Optics

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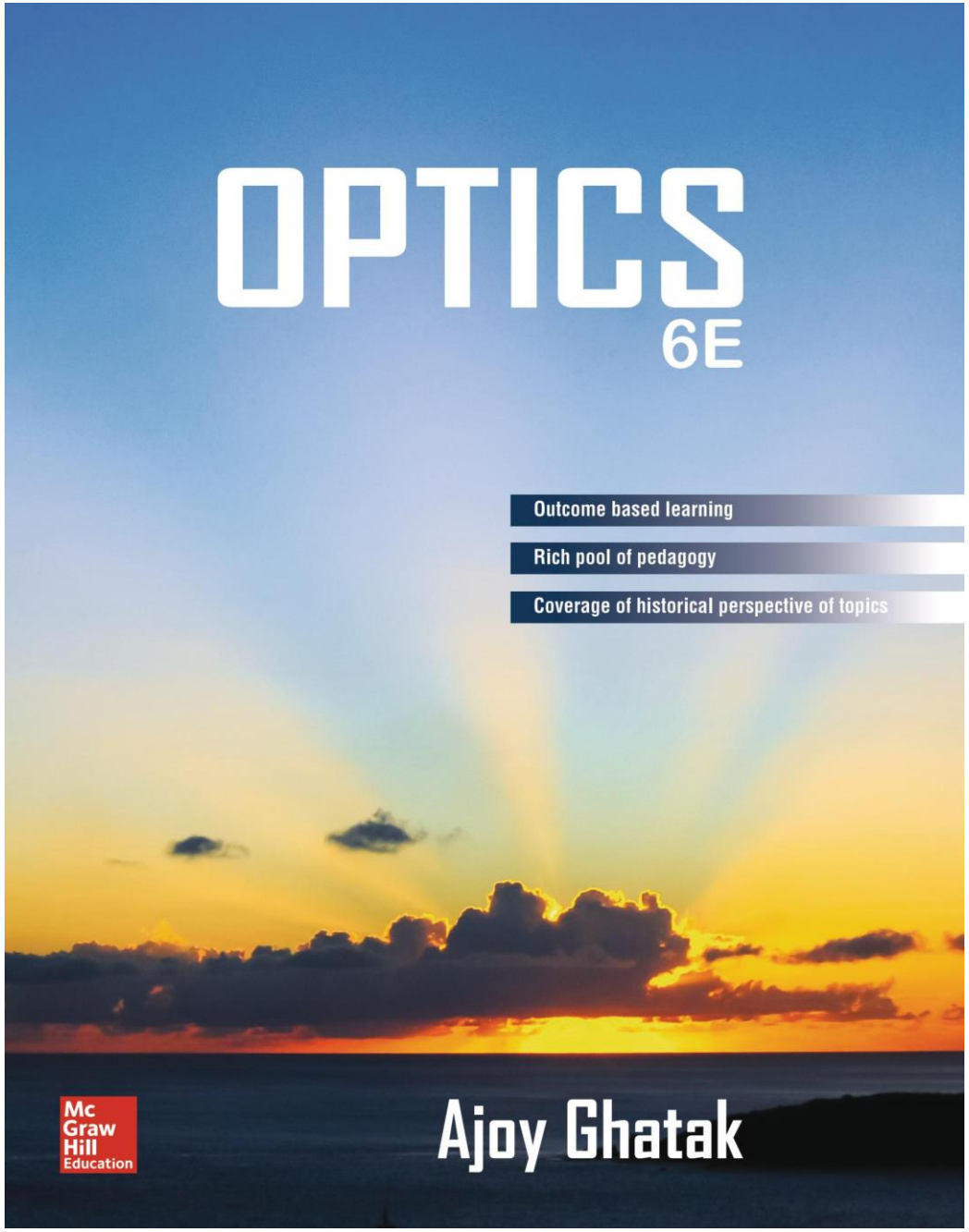
Reference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History of Optics

Chapter One

Optics is the study of light that has always fascinated humans. In his famous book, *On The Nature of Light,* Vasco Ronchi wrote:

> *Today we tend to remember only Newton and Huygens and consider them as the two great men who laid the foundations of physical optics. This is not really true and perhaps this tendency is due to the distance in time which as it increases tends to strengthen the contrast and to reduce the background. In reality, the discussion on the nature of light was fully developed even before these two men were born…*

What is Light: A Brief History

Chapter Two

The corpuscular model

The **corpuscular model of light** is a historical theory that describes light as being made up of tiny, discrete particles called corpuscles. This model was initially proposed by **Isaac Newton**. According to this model, light consists of small particles emitted by luminous objects, which travel in straight lines at high speeds.

Light as Particles: (i) Light consists of small, indivisible particles (corpuscles). (ii) These particles are emitted by sources of light and travel in straight lines.

Reflection: When light corpuscles strike a smooth surface, they are reflected, similar to how elastic balls bounce off a hard surface.

Refraction: During refraction, light corpuscles change their speed when entering a denser or rarer medium, which leads to a change in their direction.

Color: Newton suggested that the different colors of light correspond to corpuscles of different sizes.

High-Speed Motion: Corpuscles travel at extremely high speeds, which explains why light seems to travel instantaneously over short distances.

The corpuscular model

Strengths of the Model:

- It could explain basic phenomena like **reflection** and **refraction**.
- The straight-line motion of corpuscles aligned with the observation of sharp shadows.

Weaknesses of the Model:

- It struggled to explain certain phenomena, such as:
  - **Diffraction** and **interference** of light.
  - The bending of light around obstacles, which was better explained by the **wave theory of light**.
  - **Polarization**, which requires the concept of transverse waves.

The wave model

Huygens' Principle (1690): Huygens proposed that every point on a wavefront acts as a source of secondary wavelets, which combine to form the next wavefront. This principle explains reflection and refraction.

Thomas Young's Double-Slit Experiment (1801): Demonstrated **interference patterns**, providing strong evidence for the wave nature of light.

Fresnel and Diffraction: Augustin-Jean Fresnel developed mathematical theories for diffraction that aligned with wave behavior.

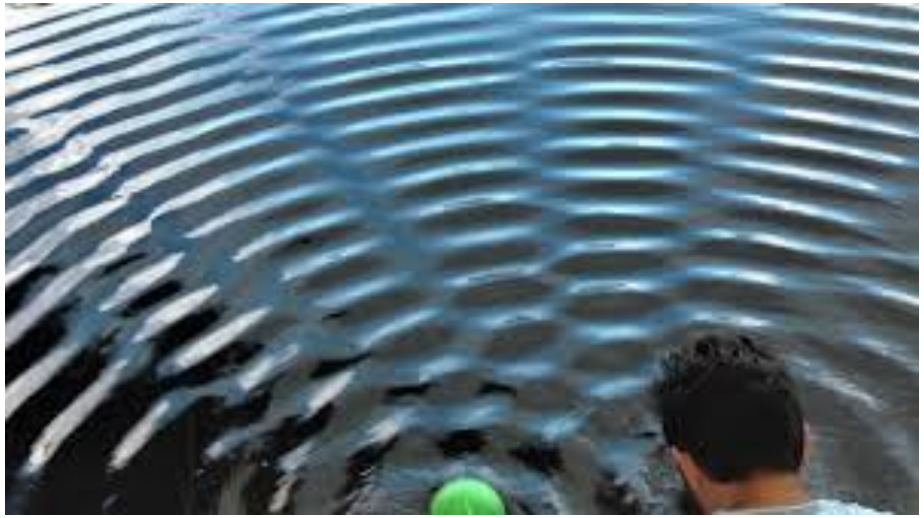
James Clerk Maxwell's Electromagnetic Theory (1864): Maxwell unified electricity and magnetism, showing that light is an **electromagnetic wave** consisting of oscillating electric and magnetic fields, propagating at the speed of light.

Two Beam Interference by Division of Wavefront

Chapter Fourteen

Interference pattern produced on the surface of water

Interference is a phenomenon where two or more waves superpose to form a resultant wave. The resultant wave's amplitude depends on the constructive or destructive interaction of the waves. Interference is a hallmark of the **wave nature of light** and other wave phenomena like sound and water waves.



Constructive Interference: Occurs when the crests of two waves align (are in phase).

Destructive Interference: Occurs when the crest of one wave aligns with the trough of another (are out of phase by 180°).

Interference of light: Young's Double-Slit Experiment

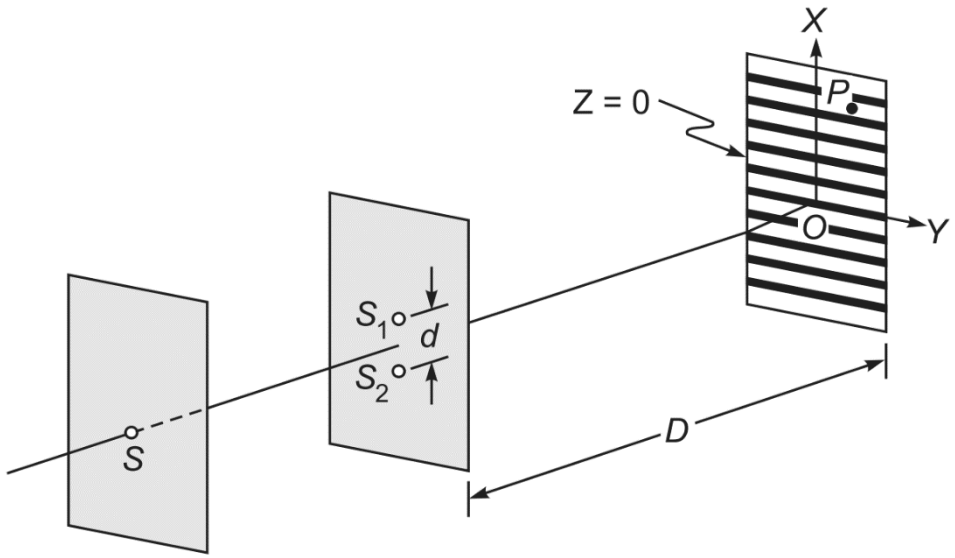


Fig. 14.6 Young's arrangement to produce interference pattern.

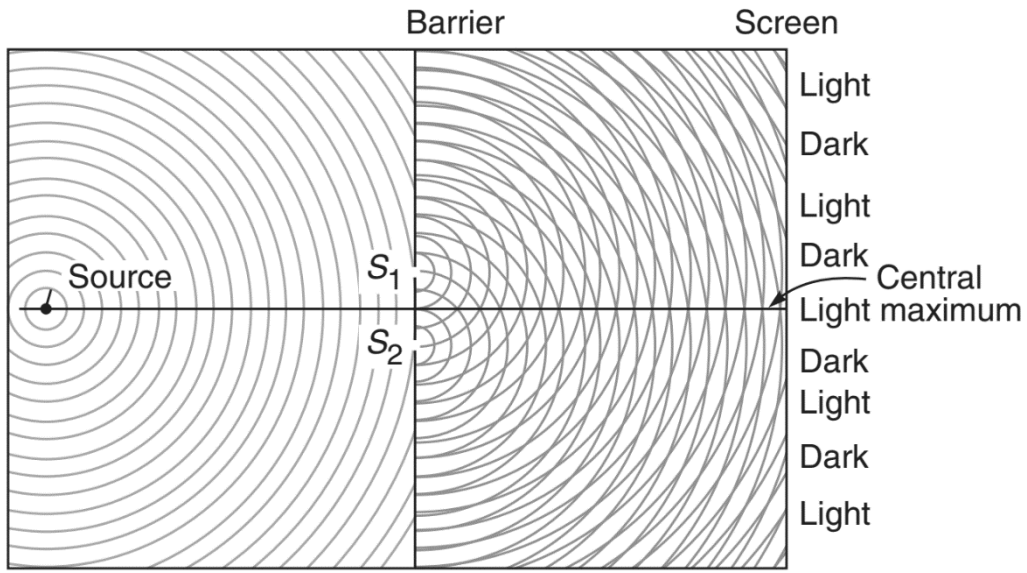


Fig. 14.7 Sections of the spherical wavefronts emanating from S, S_1 and S_2 (Adapted from Ref. 14.7; used with permission).

Interference of light: Young's Double-Slit Experiment

- Demonstrated that light passing through two closely spaced slits produces an interference pattern of bright and dark fringes on a screen.
- This pattern results from constructive and destructive interference of light waves emerging from the slits.

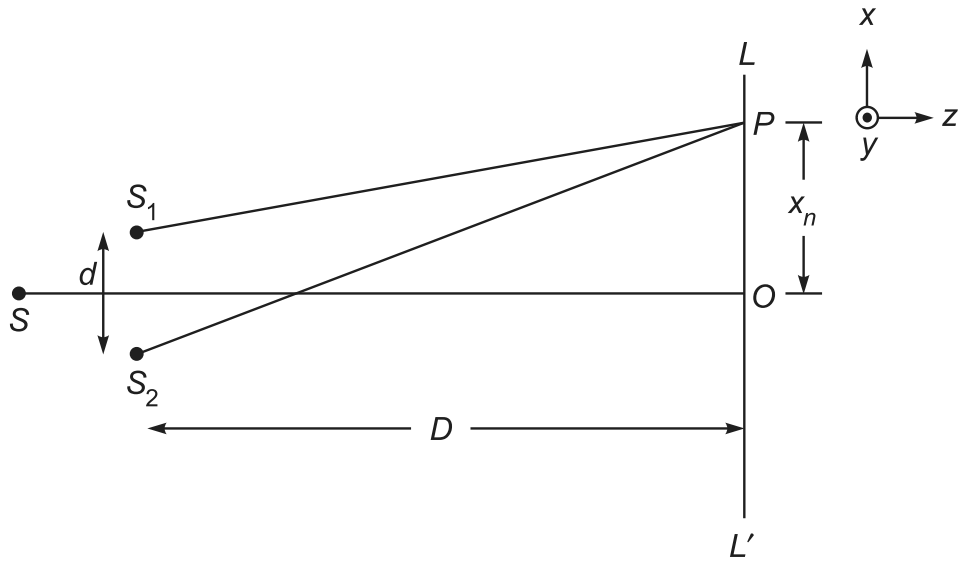


Fig. 14.8 Arrangement for producing Young's interference pattern.

Condition for bright fringes (constructive interference):

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \dots)$$

Condition for dark fringes (destructive interference):

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \dots)$$

where:

- d: Distance between the slits.
- λ: Wavelength of light.
- θ: Angle of the fringe from the central axis.

Interference of light: Young's Double-Slit Experiment

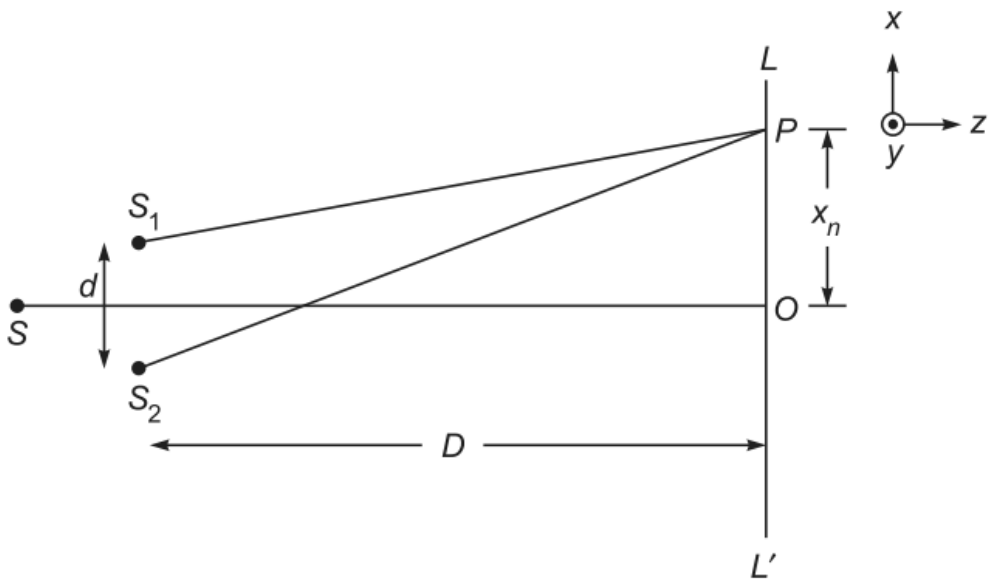


Fig. 14.8 Arrangement for producing Young's interference pattern.

For an arbitrary point P (on the line LL') to correspond to a maximum we must have

$$S_2P - S_1P = n\lambda;\ n = 0, 1, 2, 3, \ldots \qquad \textbf{(14.17)}$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x_n - \frac{d}{2}\right)^2\right] \qquad \textbf{(14.18)}$$

$$= 2x_n d$$

where $S_1S_2 = d$ and $OP = x_n$. Thus,

$$S_2P - S_1P = \frac{2x_n d}{S_2P + S_1P} \approx \frac{x_n d}{D} \qquad \textbf{(14.19)}$$

where in the last step we have replaced $S_2P + S_1P$ by $2D$ which will be valid when $D >>> d, x_n$.

Interference of light: Young's double-slit experiment

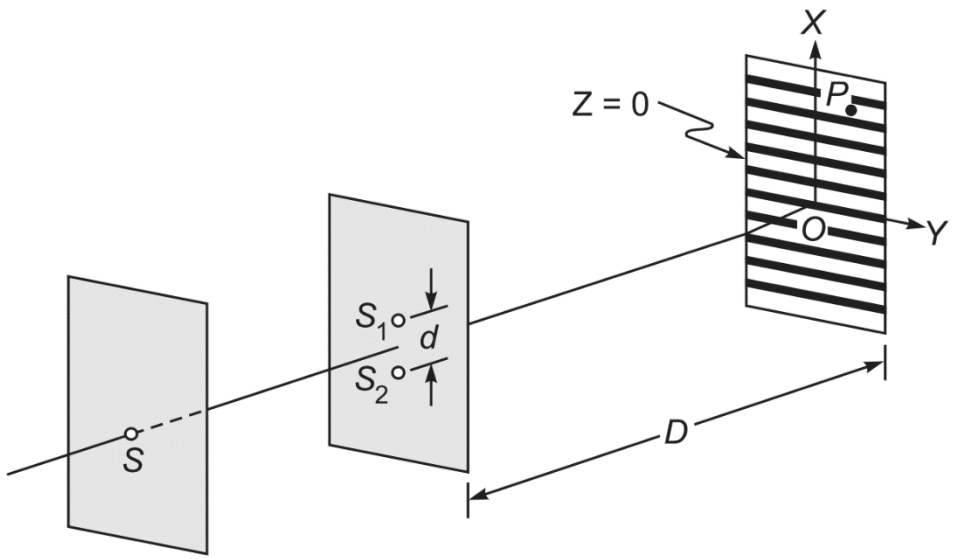


Fig. 14.6 Young's arrangement to produce interference pattern.

$$x_n = \frac{n\lambda D}{d} \qquad \textbf{(14.20)}$$

Thus, the bright (and dark) fringes are equally spaced and the distance between two consecutive bright (or dark) fringes is given by

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d} \qquad \textbf{(14.21)}$$

which is the expression for the fringe width.

