Periodic Motion

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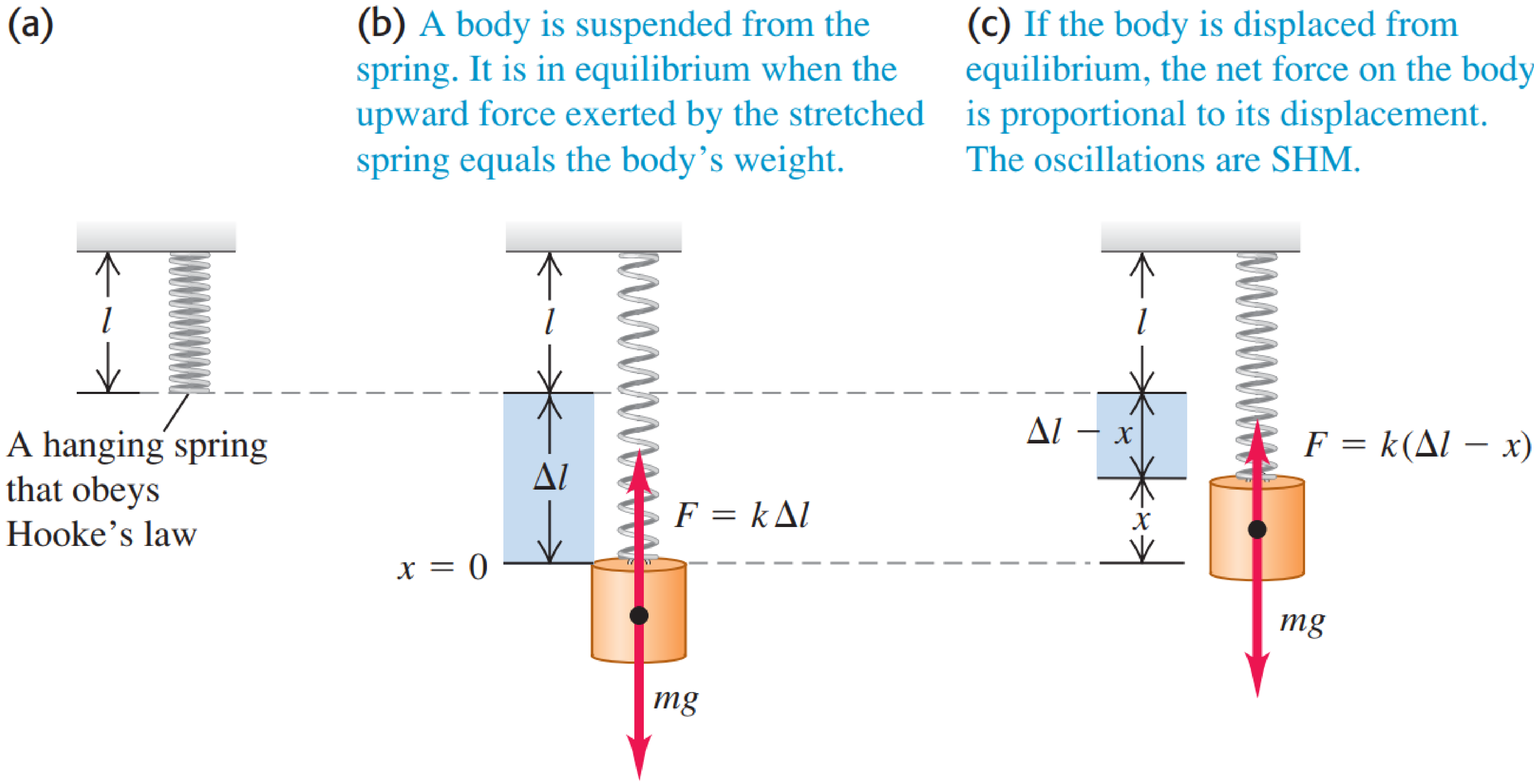
https://youtu.be/xN5CQ7YTUVE

