Dynamics of Rotational Motion

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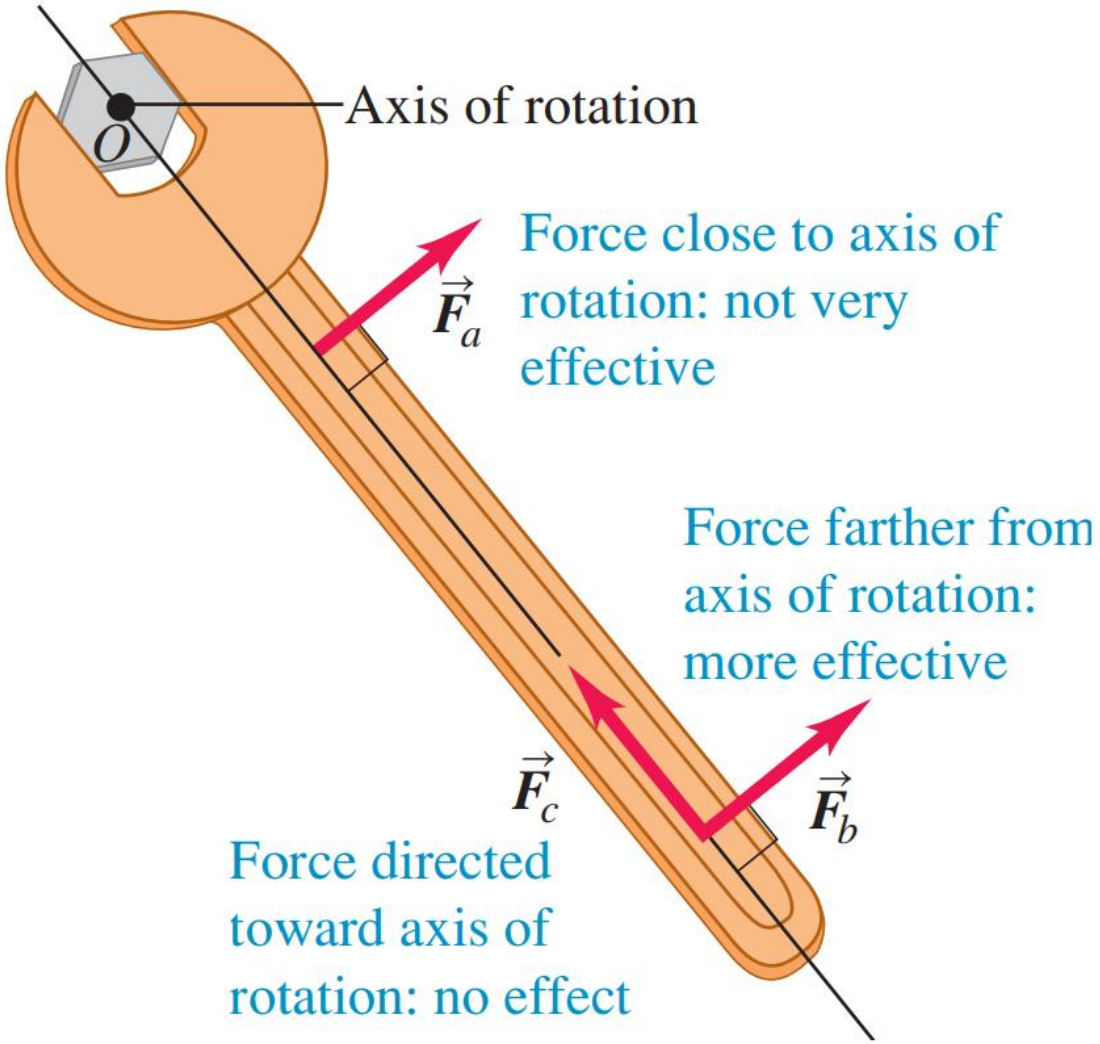
References

