# Periodic Motion

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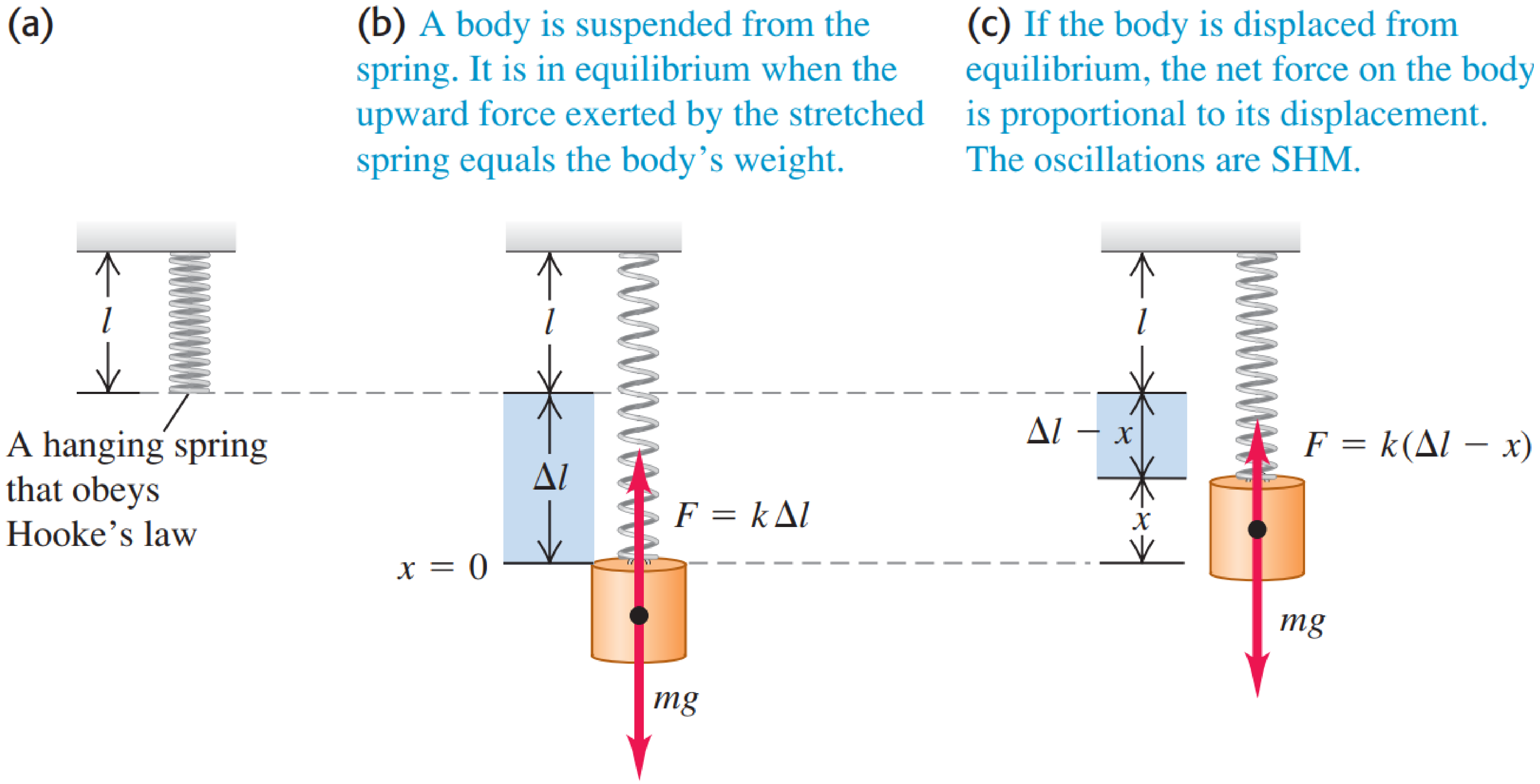
# https://youtu.be/xN5CQ7YTUVE

