Mechanics

Linear Momentum of a Particle and a System of Particles, Momentum Conservation, Application of Momentum Principle, Angular momentum of a Particle and a System of Particles, Kepler's law of Planetary Motion, Laws of Universal Gravitation, Motion of Planets and Satellites.

Fundamentals of Physics (10th Ed) – Halliday, Resnick, Walker

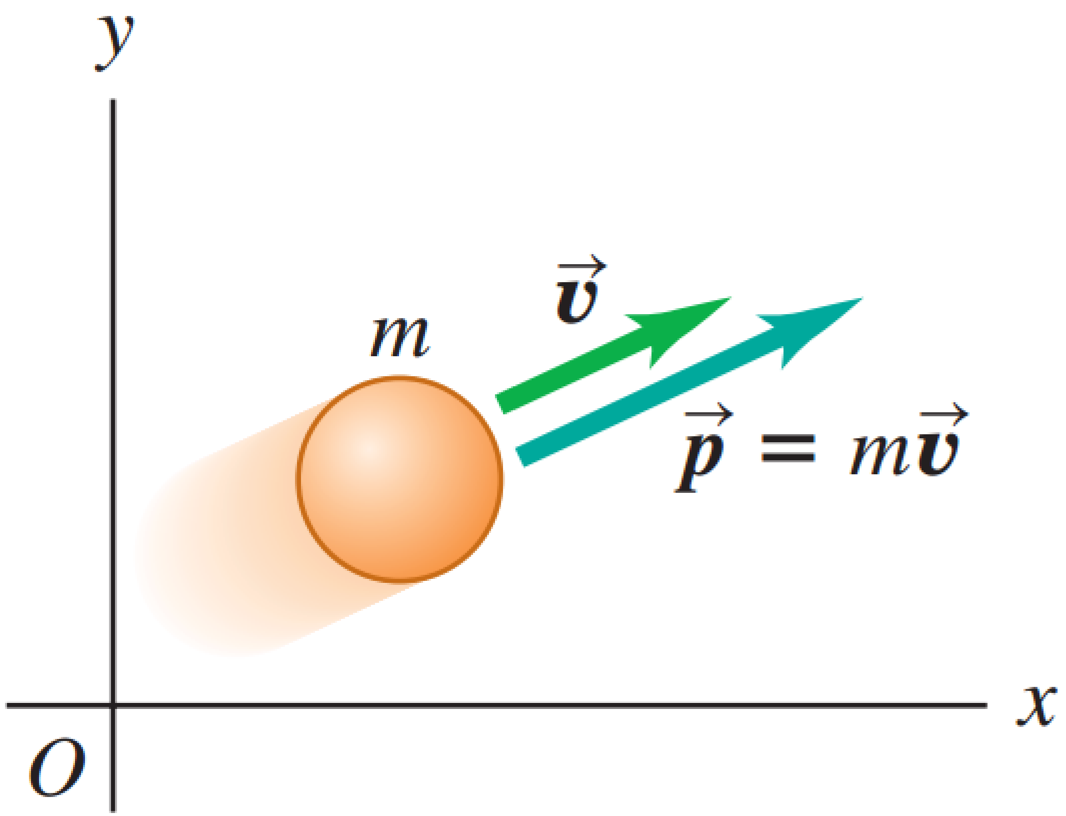
University Physics with Modern Physics (13th Ed) – Young, Freedman