University Physics with Modern Physics

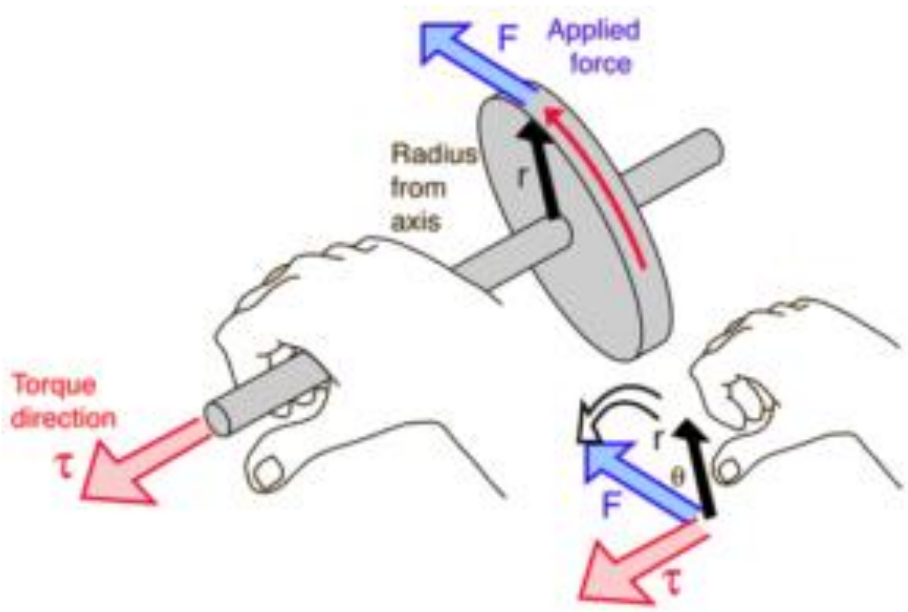
– Hugh D. Young, Roger A. Freedman

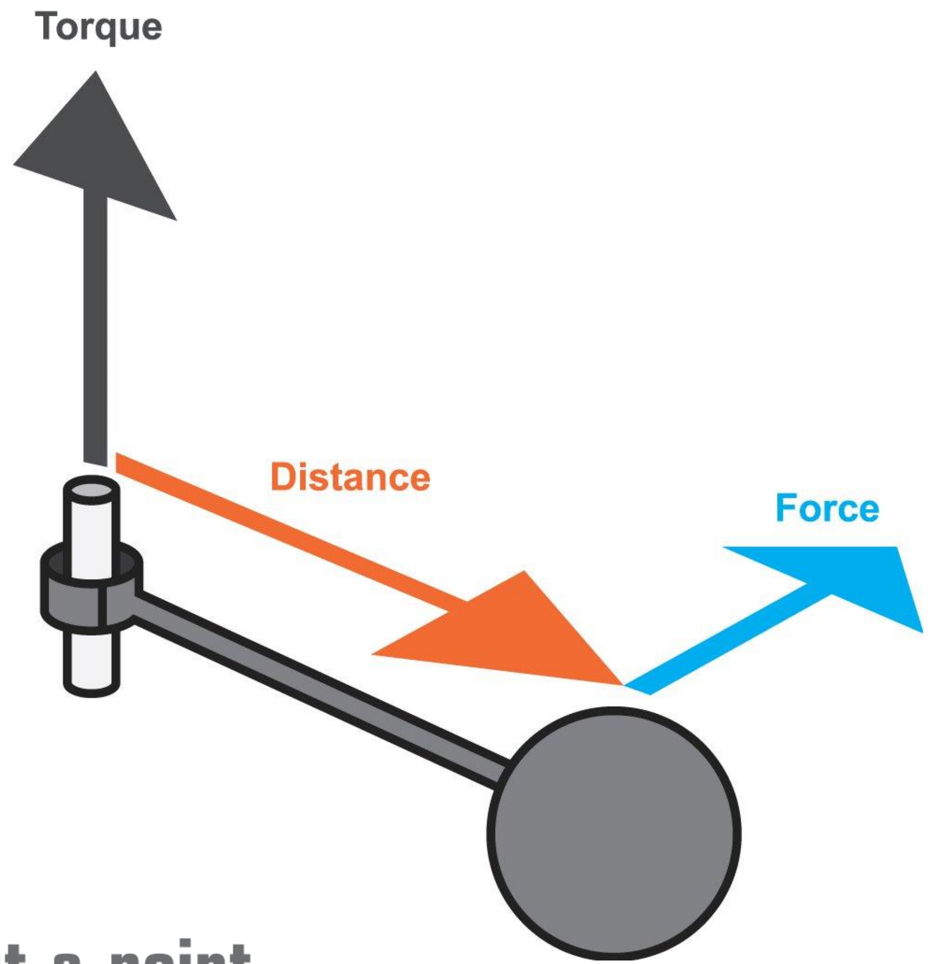
Torque

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$$



Torque

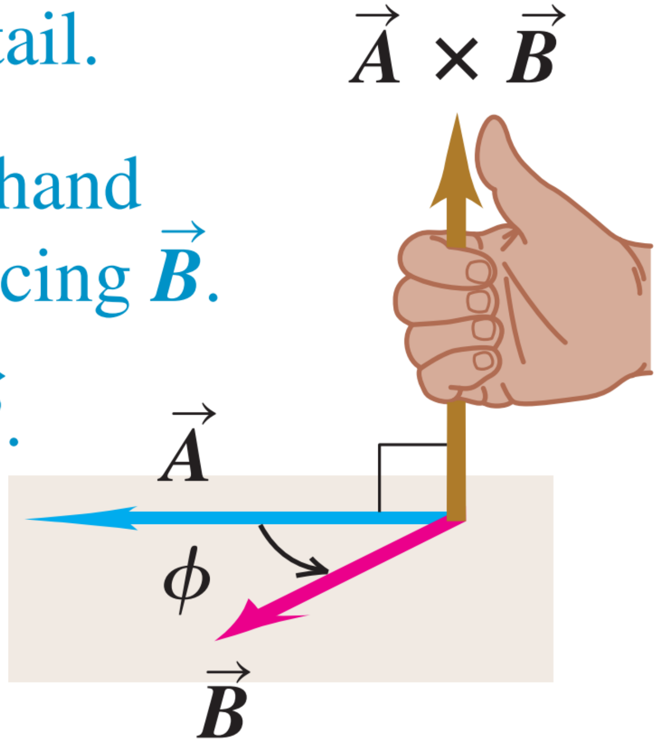




