Assignment Problems

1. Prove that the function H defined as part of SAT_H is computable in polynomial time.

2. A strong nondeterministic Turing machine(sNDTM) is an NDTM which has three possible outputs - "1", "0", "?". A sNDTM M decides a language L if

   (a) for $x \in L$ every computation of M on x yields "1" or "?" and there is at least one computation of M on x which yields "1";

   (b) for $x \notin L$ every computation of M on x yields "0" or "?" and there is at least one computation of M on x which yields "0".

   Show that L is decided by a sNDTM in polynomial time if and only if $L \in \mathsf{NP} \cap \mathsf{coNP}$.

3. Prove that if $L \in \mathsf{P}$, then so is L^*.

   Hint: Use dynamic programming.

4. Prove that if a language L is in NP, then so is L^*.

5. Show that if $\mathsf{DTIME}(n) = \mathsf{NTIME}(n)$, then $\mathsf{DTIME}(n^2) = \mathsf{NTIME}(n^2)$.

6. Prove that if an unary language is NP-complete then $\mathsf{P} = \mathsf{NP}$.

7. Prove that if every unary NP-language is in P, then $\mathsf{EXP} = \mathsf{NEXP}$.

8. Define $\mathsf{UCYCLE} = \{\ G \mid G \text{ is an undirected graph with a simple cycle }\}$. Show that $\mathsf{UCYCLE} \in \mathsf{L}$.

9. Show that $2SAT$ is in NL.

10. Prove that $\mathsf{P} \neq \mathsf{SPACE}(n)$.

    Hint: This is a "trick" question. It is not known how to prove $\mathsf{P} \not\subseteq \mathsf{SPACE}(n)$ or how to prove $\mathsf{SPACE}(n) \not\subseteq \mathsf{P}$, so the proof has to start by assuming the classes are equal and then reach a contradiction, without explicitly showing a problem in one class that cannot belong to the other.

You can try the following approach: if A and B are decision problems and $\mathsf{A} \leq_p \mathsf{B}$, then $\mathsf{B} \in \mathsf{P}$ implies $\mathsf{A} \in \mathsf{P}$; what would happen if the same were true for $\mathsf{SPACE}(n)$?

Note that reaching a conclusion of $\mathsf{P} = \mathsf{NP}$ or $\mathsf{P} \neq \mathsf{NP}$ is not a contradiction as neither results are known.