References

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

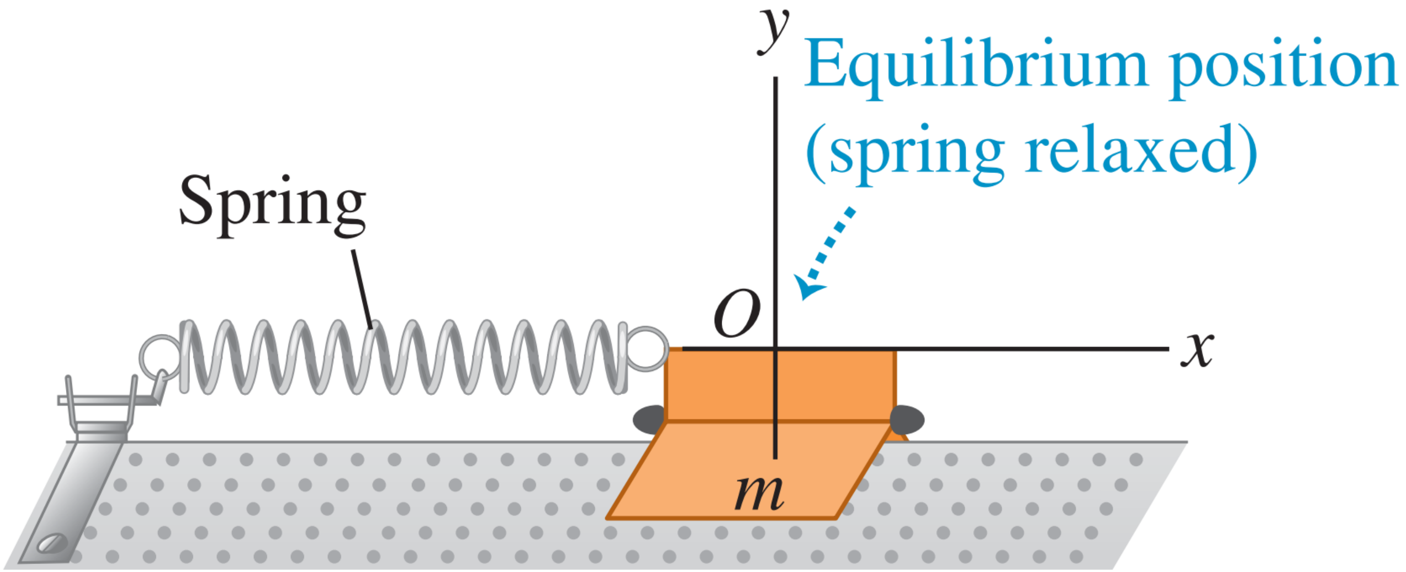
Vertical Simple Harmonic Motion



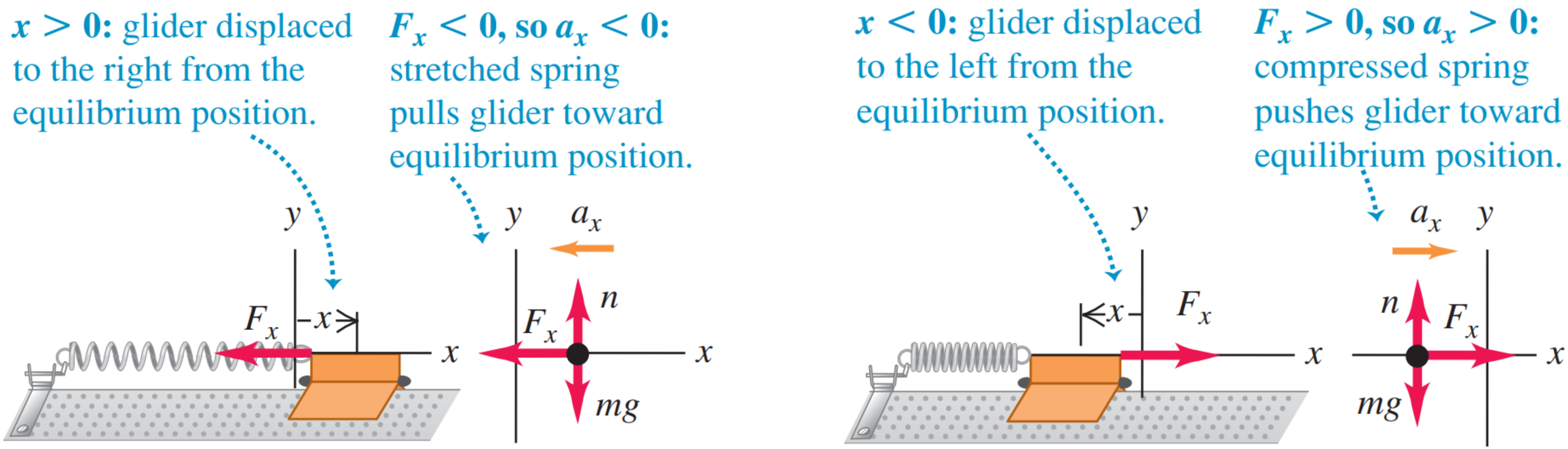
A system that can have periodic motion

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Describing oscillation



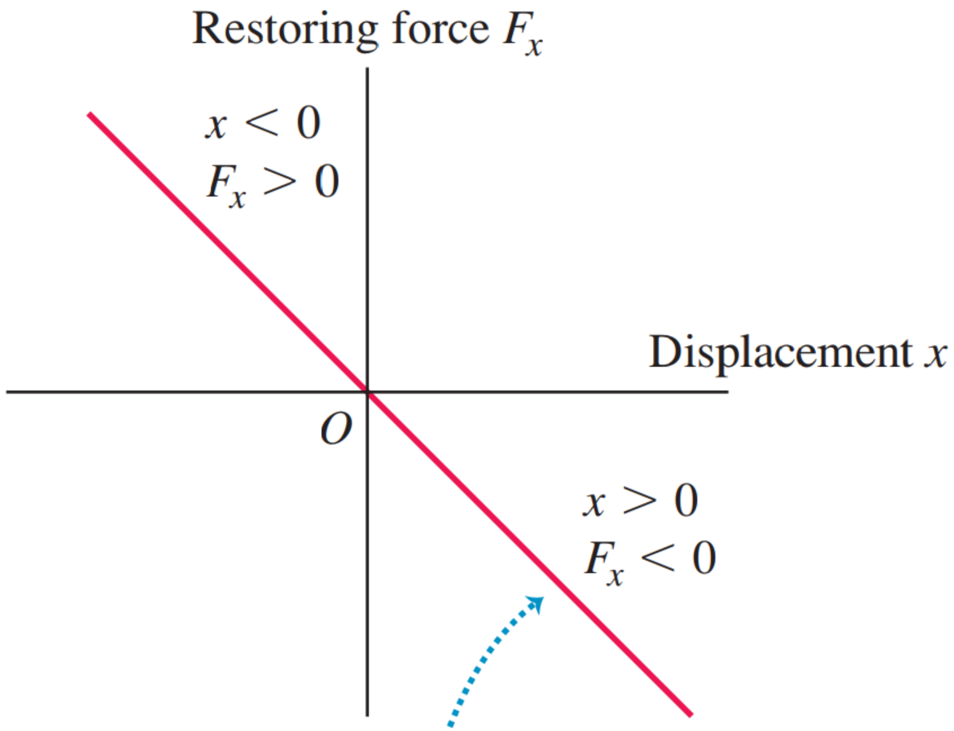
Simple Harmonic Motion (SHM)

$$F_x = -kx$$

(restoring force exerted by an ideal spring)

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \text{(simple harmonic motion)}$$

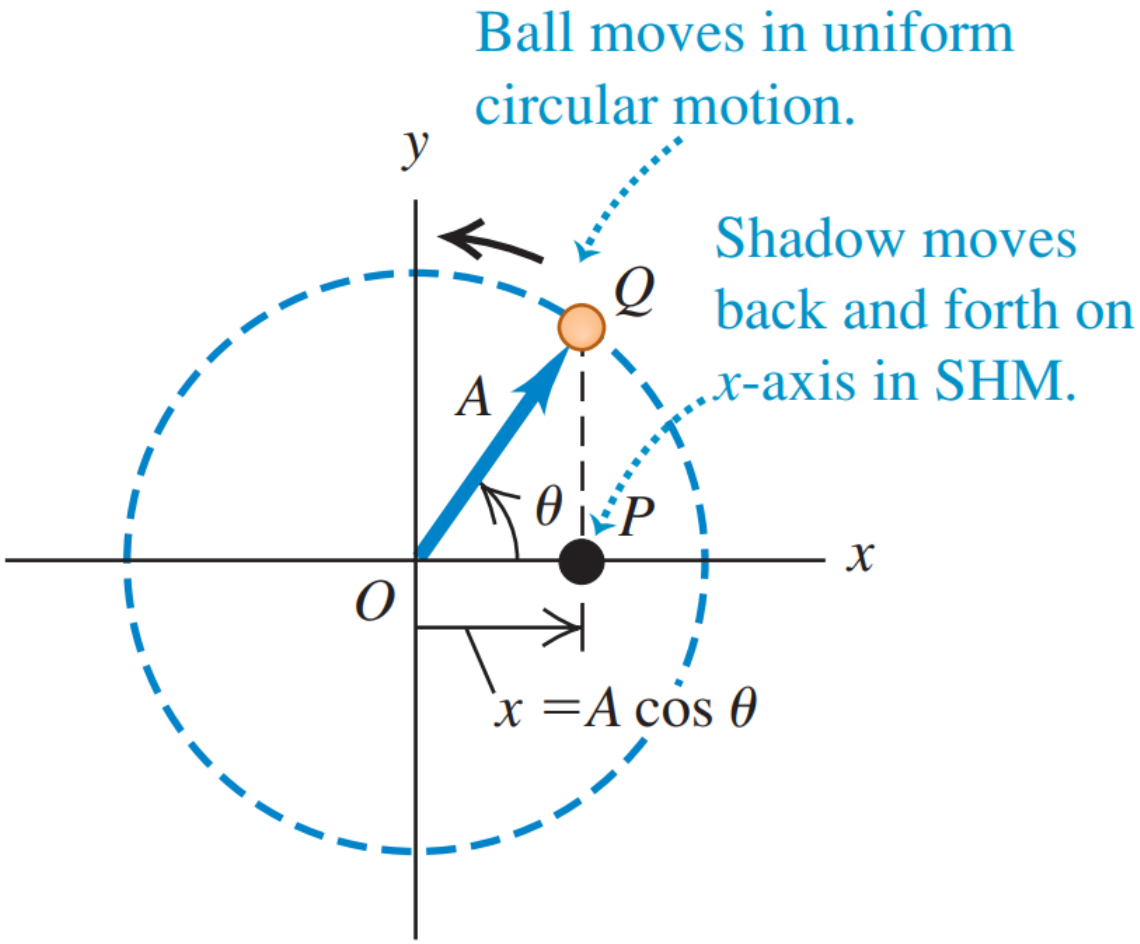
$$\omega = \sqrt{\frac{k}{m}}$$



The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of F_x versus x is a straight line.