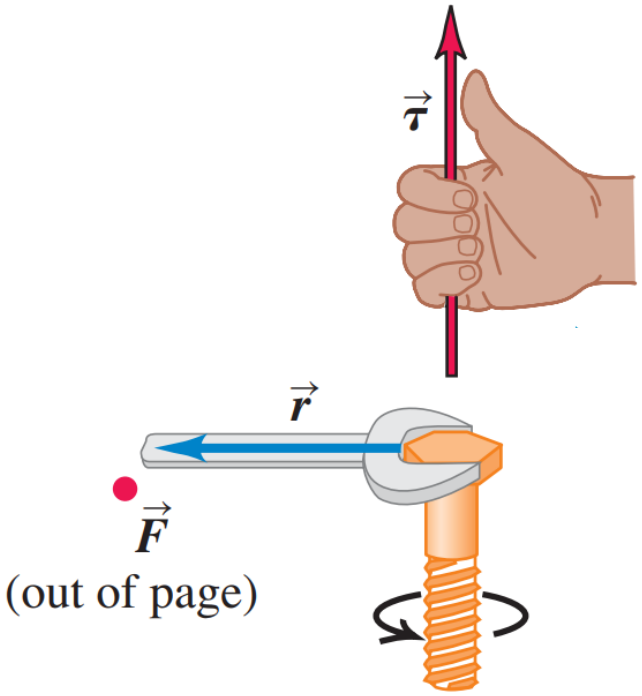
CAUTION Torque is always measured about a point

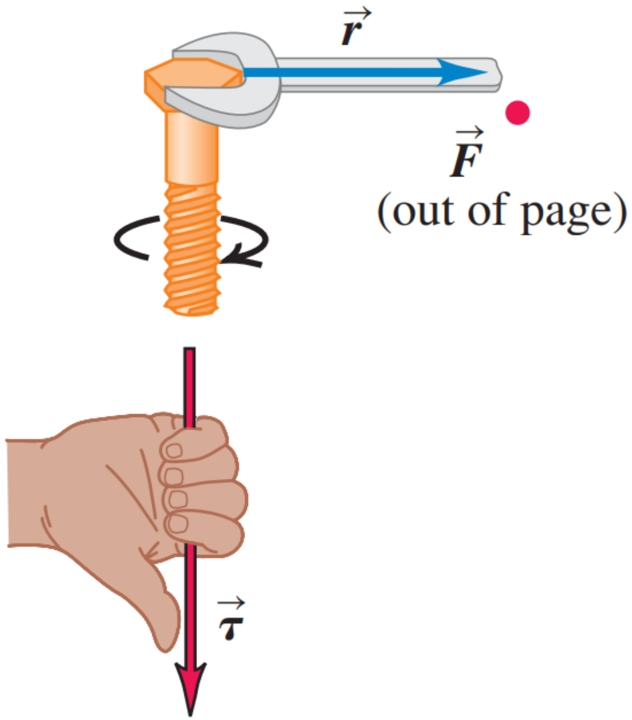
Torque is a vector quantity

1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A} \times \vec{B}$.