Linear Momentum

$$p_x = mv_x$$

$$p_y = mv_y$$

$$p_z = mv_z$$



Momentum $\vec{p}$ is a vector quantity

Newton's Second Law in Terms of Momentum

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

The Linear Momentum of a System of Particles

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$

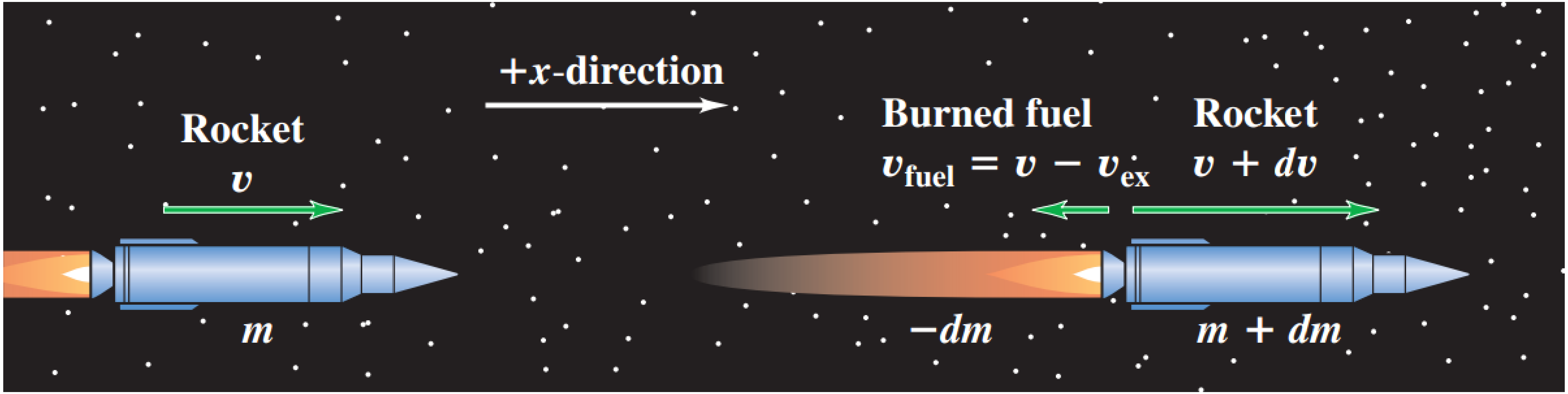
$$\boxed{\vec{P} = M\vec{v}_{\text{com}}}$$

Conservation of Linear Momentum

$$\begin{pmatrix}\text{total linear momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum} \\ \text{at some later time } t_f\end{pmatrix}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