Simple Harmonic Motion (SHM)

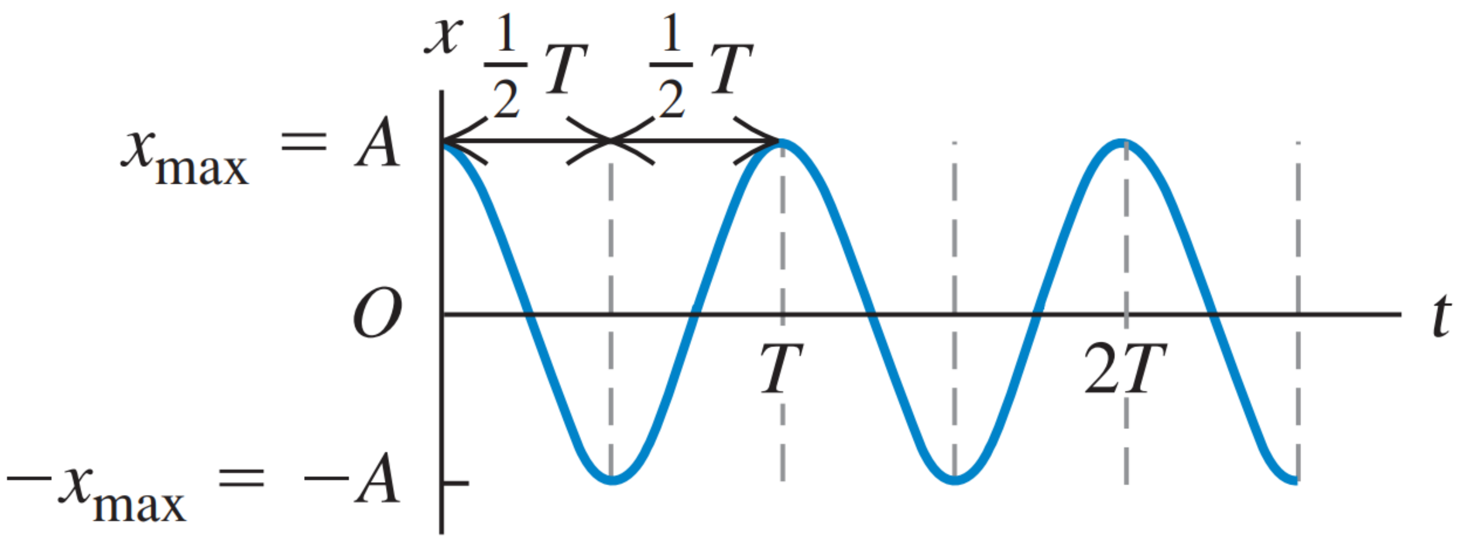
$$\boxed{x = A\cos\theta}$$



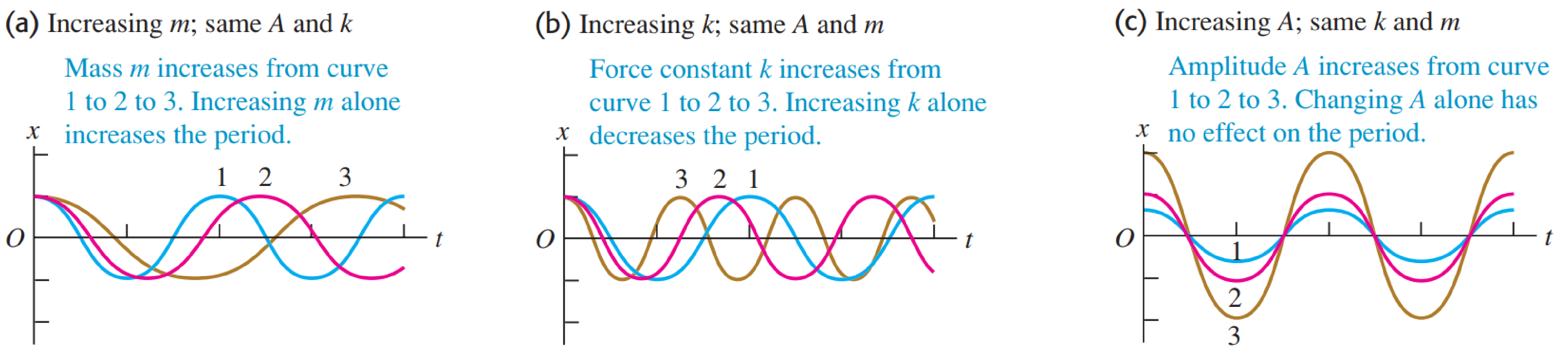
Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi) \qquad \text{(displacement in SHM)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(simple harmonic motion)}$$



Simple Harmonic Motion (SHM)



$$x = A\cos(\omega t + \phi)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi) \qquad \text{(displacement in SHM)}$$

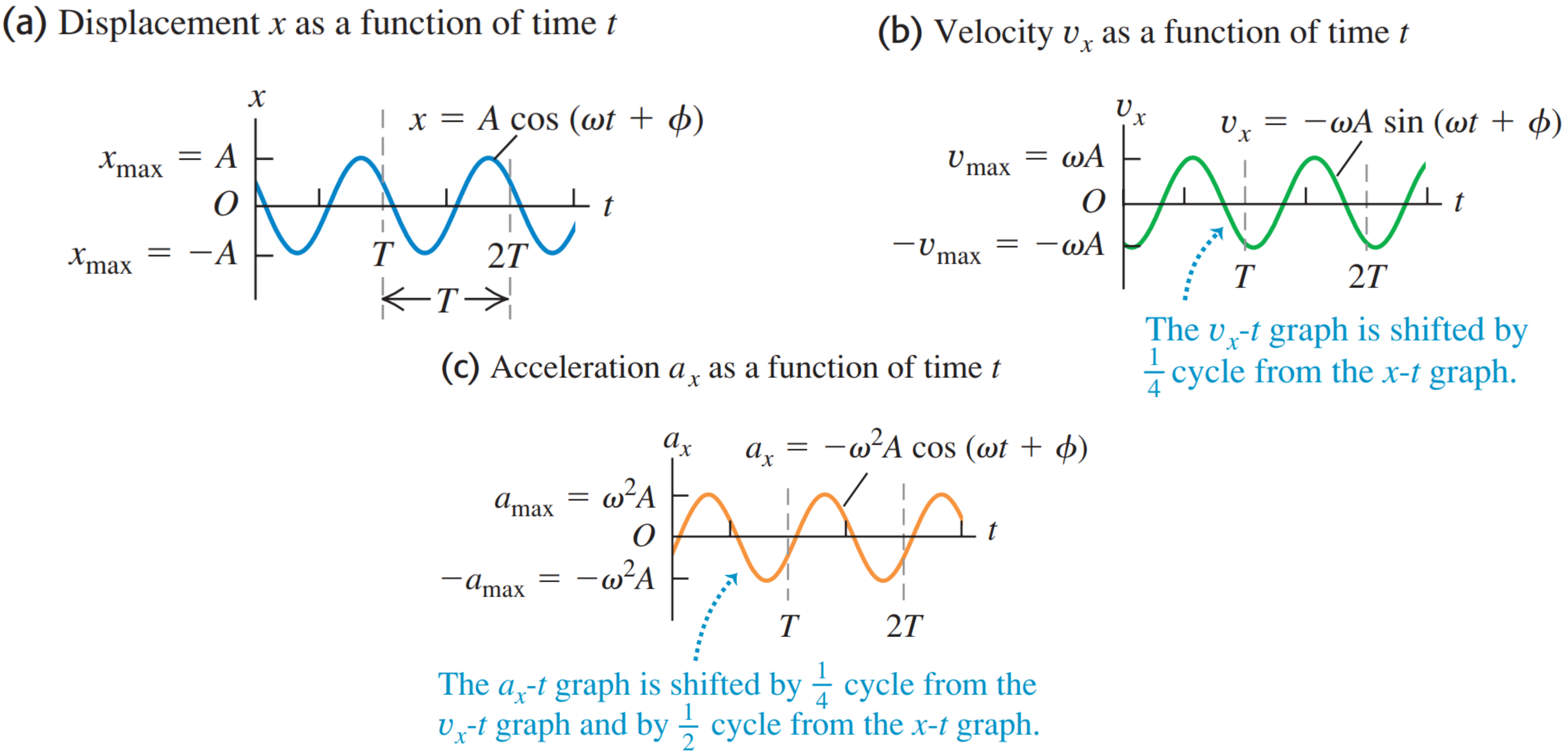
$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \qquad \text{(velocity in SHM)}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) \qquad \text{(acceleration in SHM)}$$