# References

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

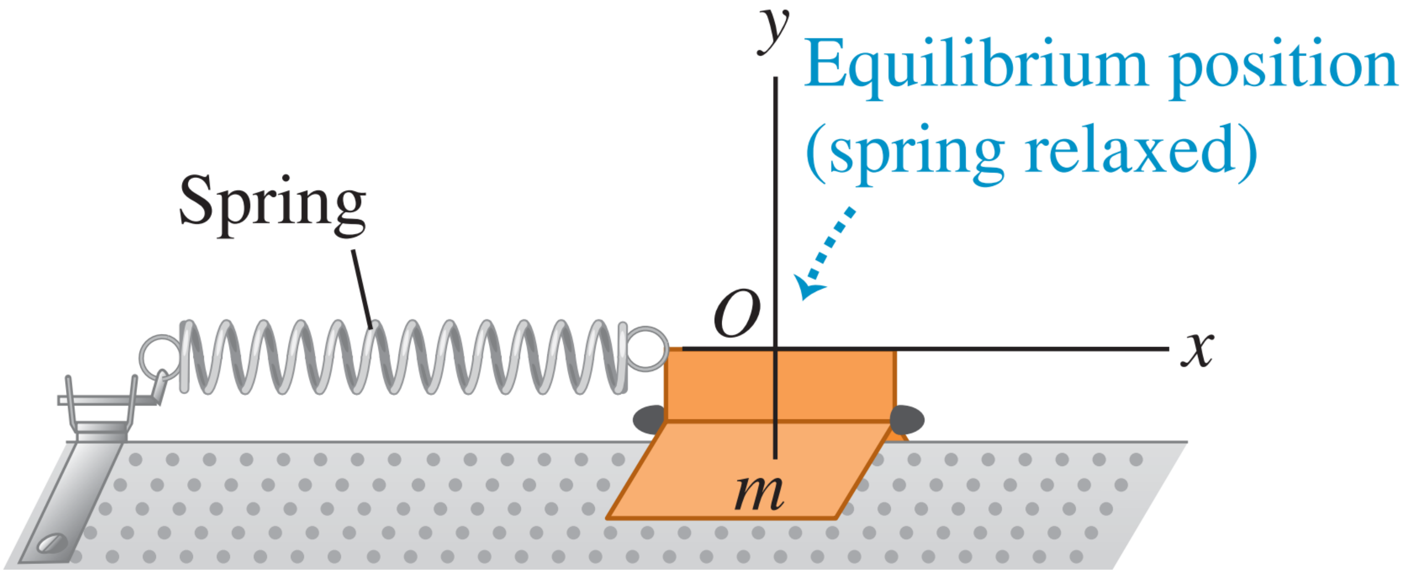
# Vertical Simple Harmonic Motion



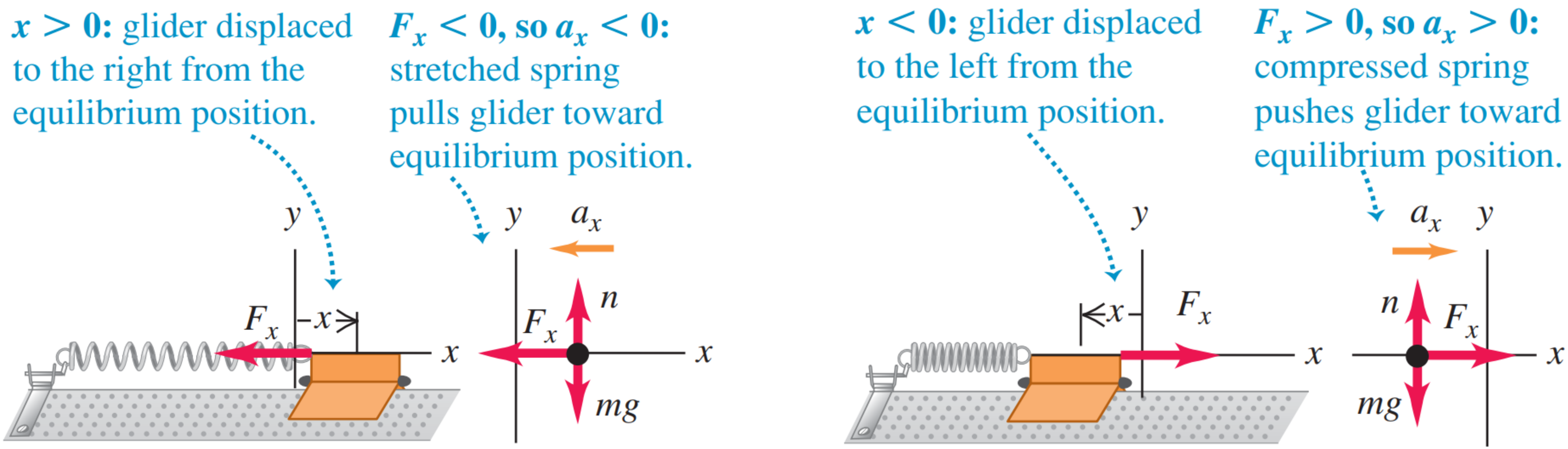
# A system that can have periodic motion

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



# Describing oscillation



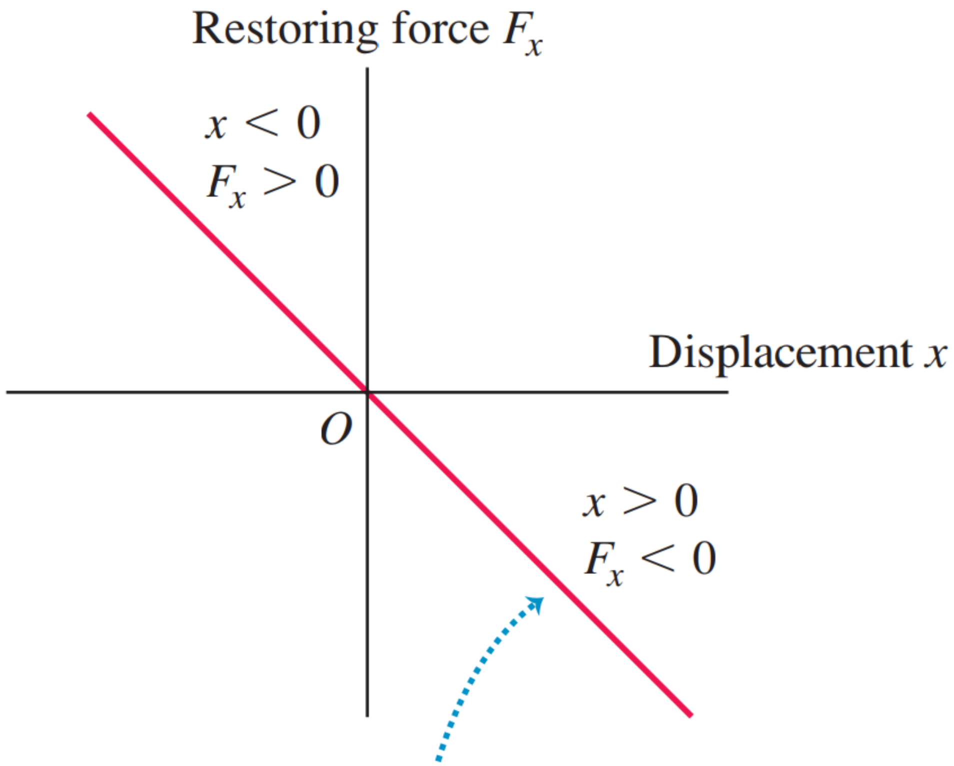
# Simple Harmonic Motion (SHM)

$$F_x = -kx$$

(restoring force exerted by an ideal spring)

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \text{(simple harmonic motion)}$$

