

Lecture 6

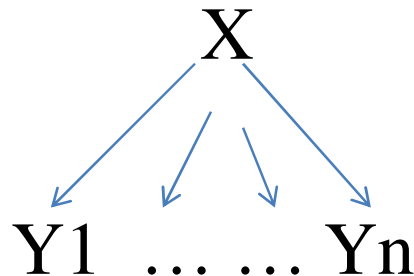
Syntax Analysis II

Derivation

- A derivation is the sequence of productions

$S \rightarrow \dots \rightarrow \dots \rightarrow \dots \rightarrow \dots$

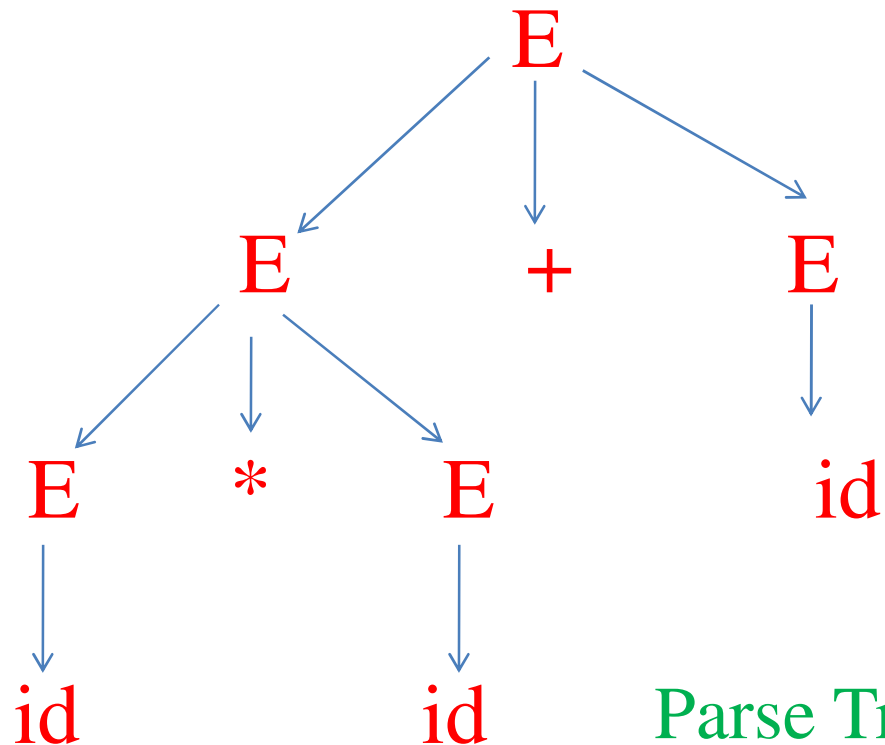
- A derivation can be drawn as a tree
 - Start symbol is the tree's root
 - For a production $X \rightarrow Y_1 \dots \dots Y_n$ add children $Y_1 \dots \dots Y_n$ to node X



Example: Left-most derivation

- Grammar: $E \rightarrow E + E \mid E * E \mid (E) \mid id$
- String : $id * id + id$

E
 $\rightarrow E + E$
 $\rightarrow E * E + E$
 $\rightarrow id * E + E$
 $\rightarrow id * id + E$
 $\rightarrow id * id + id$



Parse Tree

- A parse tree has
 - Terminals at the leaves
 - Non-terminal at the interior nodes
- An in-order traversal of the leaves is the original input
- The parse tree shows the association of operations, the input string does not

Right-most derivation

- The example is a left-most derivation
 - At each step, replace the left most non terminals

$$\begin{aligned} & E \\ \rightarrow & E + E \\ \rightarrow & E * E + E \\ \rightarrow & id * E + E \\ \rightarrow & id * id + E \\ \rightarrow & id * id + id \end{aligned}$$