$$\boxed{v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}}$$

$$\boxed{a_x = -\omega^2 x = -\frac{k}{m}x}$$

Simple Harmonic Motion (SHM)

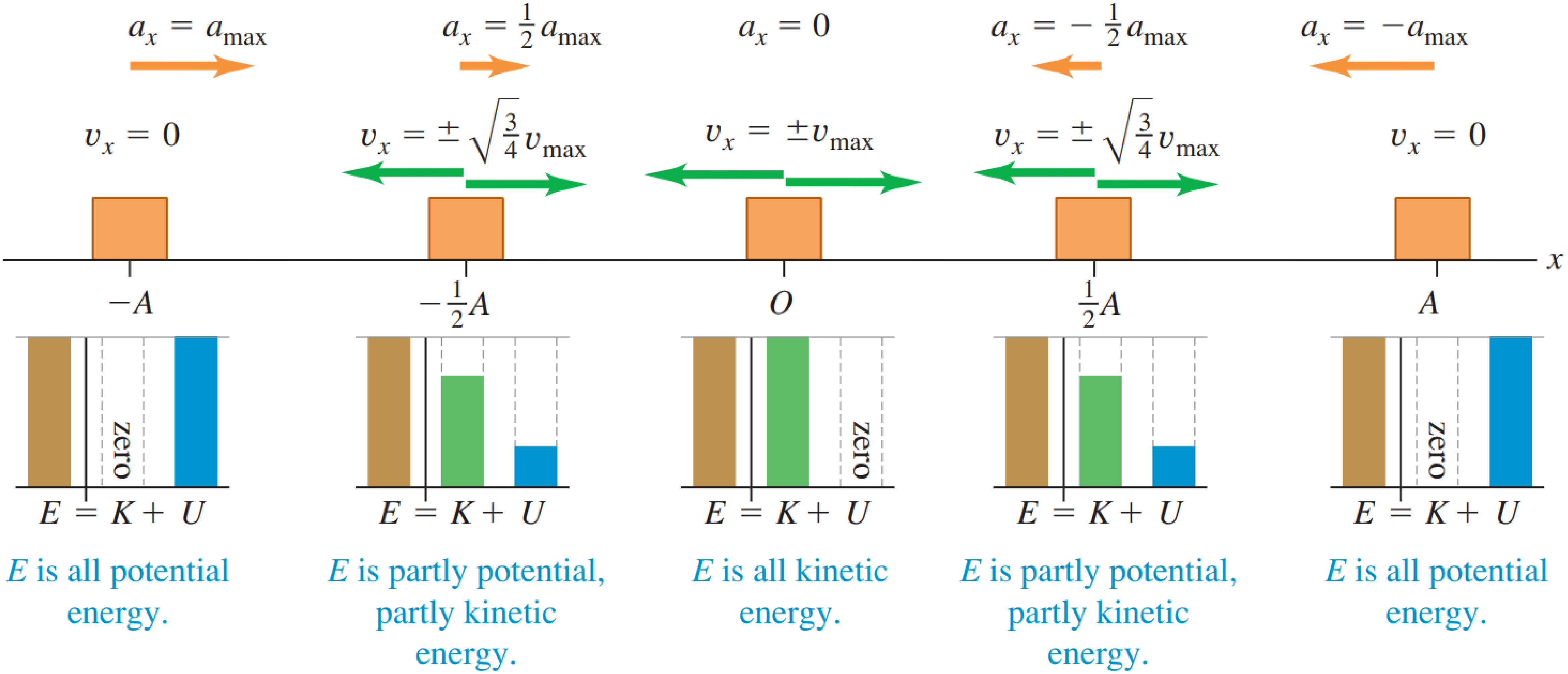


Energy in Simple Harmonic Motion

$$E = \tfrac{1}{2}mv_x{}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \qquad \text{(total mechanical energy in SHM)}$$

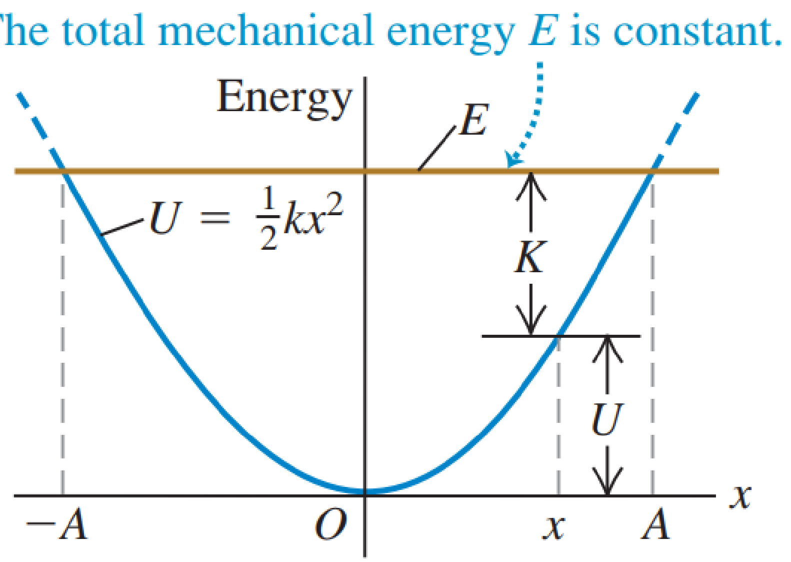
$$\begin{aligned} E &= \tfrac{1}{2}mv_x{}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}m[-\omega A\sin(\omega t + \phi)]^2 + \tfrac{1}{2}k[A\cos(\omega t + \phi)]^2 \\ &= \tfrac{1}{2}kA^2\sin^2(\omega t + \phi) + \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2 \end{aligned}$$