The intensity distribution

Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields produced at the point P by S_1 and S_2 respectively (see Fig. 14.8). The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ will, in general, have different directions and different magnitudes. However, if the distances S_1P and S_2P are very large in comparison to the distance S_1S_2, the two fields will almost be in the same direction. Thus, we may write

$$\left.\begin{aligned} \mathbf{E}_1 &= \hat{\mathbf{i}}\, E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) \\ \text{and} \quad \mathbf{E}_2 &= \hat{\mathbf{i}}\, E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right) \end{aligned}\right\} \qquad \textbf{(14.25)}$$

where $\hat{\mathbf{i}}$ represents the unit vector along the direction of either of the electric fields. The resultant field will be given by

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= \hat{\mathbf{i}}\left[E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)\right. \\ &\qquad \left. + E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right] \end{aligned} \qquad \textbf{(14.26)}$$

The intensity (I) will be proportional to the square of the electric field and will be given by

$$I = KE^2 \qquad \textbf{(14.27)}$$

or

$$\begin{aligned} I = K\Bigg[& E_{01}^2 \cos^2\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + \\ & E_{02}^2 \cos^2\left(\frac{2\pi}{\lambda} S_2P - \omega t\right) + \\ & E_{01} E_{02} \left\{\cos\left[\frac{2\pi}{\lambda}(S_2P - S_1P)\right] + \right. \\ & \left.\cos\left[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\right]\right\}\Bigg] \end{aligned} \qquad \textbf{(14.28)}$$

where K is a proportionality constant.

$$\boxed{\langle \cos^2(\omega t - \theta)\rangle \approx \frac{1}{2}} \qquad \boxed{I_1 = \frac{1}{2} K E_{01}^2}$$

The intensity distribution

Thus the intensity, that a detector will record, will be given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \qquad \textbf{(14.30)}$$

where

$$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P) \qquad \textbf{(14.31)}$$

represents the phase difference between the displacements reaching the point P from S_1 and S_2. Further

$$I_1 = \frac{1}{2}KE_{01}^2$$

represents the intensity produced by the source S_1 if no light from S_2 is allowed to fall on the screen;

The intensity distribution

(a) The maximum and minimum values of $\cos\delta$ are +1 and −1, respectively; as such the maximum and minimum values of I are given by

$$\left.\begin{aligned} I_{\max} &= (\sqrt{I_1} + \sqrt{I_2})^2 \\ I_{\min} &= (\sqrt{I_1} - \sqrt{I_2})^2 \end{aligned}\right\} \quad \textbf{(14.32)}$$

and

The maximum intensity occurs when

$$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots$$

or

$$S_2P \sim S_1P = n\lambda,$$

and the minimum intensity occurs when

$$\delta = (2n+1)\pi\,;\; n = 0, 1, 2, \ldots$$

or

$$S_2P \sim S_1P = \left(n + \frac{1}{2}\right)\lambda$$

Notice that when $I_1 = I_2$, the intensity minimum is zero. In general, $I_1 \neq I_2$ and the minimum intensity is not zero.

(b) If the holes S_1 and S_2 are illuminated by different light sources (see Fig. 14.5), then the phase difference δ will remain constant for about 10^{-10} sec (see discussion in Sec. 14.3) and thus δ would also vary with time*** in a random way. If we now carry out the averaging over time scales which are of the order of 10^{-8} sec, then

$$\langle \cos\delta \rangle = 0$$

and we obtain

$$I = I_1 + I_2$$

Thus, for two incoherent sources, the resultant intensity is the sum of the intensities produced by each one of the sources independently and no interference pattern is observed.

(c) In the arrangement shown in Fig. 14.6, if the distances S_1P and S_2P are large in comparison to d, then

$$I_1 \approx I_2 = I_0 \text{ (say)}$$

and

$$I = 2I_0 + 2I_0\cos\delta = 4I_0\cos^2\frac{\delta}{2} \quad \textbf{(14.33)}$$

The intensity distribution

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

The intensity distribution (which is often termed as the $\cos^2$ pattern) is shown in Fig. 14.9. The actual fringe pattern (as it will appear on the screen) is shown in Fig. 14.10. Figures 14.10 (a) and (b) correspond to d = 0.005 mm ($\beta \approx$ 5 mm) and d = 0.025 mm ($\beta \approx$ 1 mm), respectively. Both figures correspond to D = 5 cm and $\lambda = 5 \times 10^{-5}$ cm. The values of the parameters are such that one can see the hyperbolic nature of the fringe pattern in Fig. 14.10(a).

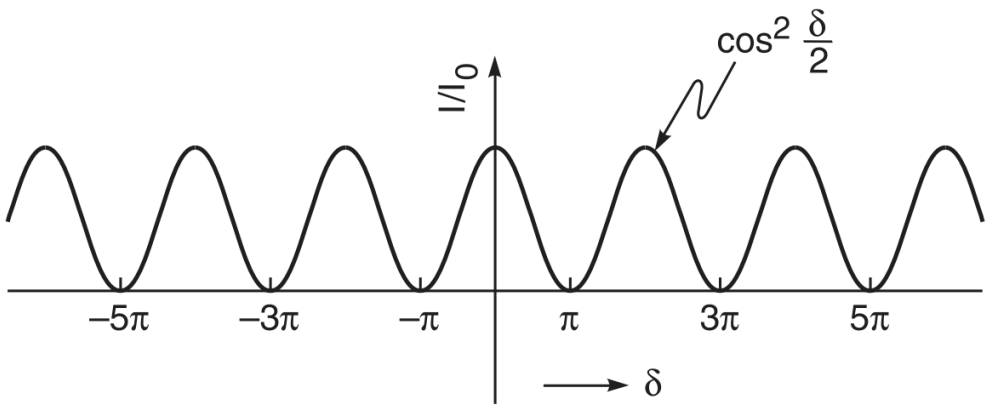
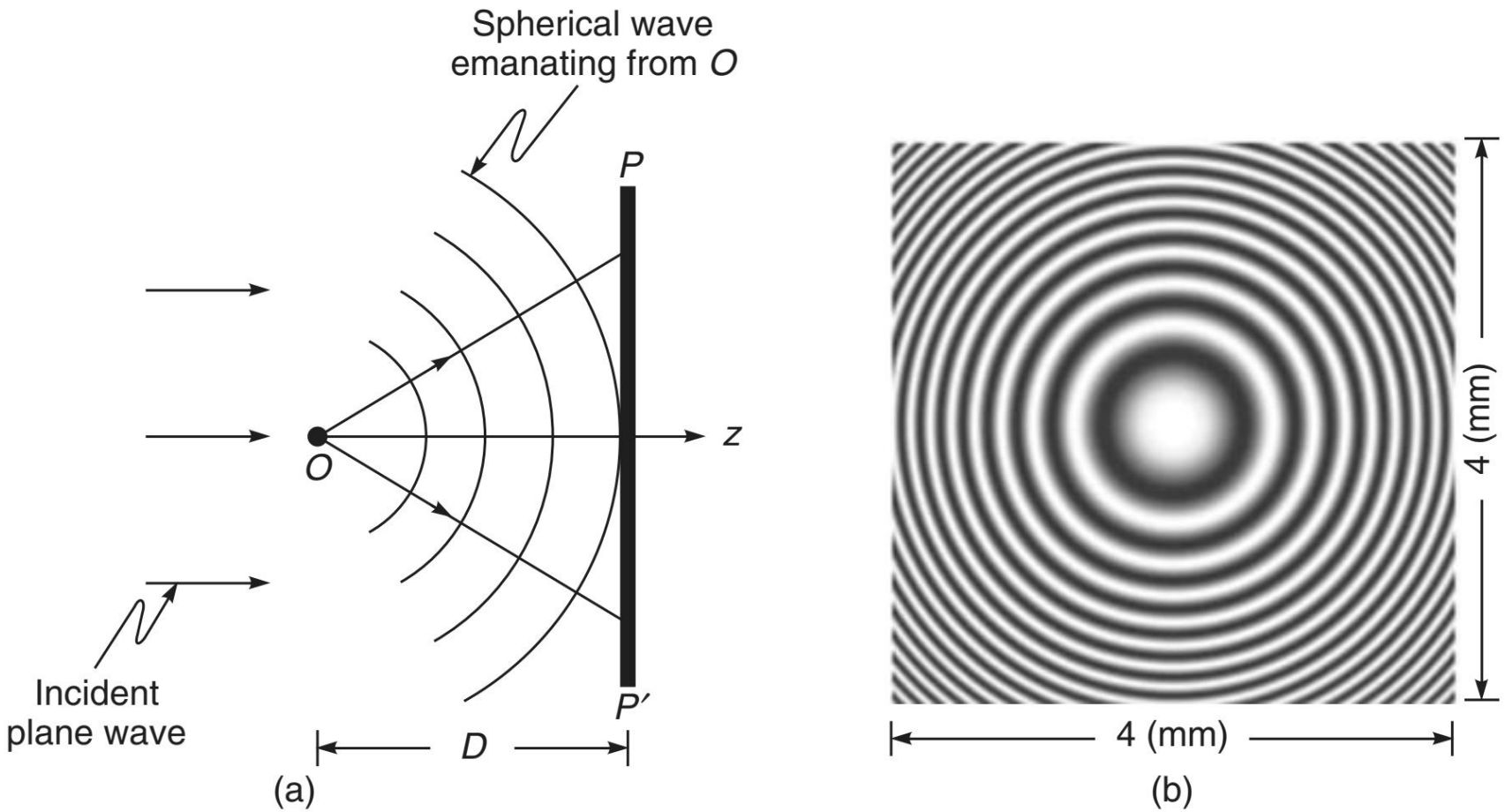


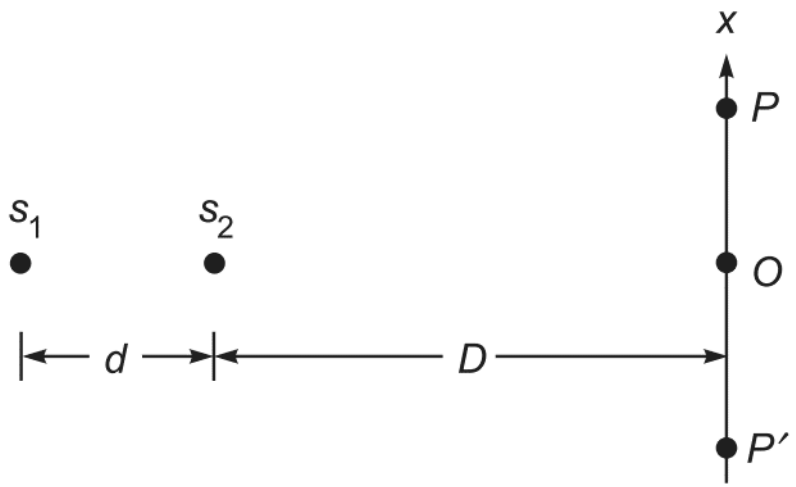
Fig. 14.9 The variation of intensity with δ.

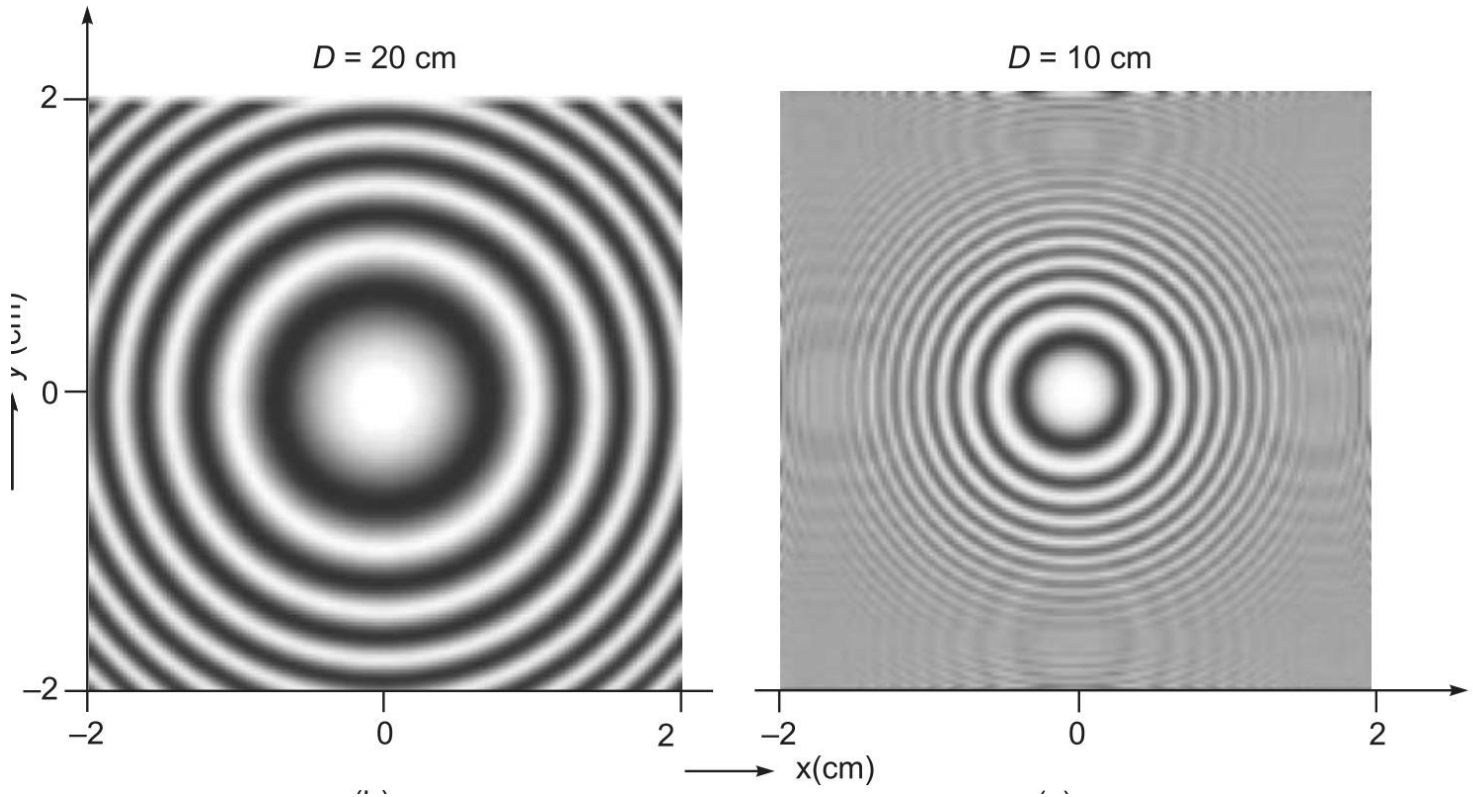
Interference pattern



(a) Superposition of a plane wave and a spherical wave emanating from the point O; (b) shows the interference fringes observed on the screen PP'.

Interference pattern





Conditions for interference

- **Coherent Sources**: Waves must have a constant phase difference and the same frequency.
- **Same Wavelength or Frequency**: Ensures stable and predictable patterns.
- **Superposition**: Waves must overlap in space and time.
- **Path Difference**: Determines constructive ($m\lambda$) or destructive ($(m + \frac{1}{2})\lambda$) interference.
- **Monochromatic Light**: Produces clearer patterns for optical interference.
- **Parallel Wavefronts**: Ensures proper overlap of the waves.
- **Finite Path Differences**: Coherence is maintained within the source's coherence length.

Coherence of Light

Coherence of light refers to the property of a light wave where the phase relationship between different points on the wave is well-defined and predictable over space and/or time. It is essential for producing stable interference patterns.

Temporal Coherence:

- Consistency of phase over time at a single point.
- Determines interference over a time interval or path difference.
- **Coherence Time** (t_c): Time phase remains predictable ($t_c = 1/\Delta f$).
- **Coherence Length** (L_c): Distance phase remains predictable ($L_c = c/\Delta f$).

Spatial Coherence:

- Uniformity of phase across different points in space.
- Determines interference over an area or aperture.

Coherence of Light

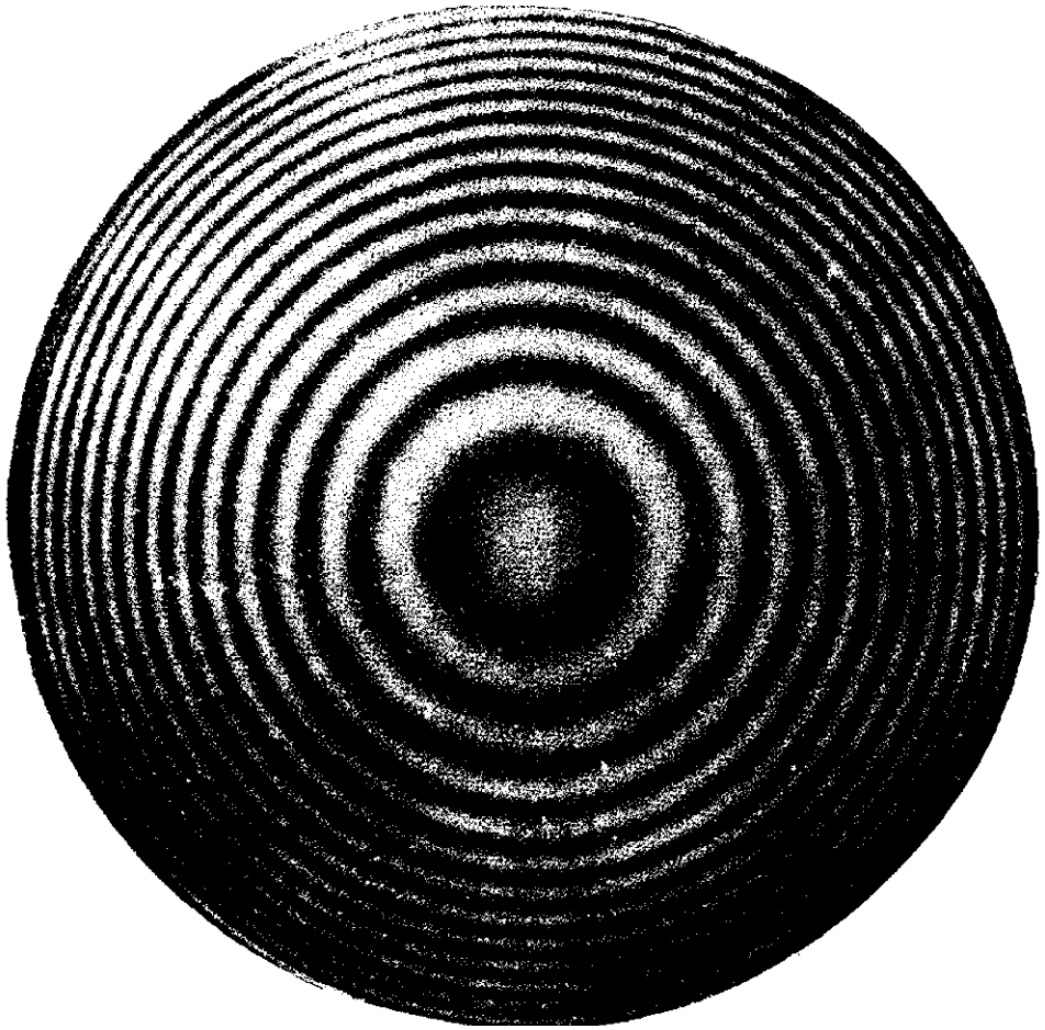
Coherence of Light Sources:

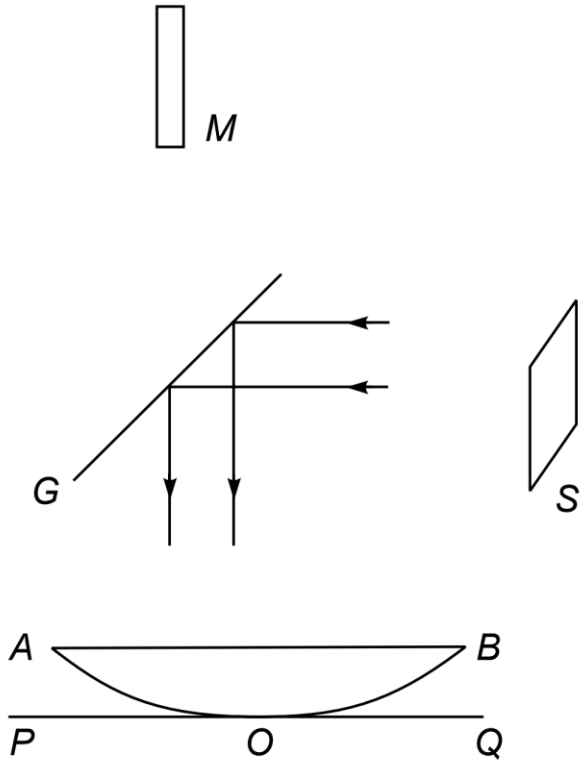
- **Highly Coherent**: Lasers (high temporal & spatial coherence).
- **Partially Coherent**: LEDs, sunlight, incandescent lamps (limited coherence).
- **Incoherent**: Rapidly fluctuating phase relationships (e.g., some natural light).

