Linear momentum

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Lets watch this video

Newton's Cradle - Incredible Science

https://www.youtube.com/watch?v=0LnbyjOyEQ8

Newton's Second Law in Terms of Momentum

Newton's second law of motion: If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force vector.

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \qquad \text{(Newton's second law in terms of momentum)}$$

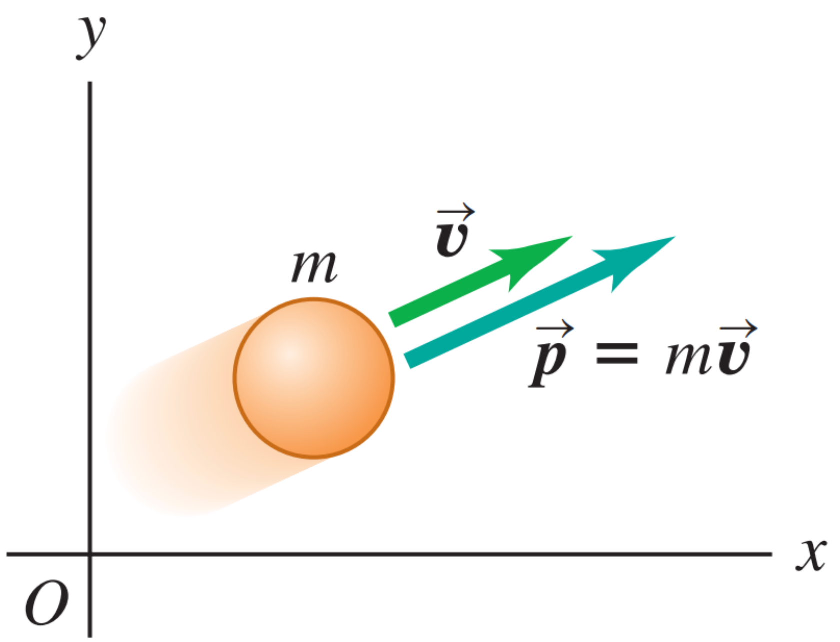
$$\vec{p} = m\vec{v} \qquad \text{(definition of momentum)}$$