Energy in Simple Harmonic Motion

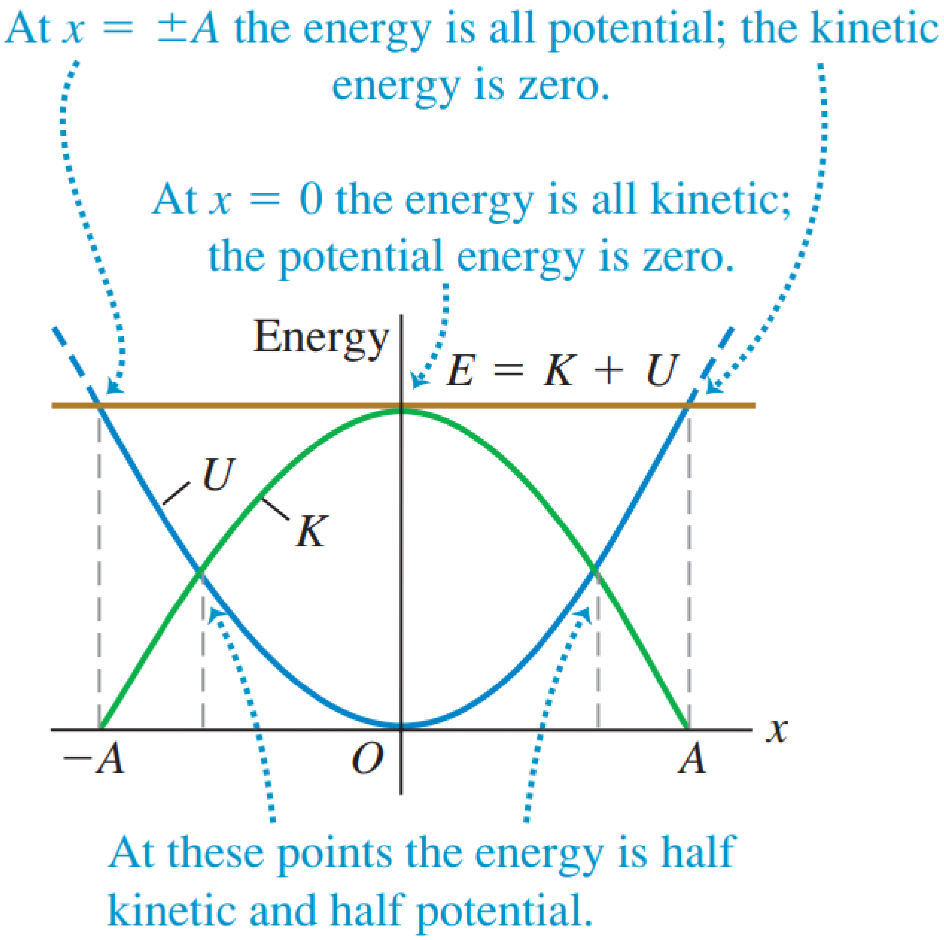


Energy in Simple Harmonic Motion

(a) The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x



(b) The same graph as in (a), showing kinetic energy K as well

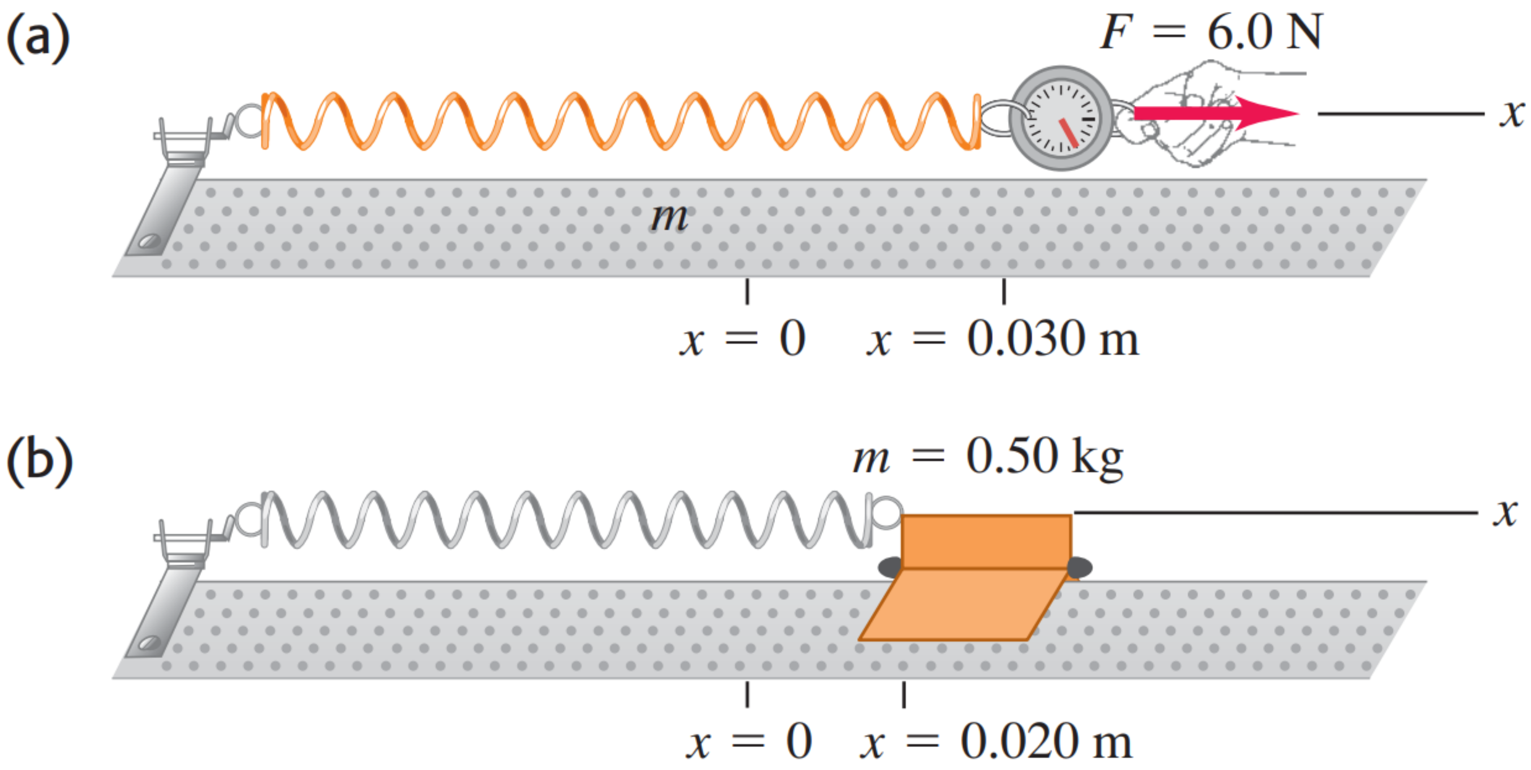


Angular frequency, frequency, and period in SHM

A spring is mounted horizontally, with its left end fixed. A spring balance attached to the free end and pulled toward the right (Fig. 14.8a) indicates that the stretching force is proportional to the displacement, and a force of 6.0 N causes a displacement of 0.030 m. We replace the spring balance with a 0.50-kg glider, pull it 0.020 m to the right along a frictionless air track, and release it from rest (Fig. 14.8b). (a) Find the force constant k of the spring. (b) Find the angular frequency ω, frequency f, and period T of the resulting oscillation.

Angular frequency, frequency, and period in SHM

14.8 (a) The force exerted *on* the spring (shown by the vector F) has x-component $F_x = +6.0$ N. The force exerted *by* the spring has x-component $F_x = -6.0$ N. (b) A glider is attached to the same spring and allowed to oscillate.



