Lecture notes: 02

Average and RMS voltage

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December 9, 2020
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Contents

1 Alternating current (AC)

Alternating current (AC) describes the flow of charge that changes direction periodically. As a result, the voltage level also reverses along with the current. AC is used to deliver power to houses, office buildings, etc. The AC voltage of a periodic waveform may be written as

$$V(t) = V_m \sin(\omega t), \tag{1}$$

where $\omega = 2\pi/T$ is the angular frequency of the waveform or voltage and T is the time period of the voltage.

2 Average of $V(t)$ over time T

The average value of $V(t)$ over the time period T is defined as

$$\overline{V} = \frac{1}{T}\int_0^T V(t)\mathrm{d}t. \tag{2}$$

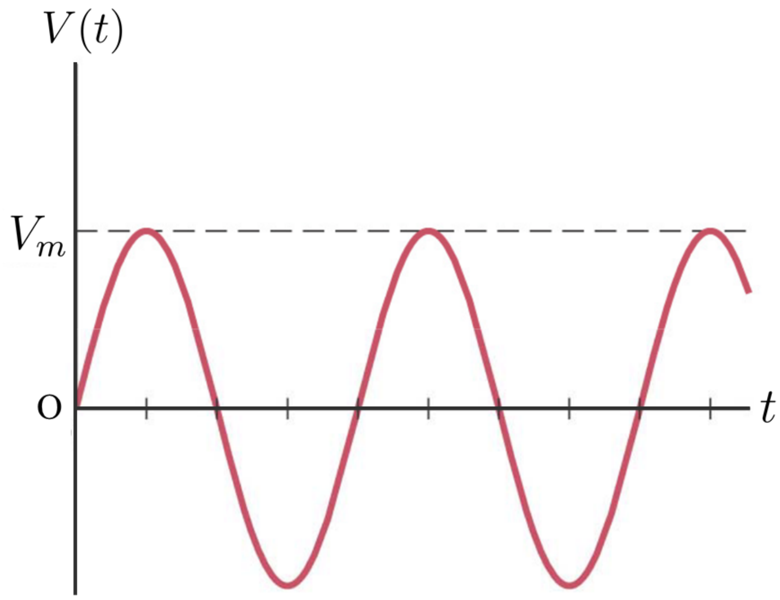

Figure 1: AC voltage $V(t)$

Hence

$$
\begin{aligned}
\overline{V} &= \frac{V_m}{T}\int_0^T \sin(\omega t)\mathrm{d}t \\
&= \frac{V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^T \\
&= \frac{V_m}{\omega T}\left\{-\cos(\omega T)+\cos 0\right\} \\
&= \frac{V_m}{2\pi}\left\{-\cos(2\pi)+\cos 0\right\} \\
&= \frac{V_m}{2\pi}(-1+1) \\
&= 0. \qquad (3)
\end{aligned}
$$

Therefor, the average value of an AC voltage over the time period of the oscillation is zero.

3 Average of $V(t)$ over time $T/2$

Since the average value of the AC voltage over the time period is zero, we may calculate the average value over the half of the time period using a similar definition as (2).

Therefore

$$
\begin{aligned}
V_{\text{avg}} &= \frac{1}{T/2}\int_0^{T/2} V(t)\mathrm{d}t \\
&= \frac{2V_m}{T}\int_0^{T/2} \sin(\omega t)\mathrm{d}t \\
&= \frac{2V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^{T/2} \\
&= \frac{2V_m}{\omega T}\{-\cos(\omega T/2)+\cos 0\} \\
&= \frac{2V_m}{2\pi}\{-\cos(\pi)+\cos 0\} \\
&= \frac{V_m}{\pi}(+1+1) \\
&= \frac{2}{\pi}V_m \\
&\approx 0.637\,V_m. \qquad (4)
\end{aligned}
$$

4 The RMS value of $V(t)$

The term "RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, is equivalent to a DC voltage that would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor.

RMS is not an "Average" voltage, and its mathematical relationship to peak voltage varies depending on the type of waveform. The RMS value is the square root of the mean (average) value of the squared function of the instantaneous values. For the voltage $V(t)$ given in (1) it can be written as

$$
V_{\text{rms}} = \left[\frac{1}{T}\int_0^{T} V^2(t)\mathrm{d}t\right]^{1/2}. \qquad (5)
$$

Hence

$$
\begin{aligned}
V_{\text{rms}}^2 &= \frac{V_m^2}{T}\int_0^T \sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T 2\sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T \{1-\cos(2\omega t)\}\,\mathrm{d}t \\
&= \frac{V_m^2}{T2}\int_0^T \mathrm{d}t - \frac{V_m^2}{T}\int_0^T \cos(2\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\Big[T\Big]_0^T - \frac{V_m^2}{2T}\left[\frac{\sin(2\omega t)}{2\omega}\right]_0^T \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(2\omega T)-\sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(4\pi)-\sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}(0-0) \\
&= \frac{V_m^2}{2}. \qquad (6)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
V_{\text{rms}} &= \frac{V_m}{\sqrt{2}} \\
&\approx 0.707\,V_m. \qquad (7)
\end{aligned}
$$

Important relations to remember

A few handy things to keep in mind about RMS values that apply when dealing with a sine wave, are as follows:

- $V_{\text{rms}} \approx 0.707\times$ peak AC voltage = 70.7 % of peak voltage
- Peak AC voltage $\approx 1.414\ \times V_{\text{rms}}$ = 141.1 % of V_{rms}
- $V_{\text{avg}} \approx 0.637\times$ peak AC voltage = 63.7 % of peak voltage
- $\dfrac{V_{\text{rms}}}{V_{\text{avg}}} = \dfrac{\pi}{2\sqrt{2}} \approx 1.11$