Momentum is a vector quantity

$$p_x = mv_x$$

$$p_y = mv_y$$

$$p_z = mv_z$$



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

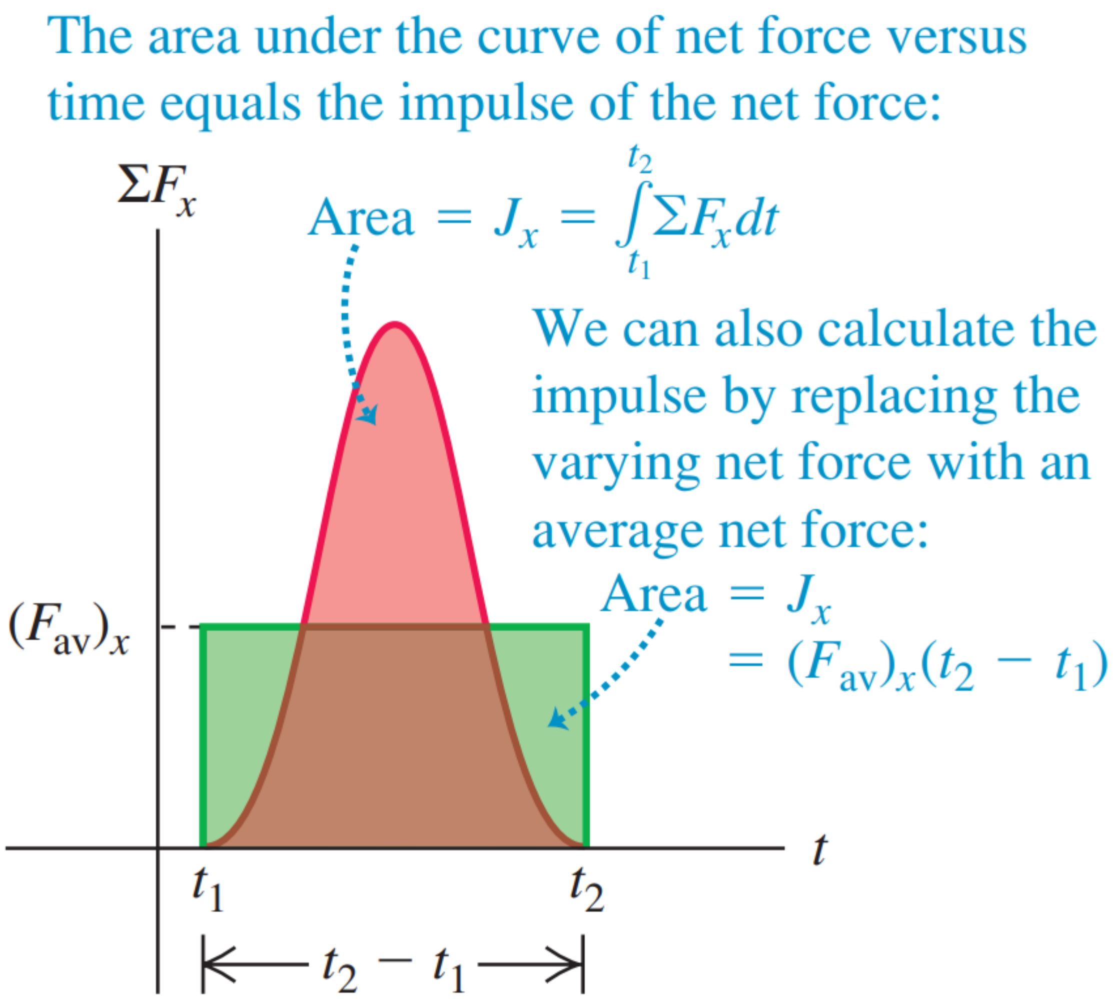
Impulse–momentum theorem

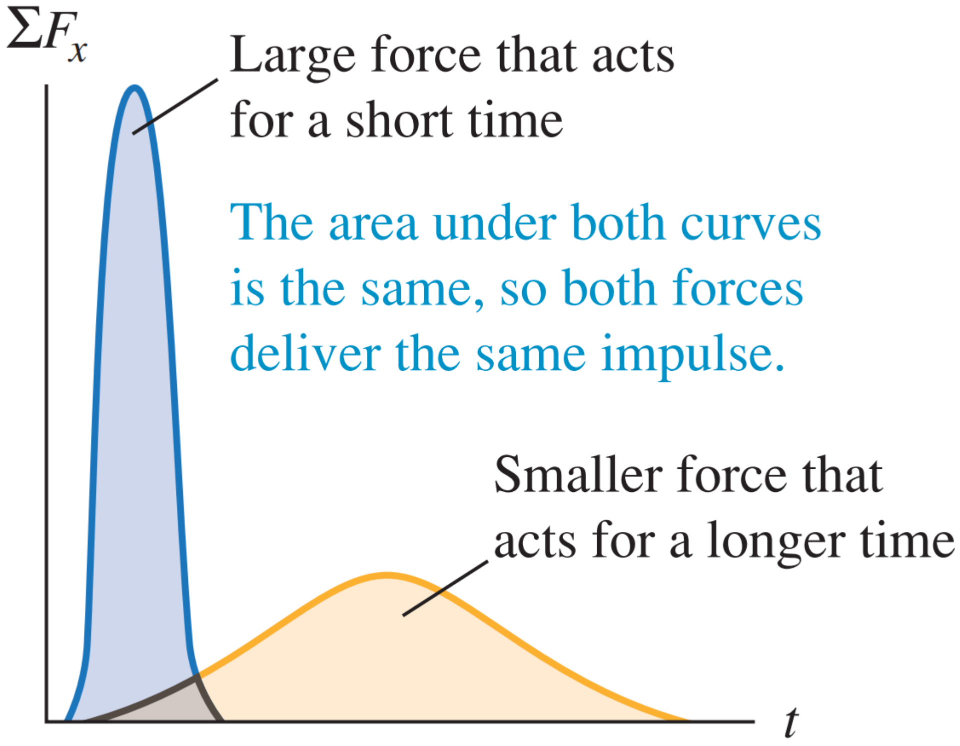
The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad \text{(impulse–momentum theorem)}$$