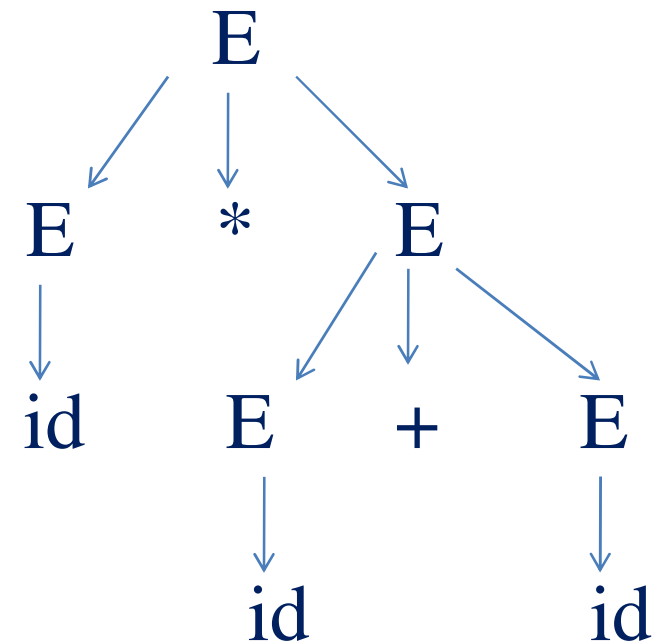
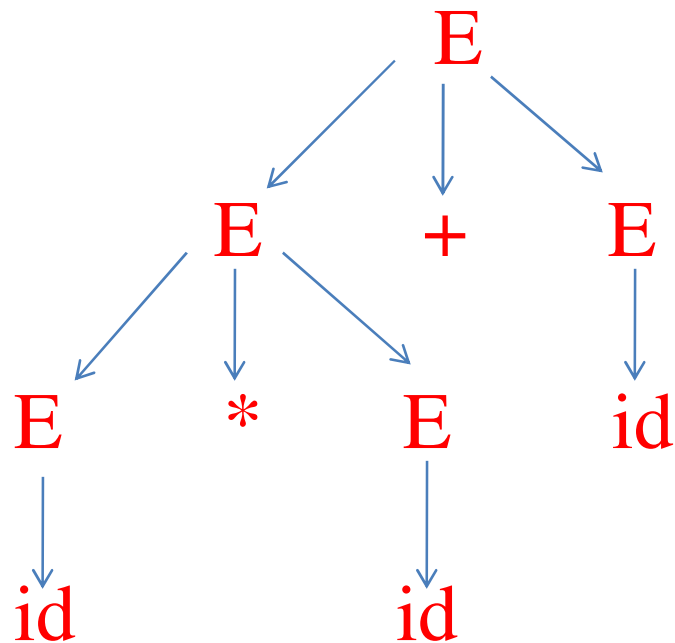
- There is a equivalent notion of right-most derivation

Parse Tree Derivations

- Note that the left-most and right-most derivation have the same parse tree
- Every parse tree has the left-most and right-most derivations

Ambiguity

- Grammar: $E \rightarrow E + E \mid E * E \mid (E) \mid id$
- String : $id * id + id$
- This string has two parse trees



Ambiguous Grammar

- A grammar is ambiguous if it has more than one parse tree for some string
 - Equivalently, there is more than one right-most or left-most derivation for some string
- Ambiguity means that some programs are ill-defined

How to handle Ambiguous Grammar

- Several ways to handle
- Rewrite the grammar unambiguously

$$E \rightarrow E' + E \mid E'$$

$$E' \rightarrow id * E' \mid id \mid (E) * E \mid (E)$$

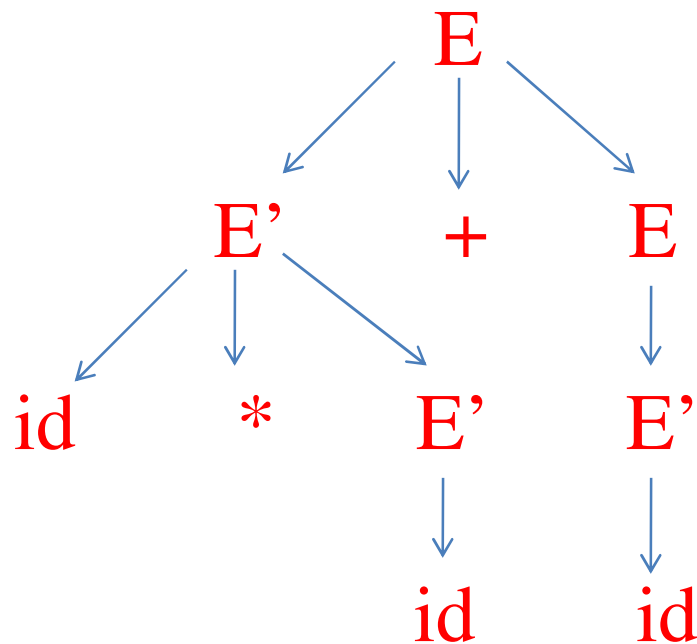
- Enforce precedence of $*$ over $+$

How to handle Ambiguous Grammar

- Grammar: $E \rightarrow E' + E \mid E'$
 $E' \rightarrow id * E' \mid id \mid (E) * E' \mid (E)$