Angular frequency, frequency, and period in SHM

EXECUTE: (a) When $x = 0.030$ m, the force the spring exerts on the spring balance is $F_x = -6.0$ N. From Eq. (14.3),

$$k = -\frac{F_x}{x} = -\frac{-6.0\ \text{N}}{0.030\ \text{m}} = 200\ \text{N/m} = 200\ \text{kg/s}^2$$

(b) From Eq. (14.10), with $m = 0.50$ kg,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{20\ \text{rad/s}}{2\pi\ \text{rad/cycle}} = 3.2\ \text{cycle/s} = 3.2\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{3.2\ \text{cycle/s}} = 0.31\ \text{s}$$

Velocity, acceleration, and energy in SHM

(a) Find the maximum and minimum velocities attained by the oscillating glider of Example 14.2. (b) Find the maximum and minimum accelerations. (c) Find the velocity v_x and acceleration a_x when the glider is halfway from its initial position to the equilibrium position $x = 0$. (d) Find the total energy, potential energy, and kinetic energy at this position.

Velocity, acceleration, and energy in SHM

EXECUTE: (a) From Eq. (14.22), the velocity v_x at any displacement x is

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

The glider's maximum *speed* occurs when it is moving through $x = 0$:

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{200\ \mathrm{N/m}}{0.50\ \mathrm{kg}}}(0.020\ \mathrm{m}) = 0.40\ \mathrm{m/s}$$

Its maximum and minimum (most negative) *velocities* are $+0.40\ \mathrm{m/s}$ and $-0.40\ \mathrm{m/s}$, which occur when it is moving through $x = 0$ to the right and left, respectively.

Velocity, acceleration, and energy in SHM

(b) From Eq. (14.4), $a_x = -(k/m)x$. The glider's maximum (most positive) acceleration occurs at the most negative value of x, $x = -A$:

$$a_{\text{max}} = -\frac{k}{m}(-A) = -\frac{200\ \text{N/m}}{0.50\ \text{kg}}(-0.020\ \text{m}) = 8.0\ \text{m/s}^2$$

The minimum (most negative) acceleration is $a_{\text{min}} = -8.0\ \text{m/s}^2$, which occurs at $x = +A = +0.020$ m.

Velocity, acceleration, and energy in SHM

(c) The point halfway from $x = x_0 = A$ to $x = 0$ is $x = A/2 = 0.010\ \text{m}$. From Eq. (14.22), at this point

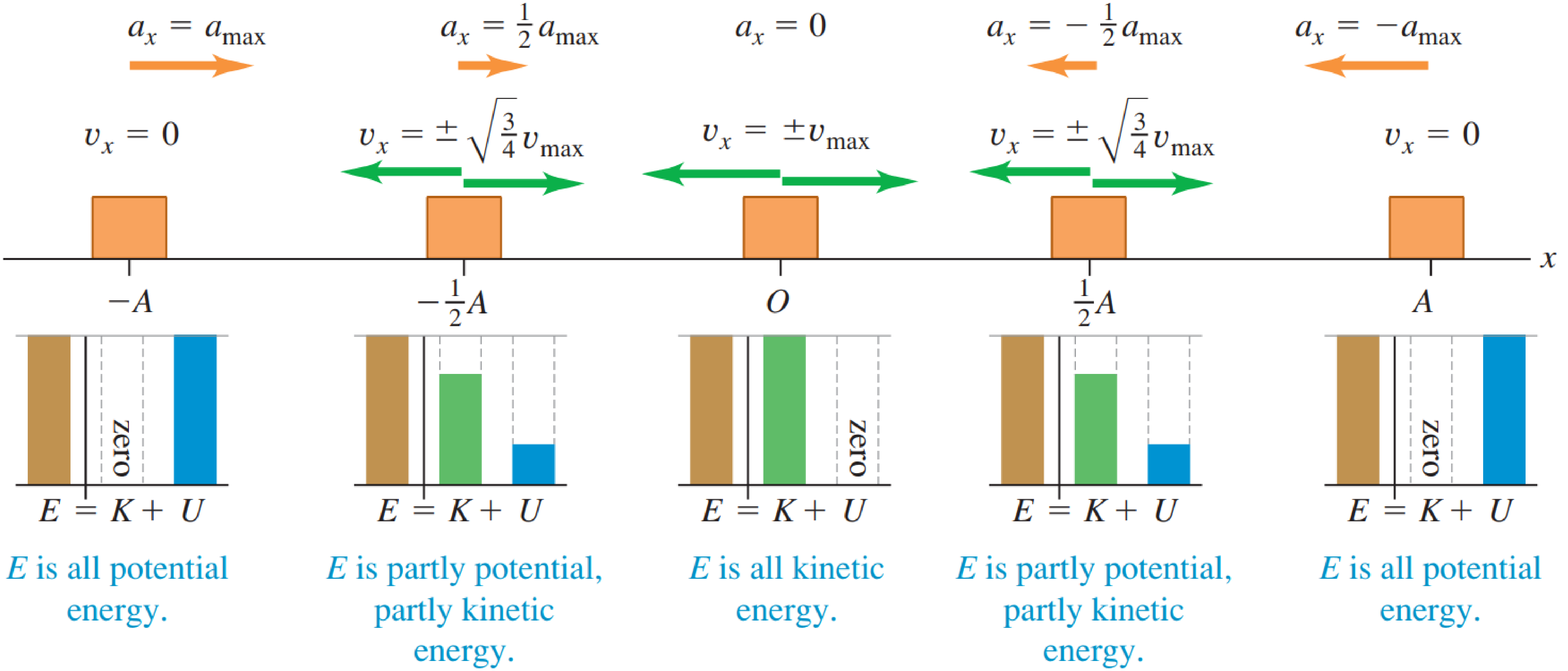
$$v_x = -\sqrt{\frac{200\ \text{N/m}}{0.50\ \text{kg}}}\sqrt{(0.020\ \text{m})^2 - (0.010\ \text{m})^2} = -0.35\ \text{m/s}$$

We choose the negative square root because the glider is moving from $x = A$ toward $x = 0$. From Eq. (14.4),

$$a_x = -\frac{200\ \text{N/m}}{0.50\ \text{kg}}(0.010\ \text{m}) = -4.0\ \text{m/s}^2$$

Figure 14.14 shows the conditions at $x = 0$, $\pm A/2$, and $\pm A$.