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} \, dt \qquad \text{(general definition of impulse)}$$

Impulse–momentum theorem





Momentum and Kinetic Energy

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

$$W_{tot} = K_2 - K_1$$

$m_1 = 0.50\ \text{kg}$ $\quad$ $m_2 = 0.10\ \text{kg}$

$v_1 = 4.0\ \text{m/s}$ $\quad$ $v_2 = 20\ \text{m/s}$

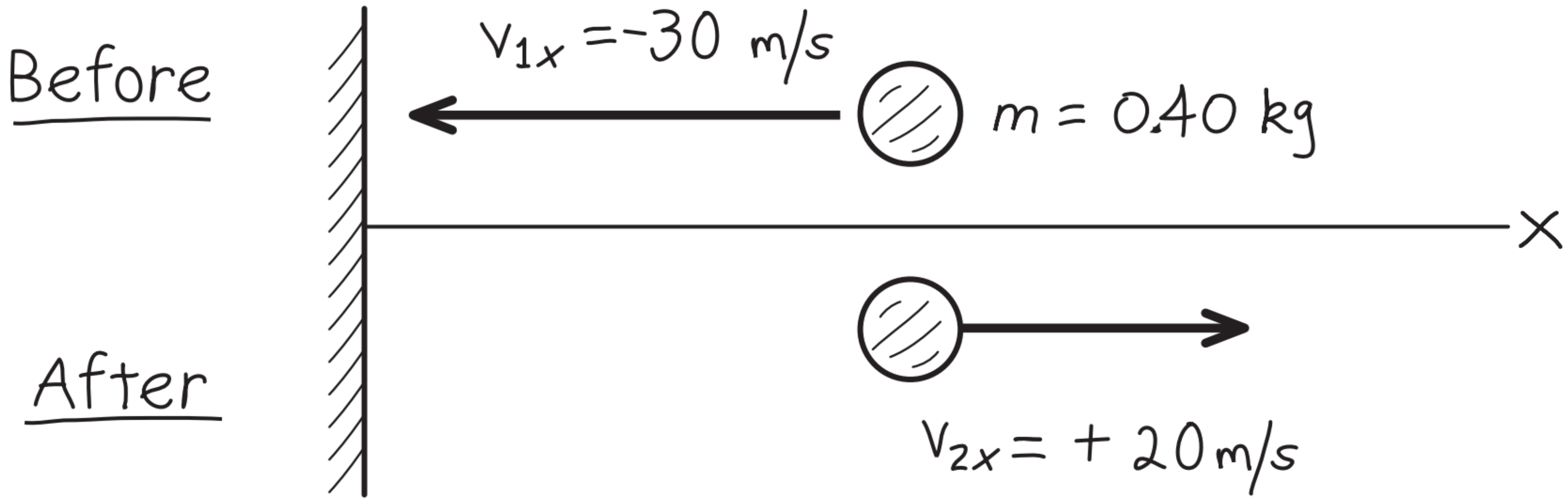
$$p_1 = p_2 = 2.0\ \text{kg-m/s}$$

$K_1 = 4.0\ \text{J}$ $\quad$ $K_2 = 20\ \text{J}$

A ball hits a wall

You throw a ball with a mass of 0.40 kg against a brick wall. It hits the wall moving horizontally to the left at 30 m/s and rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.