Torque is a vector quantity

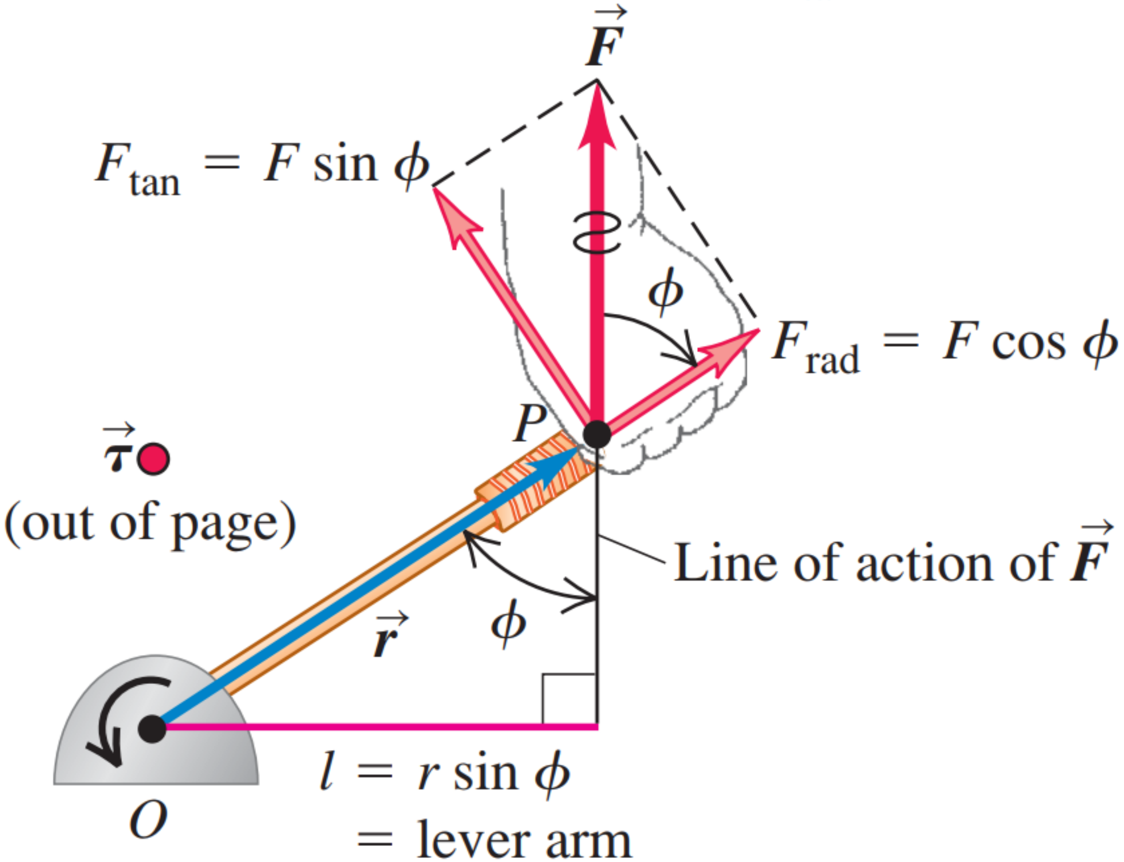




Three ways to calculate torque

$$\tau = Fl = rF\sin\phi = F_{\tan}r$$

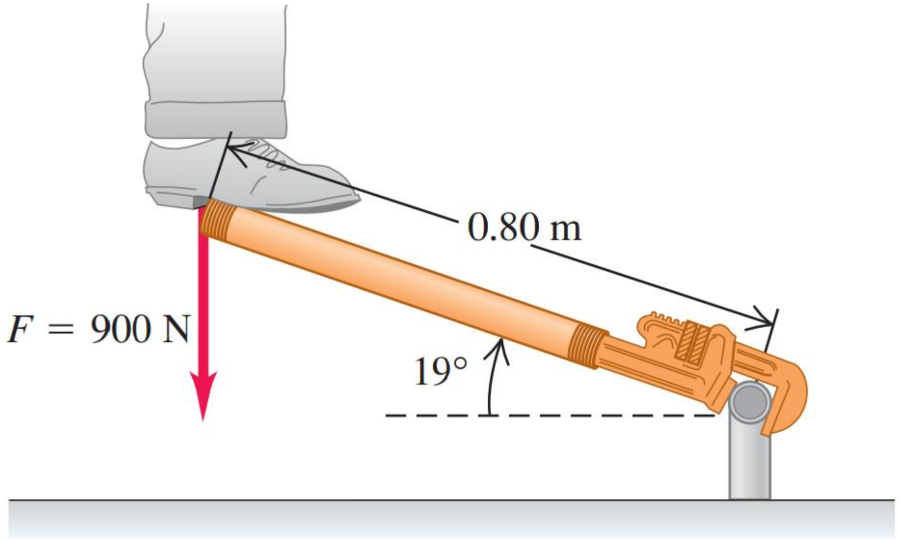
(magnitude of torque)