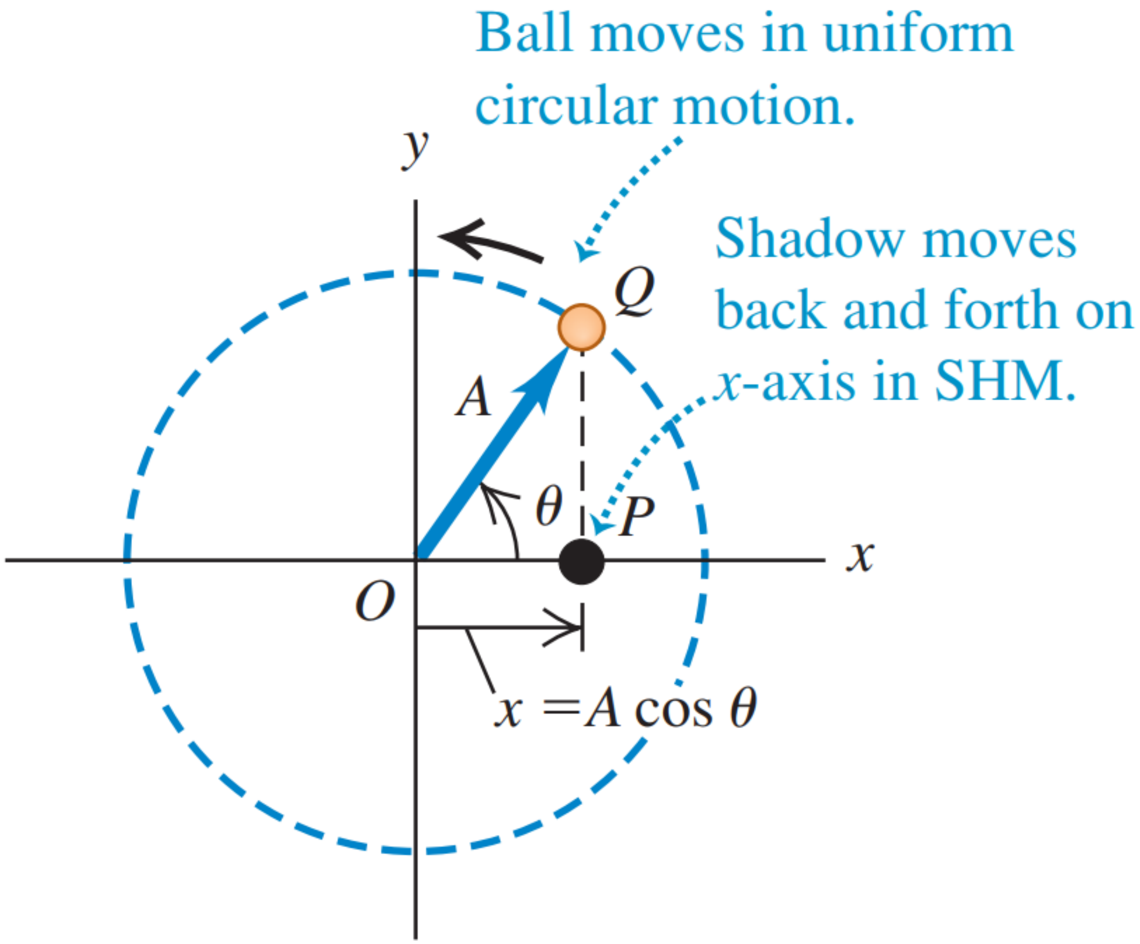
$$\omega = \sqrt{\frac{k}{m}}$$




The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of $F_x$ versus $x$ is a straight line.