Applications:

- Temporal Coherence: Spectroscopy, optical coherence tomography.
- Spatial Coherence: Interference experiments, holography, imaging.

Newton's Ring





An arrangement for observing Newton's rings. Light from an extended source S is allowed to fall on a thin film formed between the plano-convex lens AOB and the plane glass plate POQ. M represents a traveling microscope.

Newton's Ring: Determination of Wavelength of Monochromatic Light

The diameter of the nth dark ring is given by:

$$D_n^2 = 4n\lambda R$$

Similarly, the diameter of the $(n + p)^{th}$ dark ring is given by:

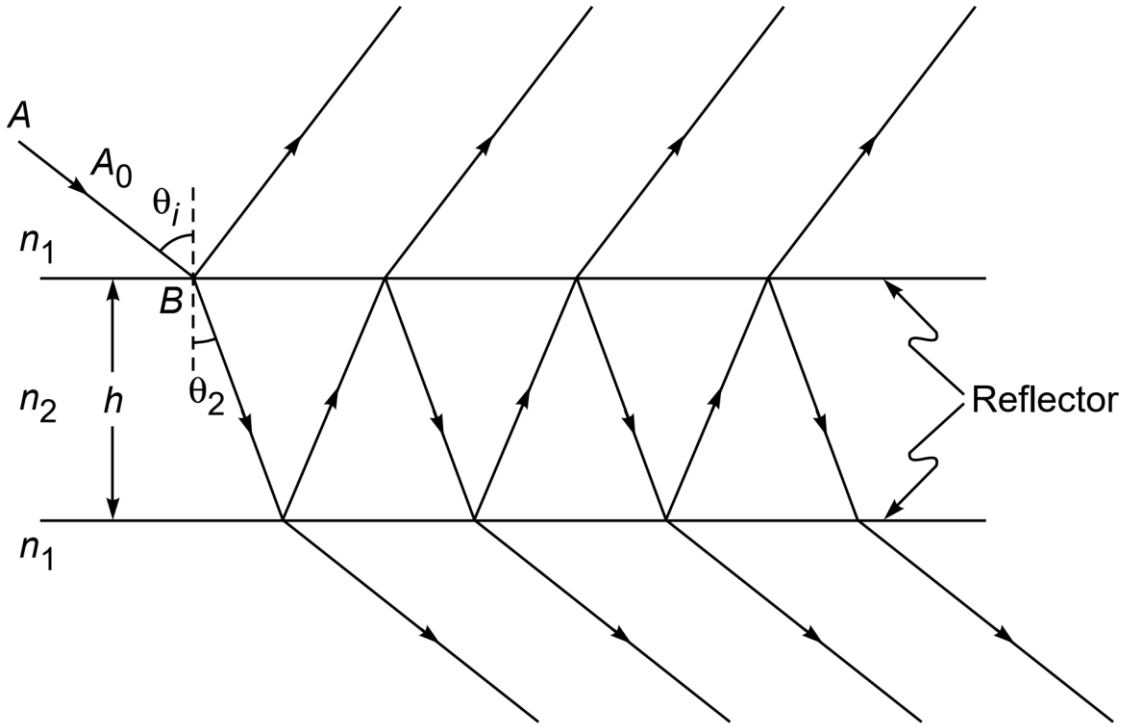
$$D_n^2 = 4(n+p)\lambda R$$

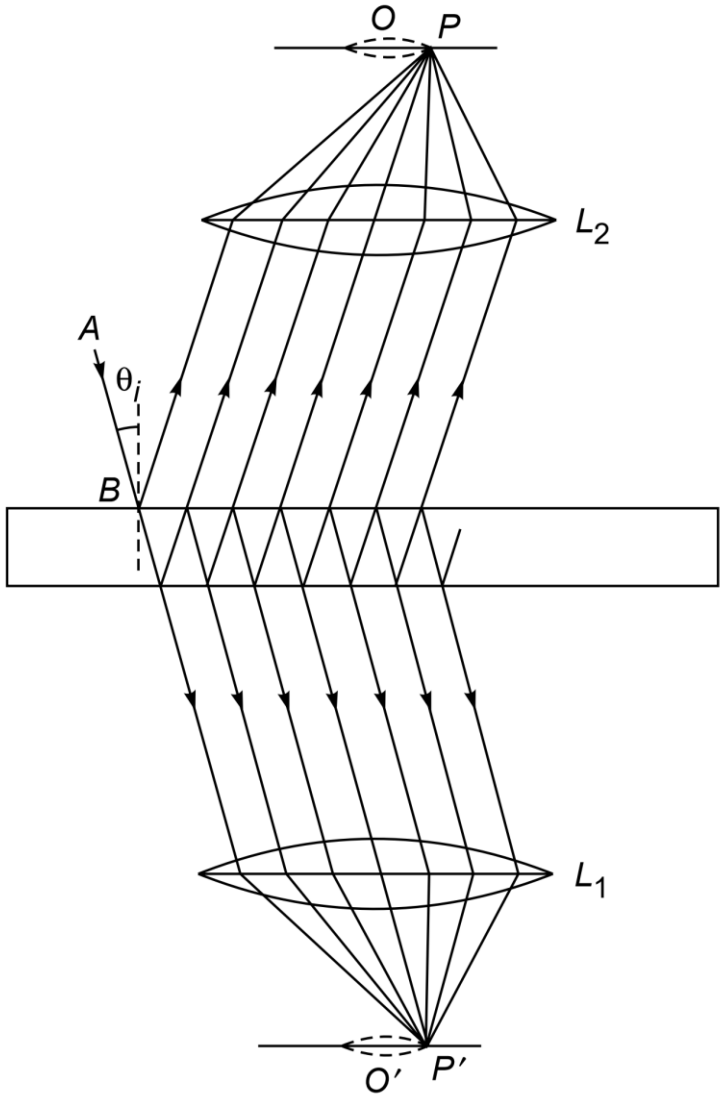
Subtracting

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

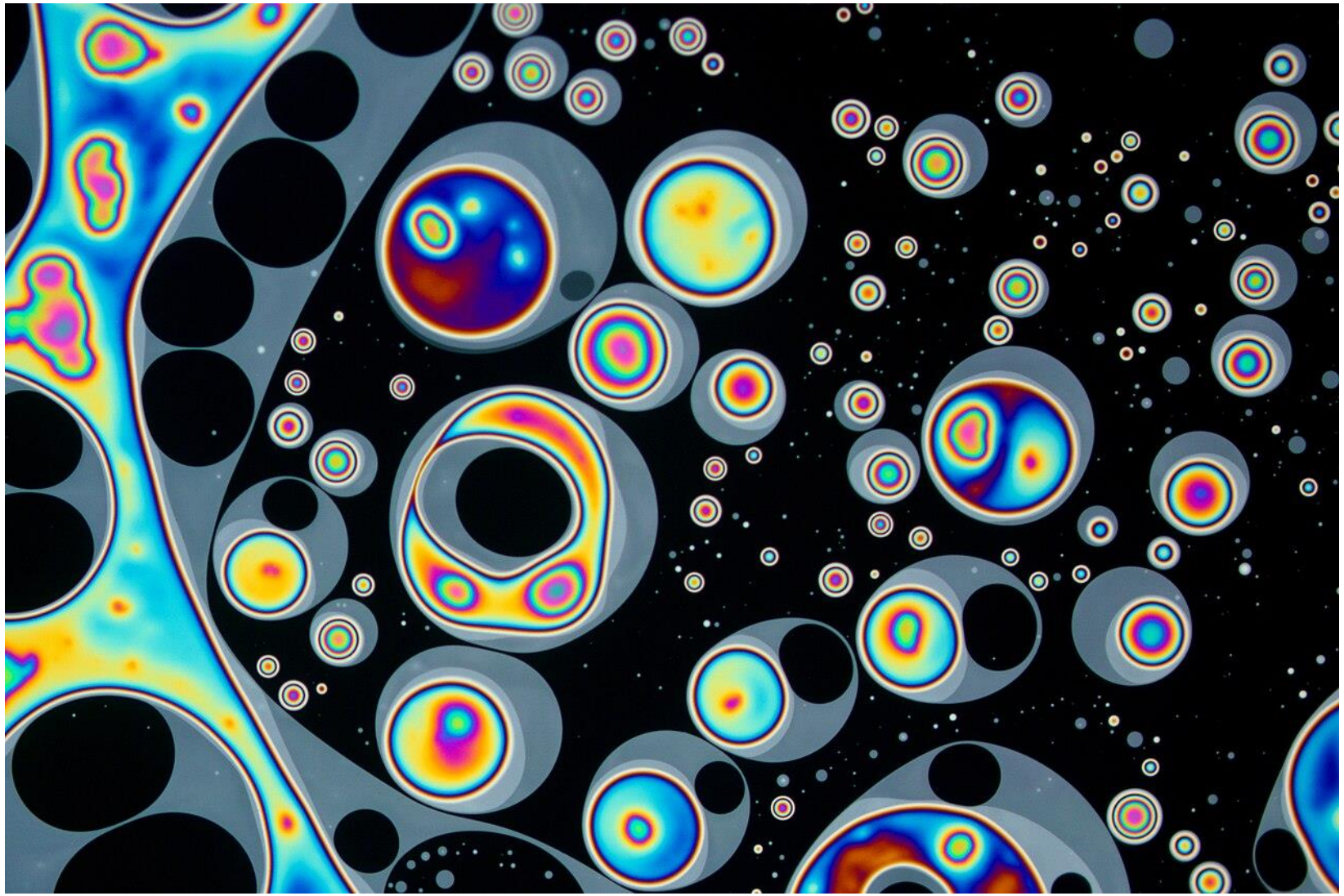
where, p is an integer.

Thin film interference





Thin film interference



Thin film interference

Diffraction

Part 4

Diffraction

Diffraction is a phenomenon that occurs when a wave encounters an obstacle, edge, or slit that is comparable in size to its wavelength. Instead of continuing in a straight line, the wave bends around the obstacle or spreads out as it passes through the slit. The effect is most noticeable when the size of the obstacle or slit is comparable to the wavelength of the wave.

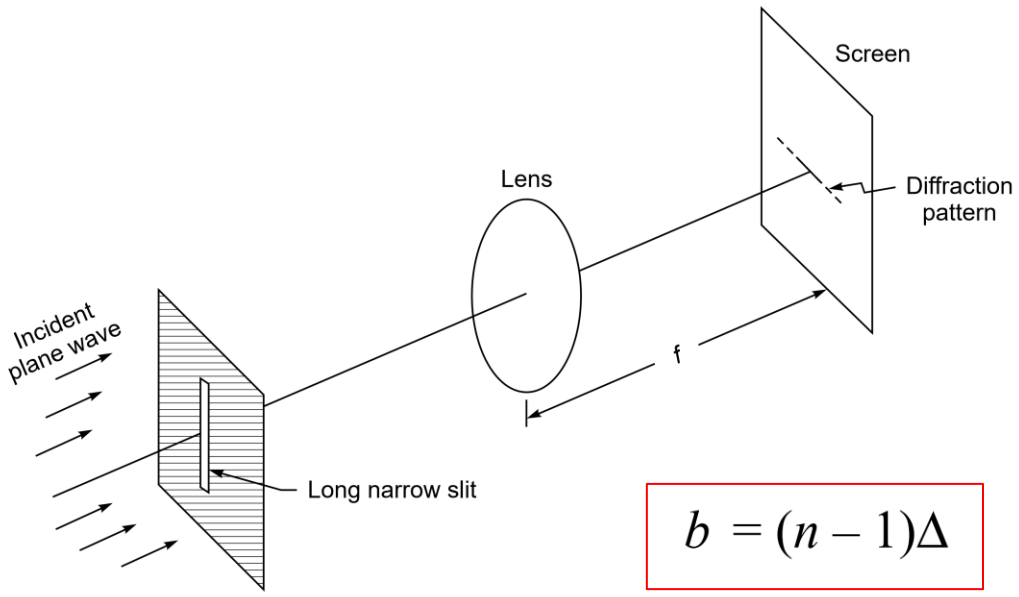
Types of Diffraction:

- **Fresnel Diffraction**: Occurs when the source of waves or the observation screen is at a finite distance.
- **Fraunhofer Diffraction**: Occurs when both the source and the observation screen are at infinite distances, typically requiring lenses to focus the wave.

Fraunhofer Diffraction: I

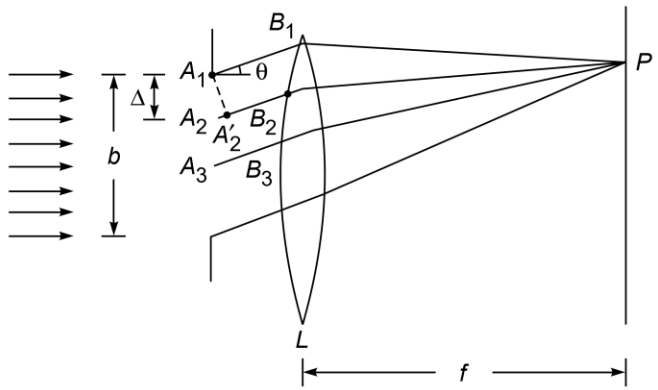
Chapter Eighteen

Single-slit diffraction pattern



$$b = (n-1)\Delta$$

(a)



(b)

Now, at point P, the amplitudes of the disturbances reaching from $A_1, A_2, \ldots$ will be very nearly the same because point P is at a distance which is very large in comparison to b [see Fig. 18.3(b)]. However, because of even slightly different path lengths to point P, the field produced by A_1 will differ in phase from the field produced by A_2.

For an incident plane wave, points $A_1, A_2, \ldots$ are in phase, and, therefore, the additional path traversed by the disturbance emanating from point A_2 will be A_2A_2', where A_2' is the foot of the perpendicular drawn from A_1 on A_2B_2. This follows from the fact that the optical paths A_1B_1P and $A_2'B_2P$ are the same. If the diffracted rays make an angle θ with the normal to the slit, then the path difference is

$$A_2A_2' = \Delta \sin\theta$$

The corresponding phase difference ϕ is given by

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta \qquad \textbf{(2)}$$

Single-slit diffraction pattern

Thus, if the field at point P due to the disturbance emanating from point A_1 is $a \cos \omega t$, then the field due to the disturbance emanating from A_2 is $a \cos(\omega t - \phi)$. Now the difference in the phases of the disturbance reaching from points A_2 and A_3 will also be ϕ, and thus the resultant field at point P is given by

$$E = a[\cos \omega t + \cos(\omega t - \phi) + \cdots + \cos[(\omega t - (n-1)\phi)] \quad \textbf{(3)}$$

where

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta$$

$$\cos \omega t + \cos(\omega t - \phi) + \cdots + \cos[\omega t - (n-1)\phi]$$

$$= \frac{\sin(n\phi/2)}{\sin(\phi/2)} \cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \quad \textbf{(4)}$$

Thus

$$E = E_0 \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right] \quad \textbf{(5)}$$

where the amplitude E_θ of the resultant field is given by[1]

$$E_\theta = a\frac{\sin(n\phi/2)}{\sin(\phi/2)} \quad \textbf{(6)}$$

In the limit of $n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to b$, we have

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta \sin\theta \to \frac{\pi}{\lambda} b \sin\theta$$

Further

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta = \frac{2\pi}{\lambda}\frac{b\sin\theta}{n}$$

will tend to zero, and, so we may write

$$E_\theta \approx \frac{a \sin(n\phi/2)}{\phi/2}$$

$$= na\,\frac{\sin(\pi b \sin\theta/\lambda)}{(\pi b \sin\theta/\lambda)}$$

$$= A\,\frac{\sin\beta}{\beta} \quad \textbf{(7)}$$

where[2]

$$A = na$$

Single-slit diffraction pattern

$$E_\theta \approx \frac{a \sin(n\phi/2)}{\phi/2}$$

$$= na \frac{\sin(\pi b \sin\theta/\lambda)}{(\pi b \sin\theta/\lambda)}$$

$$= A \frac{\sin\beta}{\beta} \qquad \textbf{(7)}$$

where[2]

$$A = na$$

and

$$\beta = \frac{\pi b \sin\theta}{\lambda} \qquad \textbf{(8)}$$

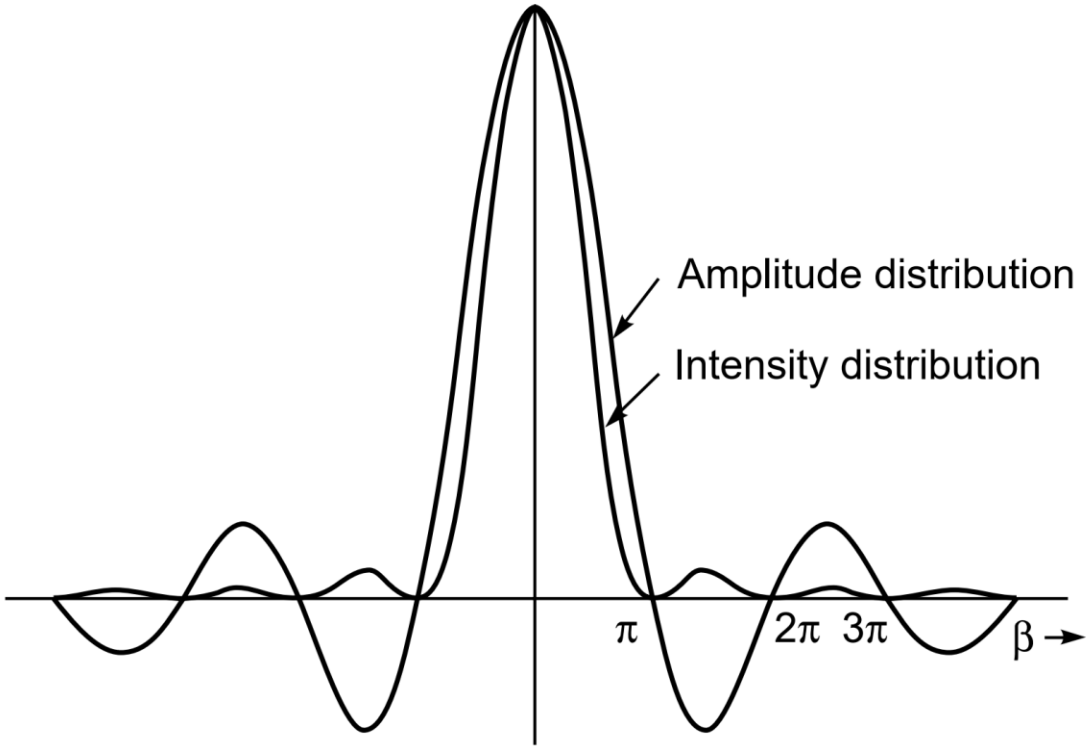
Thus

$$E = A \frac{\sin\beta}{\beta} \cos(\omega t - \beta) \qquad \textbf{(9)}$$

The corresponding intensity distribution is given by

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \qquad \textbf{(10)}$$

where I_0 represents the intensity at $\theta = 0$.