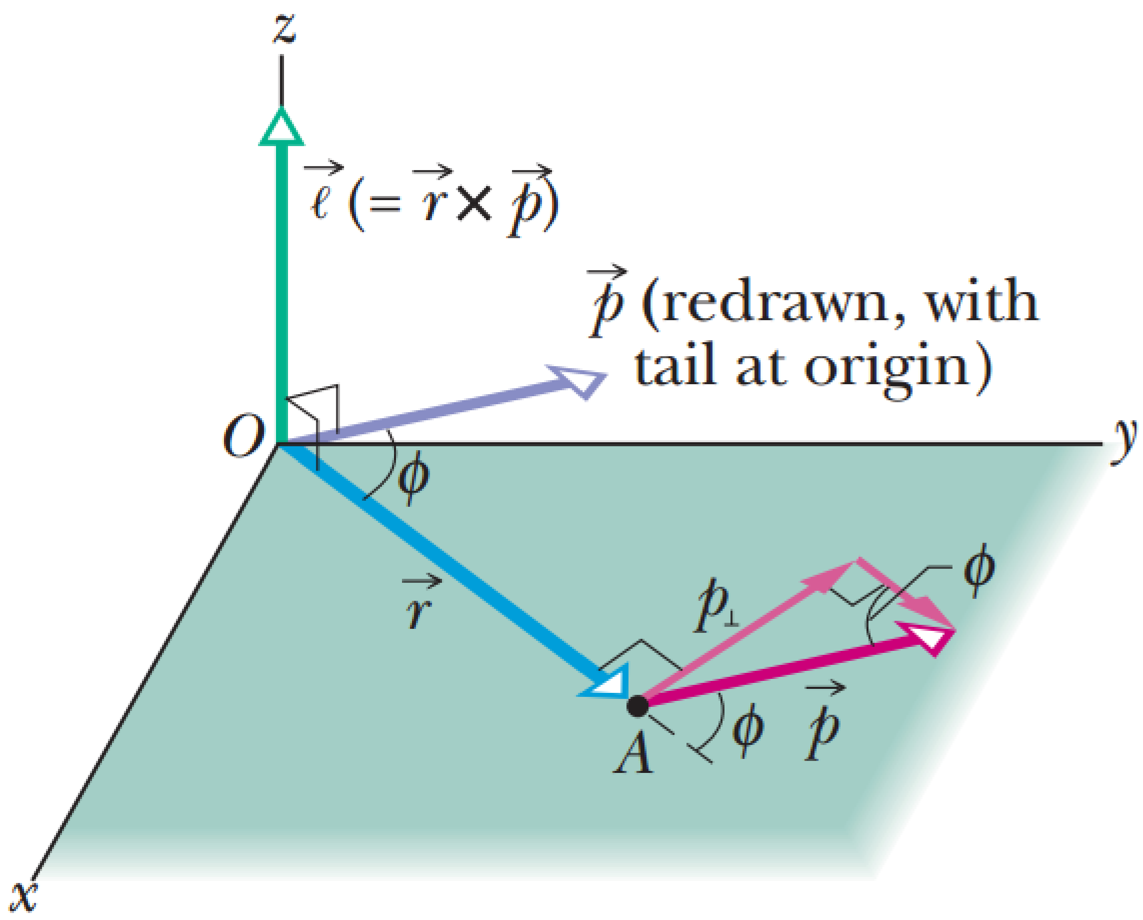


At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