Velocity, acceleration, and energy in SHM



Velocity, acceleration, and energy in SHM

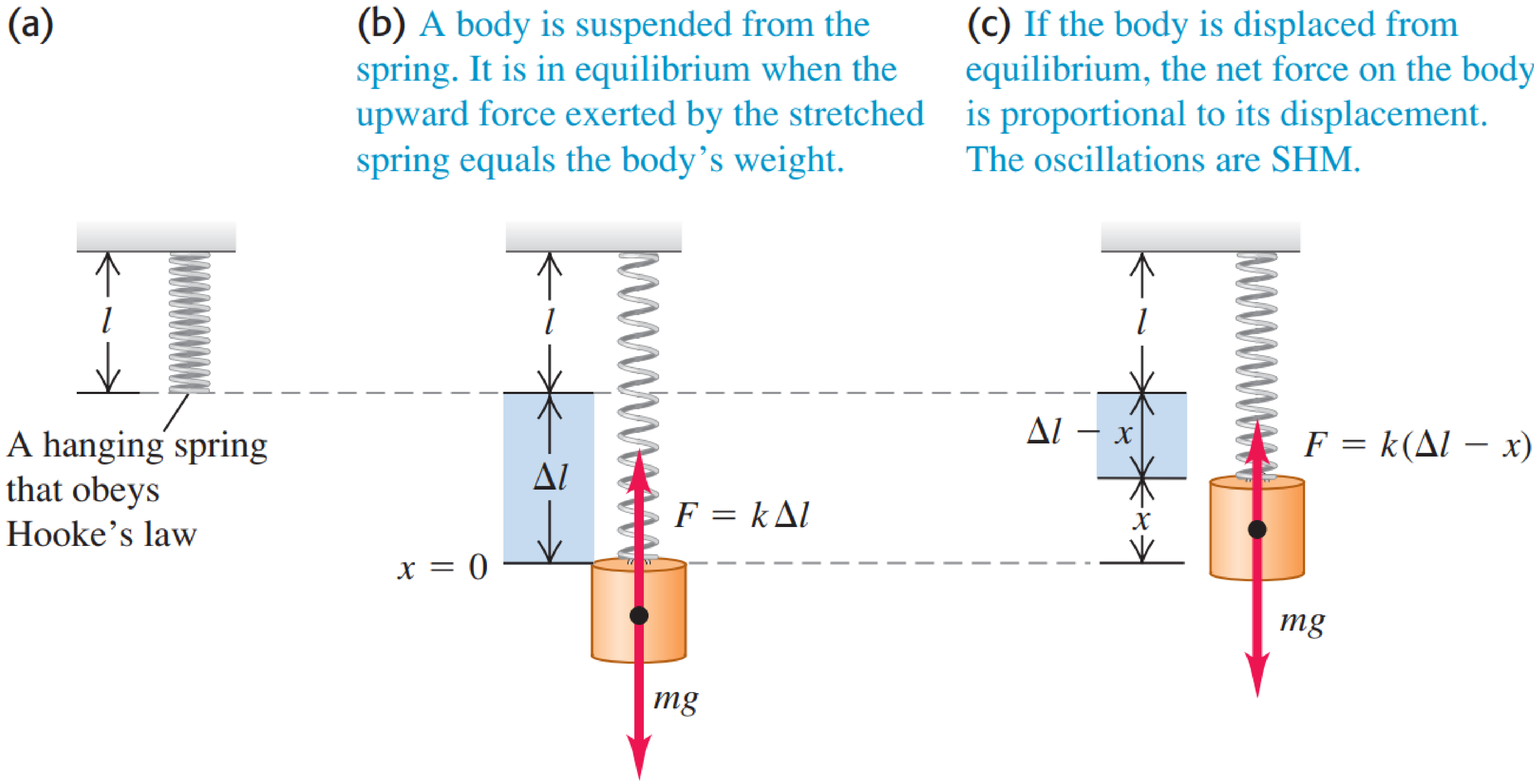
(d) The energies are

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200\ \text{N/m})(0.020\ \text{m})^2 = 0.040\ \text{J}$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\ \text{N/m})(0.010\ \text{m})^2 = 0.010\ \text{J}$$

$$K = \tfrac{1}{2}mv_x{}^2 = \tfrac{1}{2}(0.50\ \text{kg})(-0.35\ \text{m/s})^2 = 0.030\ \text{J}$$

Vertical Simple Harmonic Motion



Readings

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Chapter 14: Periodic Motion

The Principle of Superposition for Waves

Suppose that two waves travel simultaneously along the same stretched string. Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

This summation of displacements along the string means that

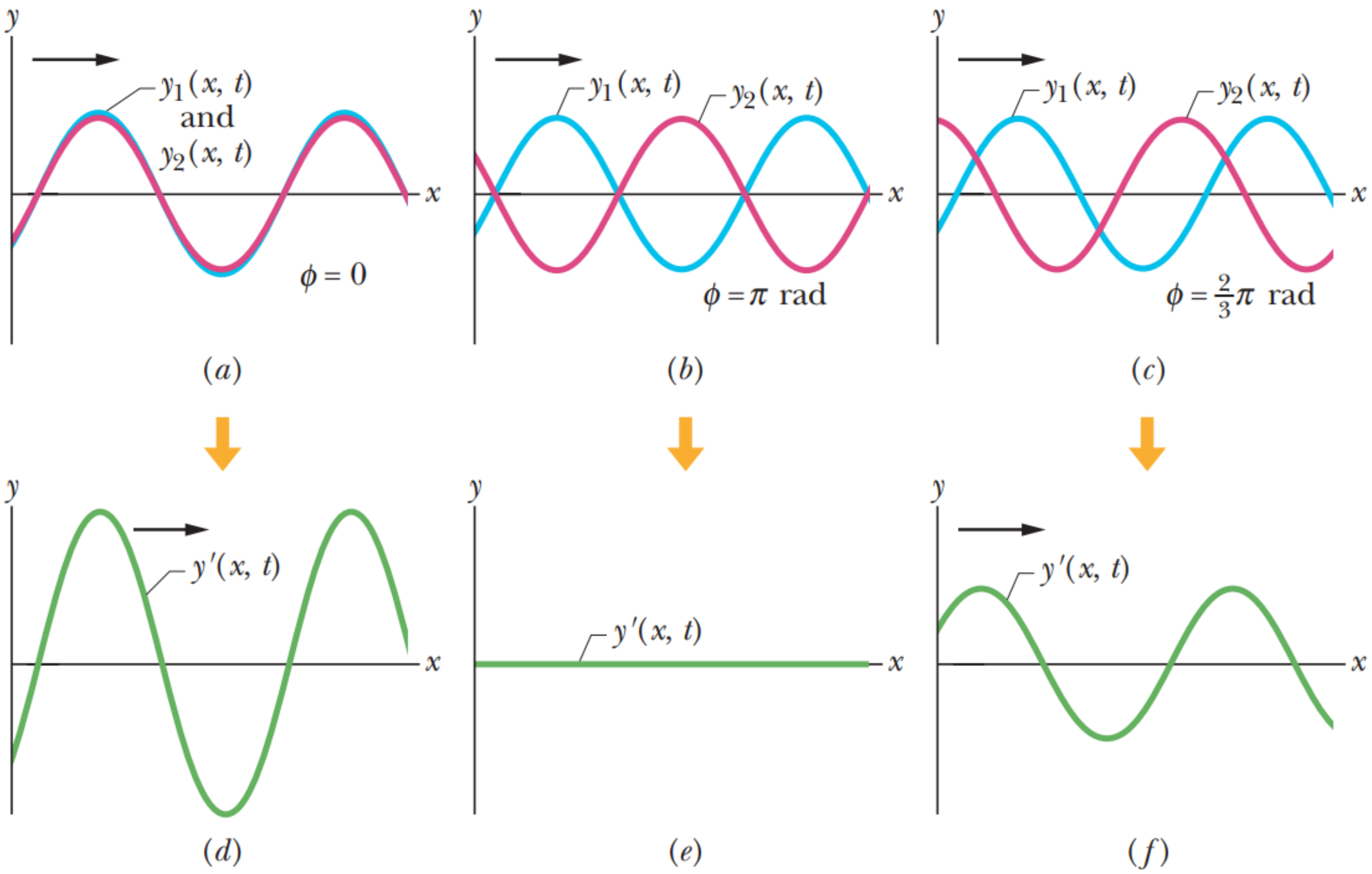
Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

This is another example of the **principle of superposition,** which says that when several effects occur simultaneously, their net effect is the sum of the individual effects. (We should be thankful that only a simple sum is needed. If two effects somehow amplified each other, the resulting nonlinear world would be very difficult to manage and understand.)