Positions of Maxima and Minima

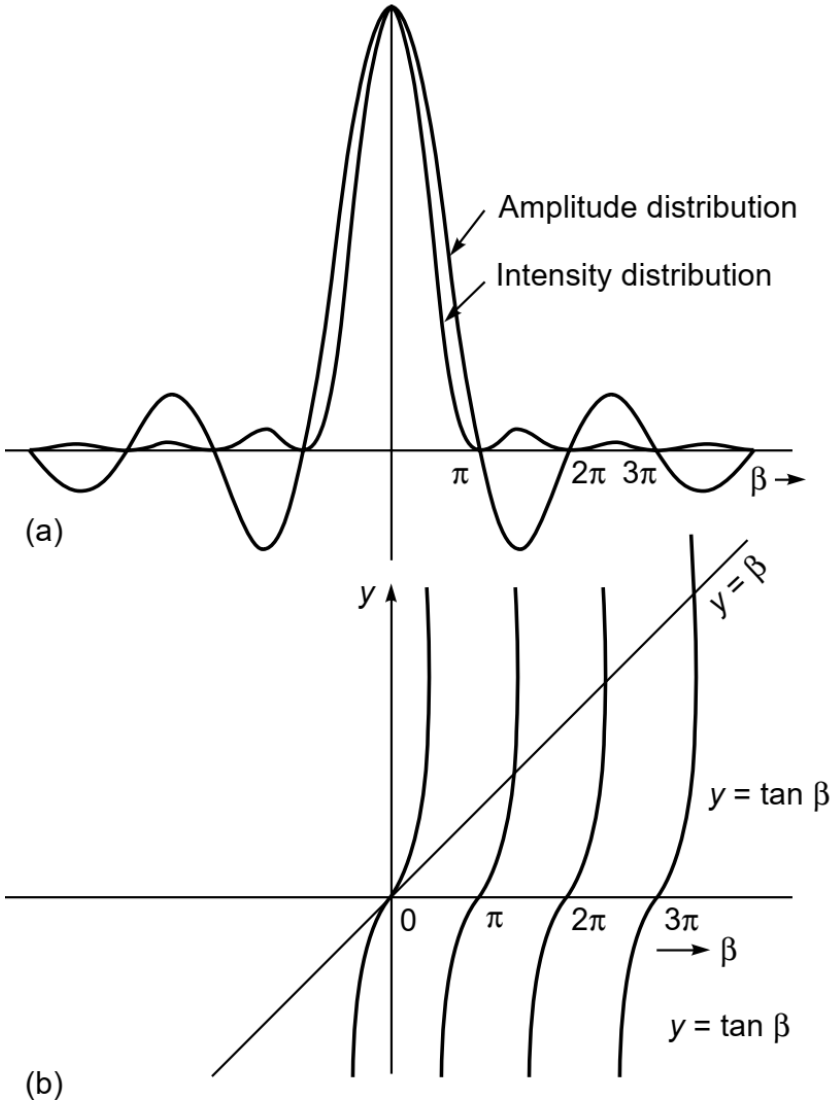
The variation of the intensity with β is shown in Fig. 18.4(a). It is obvious from Eq. (10) that the intensity is zero when

$$\beta = m\pi \qquad m \neq 0 \tag{11}$$

When $\beta = 0$, $(\sin\beta)/\beta = 1$ and $I = I_0$, which corresponds to the maximum of the intensity. Substituting the value of β, one obtains

$$b \sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3, \ldots \text{(minima)} \tag{12}$$

as the conditions for minima. The first minimum occurs at $\theta = \pm \sin^{-1}(\lambda/b)$; the second minimum occurs at $\theta = \pm \sin^{-1}(2\lambda/b)$, etc. Since $\sin\theta$ cannot exceed unity, the maximum value of m is the integer which is less than (and closest to) b/λ.



Single-slit diffraction pattern

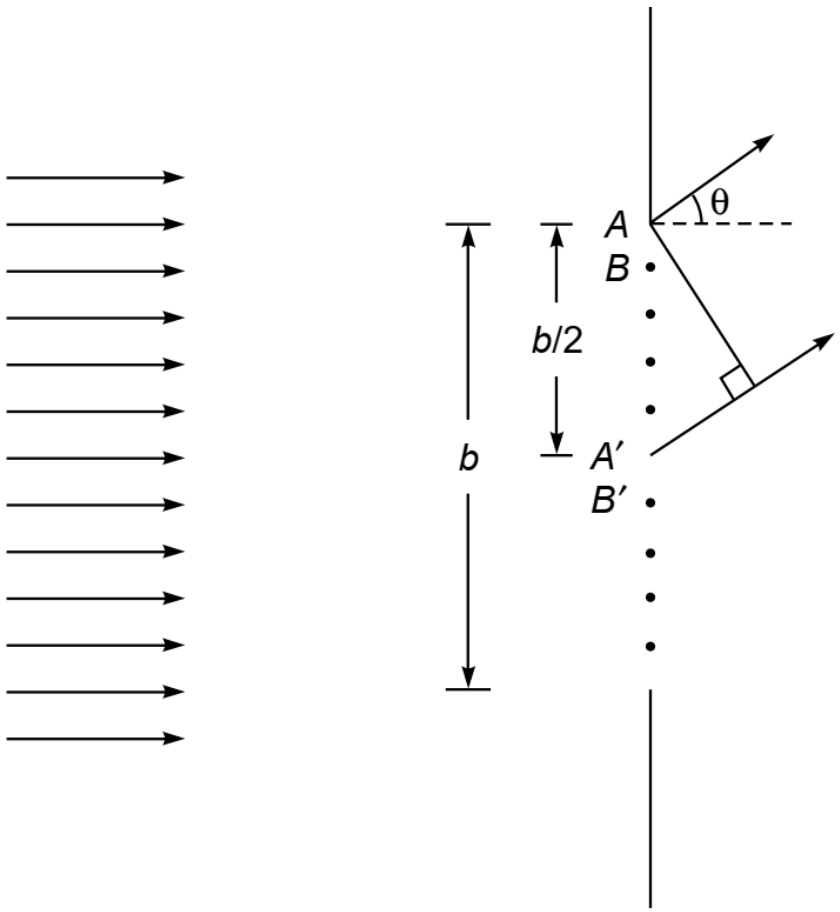
The positions of minima can be directly obtained by simple qualitative arguments. Let us consider the case $m = 1$. The angle θ satisfies the equation

$$b \sin \theta = \lambda \tag{13}$$

We divide the slit into two halves as shown in Fig. 18.5. Consider two points A and A' separated by a distance $b/2$. Clearly the path difference between the disturbances (reaching the point P) emanating from A and A' is $(b/2) \sin \theta$, which in this case is $\lambda/2$. The corresponding phase difference will be π, and the resultant disturbance will be zero. Similarly, the disturbance from point B will be canceled by the disturbance reaching from point B'. Thus the resultant disturbance due to the upper half of the slit will be canceled by the disturbances reaching from the lower half, and the resultant intensity will be zero. In a similar manner when

$$b \sin \theta = 2\lambda \tag{14}$$

we divide the slit into four parts; the first and second quarters cancel each other, and the third and fourth quarters cancel each other. Similarly when $m = 3$, the slit is divided into six parts, and so on.



18.5 The slit is divided into two halves for deriving the condition for the first minimum.

Positions of Maxima and Minima

To determine the positions of maxima, we differentiate Eq. (10) with respect to β and set it equal to zero. Thus

$$\frac{dI}{d\beta} = I_0\left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right) = 0$$

or

$$\sin\beta(\beta - \tan\beta) = 0 \qquad \textbf{(15)}$$

The condition $\sin\beta = 0$, or $\beta = m\pi$ ($m \neq 0$), corresponds to minima. The conditions for maxima are roots of the transcendental equation

$$\tan\beta = \beta \qquad \text{(maxima)} \qquad \textbf{(16)}$$

The root $\beta = 0$ corresponds to the central maximum. The other roots can be found by determining the points of intersections of the curves $y = \beta$ and $y = \tan\beta$ [see Fig. 18.4(b)]. The intersections occur at $\beta = 1.43\pi$, $\beta = 2.46\pi$, etc., and are known as the first maximum, the second maximum, etc. Since

$$\left(\frac{\sin 1.43\pi}{1.43\pi}\right)^2$$

is about 0.0496, the intensity of the first maximum is about 4.96% of the central maximum. Similarly, the intensities of the second and third maxima are about 1.68% and 0.83% of the central maximum, respectively.

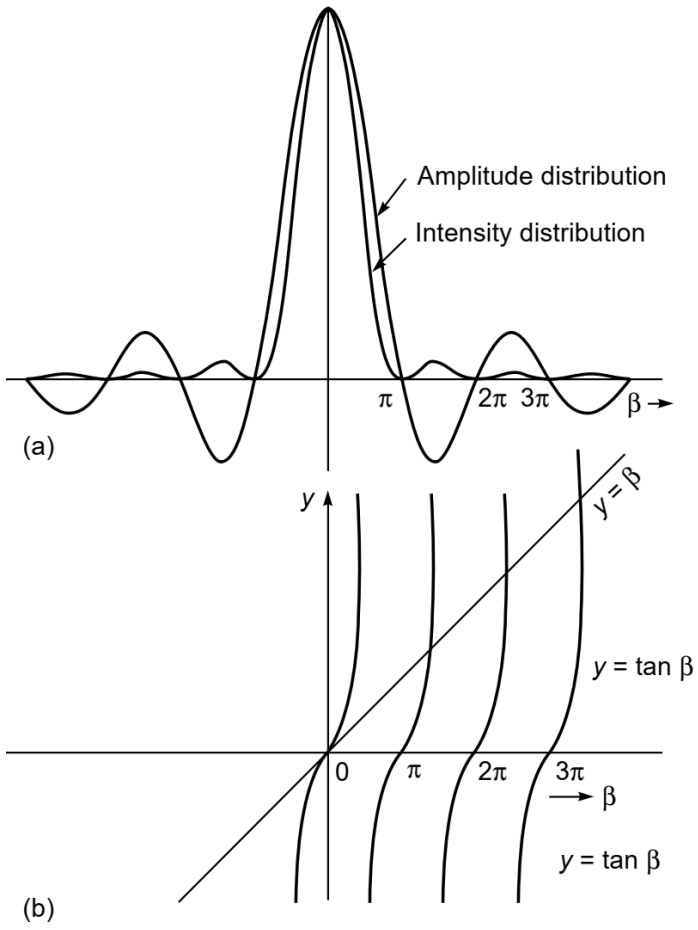


Fig. 18.4 (a) The intensity distribution corresponding to the single-slit Fraunhofer diffraction pattern. (b) Graphical method for determining the roots of the equation $\tan\beta = \beta$.

N-slit Fraunhofer diffraction pattern

We next consider the diffraction pattern produced by N parallel slits, each of width b; the distance between two consecutive slits is assumed to be d.

As before, we assume that each slit consists of n equally spaced point sources with spacing Δ (see Fig. 18.33). Thus the field at an arbitrary point P will essentially be a sum of N terms:

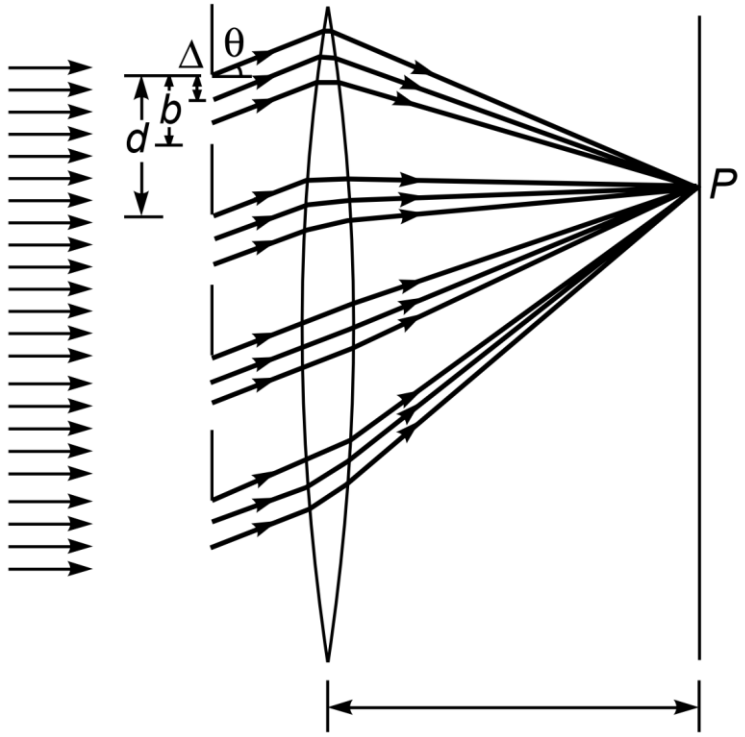
$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) + A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

$$+ \cdots + A\frac{\sin\beta}{\beta}\cos[\omega t - \beta - (N-1)\Phi_1] \qquad \textbf{(48)}$$

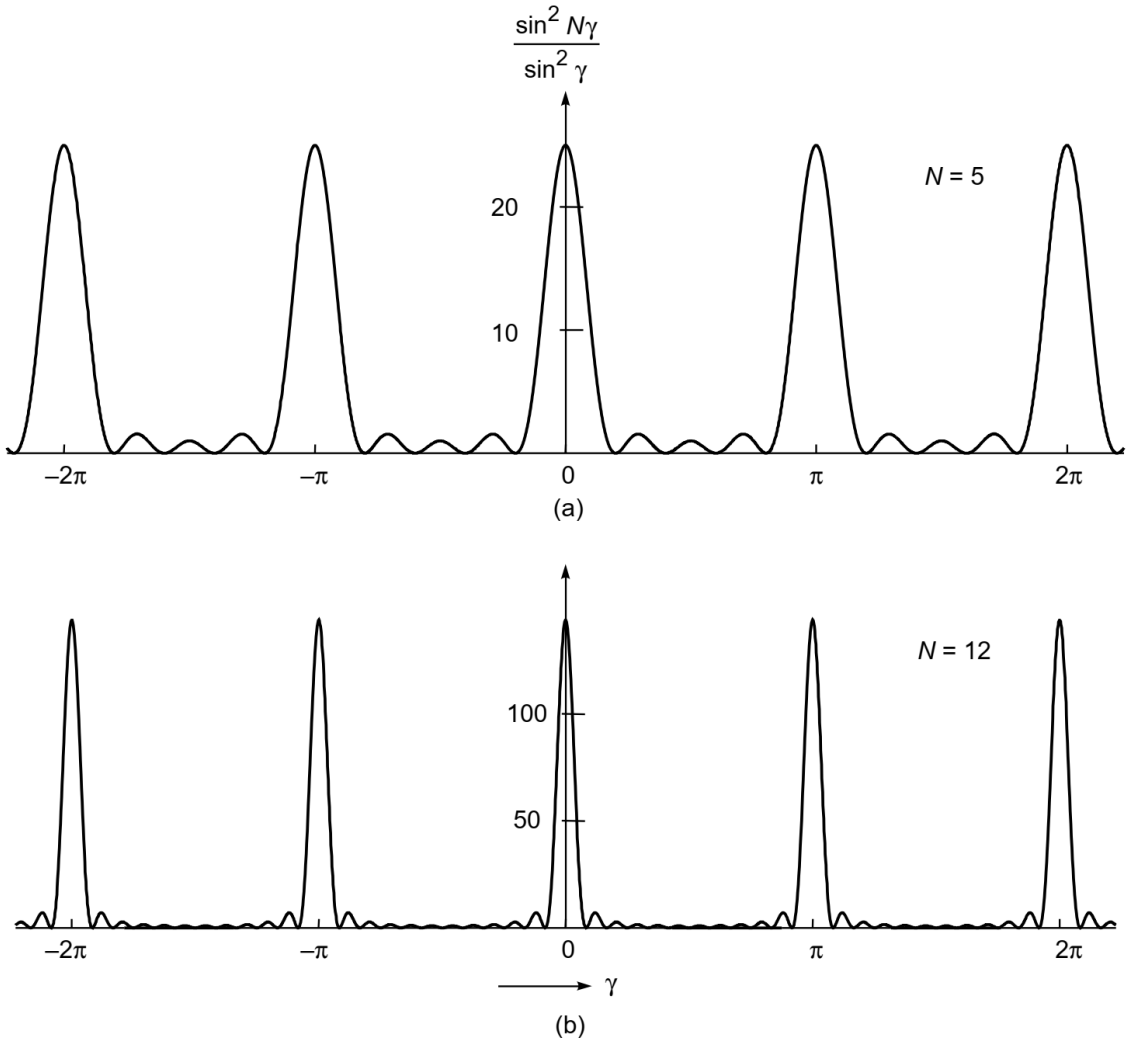
$$= \frac{A\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}\cos\left[\omega t - \beta - \frac{1}{2}(N-1)\Phi_1\right] \qquad \textbf{(49)}$$

where

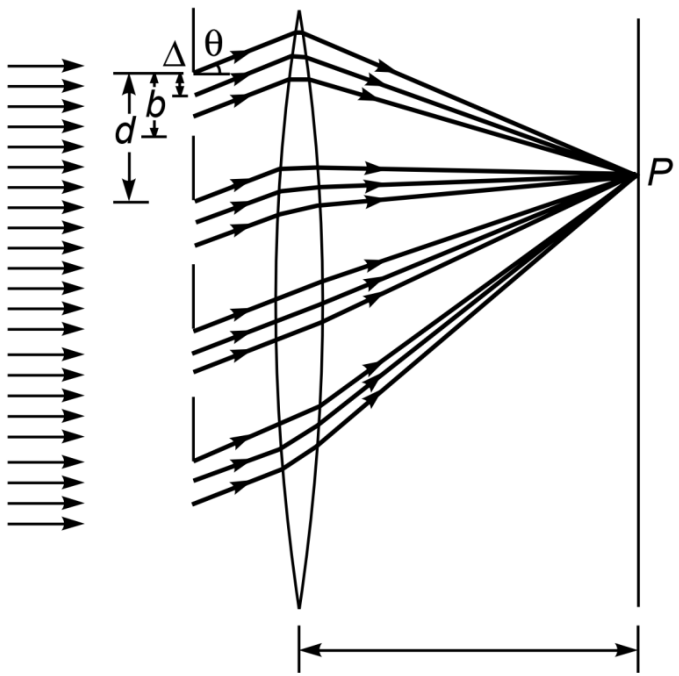
$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda}d\sin\theta$$