- String : $id * id + id$

Enforce **precedence of * over +** by divide the productions in two classes; one handle + and one handles *; so one non terminals for each operators



$E \rightarrow E' + E \rightarrow E' + E' + E \rightarrow$
 $E' + E' + E' + E \rightarrow E' + \dots + E'$

Handles +

$E' \rightarrow id * E' \rightarrow id * id * E' \rightarrow$
 $id * id * id * E' \rightarrow id * \dots * id$

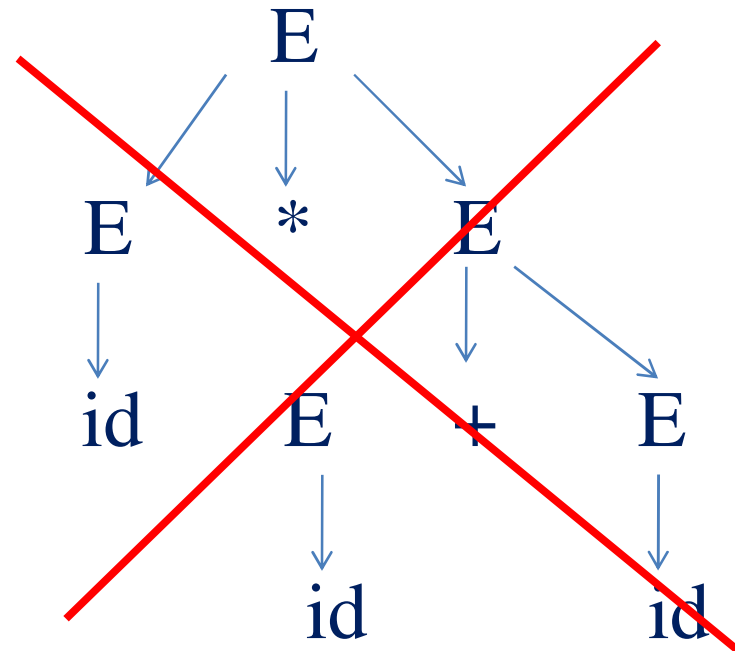
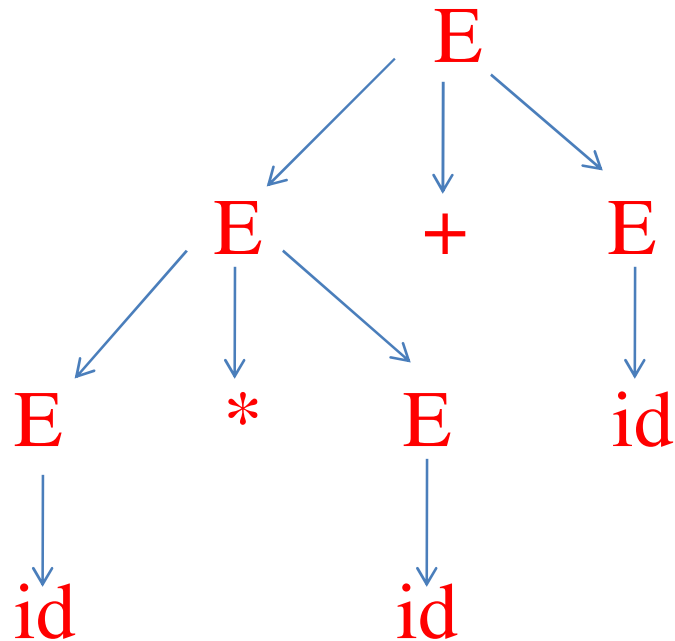
Handles *

How it works

- The grammar has two separate groups of productions
- All the pluses (+) must be generated before any of the times (*); so time(s) (*) are nested more deeply inside the parse tree; pluses (+) are generated in the outermost levels and times (*) are generated inside the pluses (+).
- So the grammar enforces that the time (*) has higher precedence than the plus (+)