# Simple Harmonic Motion (SHM)

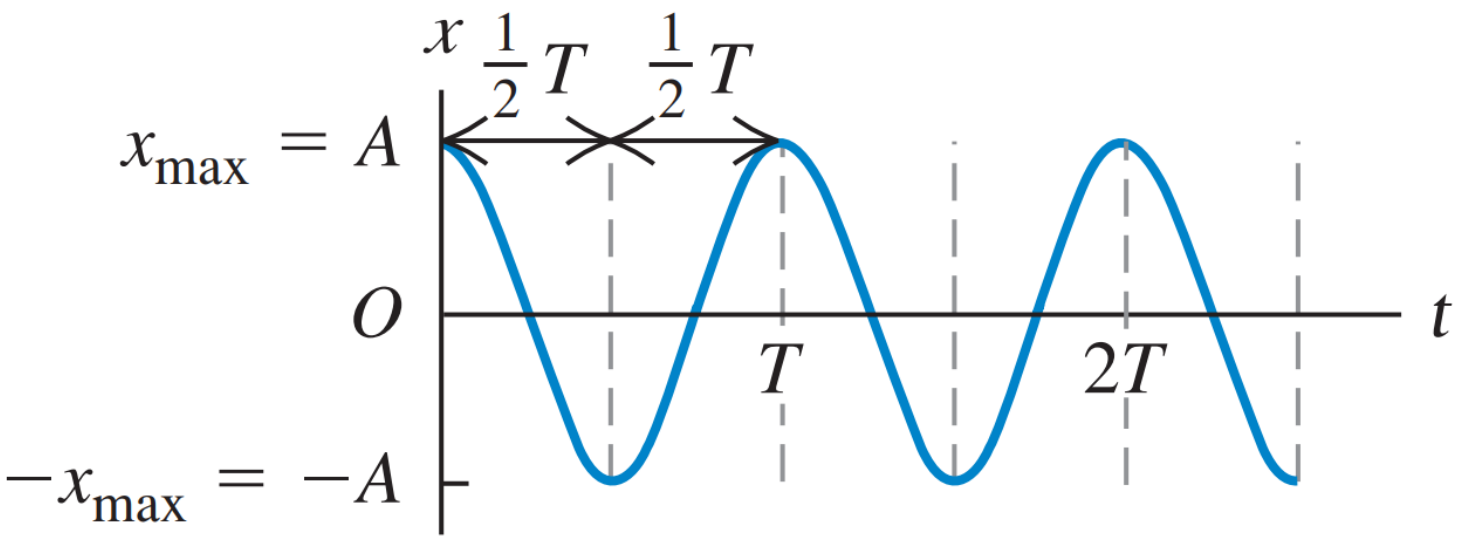
$$\boxed{x = A\cos\theta}$$



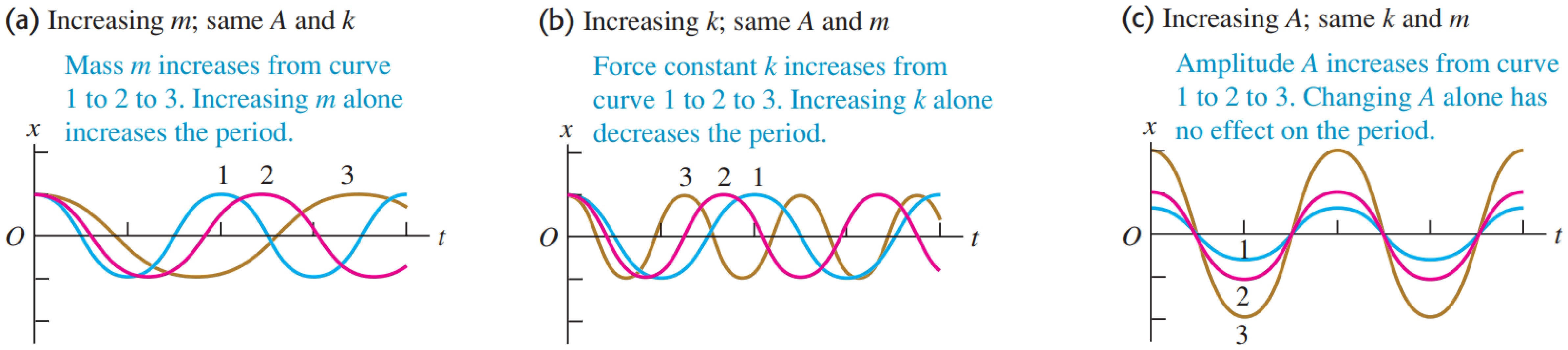
# Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi) \qquad \text{(displacement in SHM)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(simple harmonic motion)}$$



# Simple Harmonic Motion (SHM)




$$x = A\cos(\omega t + \phi)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

# Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi) \qquad \text{(displacement in SHM)}$$

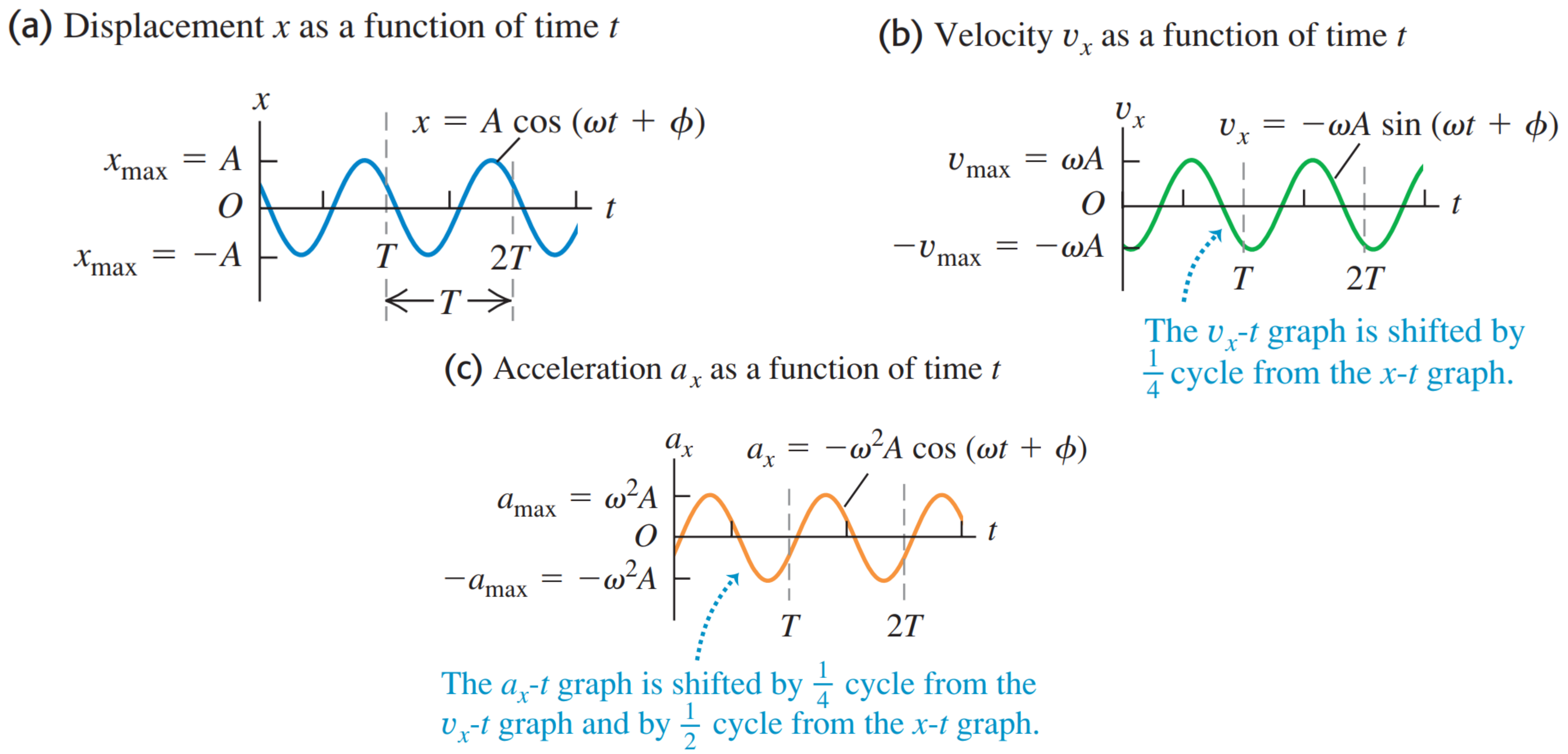
$$v_x = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) \qquad \text{(velocity in SHM)}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) \qquad \text{(acceleration in SHM)}$$