N-slit Fraunhofer diffraction pattern



18.34 The variation of the function $(\sin^2 N\gamma)/\sin^2\gamma$ with γ for $N = 5$ and 12. As N becomes larger, the function becomes more and more sharply peaked at $\gamma = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.



$$I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

Positions of Maxima and Minima

When the value of N is very large, one obtains intense maxima at $\gamma \simeq m\pi$, i.e., when

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, \ldots \tag{51}$$

This can be easily seen by noting that

$$\lim_{\gamma \to m\pi} \frac{\sin N\gamma}{\sin \gamma} = \lim_{\gamma \to m\pi} \frac{N \cos N\gamma}{\cos \gamma} = \pm N$$

thus, the resultant amplitude and the corresponding intensity distributions are given by

$$E = N \frac{A \sin \beta}{\beta} \tag{52}$$

and

$$I = N^2 I_0 \frac{\sin^2 \beta}{\beta^2} \tag{53}$$

where

$$\beta = \frac{\pi b \sin \theta}{\lambda} = \frac{\pi b}{\lambda} \frac{m\lambda}{d} = \frac{\pi b m}{d} \tag{54}$$

Such maxima are known as principal maxima.

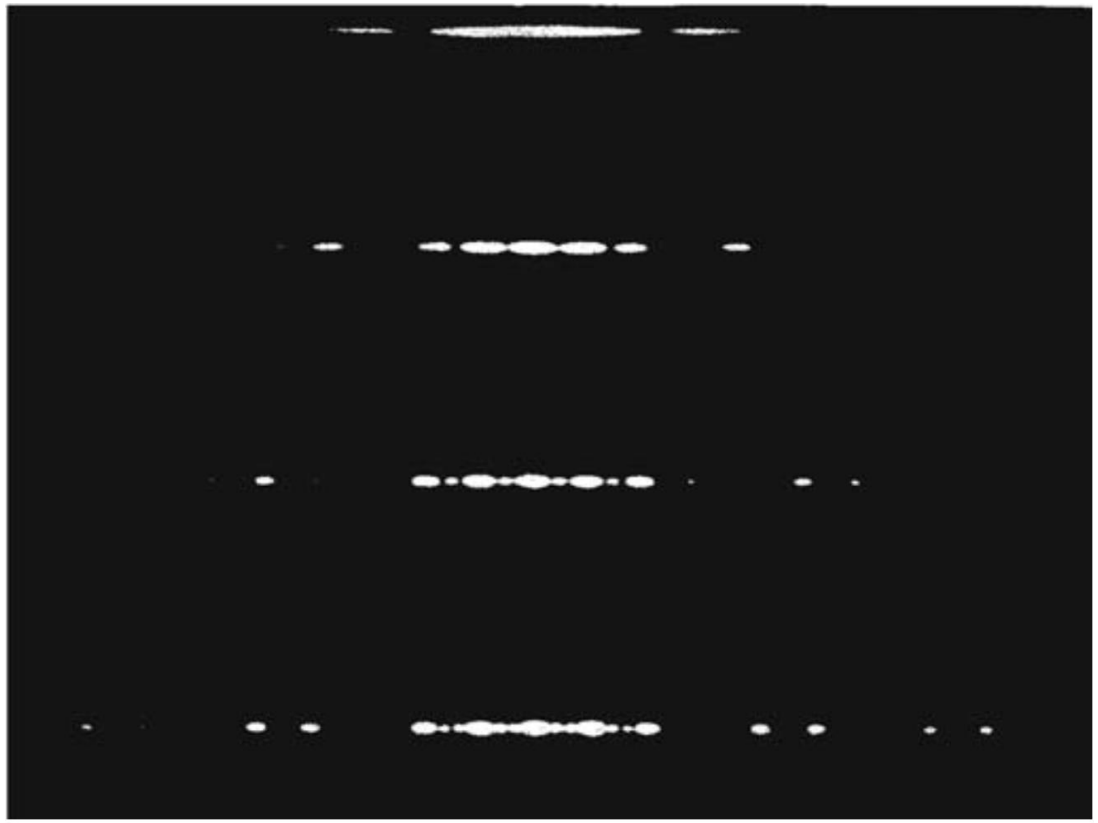


Fig. 18.35 The multiple-slit Fraunhofer diffraction patterns corresponding to b = 0.0044 cm, d = 0.0132 cm, and $\lambda = 6.328 \times 10^{-5}$ cm. The number of slits is 1, 2, 3, and 4 respectively (After Ref. 17; used with permission).

The diffraction grating

It consists of a large number of equally spaced parallel slits or grooves, which cause the incoming light to diffract and interfere, producing a pattern of maxima and minima.

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, \ldots.$$

d: The spacing between adjacent slits (grating spacing)

θ: The diffraction angle at which light is diffracted

m: The order of the diffraction maximum ($m = 0, 1, 2, \ldots$)

λ: The wavelength of the light

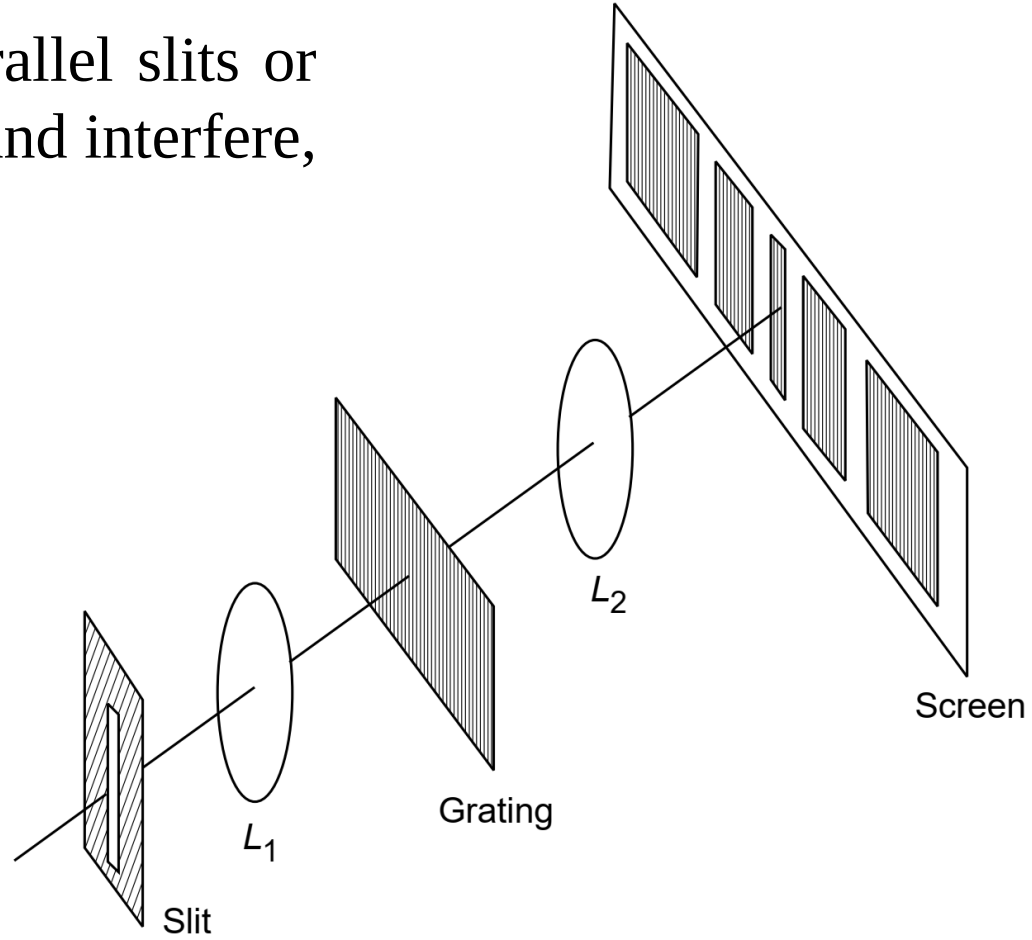


Fig. 18.38 If instead of a point source we have a slit in the focal plane of L_1, then we will obtain bands on the focal plane of L_2.

Example 18.10 Consider a diffraction grating with 15,000 lines per inch. (a) Show that if we use a white light source, the second- and third-order spectra overlap. (b) What will be the angular separation of the D_1 and D_2 lines of sodium in the second-order spectra?

Solution: (a) The grating element is

$$d = \frac{2.54}{15{,}000} = 1.69 \times 10^{-4} \text{ cm}$$

Let $\theta_{m\nu}$ and θ_{mr} represent the angles of diffraction for the mth-order spectrum corresponding to the violet and red colors, respectively. Thus

$$\theta_{2\nu} = \sin^{-1}\left(\frac{2\times 4\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.473 \approx 28.2°$$

$$\theta_{2r} = \sin^{-1}\left(\frac{2\times 7\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.828 \approx 55.90°$$

and

$$\theta_{3\nu} = \sin^{-1}\left(\frac{3\times 4\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.710 \approx 45.23°$$

where we have assumed the wavelengths of the violet and red colors to be 4×10^{-5} and 7×10^{-5} cm, respectively. Since $\theta_{2r} > \theta_{3\nu}$, the second- and third-order spectra will overlap. Further since $\sin \theta_{3r} > 1$, the third-order spectrum for the red color will not be observed.

(b) Since $d \sin \theta = m\lambda$, we have for small $\Delta\lambda$:

$$(d \cos \theta)\, \Delta\theta = m(\Delta\lambda)$$

or

$$\Delta\theta = \frac{m\Delta\lambda}{d\left\{1-(m\lambda/d)^2\right\}^{1/2}}$$

$$\simeq \frac{2\times 6\times 10^{-8}}{1.69\times 10^{-4}\left[1-\left(2\times 6\times 10^{-5}/1.69\times 10^{-4}\right)^2\right]^{1/2}}$$

$$\simeq 0.0010 \text{ rad} \simeq 3.44 \text{ minutes}$$

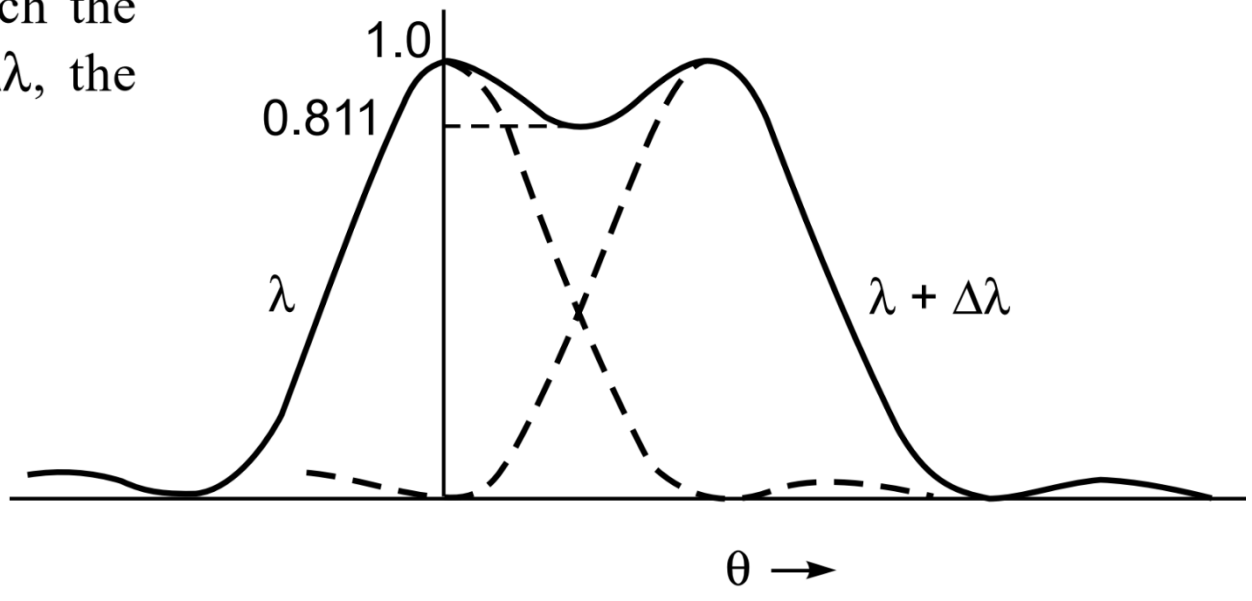
Thus, if we are using telescope of angular magnification 10, the two lines will appear to have an angular separation of 34.4 minutes.

Resolving power of a grating

In the case of a grating, the resolving power refers to the power of distinguishing two nearby spectral lines and is defined by the

$$R = \frac{\lambda}{\Delta\lambda} \qquad \textbf{(66)}$$

where $\Delta\lambda$ is the separation of two wavelengths which the grating can just resolve; the smaller the value of $\Delta\lambda$, the larger the resolving power.



XRD (X-ray diffraction)

$$2d_{hkl} \sin \theta = m\lambda$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

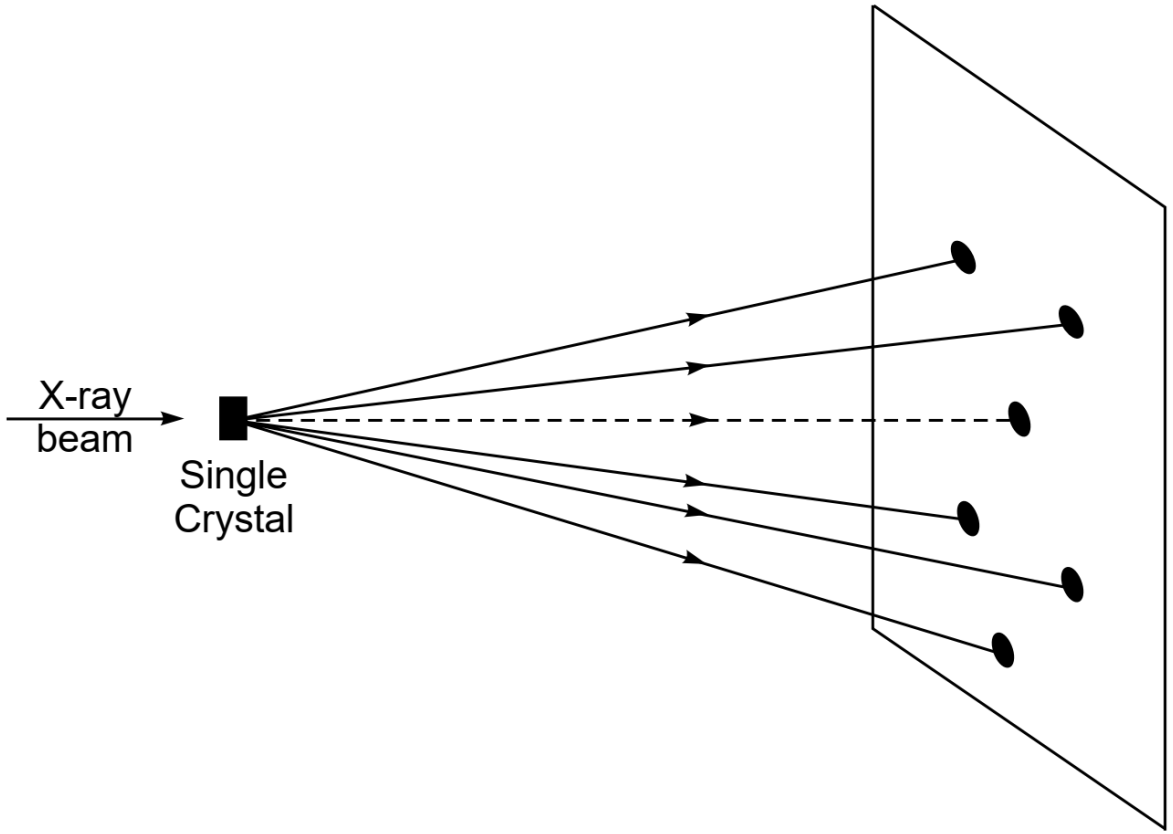


Fig. 18.50 When a polychromatic X-ray beam falls on a single crystal, one obtains Laue spots. Each set of planes chooses its own wavelength to satisfy the Bragg relation given by Eq. (84).

Polarization and Double Refraction

Chapter Twenty-Two

What is Natural Light?

Natural light refers to the light that comes from natural sources like the Sun, stars, lightning, or fire. It is a form of unpolarized light, meaning that the electric field vectors of the light waves oscillate in all possible directions perpendicular to the direction of propagation.

Characteristics of Natural Light

1. **Unpolarized**: In its natural state, the oscillations of the electric field are random and occur in all directions perpendicular to the wave's travel.
2. **Broad Spectrum**: Natural light contains a wide range of wavelengths, encompassing visible light, infrared, and ultraviolet radiation.
3. **Intensity and Color Dependence**: The intensity and color of natural light vary based on the source and atmospheric conditions (e.g., sunlight changes with time of day and weather).
4. **Abundance**: Natural light is the primary source of illumination for the Earth.