A ball hits a wall



A ball hits a wall

$$p_{1x} = mv_{1x} = (0.40\ \text{kg})(-30\ \text{m/s}) = -12\ \text{kg} \cdot \text{m/s}$$
$$p_{2x} = mv_{2x} = (0.40\ \text{kg})(+20\ \text{m/s}) = +8.0\ \text{kg} \cdot \text{m/s}$$

$$\begin{aligned} J_x &= p_{2x} - p_{1x} \\ &= 8.0\ \text{kg} \cdot \text{m/s} - (-12\ \text{kg} \cdot \text{m/s}) = 20\ \text{kg} \cdot \text{m/s} = 20\ \text{N} \cdot \text{s} \end{aligned}$$

$$(F_{\text{av}})_x = \frac{J_x}{\Delta t} = \frac{20\ \text{N} \cdot \text{s}}{0.010\ \text{s}} = 2000\ \text{N}$$

Principle of conservation of momentum

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

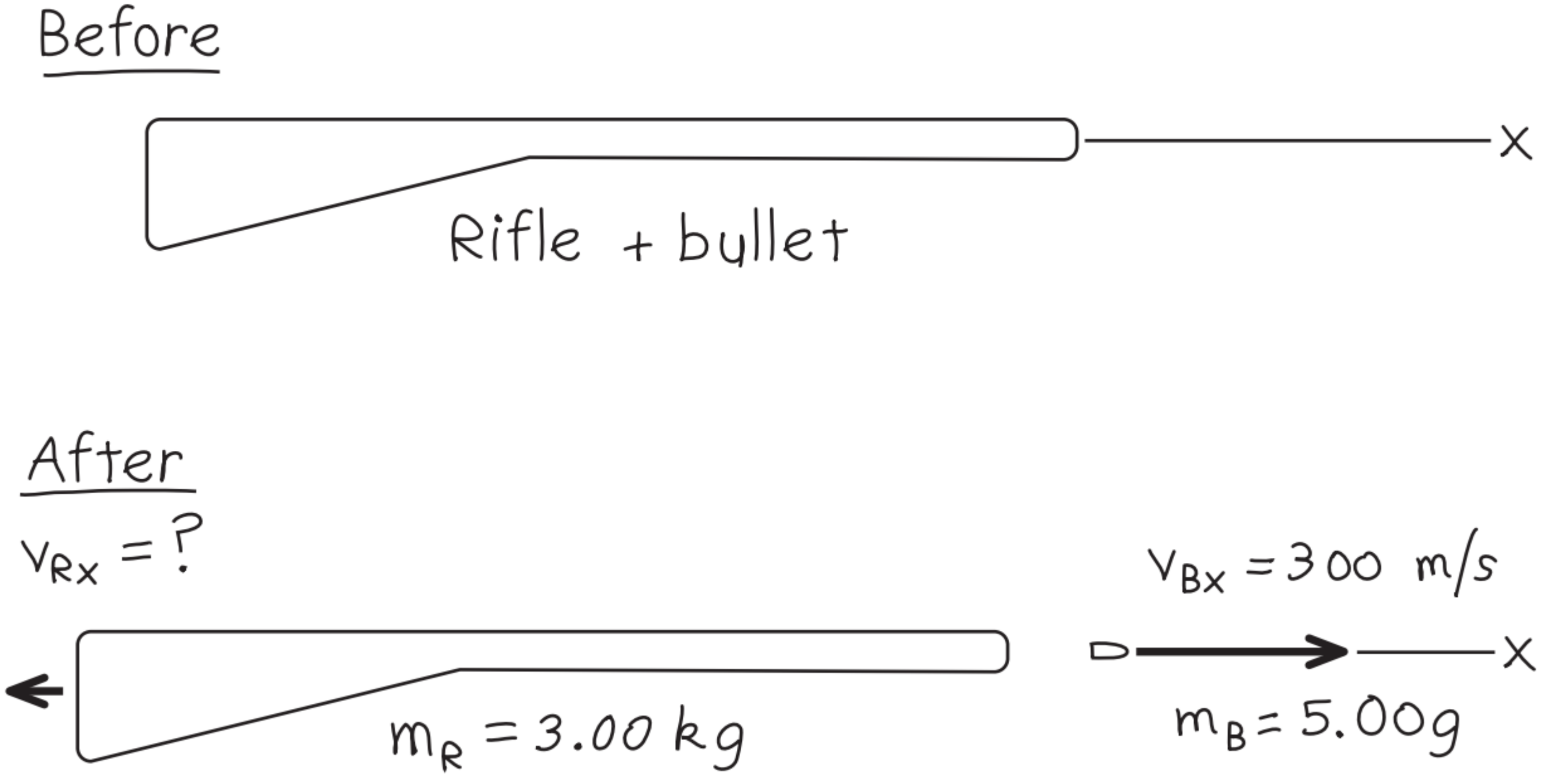
$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots \quad \text{(total momentum of a system of particles)}$$