$$\boxed{v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}}$$

$$\boxed{a_x = -\omega^2 x = -\frac{k}{m}x}$$

# Simple Harmonic Motion (SHM)

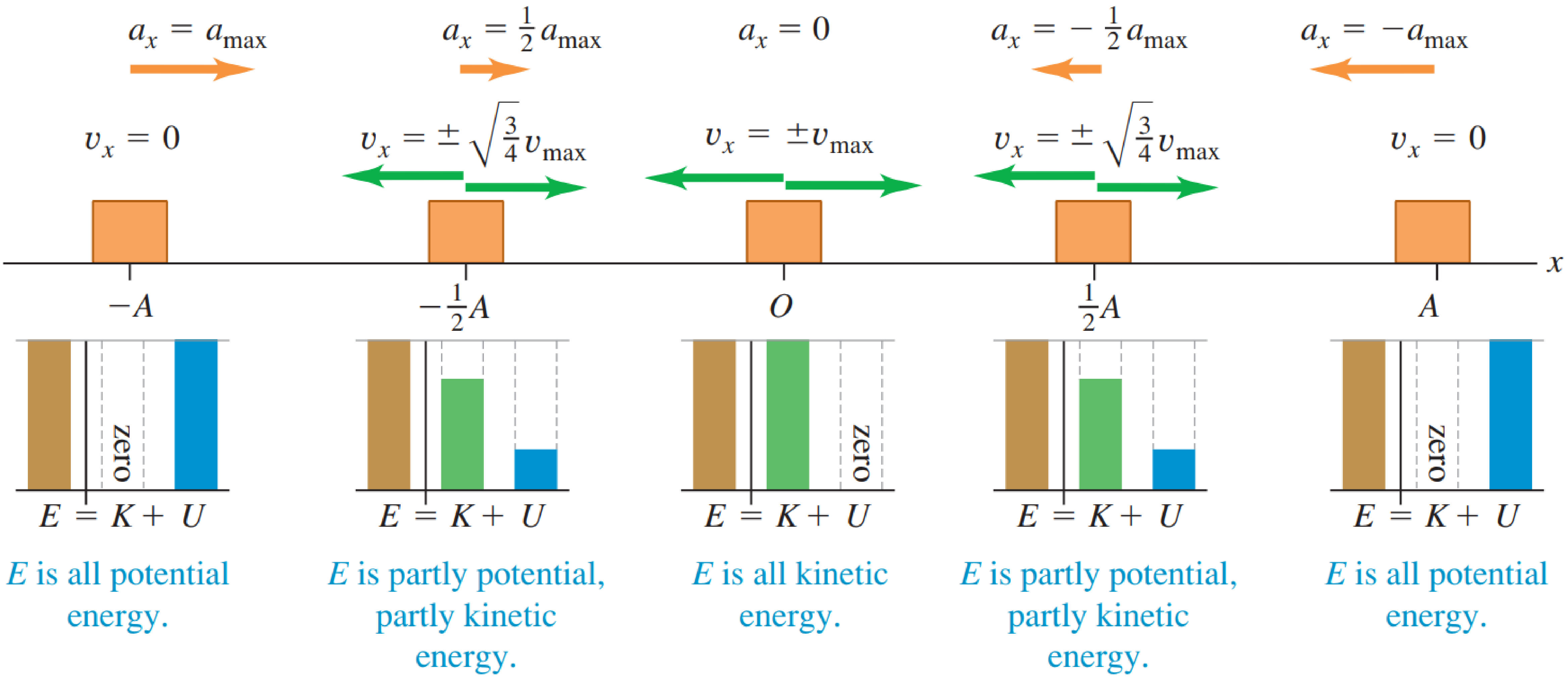


# Energy in Simple Harmonic Motion

$$E = \tfrac{1}{2}mv_x{}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \qquad \text{(total mechanical energy in SHM)}$$

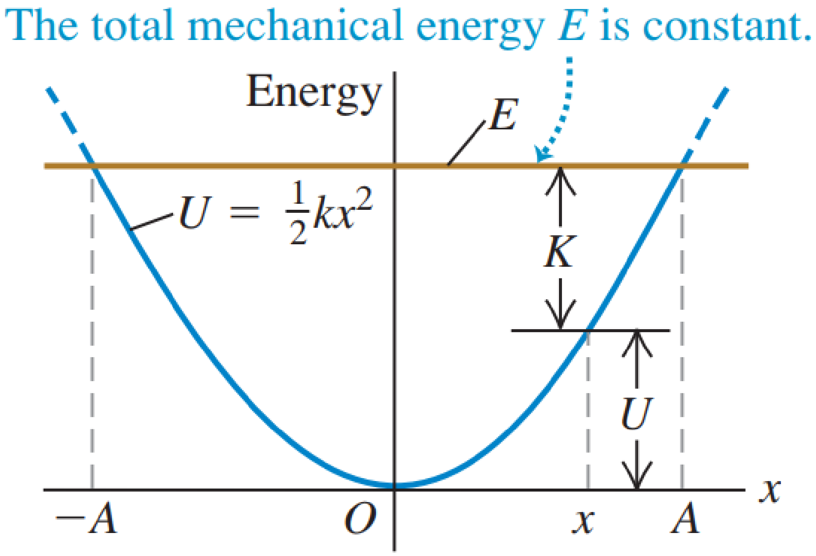
$$\begin{aligned} E &= \tfrac{1}{2}mv_x{}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}m[-\omega A\sin(\omega t + \phi)]^2 + \tfrac{1}{2}k[A\cos(\omega t + \phi)]^2 \\ &= \tfrac{1}{2}kA^2\sin^2(\omega t + \phi) + \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2 \end{aligned}$$