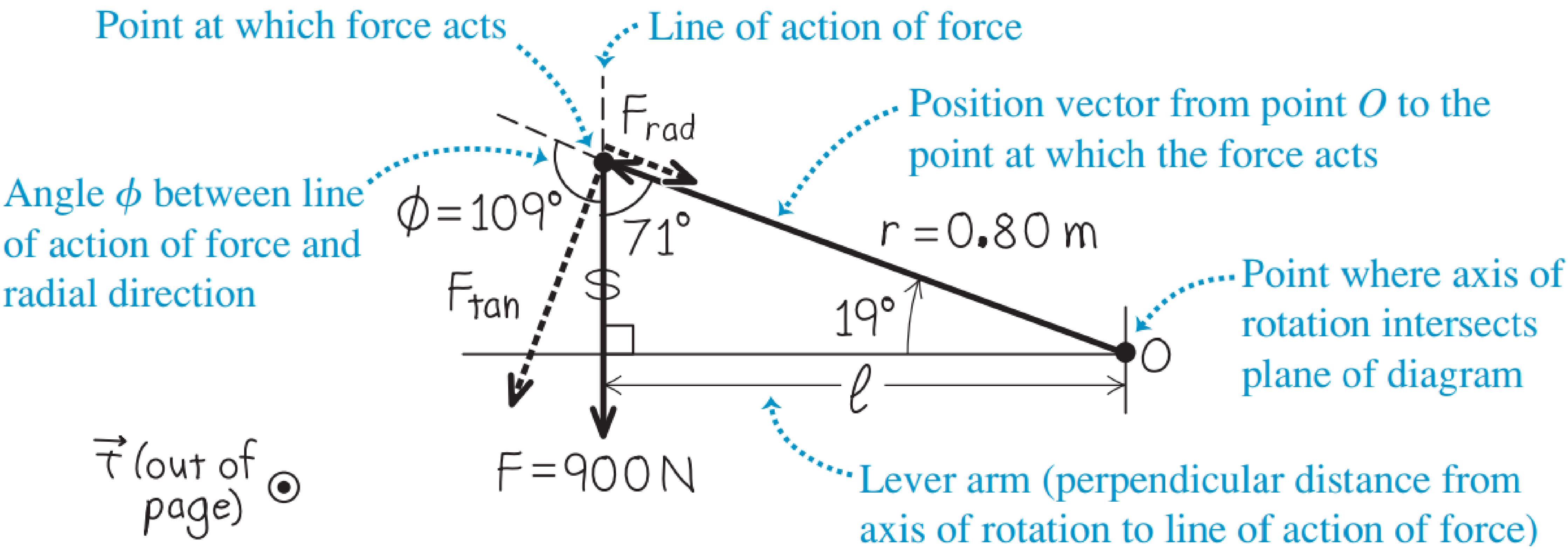
The SI unit of torque is the newton-meter.

Applying a torque

To loosen a pipe fitting, a weekend plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his full 900-N weight at a point 0.80 m from the center of the fitting (Fig. 10.5a). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.



Applying a torque



Applying a torque

EXECUTE: To use Eq. (10.1), we first calculate the lever arm l. As Fig. 10.5b shows,

$$l = r\sin\phi = (0.80\ \text{m})\sin 109° = (0.80\ \text{m})\sin 71° = 0.76\ \text{m}$$

Then Eq. (10.1) tells us that the magnitude of the torque is

$$\tau = Fl = (900\ \text{N})(0.76\ \text{m}) = 680\ \text{N}\cdot\text{m}$$

We get the same result from Eq. (10.2):

$$\tau = rF\sin\phi = (0.80\ \text{m})(900\ \text{N})(\sin\ 109°) = 680\ \text{N}\cdot\text{m}$$

Applying a torque

Alternatively, we can find $F_{\tan}$, the tangential component of $\vec{F}$ that acts perpendicular to $\vec{r}$. Figure 10.5b shows that this component is at an angle of $109° - 90° = 19°$ from $\vec{F}$, so $F_{\tan} = F \sin\phi = F(\cos 19°) = (900\ \text{N})(\cos 19°) = 851\ \text{N}$. Then, from Eq. 10.2,