$$P_x = p_{Ax} + p_{Bx} + \cdots$$
$$P_y = p_{Ay} + p_{By} + \cdots$$
$$P_z = p_{Az} + p_{Bz} + \cdots$$

Recoil of a rifle

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet and rifle?

Recoil of a rifle



Recoil of a rifle

$$P_x = 0 = m_B v_{Bx} + m_R v_{Rx}$$

$$v_{Rx} = -\frac{m_B}{m_R} v_{Bx} = -\left(\frac{0.00500 \text{ kg}}{3.00 \text{ kg}}\right)(300 \text{ m/s}) = -0.500 \text{ m/s}$$

The final momenta and kinetic energies are

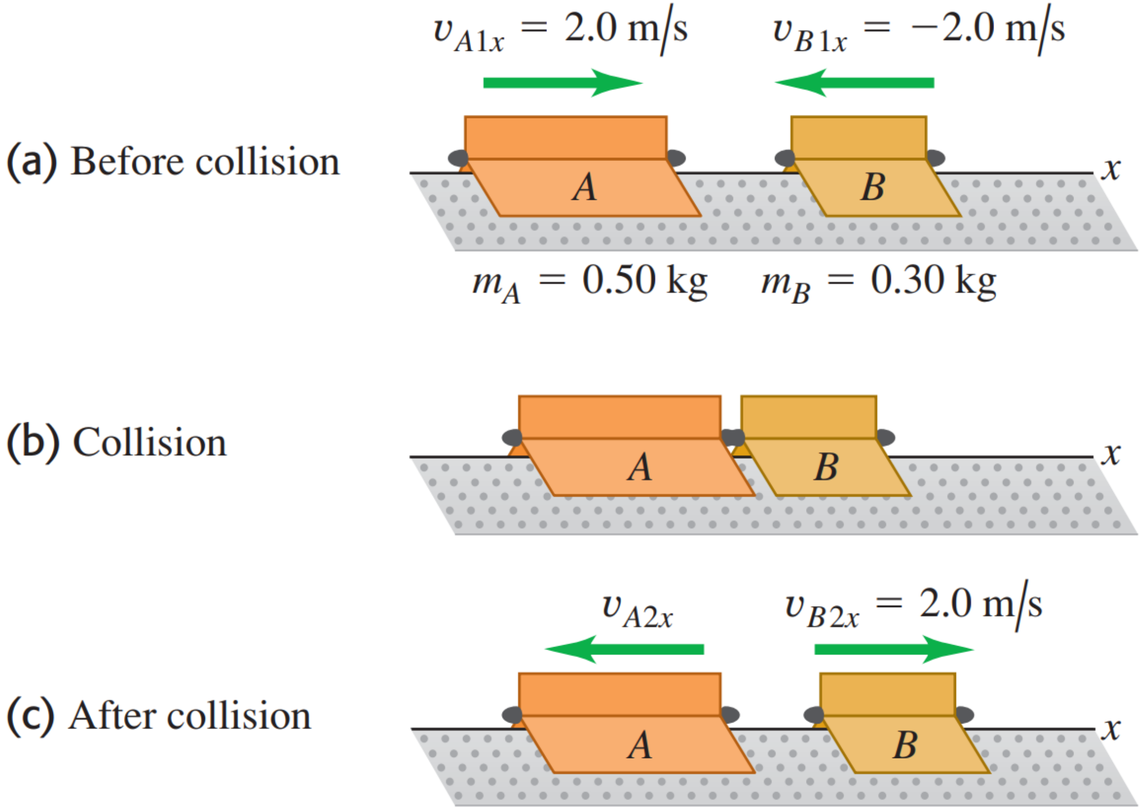
$$p_{Bx} = m_B v_{Bx} = (0.00500 \text{ kg})(300 \text{ m/s}) = 1.50 \text{ kg} \cdot \text{m/s}$$