# Energy in Simple Harmonic Motion

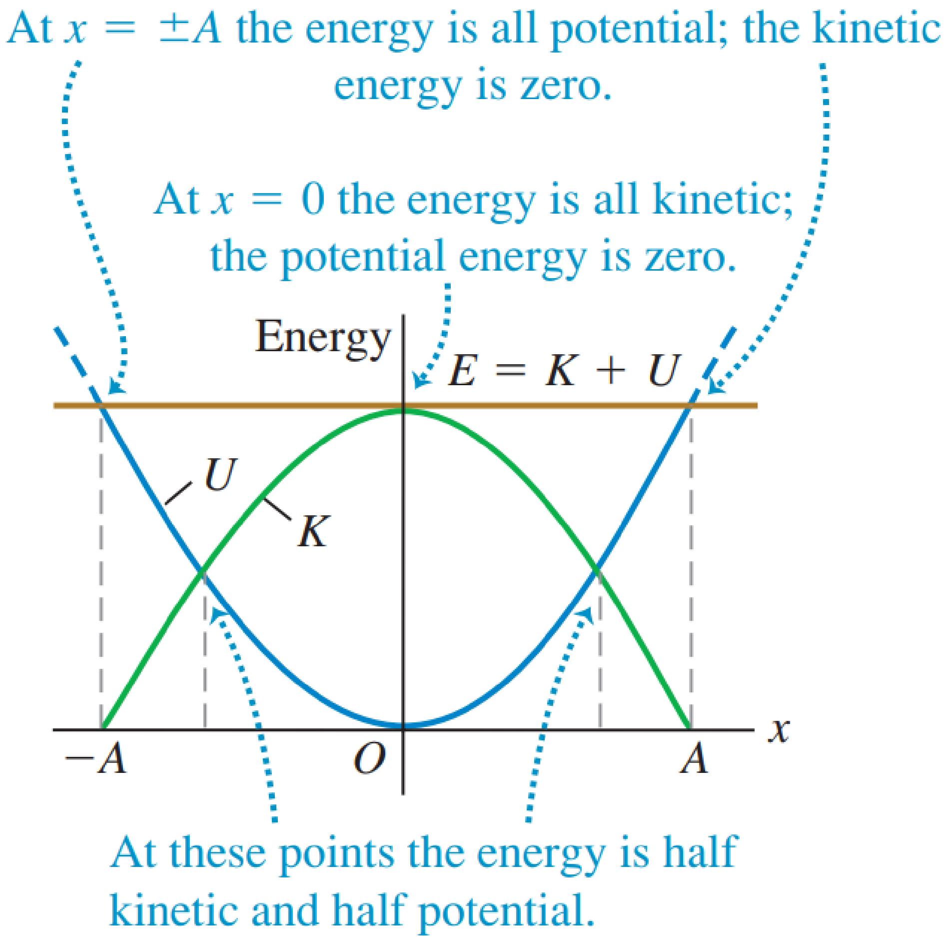


# Energy in Simple Harmonic Motion

**(a)** The potential energy $U$ and total mechanical energy $E$ for a body in SHM as a function of displacement $x$




**(b)** The same graph as in **(a)**, showing kinetic energy $K$ as well

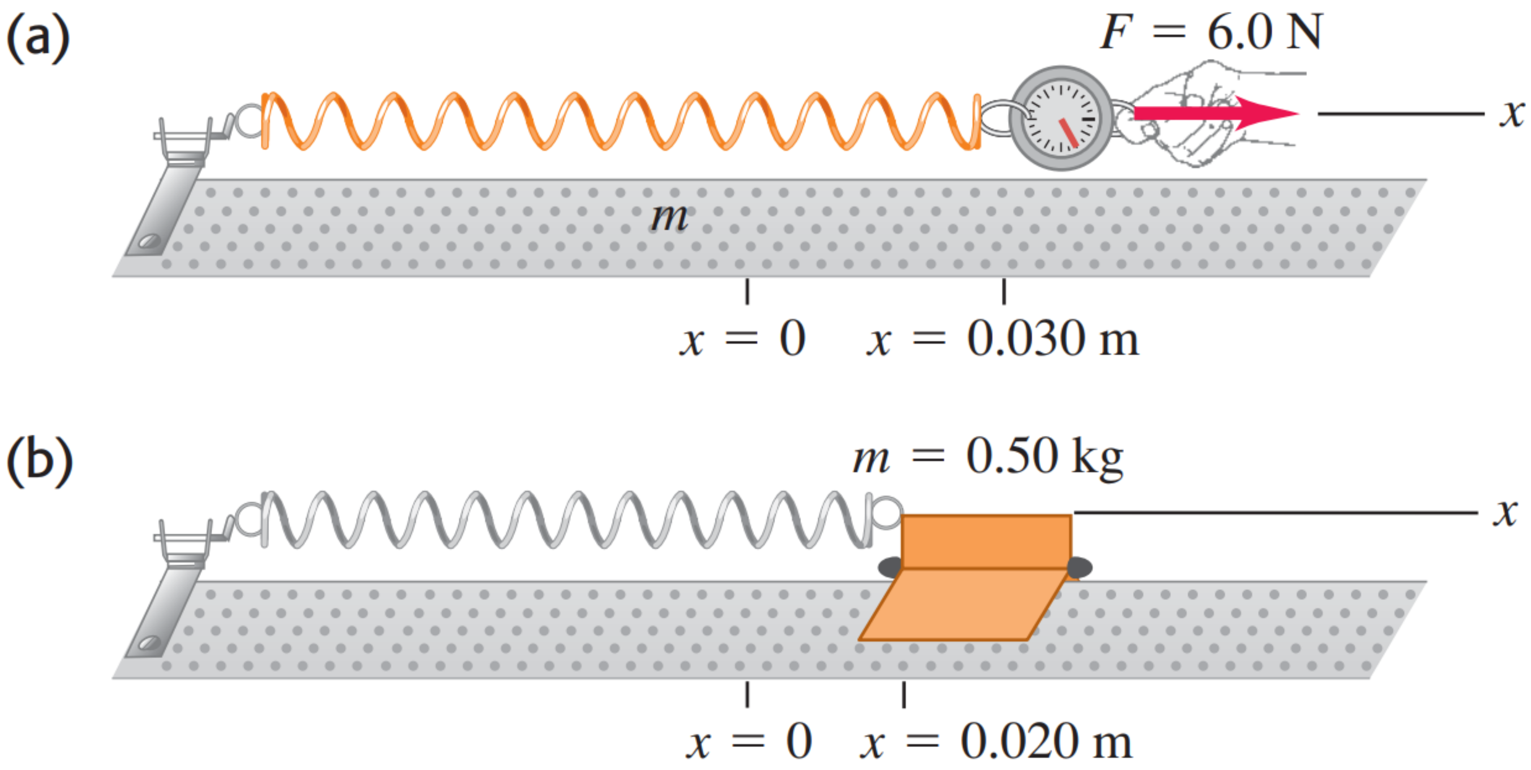


# Angular frequency, frequency, and period in SHM

A spring is mounted horizontally, with its left end fixed. A spring balance attached to the free end and pulled toward the right (Fig. 14.8a) indicates that the stretching force is proportional to the displacement, and a force of 6.0 N causes a displacement of 0.030 m. We replace the spring balance with a 0.50-kg glider, pull it 0.020 m to the right along a frictionless air track, and release it from rest (Fig. 14.8b). (a) Find the force constant $k$ of the spring. (b) Find the angular frequency $\omega$, frequency $f$, and period $T$ of the resulting oscillation.

# Angular frequency, frequency, and period in SHM

**14.8** (a) The force exerted *on* the spring (shown by the vector $F$) has $x$-component $F_x = +6.0$ N. The force exerted *by* the spring has $x$-component $F_x = -6.0$ N. (b) A glider is attached to the same spring and allowed to oscillate.



# Angular frequency, frequency, and period in SHM

**EXECUTE:** (a) When $x = 0.030$ m, the force the spring exerts on the spring balance is $F_x = -6.0$ N. From Eq. (14.3),

$$k = -\frac{F_x}{x} = -\frac{-6.0\ \text{N}}{0.030\ \text{m}} = 200\ \text{N/m} = 200\ \text{kg/s}^2$$

(b) From Eq. (14.10), with $m = 0.50$ kg,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{20\ \text{rad/s}}{2\pi\ \text{rad/cycle}} = 3.2\ \text{cycle/s} = 3.2\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{3.2\ \text{cycle/s}} = 0.31\ \text{s}$$