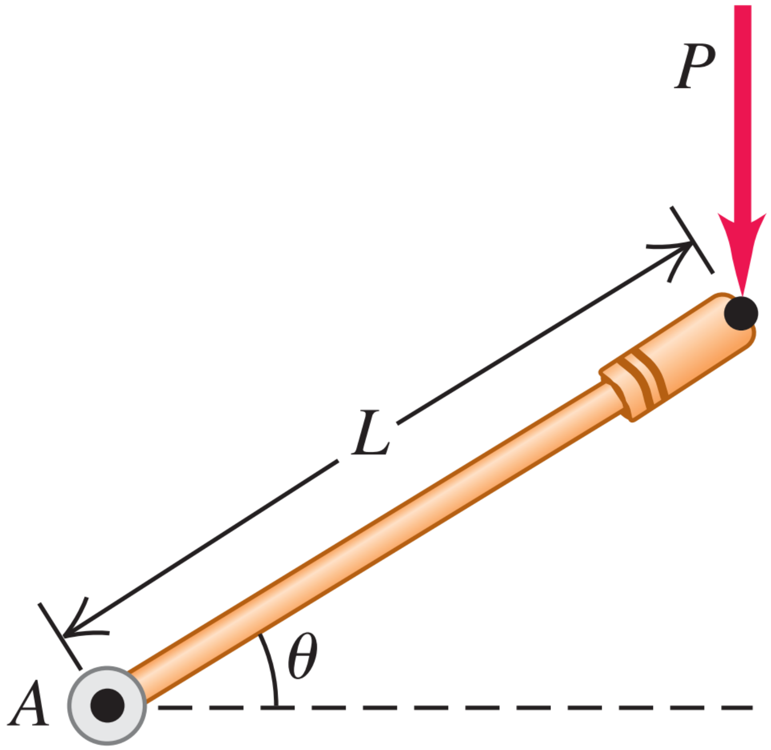
$$\tau = F_{\tan} r = (851\ \text{N})(0.80\ \text{m}) = 680\ \text{N} \cdot \text{m}$$

Curl the fingers of your right hand from the direction of $\vec{r}$ (in the plane of Fig. 10.5b, to the left and up) into the direction of $\vec{F}$ (straight down). Then your right thumb points out of the plane of the figure: This is the direction of $\vec{\tau}$.

Check yourself

The figure shows a force P being applied to one end of a lever of length L. What is the magnitude of the torque of this force about point A?

(i) $PL\sin\theta$; (ii) $PL\cos\theta$; (iii) $PL\tan\theta$.