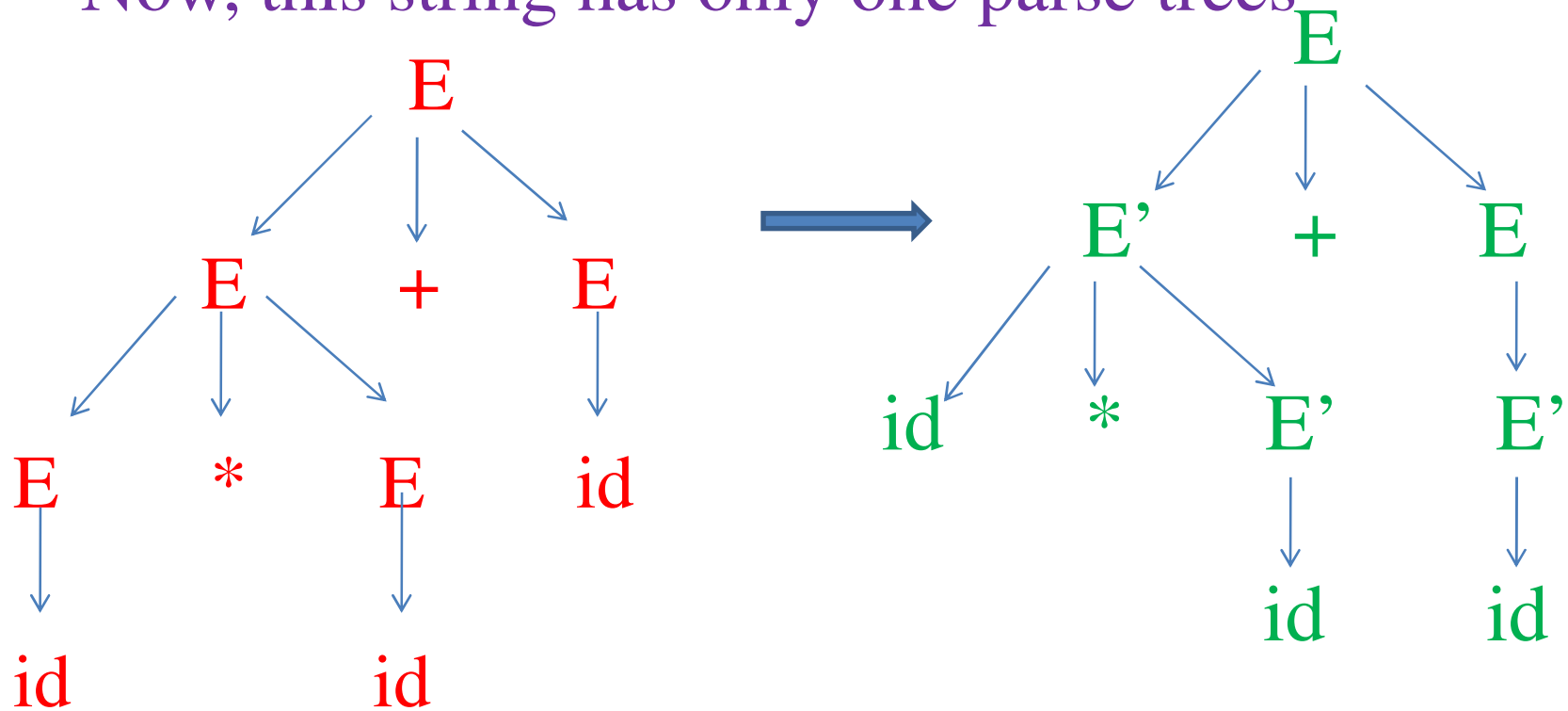
Ambiguity

- Grammar: $E \rightarrow E + E \mid E * E \mid (E) \mid id$
- String : $id * id + id$
- This string has two parse trees



Ambiguity

- Grammar: $E \rightarrow E + E \mid E * E \mid (E) \mid id$
- String : $id * id + id$
- Now, this string has only one parse trees