# Velocity, acceleration, and energy in SHM

(a) Find the maximum and minimum velocities attained by the oscillating glider of Example 14.2. (b) Find the maximum and minimum accelerations. (c) Find the velocity $v_x$ and acceleration $a_x$ when the glider is halfway from its initial position to the equilibrium position $x = 0$. (d) Find the total energy, potential energy, and kinetic energy at this position.

# Velocity, acceleration, and energy in SHM

**EXECUTE:** (a) From Eq. (14.22), the velocity $v_x$ at any displacement $x$ is

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

The glider's maximum *speed* occurs when it is moving through $x = 0$:

$$v_{\max} = \sqrt{\frac{k}{m}}\,A = \sqrt{\frac{200\ \mathrm{N/m}}{0.50\ \mathrm{kg}}}\,(0.020\ \mathrm{m}) = 0.40\ \mathrm{m/s}$$

Its maximum and minimum (most negative) *velocities* are $+0.40\ \mathrm{m/s}$ and $-0.40\ \mathrm{m/s}$, which occur when it is moving through $x = 0$ to the right and left, respectively.

# Velocity, acceleration, and energy in SHM

(b) From Eq. (14.4), $a_x = -(k/m)x$. The glider's maximum (most positive) acceleration occurs at the most negative value of $x$, $x = -A$:

$$a_{max} = -\frac{k}{m}(-A) = -\frac{200\ \text{N/m}}{0.50\ \text{kg}}(-0.020\ \text{m}) = 8.0\ \text{m/s}^2$$

The minimum (most negative) acceleration is $a_{min} = -8.0\ \text{m/s}^2$, which occurs at $x = +A = +0.020\ \text{m}$.

# Velocity, acceleration, and energy in SHM

(c) The point halfway from $x = x_0 = A$ to $x = 0$ is $x = A/2 = 0.010\ \mathrm{m}$. From Eq. (14.22), at this point

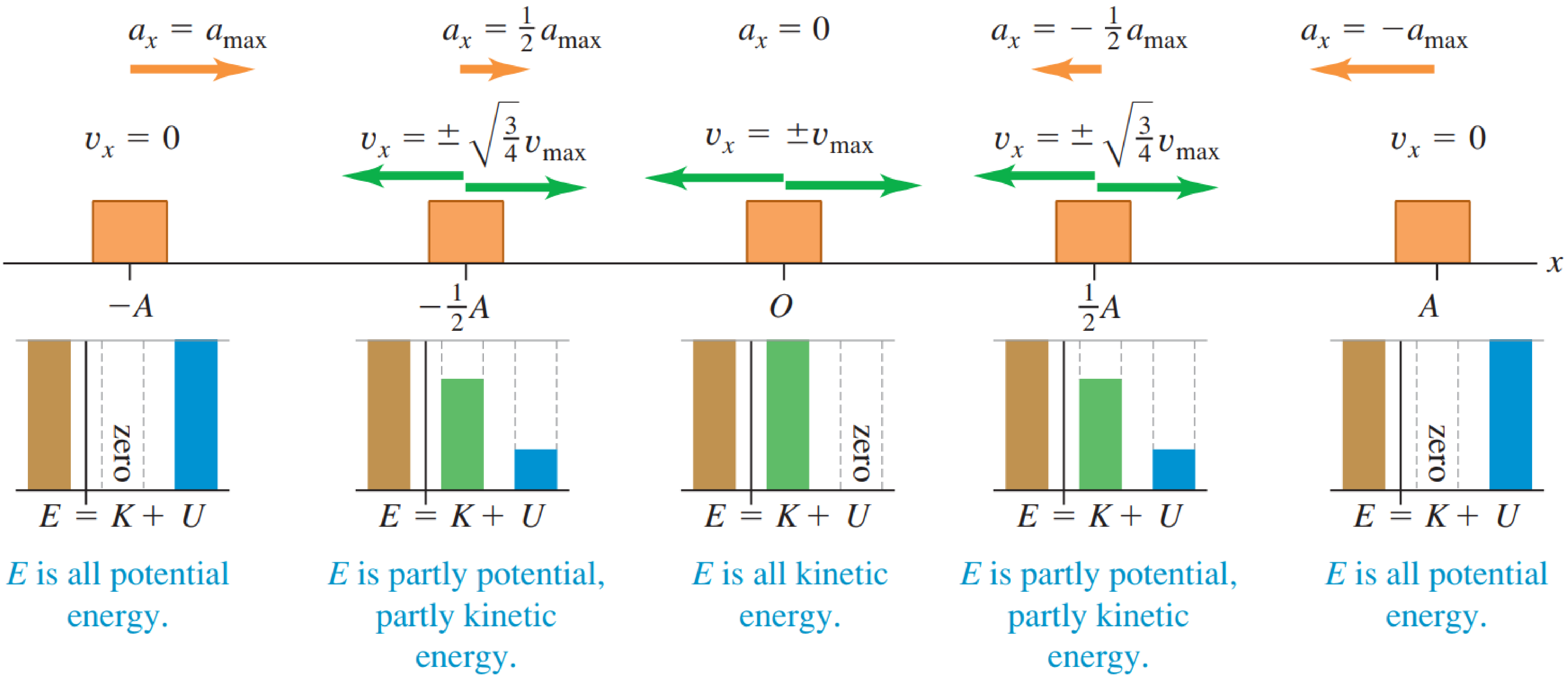
$$v_x = -\sqrt{\frac{200\ \mathrm{N/m}}{0.50\ \mathrm{kg}}}\sqrt{(0.020\ \mathrm{m})^2 - (0.010\ \mathrm{m})^2} = -0.35\ \mathrm{m/s}$$