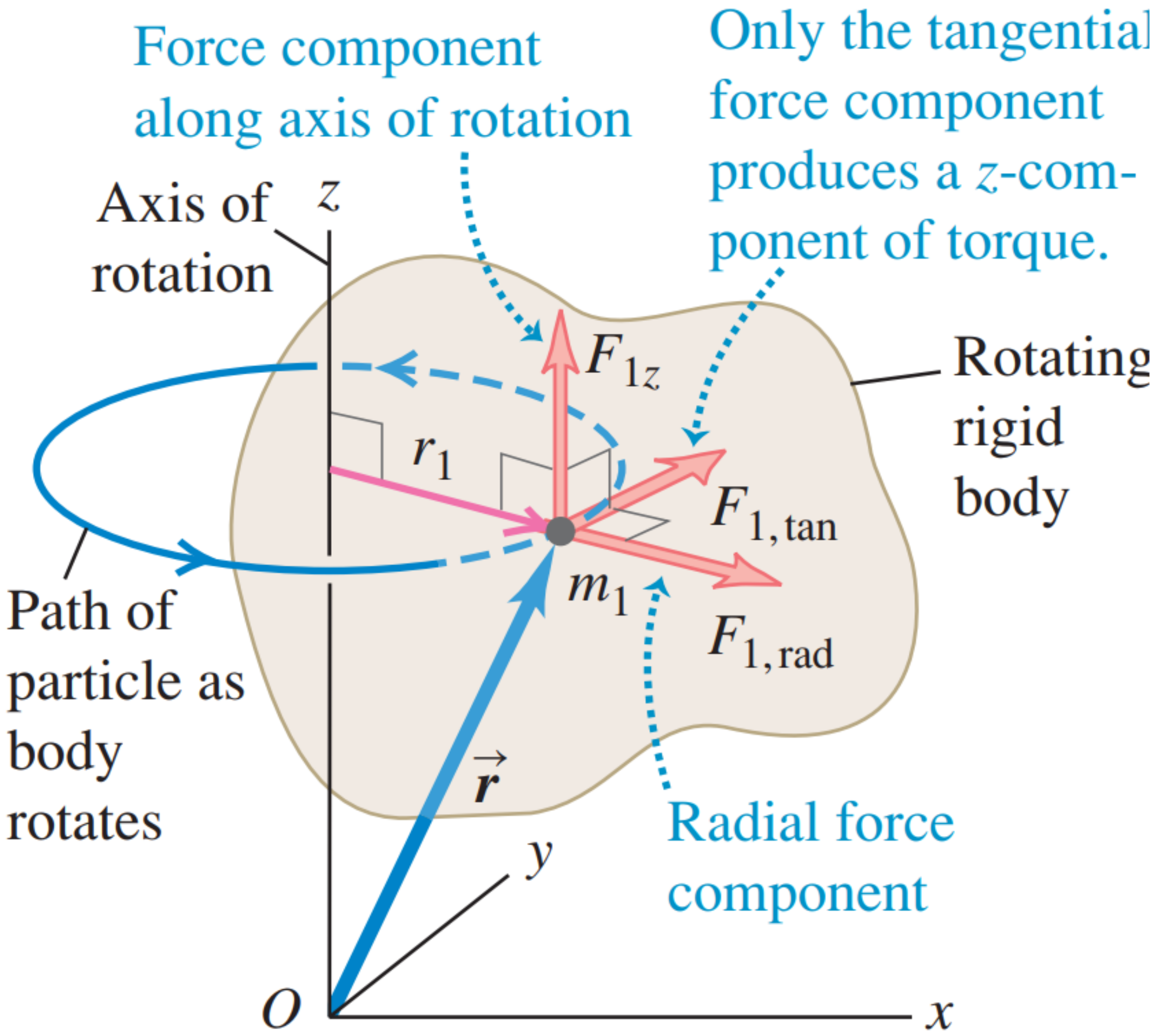


Torque and Angular Acceleration

$$F_{1,\tan} = m_1 a_{1,\tan}$$

$$F_{1,\tan} r_1 = m_1 r_1^2 \alpha_z$$

$$\tau_{1z} = m_1 r_1^2 \alpha_z$$



Torque and Angular Acceleration

$$\tau_{1z} + \tau_{2z} + \cdots = m_1 r_1^2 \alpha_z + m_2 r_2^2 \alpha_z + \cdots$$

$$\sum \tau_{iz} = \left(\sum m_i r_i^2 \right) \alpha_z$$

$$\sum \tau_z = I \alpha_z$$

(rotational analog of Newton's second law for a rigid body)

