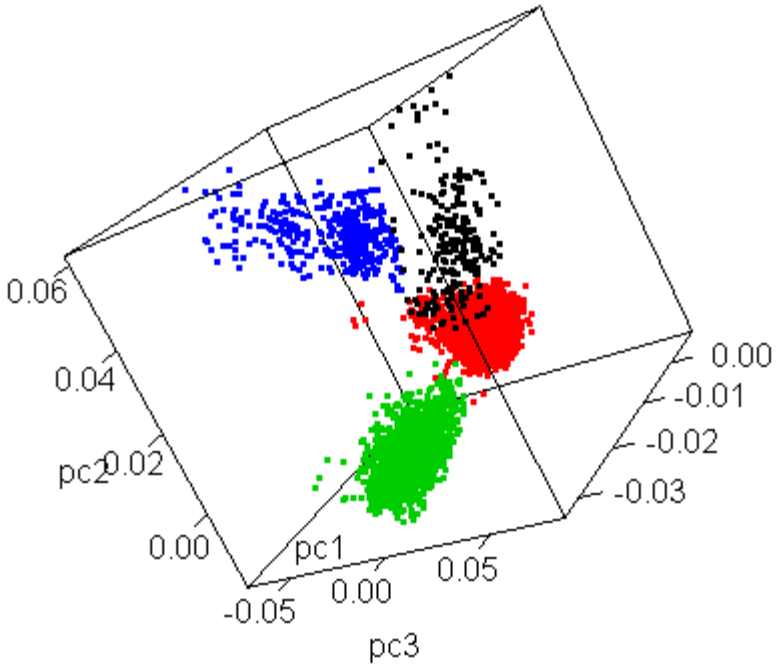
Dimensionality reduction, or dimension reduction, is the transformation of data **from a high-dimensional** space **into a low-dimensional** space so that the low-dimensional representation retains some meaningful properties of the original data, ideally close to its intrinsic dimension.



- **Applications**
  1. The most frequent use case for dimensionality reduction is **data visualization**
     - humans can only interpret on a plot the maximum of three dimensions.
  2. Dimensionality reduction **removes redundant** or highly correlated features;
  3. It also **reduces the noise** in the data
- Widely used **techniques** of dimensionality reduction
  - Principal Component Analysis (PCA)
  - Uniform Manifold Approximation and Projection (UMAP)
  - Autoencoders

Principal Components Analysis (PCA)

- Principal Components Analysis (PCA) is a technique that finds underlying variables (known as principal components) that best differentiate your data points.
- Principal components are **dimensions** along which your **data points are most spread out**.



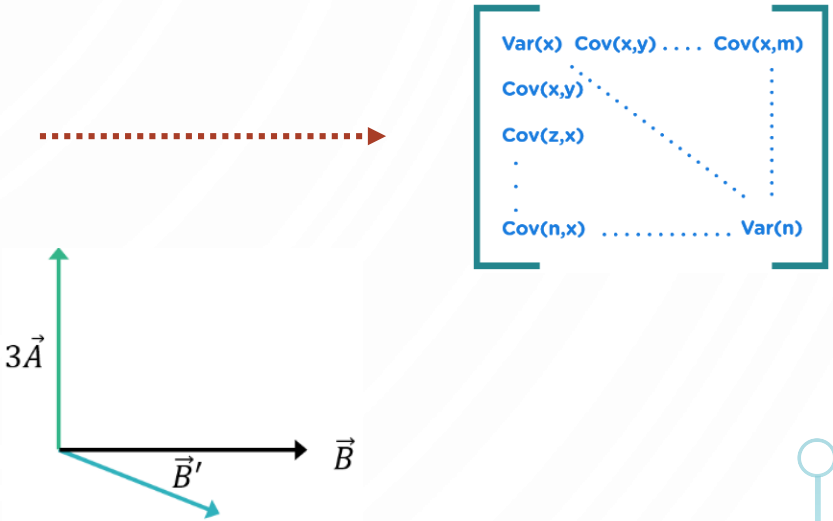
Applications of PCA

- Visualize multidimensional data.
- Reduce the number of dimensions in data.
- Help resize an image.
- Used in finance to analyze stock data and forecast returns.
- Find patterns in the high-dimensional datasets.

How PCA Works?