Angular Momentum

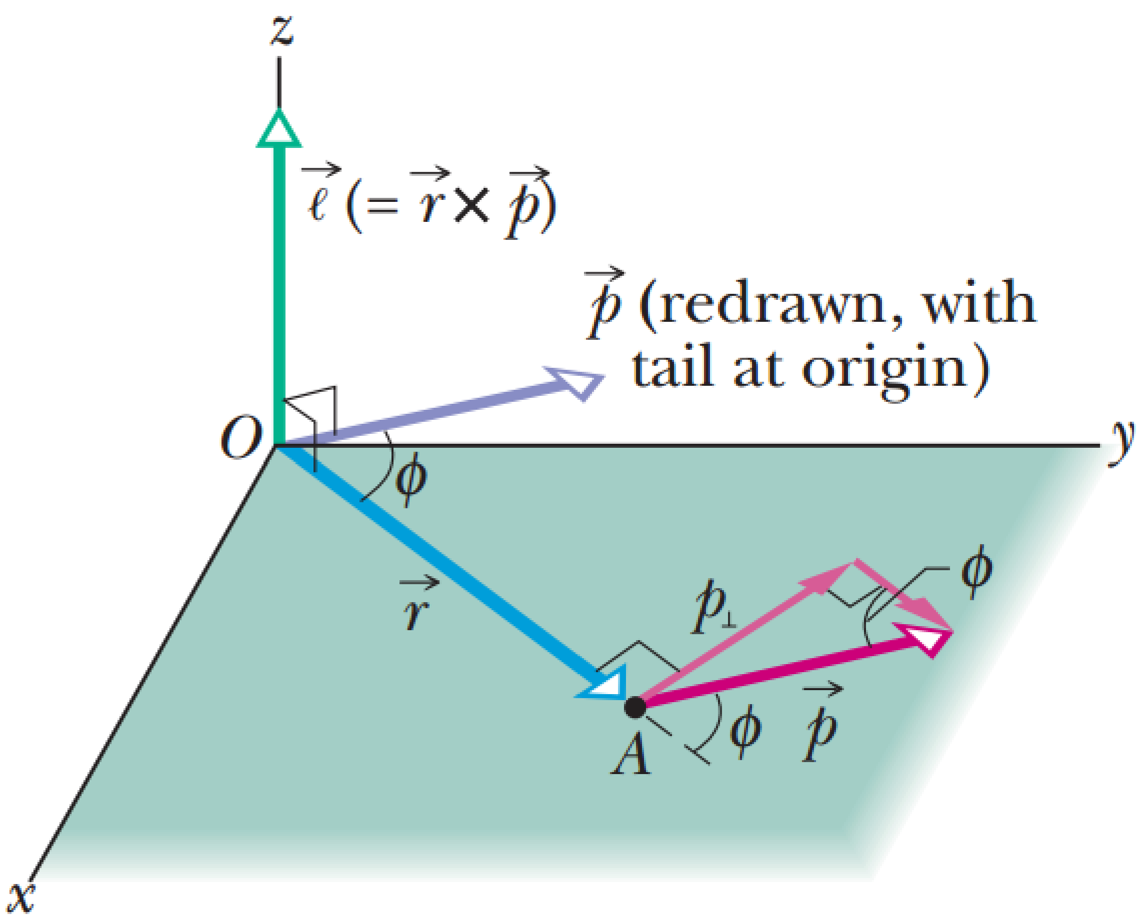
$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$