$$K_B = \tfrac{1}{2} m_B v_{Bx}{}^2 = \tfrac{1}{2}(0.00500 \text{ kg})(300 \text{ m/s})^2 = 225 \text{ J}$$

$$p_{Rx} = m_R v_{Rx} = (3.00 \text{ kg})(-0.500 \text{ m/s}) = -1.50 \text{ kg} \cdot \text{m/s}$$

$$K_R = \tfrac{1}{2} m_R v_{Rx}{}^2 = \tfrac{1}{2}(3.00 \text{ kg})(-0.500 \text{ m/s})^2 = 0.375 \text{ J}$$

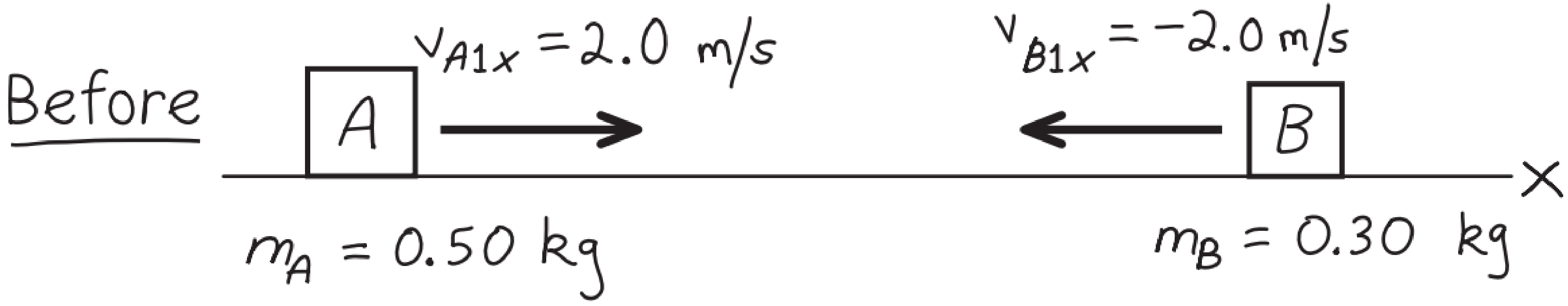
Collision along a straight line



$$
\begin{aligned}
P_x &= m_A v_{A1x} + m_B v_{B1x} \\
&= (0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s}) \\
&= 0.40\ \text{kg} \cdot \text{m/s}
\end{aligned}
$$

$$
\begin{aligned}
v_{A2x} &= \frac{P_x - m_B v_{B2x}}{m_A} = \frac{0.40\ \text{kg} \cdot \text{m/s} - (0.30\ \text{kg})(2.0\ \text{m/s})}{0.50\ \text{kg}} \\
&= -0.40\ \text{m/s}
\end{aligned}
$$