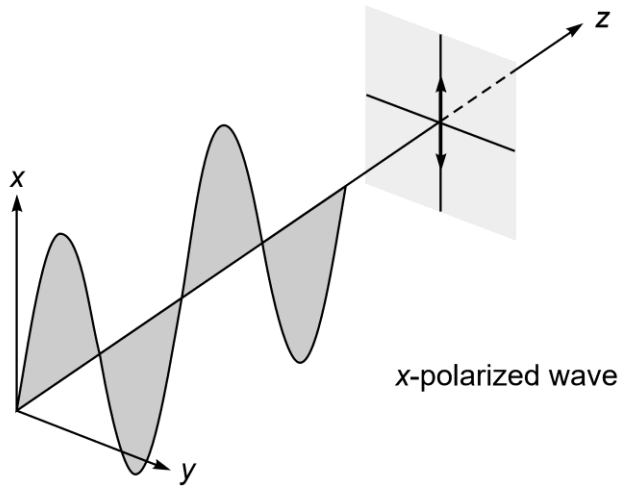
Unpolarized Light

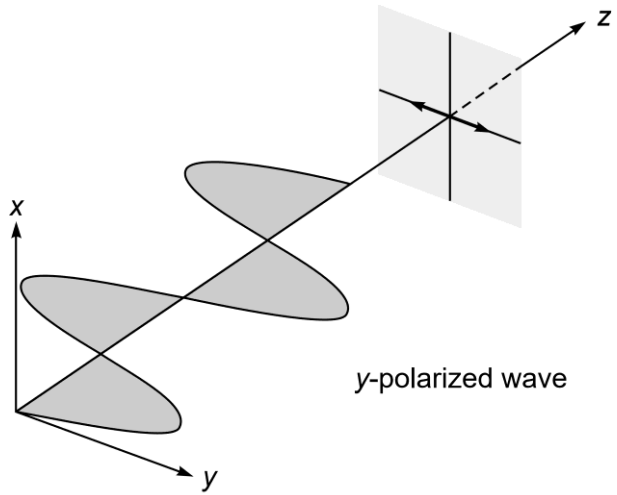
- **Definition**: Unpolarized light consists of waves with electric field vectors that oscillate randomly in all possible directions perpendicular to the direction of propagation.
- **Examples**: Sunlight, light from an incandescent bulb, and candlelight.
- **Visualization**: Imagine many individual waves with electric fields pointing in different directions at any given moment.

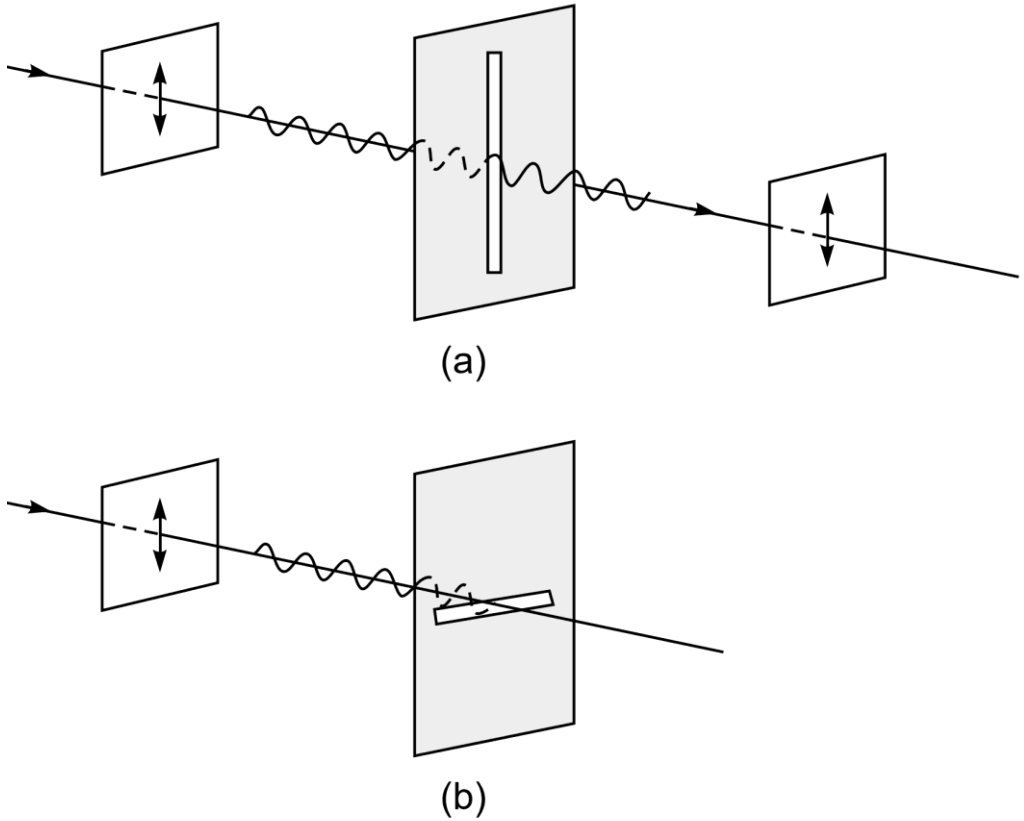
Polarized Light

- **Definition**: Polarized light consists of waves in which the electric field vectors oscillate in a specific, well-defined direction.
- **Types of Polarization**:
  1. **Linear Polarization**: The electric field oscillates in a single direction.
  2. **Circular Polarization**: The electric field rotates in a circular motion as the wave propagates.
  3. **Elliptical Polarization**: A more general case where the electric field traces out an ellipse.
- **Examples**: Light reflected off a surface (like water or glass), light passing through a polarizing filter, and laser light.

Polarized Light







22.3 If a linearly polarized transverse wave (propagating on a string) is incident on a long narrow slit, then the slit will allow only the component of the displacement, which is along the length of the slit, to pass through.

Polarization of Light

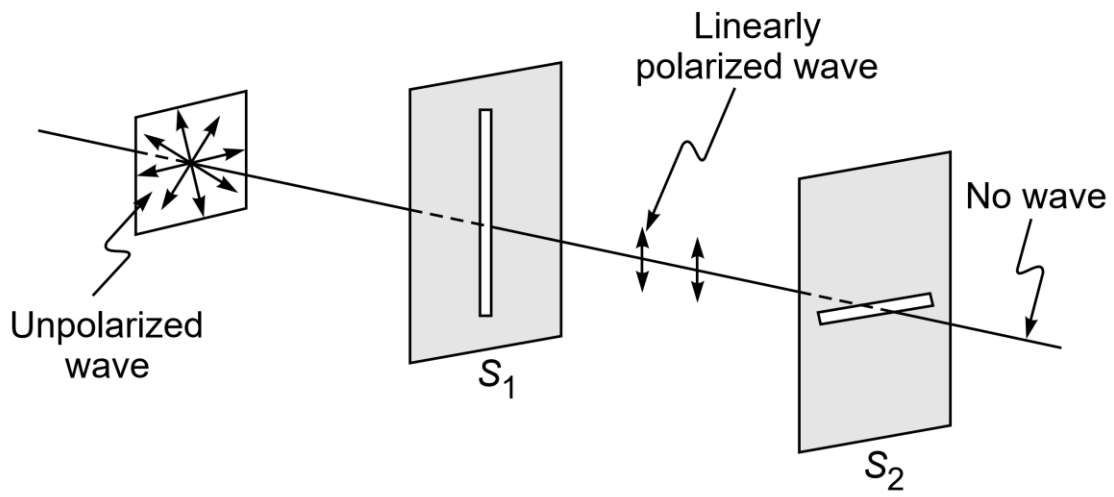
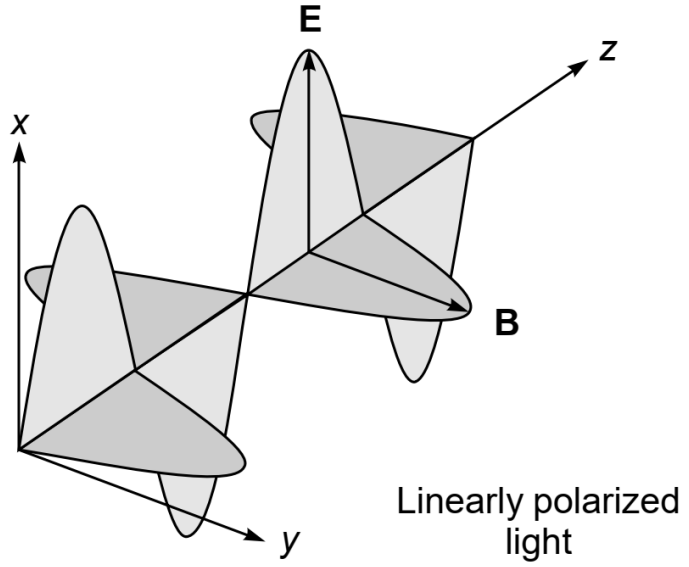


Fig. 22.4 If an unpolarized wave propagating on a string is incident on a long narrow slit S_1, then the transmitted beam is linearly polarized and its amplitude does not depend on the orientation of S_1. If this polarized wave is allowed to pass through another slit S_2, then the intensity of the emerging wave depends on the relative orientation of S_2 with respect to S_1.

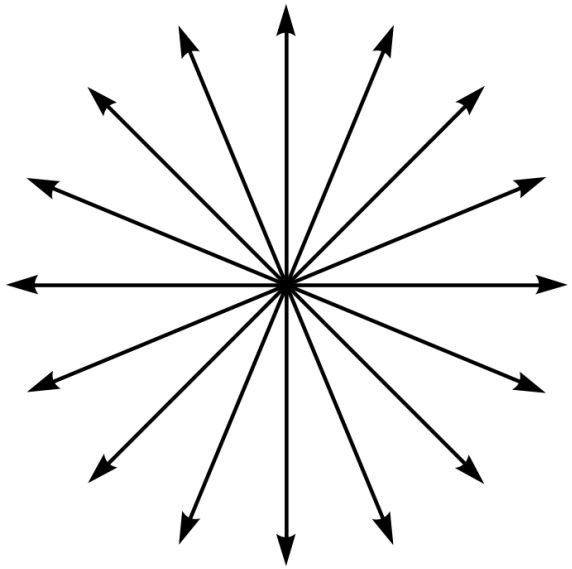


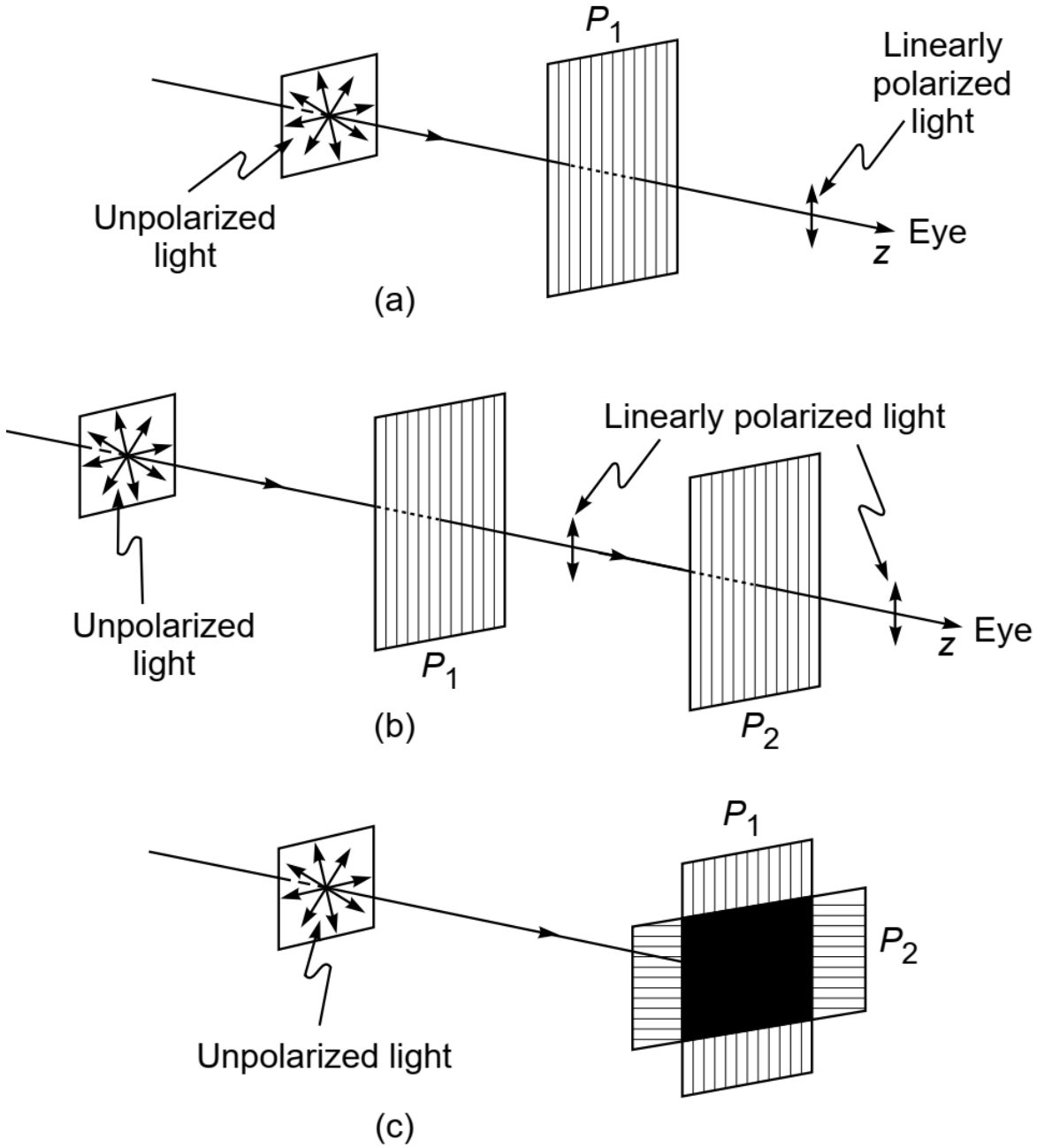
An x-polarized electromagnetic wave propagating in the z direction.

$$E_x = E_0 \cos(kz - \omega t) \qquad E_y = 0 \qquad E_z = 0$$

$$B_x = 0 \qquad B_y = B_0 \cos(kz - \omega t) \qquad B_z = 0$$

$$k = \frac{\omega}{v} = \omega\sqrt{\varepsilon\mu}$$





If an ordinary light beam is allowed to fall on a Polaroid, then the emerging beam will be linearly polarized; and if we place another Polaroid P_2, then the intensity of the transmitted light will depend on the relative orientation of P_2 with respect to P_1.

Polarization of Light

Polarization refers to the process or phenomenon in which the oscillations of the electric field vector of light are restricted to a specific direction. Light is an electromagnetic wave, and in unpolarized light, the electric field oscillates randomly in all directions perpendicular to the direction of propagation. When this randomness is eliminated or reduced, the light is said to be **polarized**.

The **plane of polarization** refers to the plane in which the electric field vector of polarized light oscillates. It provides a way to describe the orientation of the electric field in a polarized light wave.

Types of Polarization

Linear Polarization:
- The electric field oscillates in only one direction perpendicular to the wave's propagation.
- Achieved using polarizing filters or by reflection.
- Example: Light passing through a Polaroid filter.

Circular Polarization:
- The electric field vector rotates in a circular motion as the wave propagates, forming a helix.
- Achieved using a combination of polarizers and quarter-wave plates.
- Can be **right-handed** or **left-handed**, depending on the rotation direction.

Elliptical Polarization:
- The electric field traces an elliptical path.
- This is a more general form of polarization and includes linear and circular polarization as special cases.

Methods of Polarization

1. **Polarization by Transmission**:
   - When unpolarized light passes through a polarizing material (like a Polaroid sheet), only the electric field components aligned with the material's transmission axis pass through, resulting in linearly polarized light.
2. **Polarization by Reflection**:
   - Light reflecting off a non-metallic surface becomes partially polarized. At a specific angle (Brewster's angle), the reflected light is fully polarized perpendicular to the surface.
3. **Polarization by Scattering**:
   - When sunlight interacts with particles in the atmosphere, the scattered light becomes partially polarized. This is why polarized sunglasses reduce glare from the sky.
4. **Polarization by Double Refraction (Birefringence)**:
   - Certain materials, like calcite, split light into two beams with orthogonal polarization states.

Producing Plane-Polarized Light from Superposition

Plane-polarized (or linearly polarized) light can be produced by the **superposition** of two perpendicular light waves with specific phase and amplitude relationships.

$$\mathbf{E}_1 = \hat{\mathbf{x}}\, a_1 \cos(kz - \omega t + \theta_1) \quad \textbf{(11)}$$

$$\mathbf{E}_2 = \hat{\mathbf{x}}\, a_2 \cos(kz - \omega t + \theta_2) \quad \textbf{(12)}$$

where a_1 and a_2 represent the amplitudes of the waves, $\hat{\mathbf{x}}$ represents the unit vector along the x axis, and θ_1 and θ_2 are phase constants. The resultant of these two waves is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \quad \textbf{(13)}$$

which can always be written in the form

$$\mathbf{E} = \hat{\mathbf{x}}\, a \cos(kz - \omega t + \theta) \quad \textbf{(14)}$$

where

$$a = [a_1^2 + a_2^2 + 2a_1a_2 \cos(\theta_1 - \theta_2)]^{1/2} \quad \textbf{(15)}$$

represents the amplitude of the wave. Equation (14) tells us that the resultant is also a linearly polarized wave with its electric vector oscillating along the same axis.

Producing Plane-Polarized Light from Superposition

We next consider the superposition of two linearly polarized electromagnetic waves (both propagating along the z axis) but with their electric vectors oscillating along two mutually perpendicular directions. Thus, we may have

$$\mathbf{E}_1 = \hat{\mathbf{x}}\, a_1 \cos(kz - \omega t) \tag{16}$$

$$\mathbf{E}_2 = \hat{\mathbf{y}}\, a_2 \cos(kz - \omega t + \theta) \tag{17}$$

For $\theta = n\pi$, the resultant will also be a linearly polarized wave with its electric vector oscillating along a direction making a certain angle with the x axis; this angle will depend on the relative values of a_1 and a_2.