$$\ell = rmv \sin \phi.$$

Angular Momentum

$$\ell = rmv \sin \phi$$

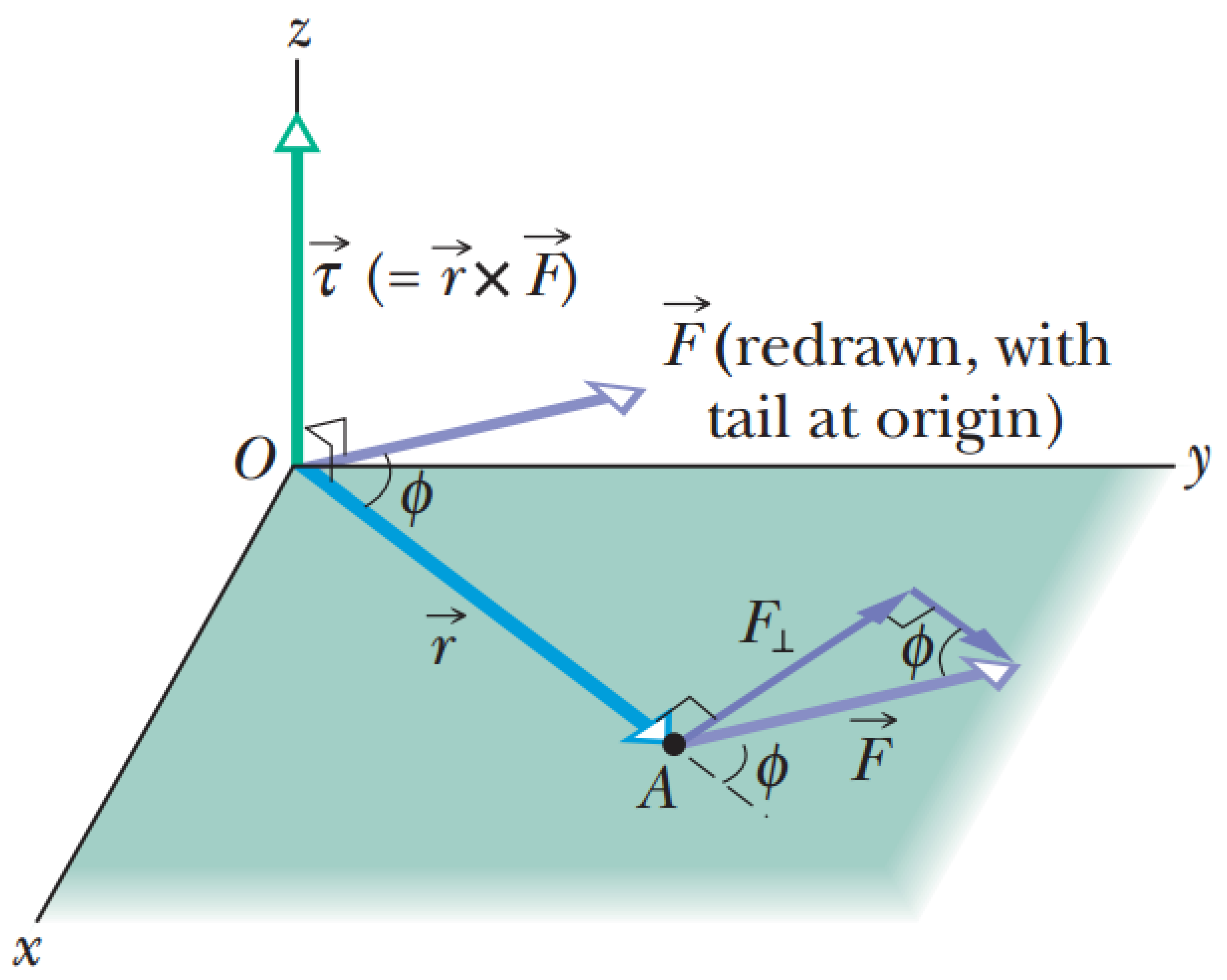


Important. Note two features here: (1) angular momentum has meaning only with respect to a specified origin and (2) its direction is always perpendicular to the plane formed by the position and linear momentum vectors $\vec{r}$ and $\vec{p}$.

Newton's Second Law in Angular Form

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \text{(single particle)}$$

$$\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt} \quad \text{(single particle)}$$



Newton's Second Law in Angular Form

$$\vec{\ell} = m(\vec{r} \times \vec{v})$$

$$\frac{d\vec{\ell}}{dt} = m\left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v}\right) = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v})$$