Example

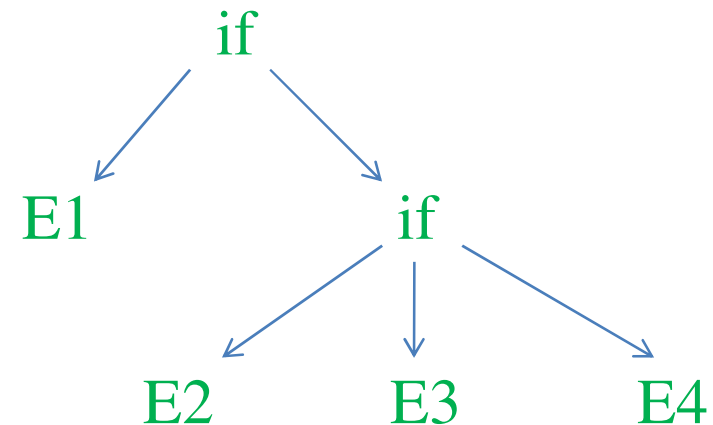
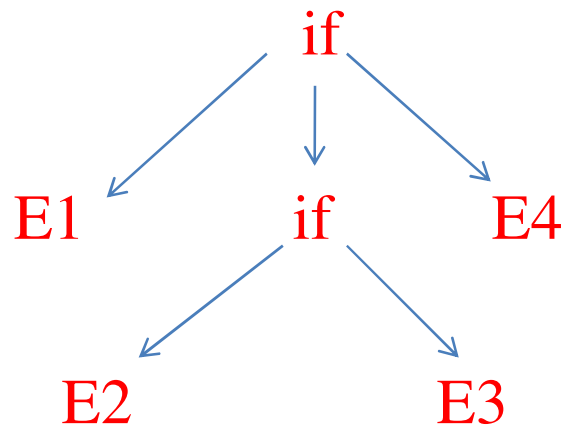
- Another Expression

$E \rightarrow \text{if } E \text{ then } E$

$\mid \text{if } E \text{ then } E \text{ else } E$

$\mid \text{OTHER}$

- The expression: *if E1 then if E2 then E3 else E4* has two separate parse trees



How to handle the Ambiguity

- The property that we want is: *else matches the closest unmatched then*
- Can be resolved as:

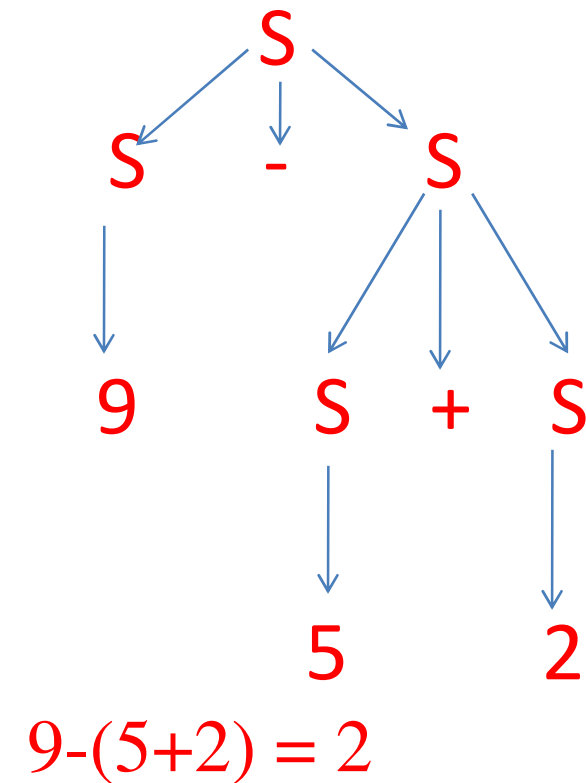
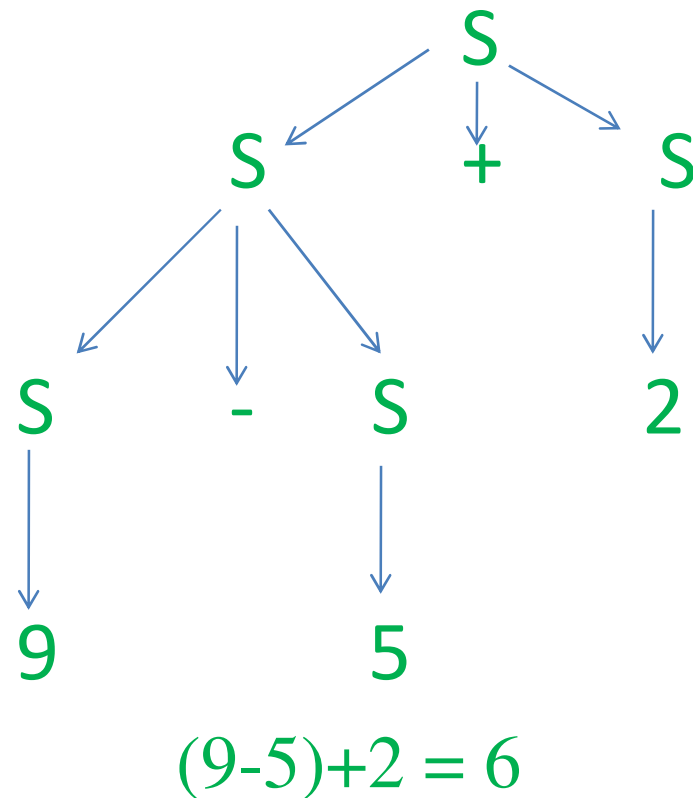
$E \rightarrow MIF \mid UIF$