To find the state of polarization of the resultant field, we consider the time variation of the resultant electric field at an arbitrary plane perpendicular to the z axis which we may, without any loss of generality, assume to be $z = 0$. If E_x and E_y represent the x and y components of the resultant field $\mathbf{E}$ $(= \mathbf{E}_1 + \mathbf{E}_2)$, then

$$E_x = a_1 \cos \omega t \tag{18}$$

and

$$E_y = a_2 \cos(\omega t - \theta) \tag{19}$$

where we have used Eqs. (16) and (17) with $z = 0$. For $\theta = n\pi$, the above equations simplify to

$$E_x = a_1 \cos \omega t$$

and

$$E_y = (-1)^n a_2 \cos \omega t \tag{20}$$

from which we obtain

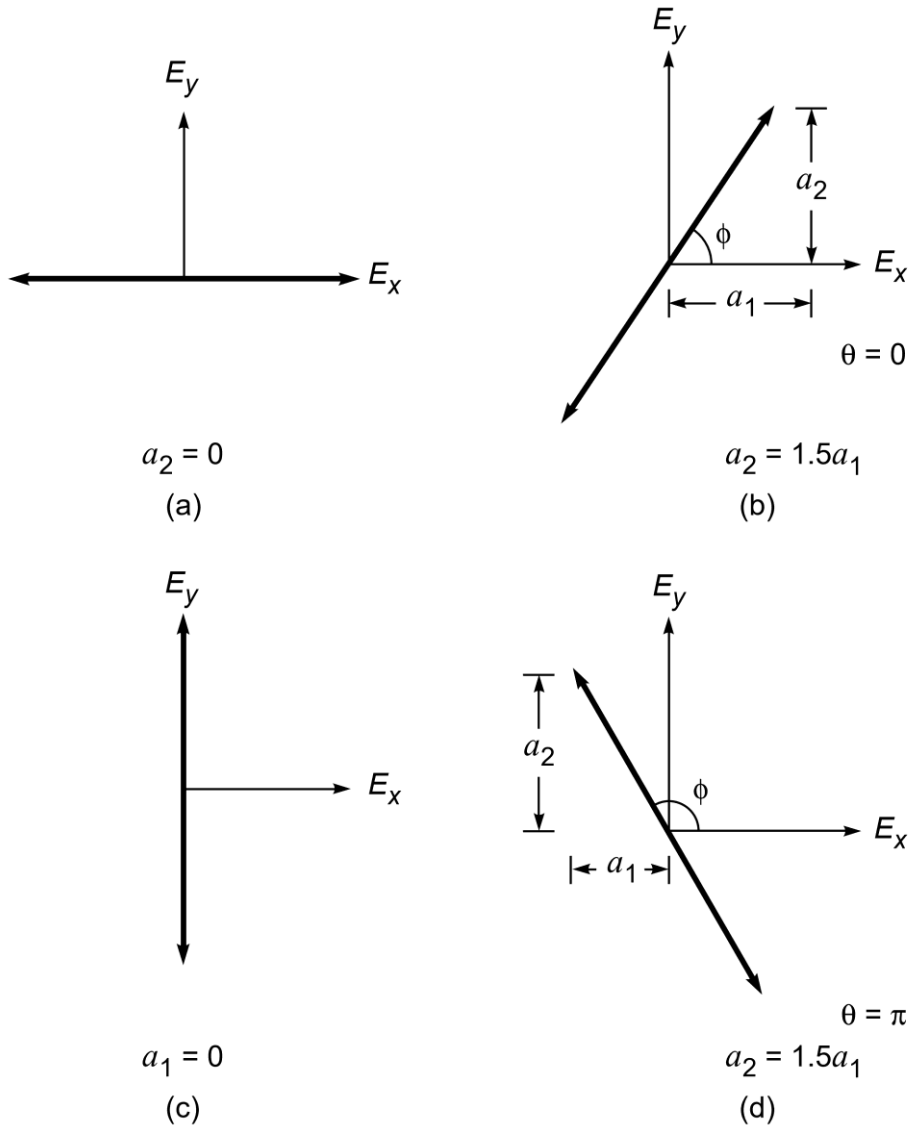
$$\frac{E_y}{E_x} = \pm \frac{a_2}{a_1} \qquad \text{(independent of } t\text{)} \tag{21}$$

where the upper and lower signs correspond to n even and n odd, respectively. In the $E_x E_y$ plane, Eq. (21) represents a straight line; the angle ϕ that this line makes with the E_x axis depends on the ratio a_2/a_1. In fact

$$\phi = \tan^{-1}\left(\pm \frac{a_2}{a_1}\right) \tag{22}$$

The condition $\theta = n\pi$ implies that the two vibrations are either in phase ($n = 0, 2, 4, \ldots$) or out of phase ($n = 1, 3, 5, \ldots$). Thus, the superposition of two linearly polarized electromagnetic waves with their electric fields at right angles to each other and oscillating in phase is again a linearly polarized wave with its electric vector, in general, oscillating in a direction which is different from the fields of either of the two waves.

Producing Plane-Polarized Light from Superposition



The superposition of two linearly polarized waves with their electric fields oscillating in phase along the x axis and the y axis. The resultant is again a linearly polarized wave with its electric vector oscillating in a direction making an angle ϕ with the x axis.

Producing Circularly-Polarized Light from Superposition

For $\theta \neq n\pi$ ($n = 0, 1, 2, \ldots$), the resultant electric vector does not, in general, oscillate along a straight line. We first consider the simple case corresponding to $\theta = \pi/2$ with $a_1 = a_2$. Thus,

$$E_x = a_1 \cos \omega t \quad \textbf{(23)}$$

$$E_y = a_1 \sin \omega t \quad \textbf{(24)}$$

If we plot the time variation of the resultant electric vector whose x and y components are given by Eqs. (23) and (24), we find that the tip of the electric vector rotates on the circumference of a circle (of radius a_1) in the counter-clockwise direction [see Fig. 22.17(c)], and the propagation is in the $+z$ direction which is coming out of the page. Such a wave is known as a right circularly polarized wave (usually abbreviated as a RCP wave).[2] That the tip of the resultant electric vector should lie on the circumference of a circle is also obvious from the fact that

$$E_x^2 + E_y^2 = a_1^2 \qquad \text{(independent of } t\text{)}$$

For $\theta = 3\pi/2$, we would have had

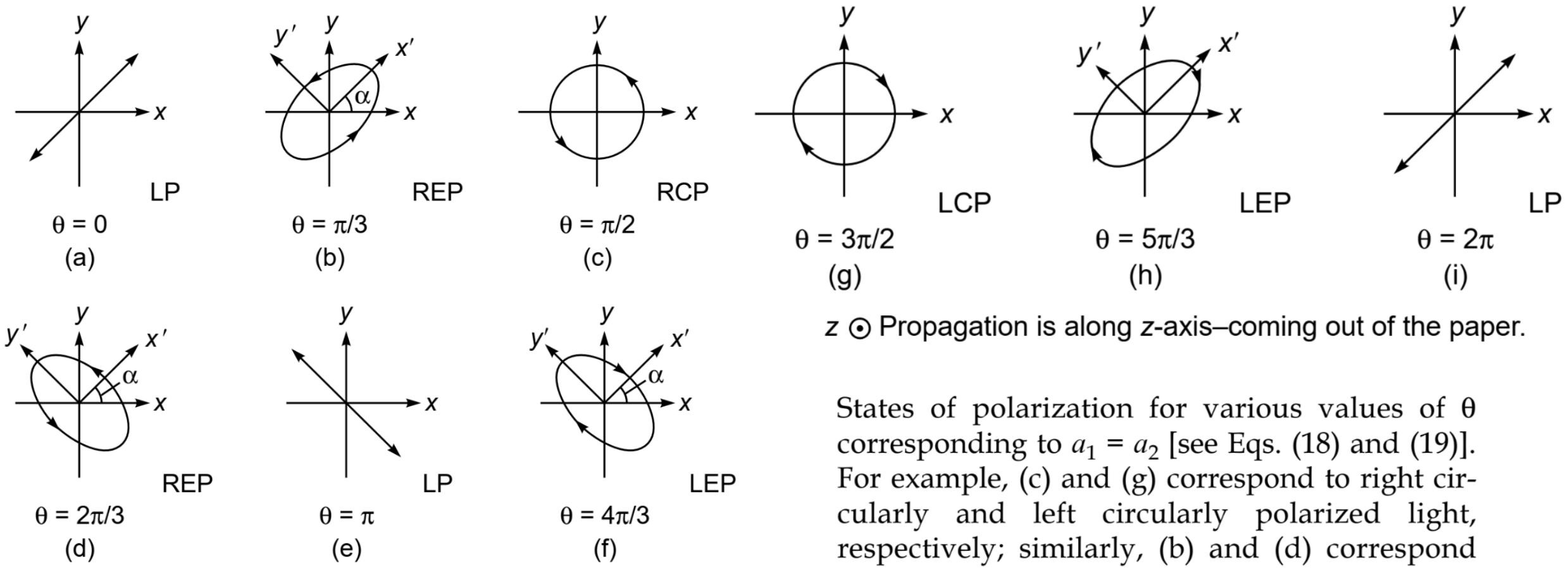
$$E_x = a_1 \cos \omega t \quad \textbf{(25)}$$

$$E_y = -a_1 \sin \omega t \quad \textbf{(26)}$$

which would also represent a circularly polarized wave; however, the electric vector will rotate in the clockwise direction [see Fig. 22.17(g)]. Such a wave is known as a left circularly polarized wave (usually abbreviated as a LCP wave).

For $\theta \neq m\pi/2$ ($m = 0, 1, 2, \ldots$), the tip of the electric vector rotates on the circumference of an ellipse (see Fig. 22.17 which corresponds to various values of θ). As can be seen from the figure, this ellipse will degenerate into a straight line or a circle when θ becomes an even or an odd multiple of $\pi/2$. In general, when $a_1 \neq a_2$, one obtains an elliptically polarized wave which degenerates into a straight line for $\theta = 0, \pi, 2\pi, \ldots$, etc.

Producing Circularly-Polarized Light from Superposition



$z \odot$ Propagation is along z-axis–coming out of the paper.

States of polarization for various values of θ corresponding to $a_1 = a_2$ [see Eqs. (18) and (19)]. For example, (c) and (g) correspond to right circularly and left circularly polarized light, respectively; similarly, (b) and (d) correspond to right elliptically polarized (REP) light, and (f) and (h) correspond to left elliptically polarized (LEP) light. The propagation is out of the page.

Producing Circularly-Polarized Light from Superposition

$$E_x = a_1 \cos \omega t$$
$$E_y = a_2 \cos (\omega t - \theta)$$

We assume that the major axis of the ellipse is along the x' or the y' axes and that the x' axis makes an angle α with the x axis [see Fig. 22.17(b)]; i.e.,

$$E_x' = E_1 \cos(\omega t - \phi) \quad \textbf{(28)}$$

and

$$E_y' = E_2 \sin(\omega t - \phi) \quad \textbf{(29)}$$

Obviously,

$$\left(\frac{E_x'}{E_1}\right)^2 + \left(\frac{E_y'}{E_2}\right)^2 = 1$$

which represents the equation of an ellipse. Now, for the rotated coordinates

$$E_x = E_x' \cos \alpha - E_y' \sin \alpha$$
$$E_y = E_x' \sin \alpha + E_y' \cos \alpha$$

If we multiply the first equation by $\cos \alpha$ and the second equation by $\sin \alpha$ and add, we get

$$E_x' = E_x \cos \alpha + E_y \sin \alpha \quad \textbf{(30)}$$

Similarly,

$$E_y' = -E_x \sin \alpha + E_y \cos \alpha \quad \textbf{(31)}$$

Substituting Eqs. (18), (19), (28), and (29) into Eqs. (30) and (31), we get

$$E_1 \cos(\omega t - \phi) = a_1 \cos \omega t \cos \alpha + a_2 \cos(\omega t - \theta) \sin \alpha$$
$$E_2 \sin(\omega t - \phi) = -a_1 \cos \omega t \sin \alpha + a_2 \cos(\omega t - \theta) \cos \alpha$$

The above equations have to be valid at all times; thus, we equate the coefficients of $\cos \omega t$ and $\sin \omega t$ on both sides of the equation to obtain

$$E_1 \cos \phi = a_1 \cos \alpha + a_2 \cos \theta \sin \alpha$$
$$E_1 \sin \phi = a_2 \sin \theta \sin \alpha$$

$$-E_2 \sin \phi = -a_1 \sin \alpha + a_2 \cos \theta \cos \alpha$$
$$E_2 \cos \phi = a_2 \sin \theta \cos \alpha$$

Producing Circularly-Polarized Light from Superposition

If we square the four equations above and add, we get

$$E_1^2 + E_2^2 = a_1^2 + a_2^2$$

which is to be expected because the total intensity of both beams should be equal. Further,

$$\frac{E_2}{E_1} = \frac{a_2 \sin\theta \cos\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \sin\alpha} \quad \textbf{(32)}$$

Thus,

$$\begin{aligned} a_2^2 \sin^2\theta \sin\alpha \cos\alpha = a_1^2 \sin\alpha \cos\alpha \\ - a_2^2 \cos^2\theta \sin\alpha \cos\alpha \\ - a_1 a_2 \cos\theta (\cos^2\alpha - \sin^2\alpha) \end{aligned}$$

Simple manipulations give

$$\tan 2\alpha = \frac{2 a_1 a_2 \cos\theta}{a_1^2 - a_2^2} \quad \textbf{(33)}$$

For $a_1 = a_2$ $\quad 2\alpha = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4}$ **(34)**

implying that the major (or minor) axis of the ellipse makes 45° with the x axis [see Fig. 22.17(b)]. Further,

$$\frac{E_2}{E_1} = \frac{\sin\theta}{1 + \cos\theta} = \tan\frac{\theta}{2} \quad \textbf{(35)}$$

Thus, for $a_1 = a_2$ and for

$$\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$$

$$\frac{E_2}{E_1} = +0.577, 1, 1.732, -1.732, -1, -0.577$$

which correspond to REP, RCP, REP, LEP, LCP, and LEP, respectively, as shown in Fig. 22.17. For example, for $\theta = 4\pi/3$

$$E_x' = E_1 \cos(\omega t - \phi)$$

$$E_y' = -1.732\, E_1 \sin(\omega t - \phi)$$

Thus, the major axis of the ellipse is along the y' axis.

Producing Circularly-Polarized Light from Superposition

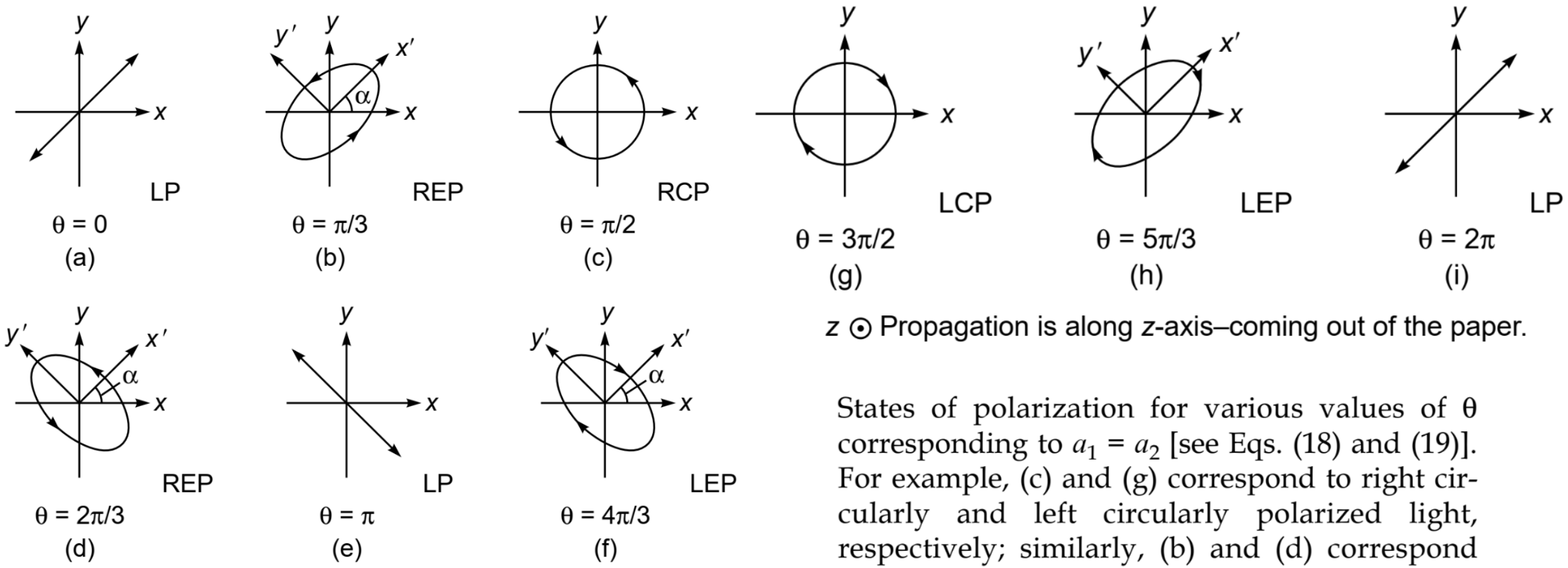
To determine the state of polarization, we may choose $t = 0$ at the instant so that ϕ may be assumed to be zero:

$$E_x' = E_1 \cos \omega t$$
$$E_y' = -1.732 E_1 \sin \omega t$$

Thus at

$t = 0$	$E_x' = E_1$	$E_y' = 0$
$t = \dfrac{\pi}{2\omega}$	$E_x' = 0$	$E_y' = -1.732 E_1$
$t = \dfrac{\pi}{\omega}$	$E_x' = -E_1$	$E_y' = 0$

Producing Circularly-Polarized Light from Superposition

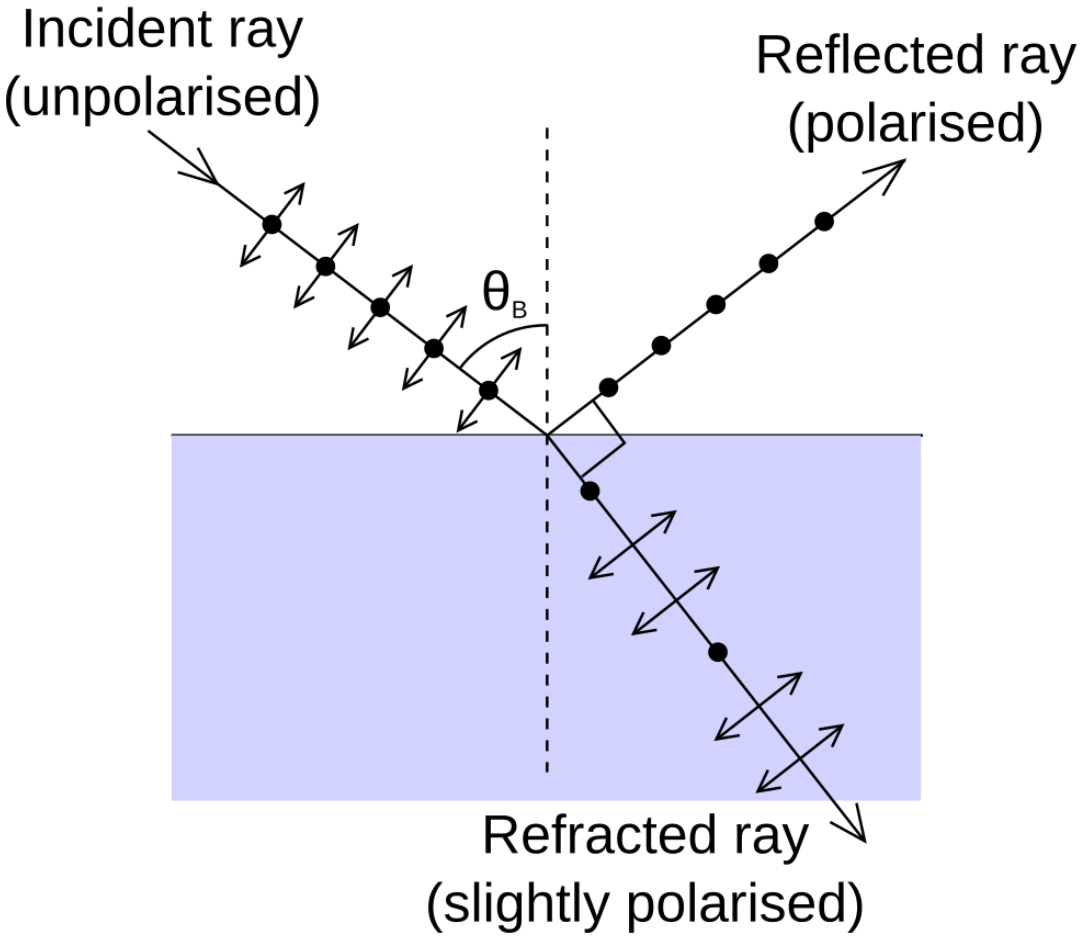


$z \odot$ Propagation is along z-axis–coming out of the paper.