$$= m(\vec{r} \times \vec{a}) = \vec{r} \times m\vec{a}$$

$$\boxed{\vec{v} \times \vec{v} = 0}$$

$$= \vec{r} \times \vec{F}_{\text{net}} = \sum(\vec{r} \times \vec{F})$$

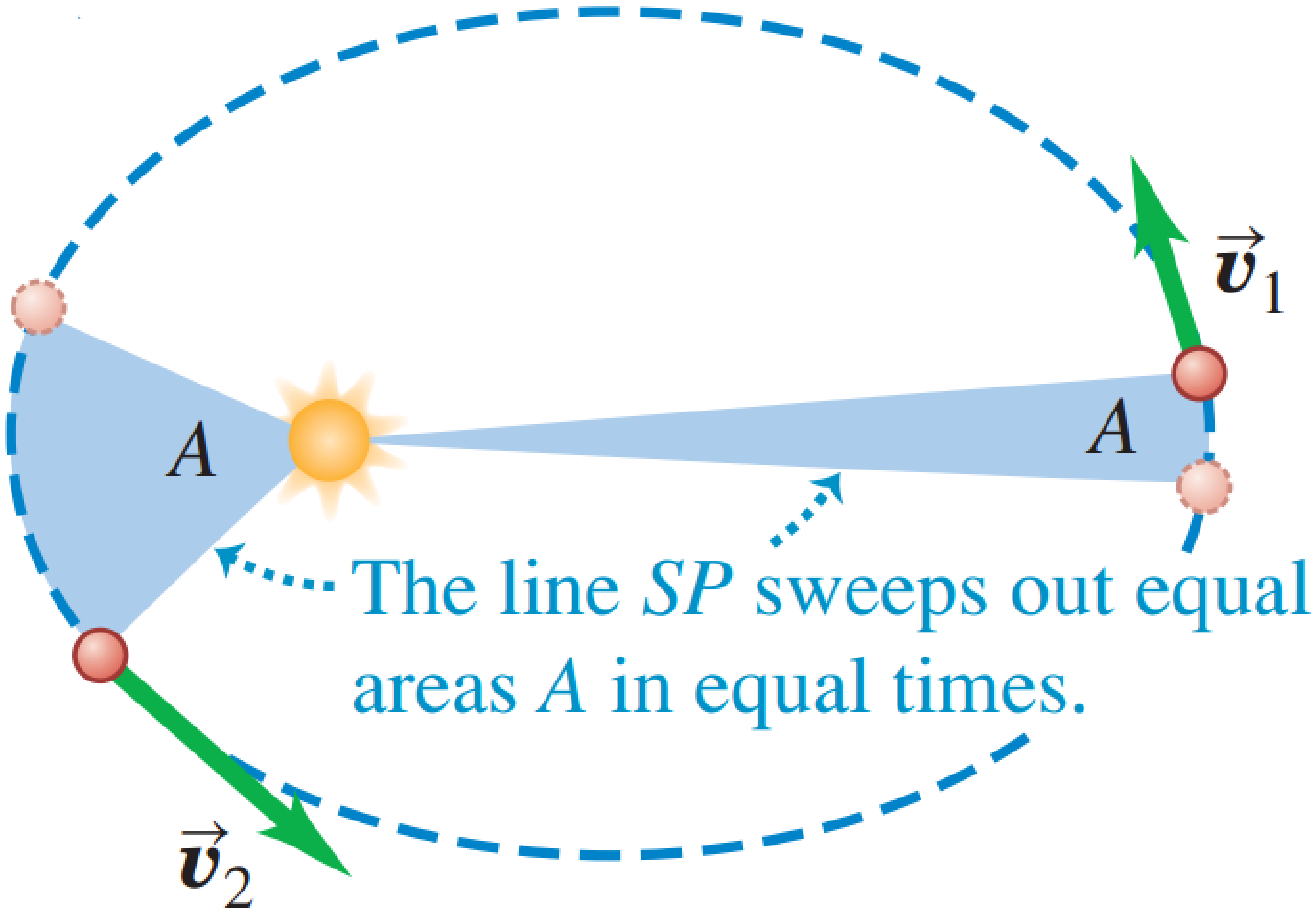
$$\boxed{\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}}$$

Planets and Satellites: Kepler's Laws

The law of orbits. All planets move in elliptical orbits with the Sun at one focus.

The law of areas. A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)

Planets and Satellites: Kepler's Laws



Planets and Satellites: Kepler's Laws

The law of periods. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad \text{(law of periods)},$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis a is substituted for r.

Satellites: Orbits and Energy

The potential energy of the system is given by $$U = -\frac{GMm}{r}$$

Here *r* is the radius of the satellite's orbit,
M and *m* are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit, we write Newton's second law *(F = ma)* as

$$\frac{GMm}{r^2} = m\frac{v^2}{r},$$

Satellites: Orbits and Energy

The kinetic energy is $$K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r},$$