A completely inelastic collision



A completely inelastic collision

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$$

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$$

$$= \frac{(0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s})}{0.50\ \text{kg} + 0.30\ \text{kg}}$$

$$= 0.50\ \text{m/s}$$

$$K_2 = \tfrac{1}{2}(m_A + m_B){v_{2x}}^2 = \tfrac{1}{2}(0.50\ \text{kg} + 0.30\ \text{kg})(0.50\ \text{m/s})^2$$

$$= 0.10\ \text{J}$$

Center of mass

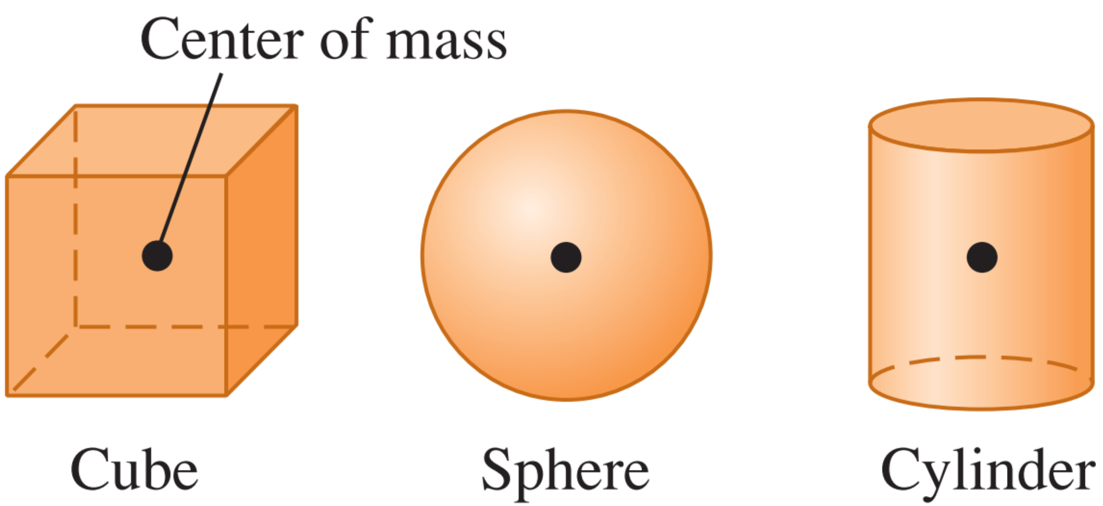
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

(center of mass)

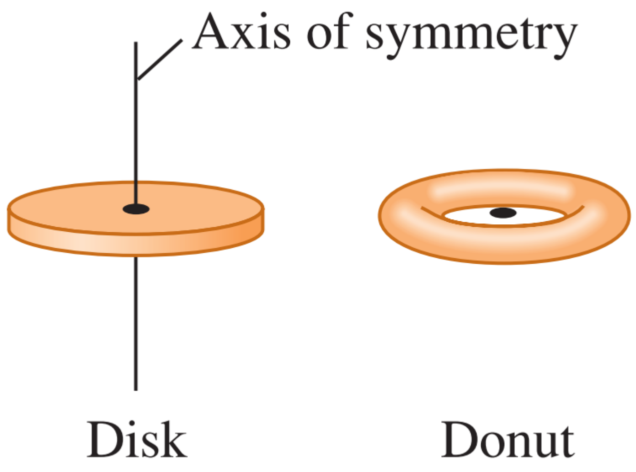
$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_iy_i}{\sum_i m_i}$$

$$\vec{\boldsymbol{r}}_{cm} = \frac{m_1\vec{\boldsymbol{r}}_1 + m_2\vec{\boldsymbol{r}}_2 + m_3\vec{\boldsymbol{r}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{\boldsymbol{r}}_i}{\sum_i m_i} \quad \text{(center of mass)}$$

Center of mass



If a homogeneous object has a geometric center, that is where the center of mass is located.