States of polarization for various values of θ corresponding to $a_1 = a_2$ [see Eqs. (18) and (19)]. For example, (c) and (g) correspond to right circularly and left circularly polarized light, respectively; similarly, (b) and (d) correspond to right elliptically polarized (REP) light, and (f) and (h) correspond to left elliptically polarized (LEP) light. The propagation is out of the page.

Brewster’s Law

Brewster's law describes the phenomenon of **polarization by reflection**. When unpolarized light strikes the surface of a transparent material at a specific angle, called the **Brewster angle**, the reflected light becomes completely **polarized** perpendicular to the plane of incidence.



Brewster's Law

1. According to Snell's law:

$$n_1 \sin\theta_B = n_2 \sin\theta_r$$

   At the Brewster angle, the angle between the reflected and refracted rays is 90°, so:

$$\theta_r = 90^\circ - \theta_B$$

2. Using the relationship $\sin(90^\circ - \theta_B) = \cos\theta_B$, Snell's law becomes:

$$n_1 \sin\theta_B = n_2 \cos\theta_B$$

3. Dividing both sides by $\cos\theta_B$, we get:

$$\tan\theta_B = \frac{n_2}{n_1}$$

Brewster's Law

- **Polarizing Sunglasses**: These block glare by filtering out the polarized light reflected from surfaces like water or roads.
- **Photography**: Polarizing filters are used to reduce reflections from glass or water, enhancing image contrast.
- **Optical Instruments**: Brewster's law is applied in laser systems to generate polarized light.
- **Material Identification**: Brewster's angle helps determine the refractive index of materials by observing the polarization of reflected light.

Polarization by Double Refraction

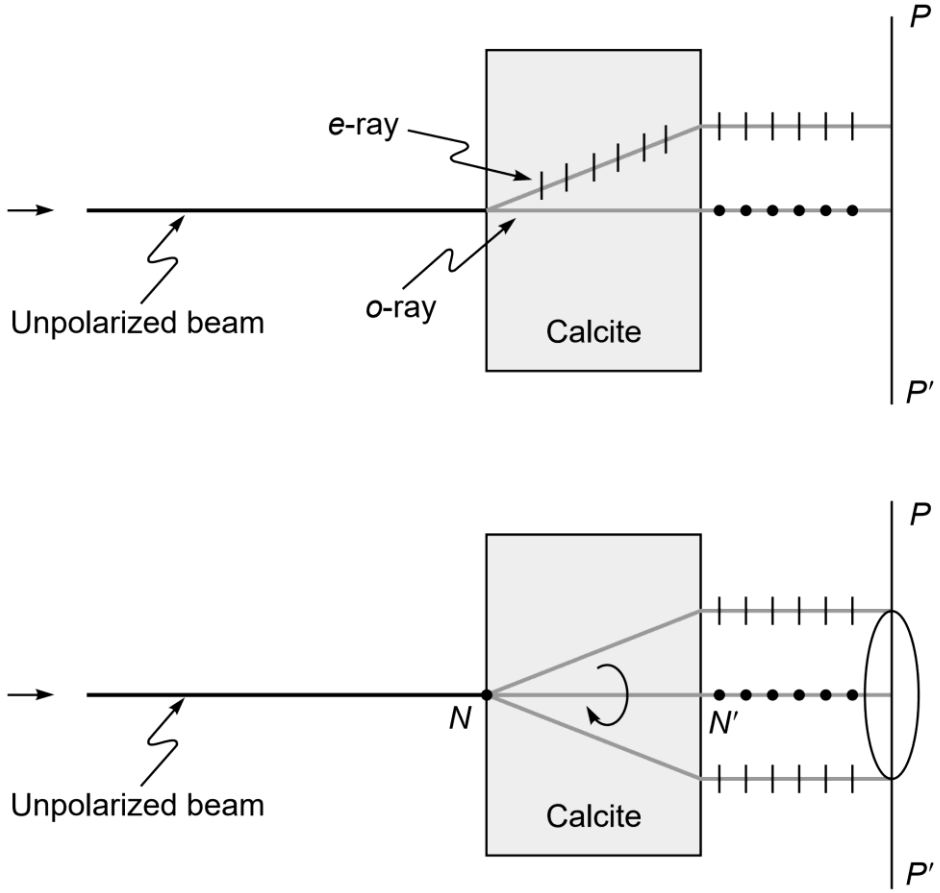


Fig. 22.18 (a) When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams. (b) If we rotate the crystal about *NN*,′ then the *e*-ray will rotate about *NN*′.

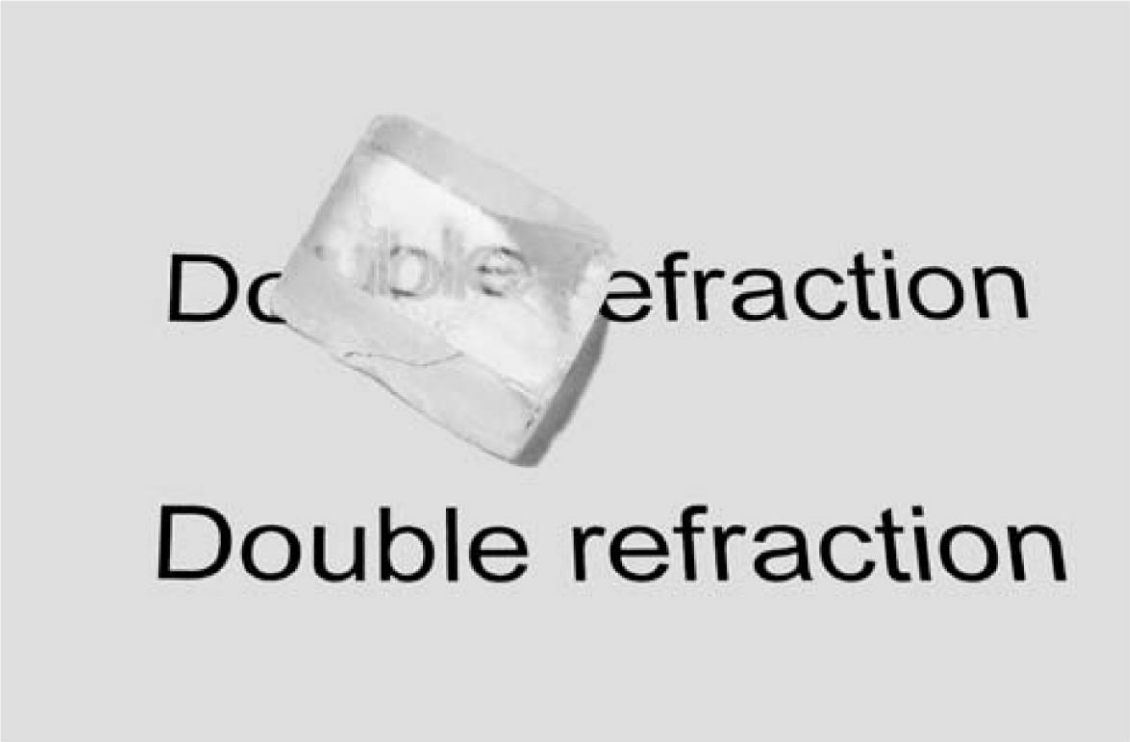


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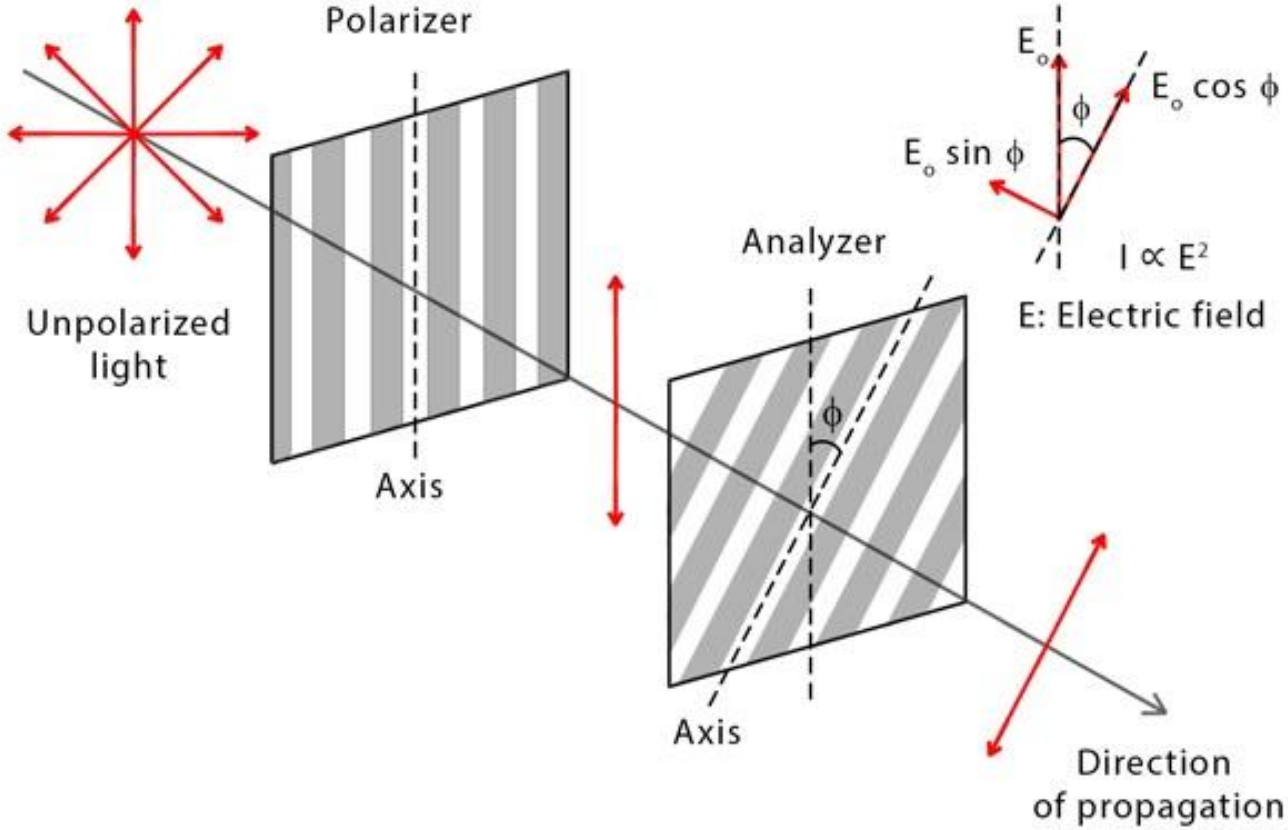
Fig. 22.19 A calcite crystal showing double refraction.

Malus' law

Malus' Law describes the intensity of light passing through a polarizing filter. It explains how the intensity of a polarized light beam changes when it passes through a polarizer. The law is important for understanding how light behaves when it interacts with polarizing materials like polarizing filters in sunglasses or cameras.



$$I = I_0 \cos^2 \phi$$

I : Intensity of light after pasing through the analyzer

I_0 : Intensity of light after pasing through the polarizer

ϕ : Angle between the axes of the polarizer and analyzer

Malus' law

Malus' Law describes the intensity of light passing through a polarizing filter. It explains how the intensity of a polarized light beam changes when it passes through a polarizer. The law is given by:

$$I = I_0 \cos^2(\theta)$$

Where:

- I is the transmitted intensity of the light after passing through the polarizer.
- I_0 is the initial intensity of the polarized light before passing through the polarizer.
- θ is the angle between the light's polarization direction and the axis of the polarizer.

Malus' law

- **Polarizing Filters in Photography**: Used to reduce reflections or increase contrast by rotating the filter relative to the polarized light from the scene.
- **Sunglasses**: Block glare from horizontal surfaces by passing only vertically polarized light.
- **Optical Instrumentation**: Helps in controlling light intensity in optical devices like lasers and light modulators.

Malus' law

Suppose a beam of light with an initial intensity of 100 W/m² is polarized at an angle of 30° to the axis of a polarizing filter. We need to calculate the transmitted intensity of the light through the polarizer.

- $I_0 = 100\,\mathrm{W/m^2}$ (initial intensity),
- $\theta = 30^\circ$ (the angle between the polarization direction and the axis of the polarizer).

Substituting these values into the equation:

$$I = 100 \times \cos^2(30^\circ)$$

First, calculate $\cos(30^\circ)$:

$$\cos(30^\circ) = \frac{\sqrt{3}}{2} \approx 0.866$$

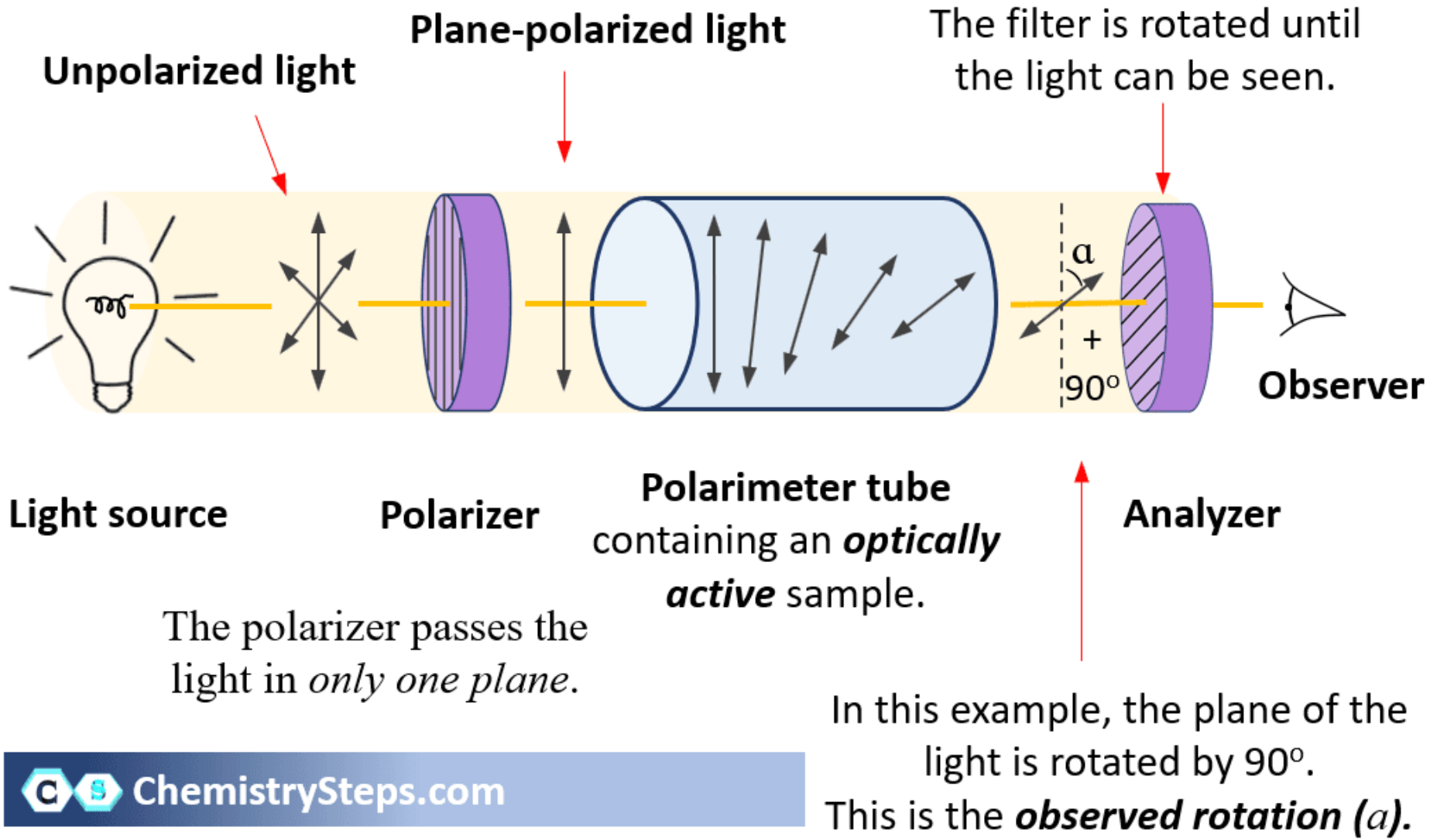
Now, square the cosine value:

$$\cos^2(30^\circ) = (0.866)^2 \approx 0.749$$

Now, calculate the transmitted intensity:

$$I = 100 \times 0.749 = 74.9\,\mathrm{W/m^2}$$

Optical activity



Optical activity

Optical activity refers to the ability of certain materials to rotate the plane of polarization of polarized light as it passes through them. This phenomenon occurs due to the interaction between light and chiral molecules in the material.

Key Concepts

- **Chirality**: For a material to exhibit optical activity, it must have a chiral structure, meaning it is not superimposable on its mirror image. Common examples include sugars and certain organic compounds.
- **Plane-Polarized Light**: Light is plane-polarized when its electric field oscillates in one direction only.
- **Rotation of the Plane of Polarization**: When plane-polarized light passes through an optically active substance, the plane of polarization rotates by an angle θ depending on the material's properties, the length of the path, and the concentration of the chiral molecules.

Optical activity

Right and Left-Handed Rotation

- **Dextrorotatory**: A substance that rotates the plane of polarization clockwise (right-handed).
- **Levorotatory**: A substance that rotates the plane of polarization counterclockwise (left-handed).

Applications of Optical Activity

- **Stereochemistry**: Optical activity is used to distinguish between enantiomers, which are mirror-image molecules.
- **Analysis of Chiral Compounds**: Optical activity is important in characterizing and identifying chiral substances.
- **Pharmaceutical Industry**: Used to differentiate between the active and inactive enantiomers of drugs.
- **Sugar Analysis**: Optical activity is used to measure the concentration of sugar solutions.