which shows us that for a satellite in a circular orbit, *K = -U/2.*

The total mechanical energy of the orbiting satellite is

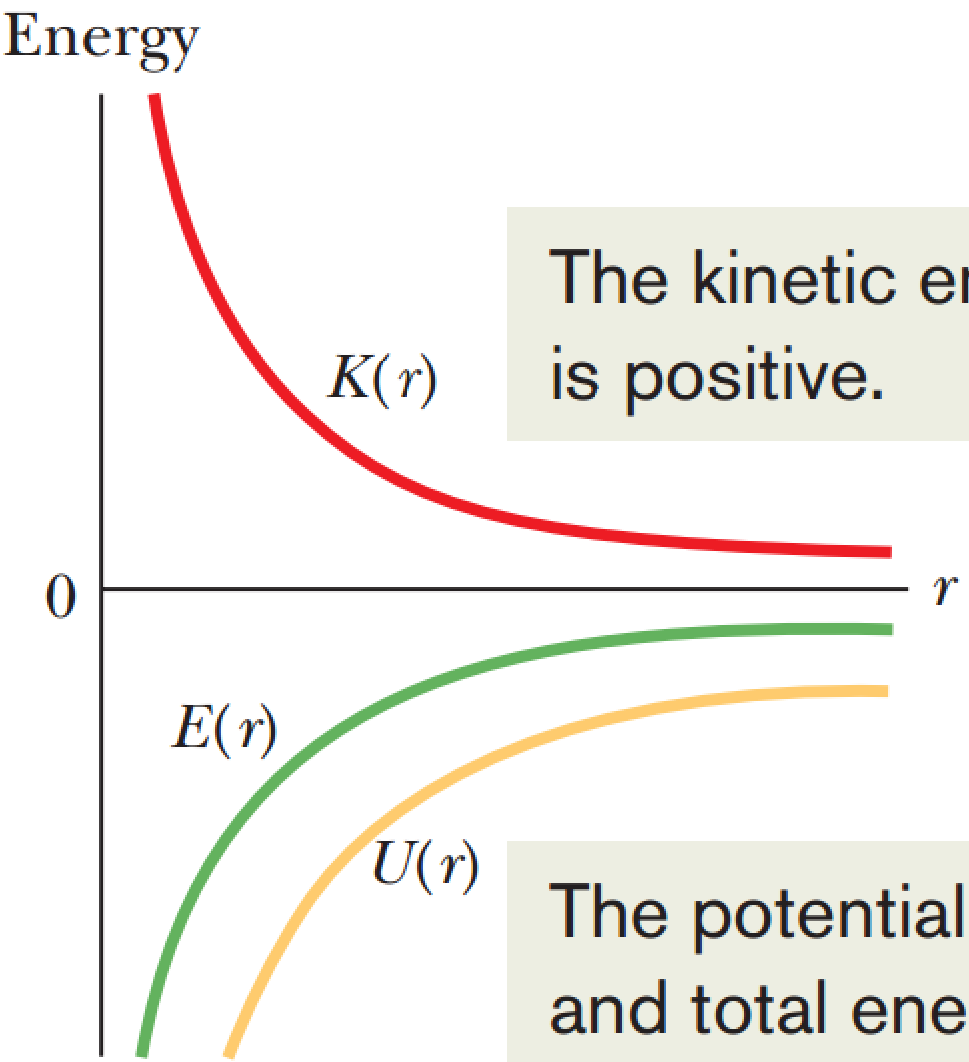
$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$\boxed{E = -K}$$

$$E = -\frac{GMm}{2r} \quad \text{(circular orbit).}$$

Satellites: Orbits and Energy

This is a plot of a satellite's energies versus orbit radius.



The kinetic energy is positive.

The potential energy and total energy are negative.

Satellites: Orbits and Energy

For a satellite in an elliptical orbit of semimajor axis *a*, we can substitute *a* for *r* to find the mechanical energy:

$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit).}$$

Satellites: Orbits and Energy

A playful astronaut releases a bowling ball, of mass $m = 7.20$ kg, into circular orbit about Earth at an altitude h of 350 km.

(a) What is the mechanical energy E of the ball in its orbit?

Satellites: Orbits and Energy

Calculations: The orbital radius must be

$$r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},$$

in which R is the radius of Earth. Then, from Eq. 13-40 with Earth mass $M = 5.98 \times 10^{24}$ kg, the mechanical energy is

$$\begin{aligned} E &= -\frac{GMm}{2r} \\ &= -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.20 \text{ kg})}{(2)(6.72 \times 10^6 \text{ m})} \\ &= -2.14 \times 10^8 \text{ J} = -214 \text{ MJ}. \qquad \text{(Answer)} \end{aligned}$$

Satellites: Orbits and Energy

Calculate the total energy of a five metric ton telecommunications satellite circulating in a geostationary orbit.

The mass of earth is 5.98×10^{24} kg.

1 metric ton = 1000 kg.

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\boxed{E = -\frac{GMm}{2r}}$$

E = -23.60 GJ