First Assignment

1. Construct a Turing machine that accepts the complement of the following language.
$$\{a^n b^n c^n \mid n \geq 0\}.$$

2. Construct a Turing machine that accepts the language
$$\{ww \mid w \in \{a, b\}^*\}.$$

3. Given that the languages L_1 and L_2 are recursive, prove that the following two languages are recursive as well –
   (a) $L_1 L_2$.
   (b) $(L_1)^*$.

4. Given that the languages L_1 and L_2 are recursively enumerable, prove that the following two languages are recursively enumerable as well –
   (a) $L_1 L_2$.
   (b) $(L_1)^*$.

5. Let L_1 and L_2 be languages over the alphabet Σ. Define the *right quotient* of L_2 by L_1 as $L_1 \backslash L_2$
$$\{w \in \Sigma^* \mid \exists y \in L_1 \text{ and } yw \in L_2\}.$$
   If L_1 and L_2 are recursively enumerable, is $L_1 \backslash L_2$ also recursively enumerable?

6. (`Hard Problem`) Consider a single tape Turing machine which has one interesting restriction on its actions – it can not overwrite those tape cells on which input symbols are written, i.e. the tape cells on which

input is written is read-only. The other tape cells are normal tape cells, i.e. symbols can be read from them and new symbols can be written onto them. Is this new model of Turing machine as powerful as the standard model of Turing machine?

7. Prove that $\mathsf{2SAT}$ is in P.

8. Prove that HALT is NP-hard.

9. Prove that $\mathsf{L_D}$ is NP-hard.

10. Can you construct a polynomial-time reduction from HALT to $\mathsf{L_D}$?

11. The $\mathsf{2-COLORING}$ problem is defined as follows.

$$\begin{aligned} \mathsf{2-COLORING} = \{ < \mathsf{G} > \ | & \text{ Vertices of the graph } \mathsf{G} \text{ can be painted} \\ & \text{with 2 colours such that adjacent vertices do not} \\ & \text{have the same colour } \}. \end{aligned}$$

Prove that $\mathsf{2-COLORING}$ is in P.

12. The CLIQUE problem is defined as follows.

$$\mathsf{CLIQUE} = \{ < \mathsf{G}, \mathsf{k} > \ | \text{ The graph } \mathsf{G} \text{ has clique of size } \mathsf{k} \, \}.$$

Prove that $\mathsf{CLIQUE} \leq_p \mathsf{INDEPENDENT\ SET}$.

13. $\mathsf{INDEPENDENT\ SET} \leq_p \mathsf{CLIQUE}$.

14. $\mathsf{VERTEX\ COVER} \leq_p \mathsf{INDEPENDENT\ SET}$.

15. $\mathsf{INDEPENDENT\ SET} \leq_p \mathsf{dHAMPATH}$.