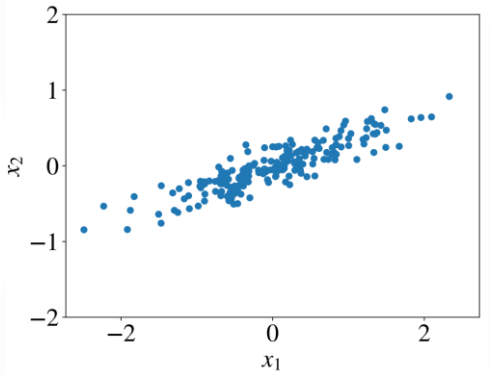
1. **Normalize** the data
   - Standardize the data before performing PCA.
   - This will ensure that each feature has a mean = 0 and variance = 1.
2. Build the **covariance matrix**
   - Construct a square matrix to express the correlation between two or more features in a multidimensional dataset.
3. Find the **Eigenvectors** and Eigenvalues
   - Calculate the eigenvectors/unit vectors and eigenvalues.
   - Eigenvalues are scalars by which we multiply the eigenvector of the covariance matrix.
4. **Sort the eigenvectors** in highest to lowest order and select the number of principal components.

$$Z = \frac{x - \mu}{\sigma}$$

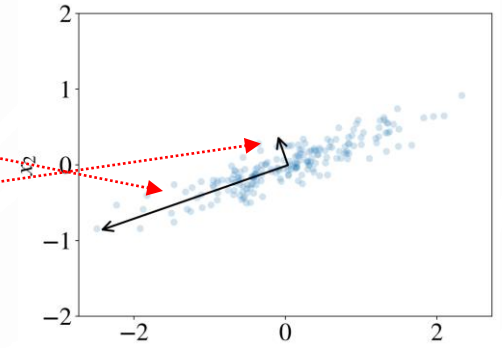


PCA Example

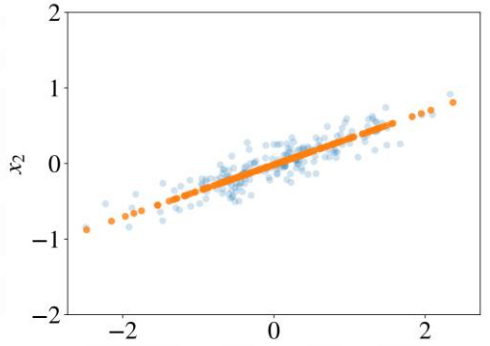
- Consider a two-dimensional data as shown in figure on the right:
  - Principal components are vectors that define a new coordinate system in which the **first axis** goes in the direction of the **highest variance** in the data.
  - The second axis is **orthogonal** to the first one and goes in the direction of the second highest variance in the data.
- If our data was three-dimensional:
  - The third axis would be orthogonal to both the first and the second axes and go in the direction of the third highest variance
- And so on for high dimensional data.