Interference of Waves

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi)$$



Interference of Waves

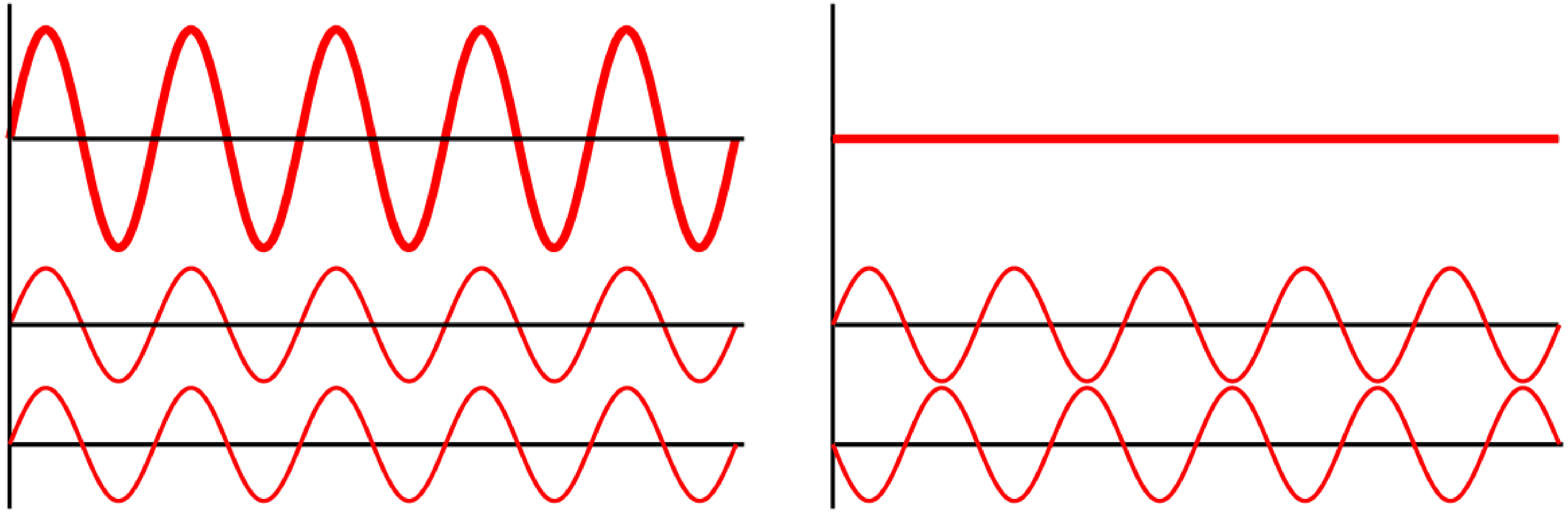
$$y'(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$

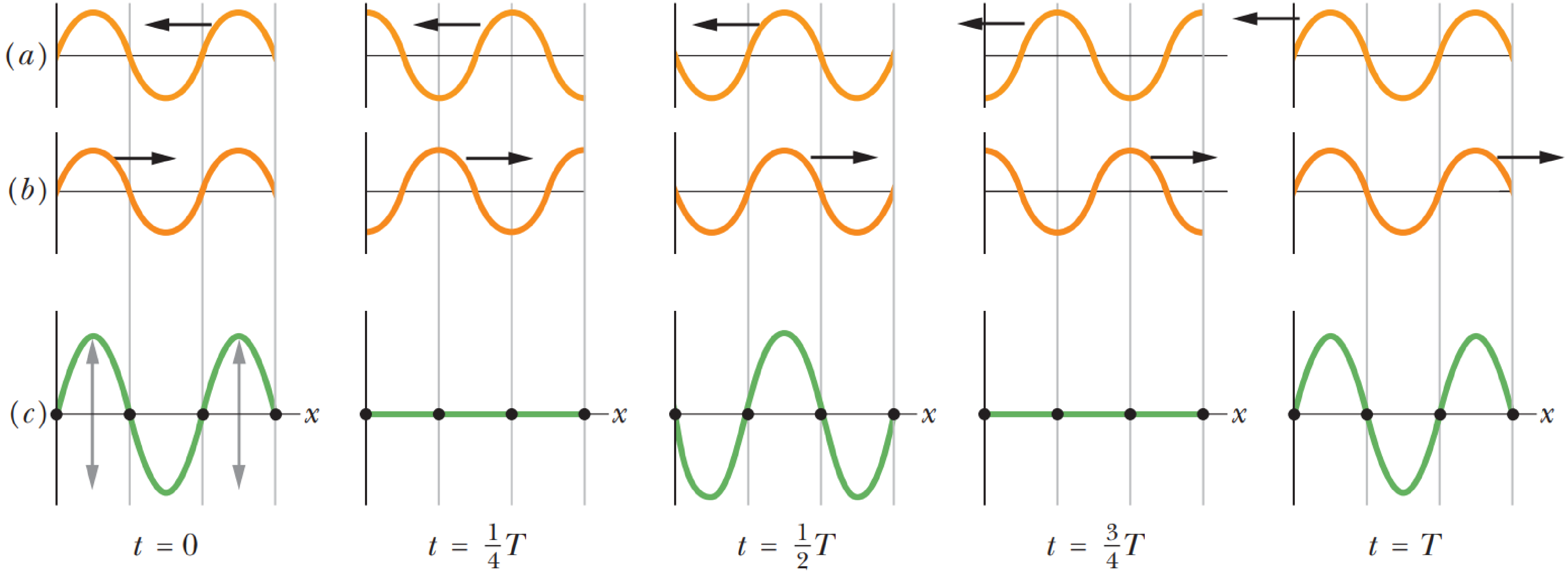
Interference of Waves

If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.



Standing Waves

As the waves move through each other, some points never move and some move the most.



Standing Waves

If two sinusoidal waves of the same amplitude and wavelength travel in *opposite* directions along a stretched string, their interference with each other produces a standing wave.

To analyze a standing wave, we represent the two waves with the equations

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

and

$$y_2(x,t) = y_m \sin(kx + \omega t).$$

The principle of superposition gives, for the combined wave,

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

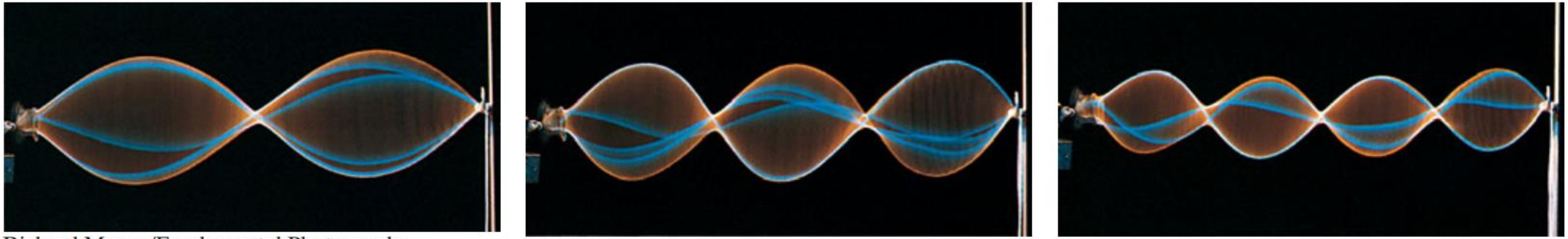
Applying the trigonometric relation

$$y'(x,t) = [2y_m \sin kx] \cos \omega t.$$

Standing Waves

$$x = n\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(nodes)}$$

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)}$$



Richard Megna/Fundamental Photographs

Well