$MIF \rightarrow \text{if } E \text{ then } MIF \text{ else } MIF$
 $\mid \text{OTHER}$

$UIF \rightarrow \text{if } E \text{ then } E$
 $\mid \text{if } E \text{ } MIF \text{ else } UIF$

Associativity of Operators

- Let a Grammar : $S \rightarrow S + S \mid S - S \mid 0 - 9$
- Input: $9 - 5 + 2$



How to handle

- When an operand like 5 has operators to its left and right, conventions are needed for deciding which operator applies to that operand.
- Rule: **operator + is associates to the left**. Arithmetic operators (+, -, *, /) are left associative.
- Assignment operator (“=”) in C is right associative.

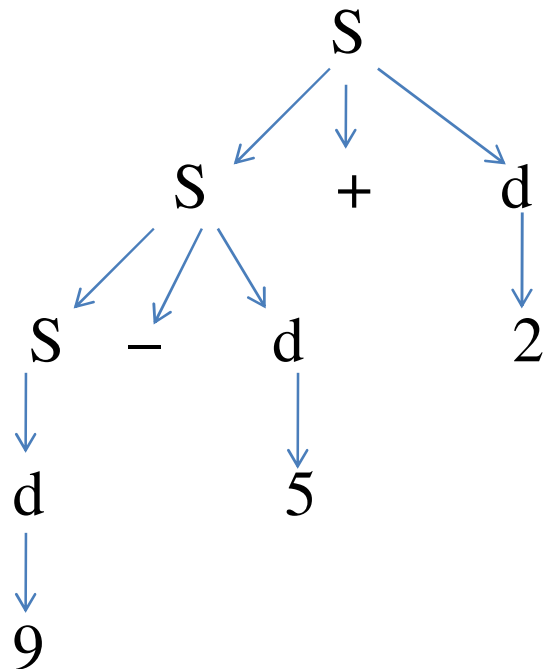
How to handle

- Left Associative

$$S \rightarrow S + d \mid S - d \mid d$$

$$d \rightarrow [0-9]$$

- Input: $9 - 5 + 2$



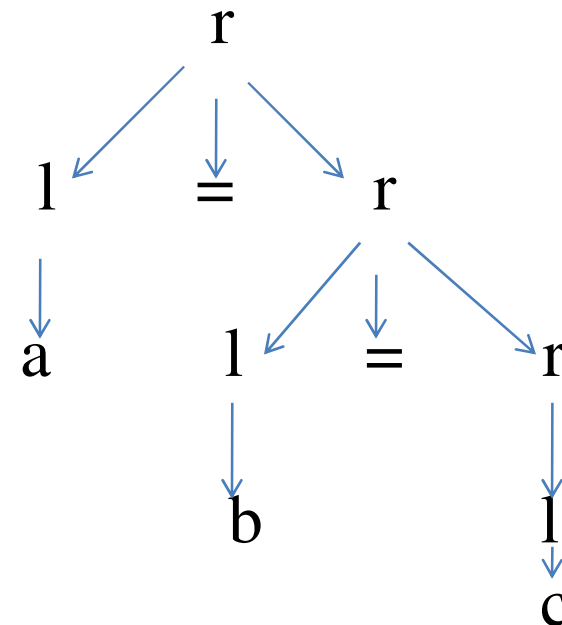
Left-most derivation

- Right Associative

$$r \rightarrow l = r \mid l$$

$$l \rightarrow [a-z]$$

- Input: $a = b = c$



Right-most derivation

Push Down Automata

- The language for balanced parenthesis, i.e. $\{(^i)^i \mid i \geq 0\}$, CFG productions are

$$S \rightarrow (S)S$$

$$S \rightarrow \varepsilon$$

- The situation can be handled if the DFA is augmented with *memory*
 - Memory implemented as a *stack*
 - Such an automata is called *Push Down Automata (PDA)*

Thanks