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

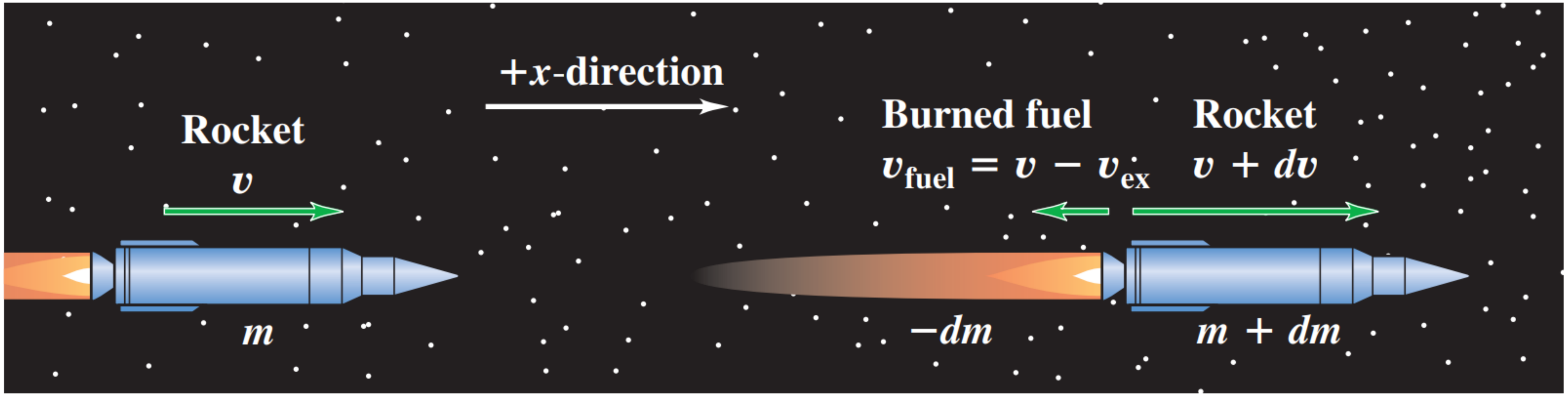
Center of mass

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P}$$

$$\sum \vec{F}_{ext} = M\vec{a}_{cm} \qquad \text{(body or collection of particles)}$$

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

Acceleration of a rocket



$$\boxed{mv = (m + dm)(v + dv) + (-dm)(v - v_{\text{ex}})}$$

$$\boxed{m\,dv = -dm\,v_{\text{ex}} - dm\,dv}$$

$$\boxed{m\frac{dv}{dt} = -v_{\text{ex}}\frac{dm}{dt}}$$

Acceleration of a rocket

$$\boxed{m\frac{dv}{dt} = -v_{\mathrm{ex}}\frac{dm}{dt}}$$

Net force or thrust on the rocket: $F = -v_{\mathrm{ex}}\dfrac{dm}{dt}$

Acceleration of the rocket: $a = \dfrac{dv}{dt} = -\dfrac{v_{\mathrm{ex}}}{m}\dfrac{dm}{dt}$

Acceleration of a rocket

The engine of a rocket in outer space, far from any planet, is turned on. The rocket ejects burned fuel at a constant rate; in the first second of firing, it ejects $\frac{1}{120}$ of its initial mass m_0 at a relative speed of $2400\ \mathrm{m/s}$. What is the rocket's initial acceleration?

The initial rate of change of mass is

$$\frac{dm}{dt} = -\frac{m_0/120}{1\ \mathrm{s}} = -\frac{m_0}{120\ \mathrm{s}}$$

Acceleration of a rocket

$$a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} = -\frac{2400\ \text{m/s}}{m_0}\left(-\frac{m_0}{120\ \text{s}}\right) = 20\ \text{m/s}^2$$

EVALUATE: The answer doesn't depend on m_0. If v_{ex} is the same, the initial acceleration is the same for a 120,000-kg spacecraft that ejects 1000 kg/s as for a 60-kg astronaut equipped with a small rocket that ejects 0.5 kg/s.

Readings

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Chapter 8: Momentum, Impulse, and Collisions

Section 8.2 Conservation of Momentum
Section 8.3 Momentum Conservation and Collisions