Original data

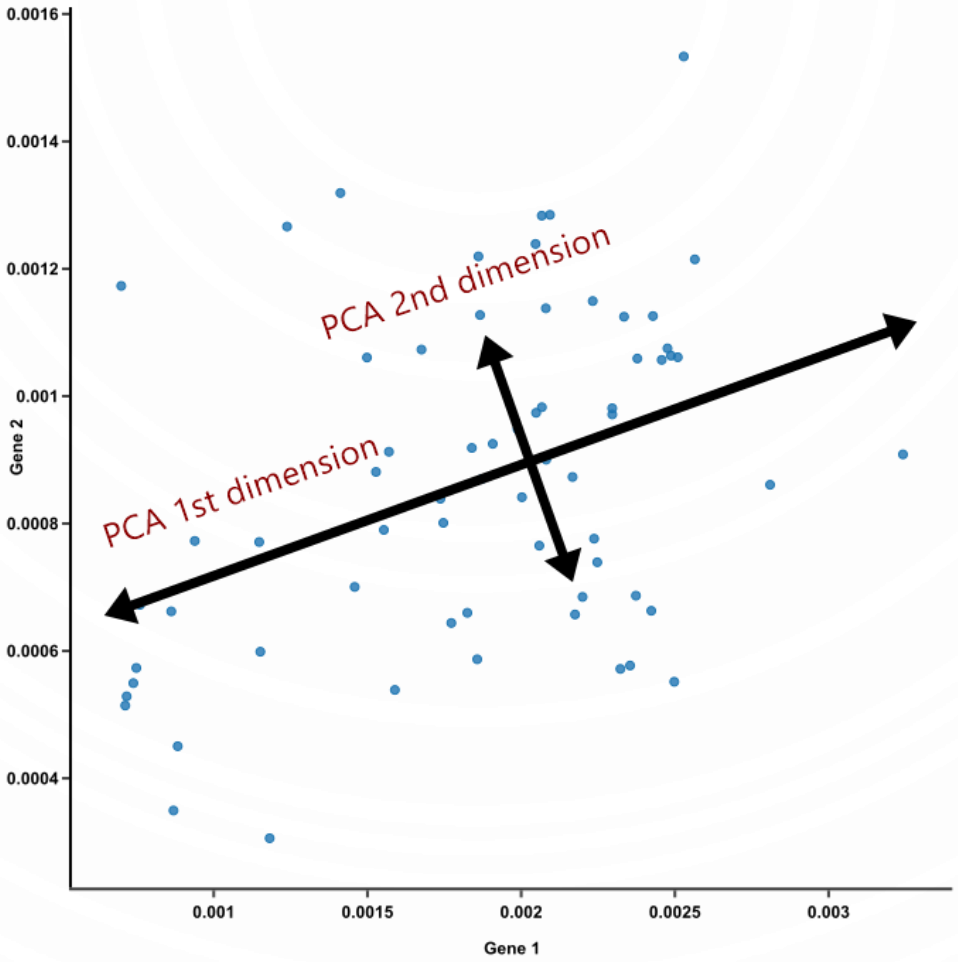


2 principal components displayed as vectors

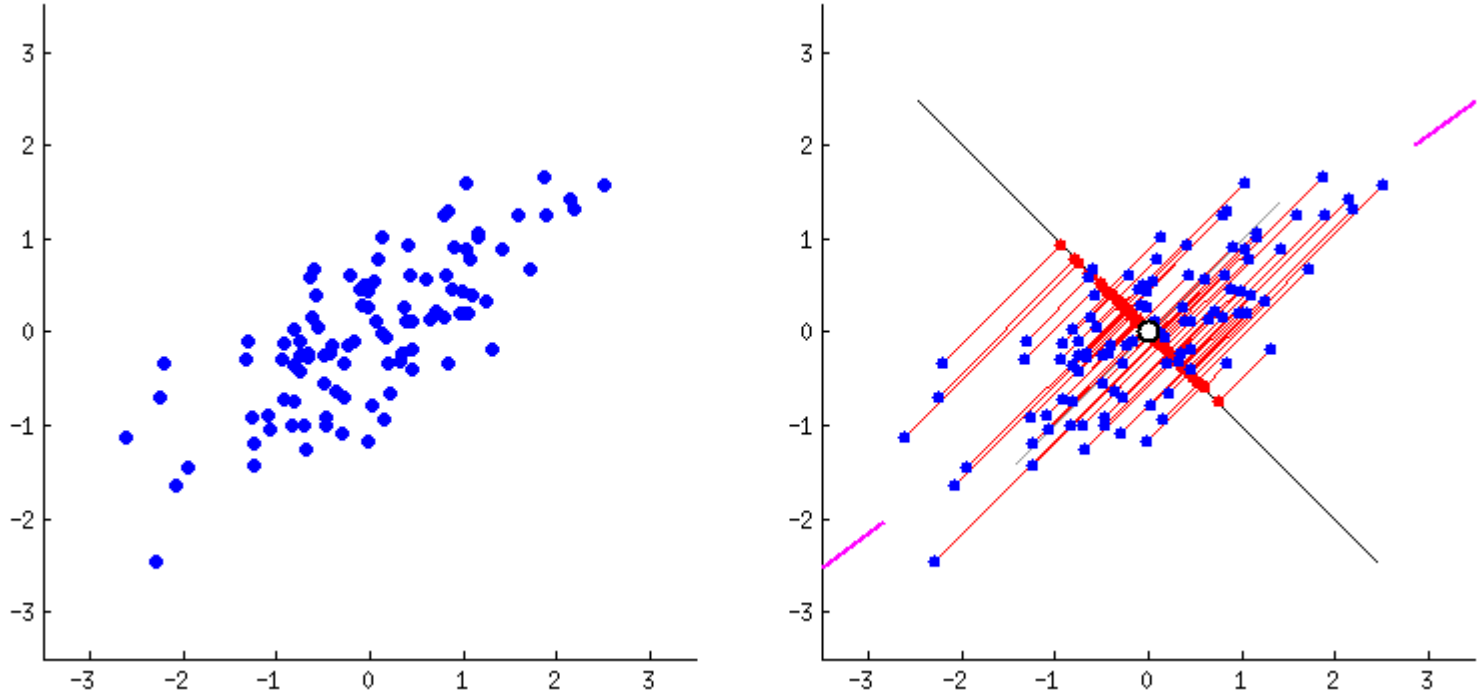


The data projected on the first principal component

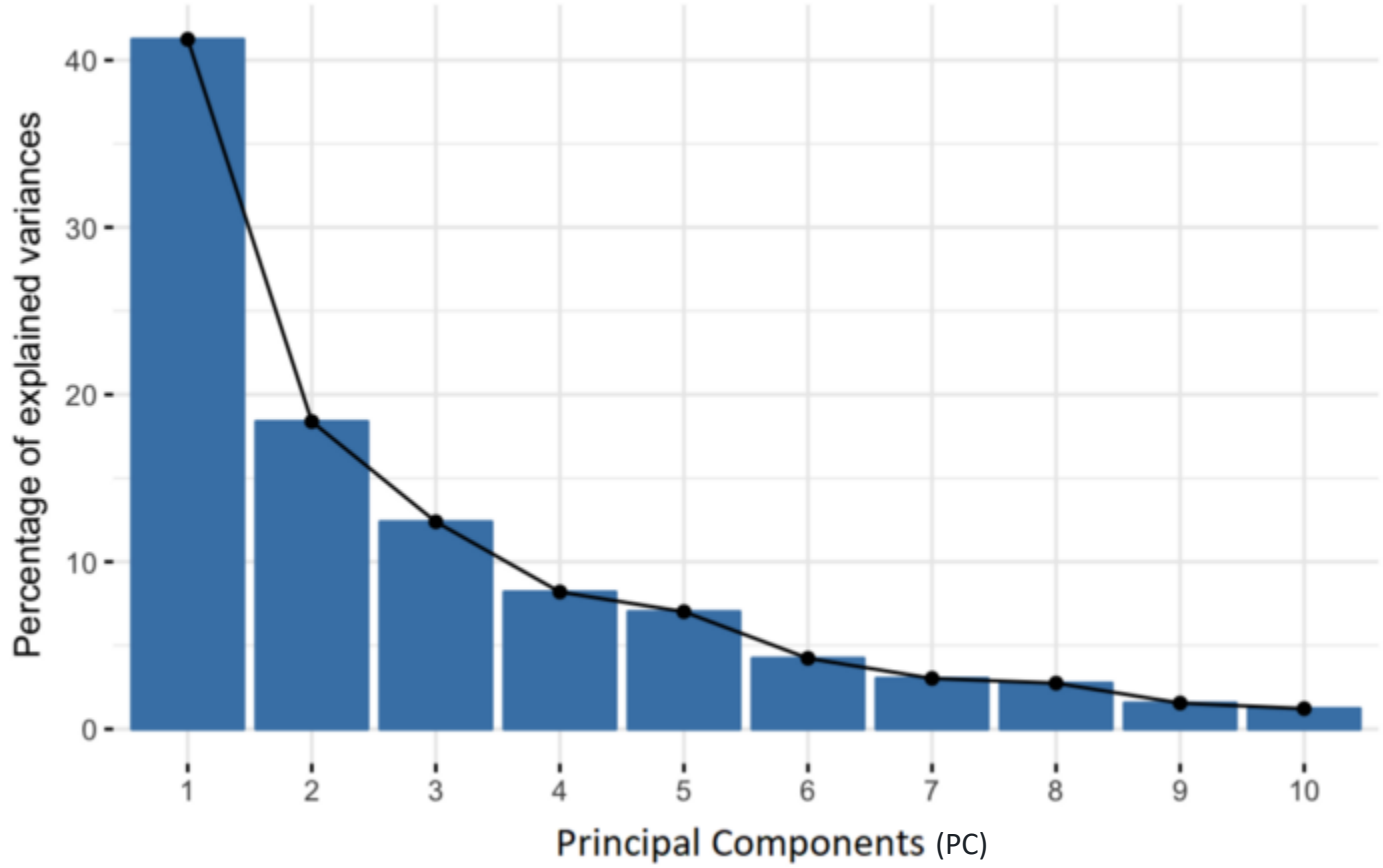
PCA Example


0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
Gene 2
0.001
0.0015
0.002
0.0025
0.003
Gene 1
PCA 2nd dimension
PCA 1st dimension

Example



Percentage of Variance (Information) for each by PC



Python Example

https://colab.research.google.com/drive/1eIddjy28cpp26pyM8qmJqdCABzG0VbBF?usp=sharing