We choose the negative square root because the glider is moving from $x = A$ toward $x = 0$. From Eq. (14.4),

$$a_x = -\frac{200\ \mathrm{N/m}}{0.50\ \mathrm{kg}}(0.010\ \mathrm{m}) = -4.0\ \mathrm{m/s^2}$$

Figure 14.14 shows the conditions at $x = 0$, $\pm A/2$, and $\pm A$.

# Velocity, acceleration, and energy in SHM



# Velocity, acceleration, and energy in SHM

(d) The energies are

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200\ \text{N/m})(0.020\ \text{m})^2 = 0.040\ \text{J}$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\ \text{N/m})(0.010\ \text{m})^2 = 0.010\ \text{J}$$

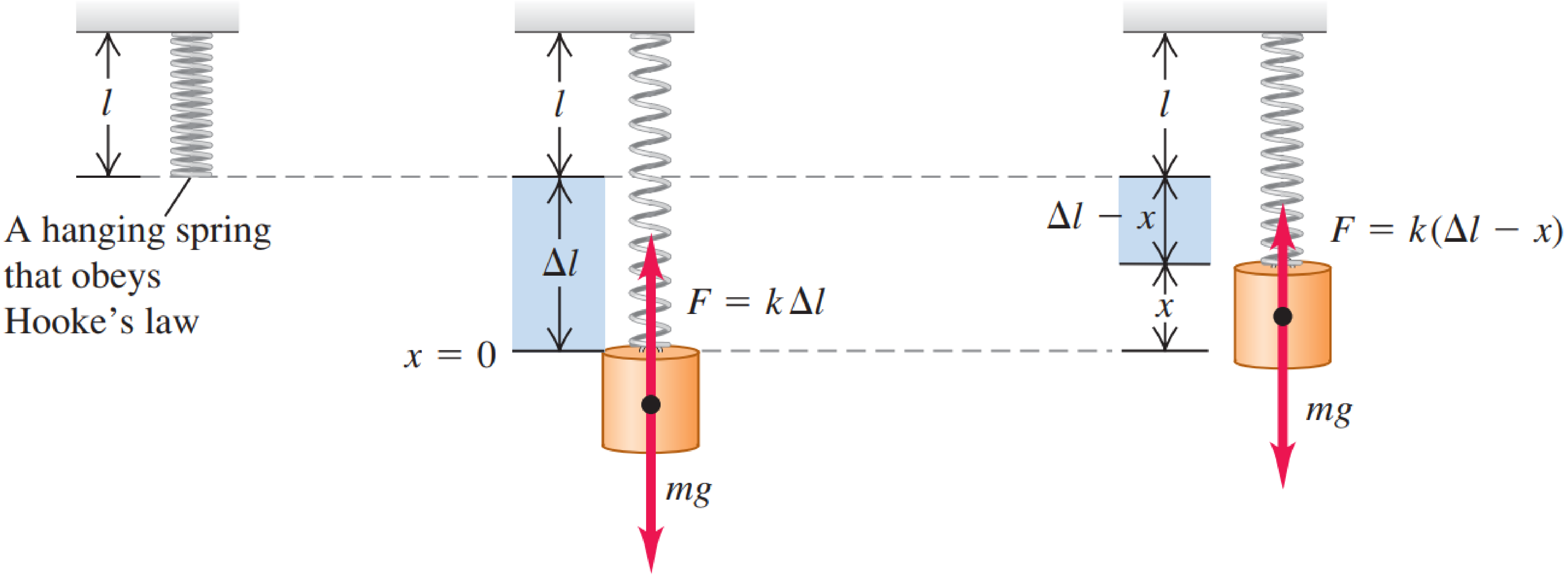
$$K = \tfrac{1}{2}mv_x{}^2 = \tfrac{1}{2}(0.50\ \text{kg})(-0.35\ \text{m/s})^2 = 0.030\ \text{J}$$

# Vertical Simple Harmonic Motion

(a)

(b) A body is suspended from the spring. It is in equilibrium when the upward force exerted by the stretched spring equals the body's weight.

(c) If the body is displaced from equilibrium, the net force on the body is proportional to its displacement. The oscillations are SHM.



# Readings

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Chapter 14: Periodic Motion