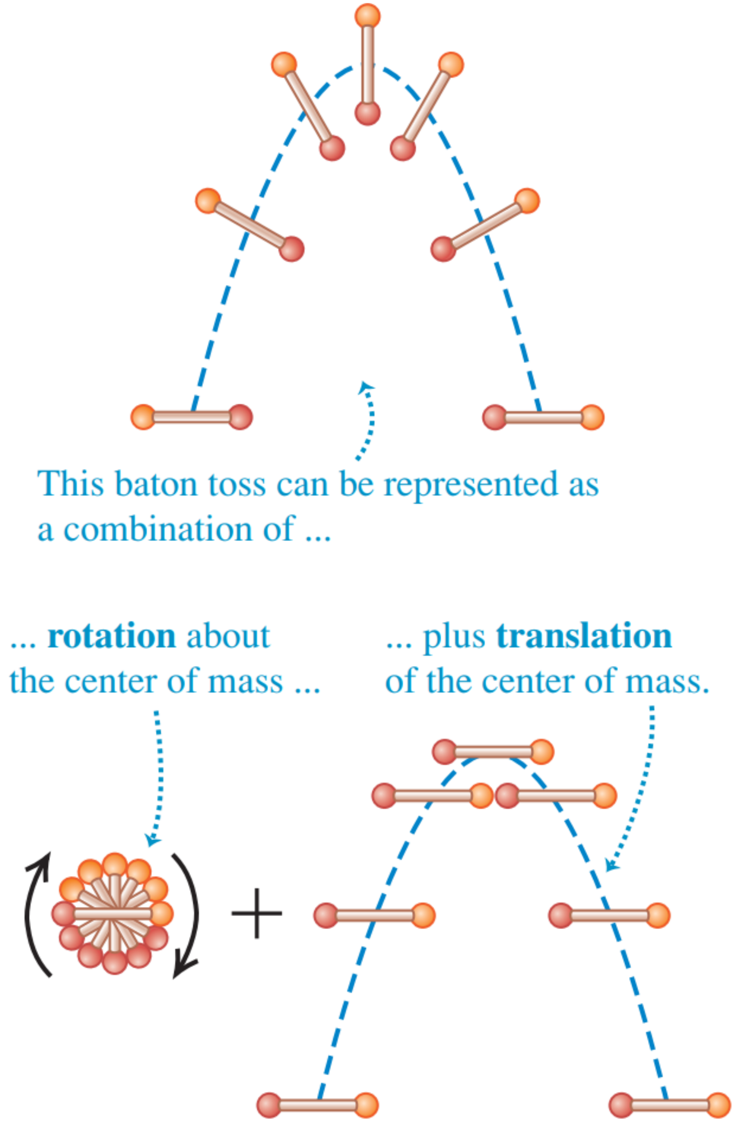
Screwdriver

$$\sum \tau_z = I\alpha_z$$

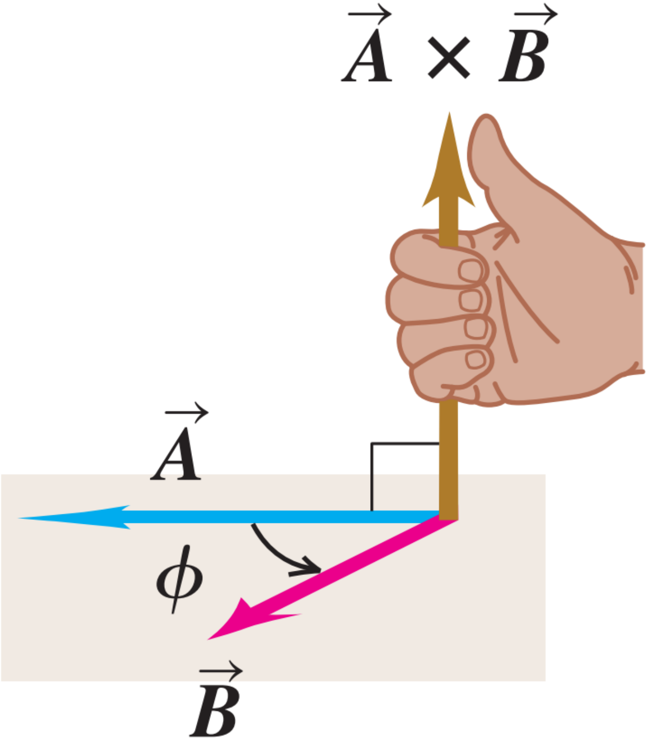
Combined Translation and Rotation

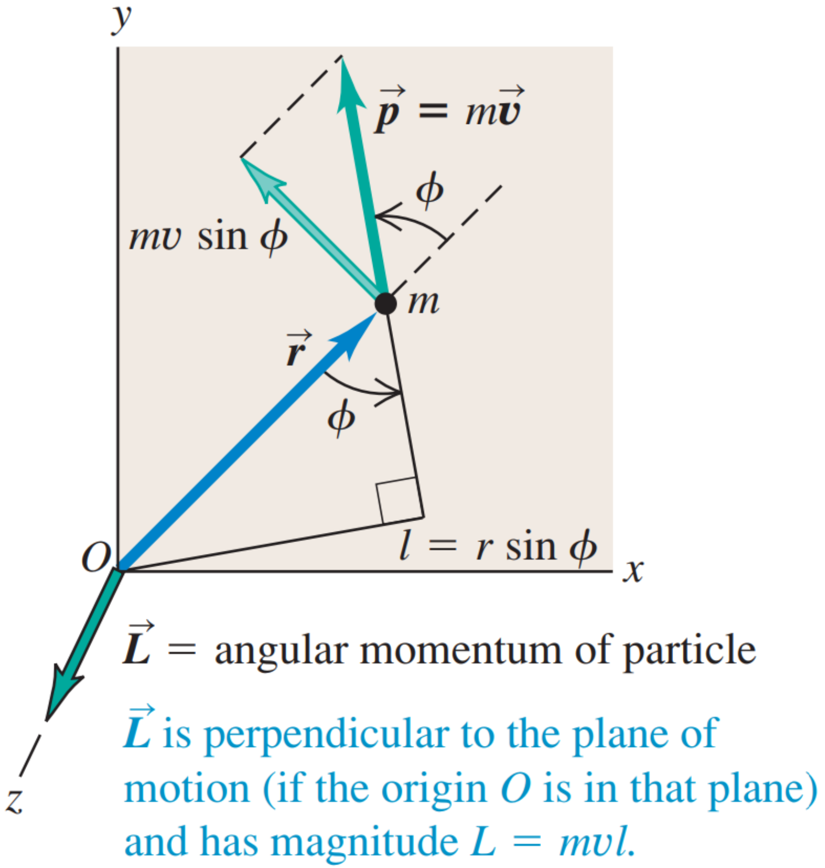
$$K = \frac{1}{2}Mv_{\mathrm{cm}}^{2} + \frac{1}{2}I_{\mathrm{cm}}\omega^{2}$$

(rigid body with both translation and rotation)



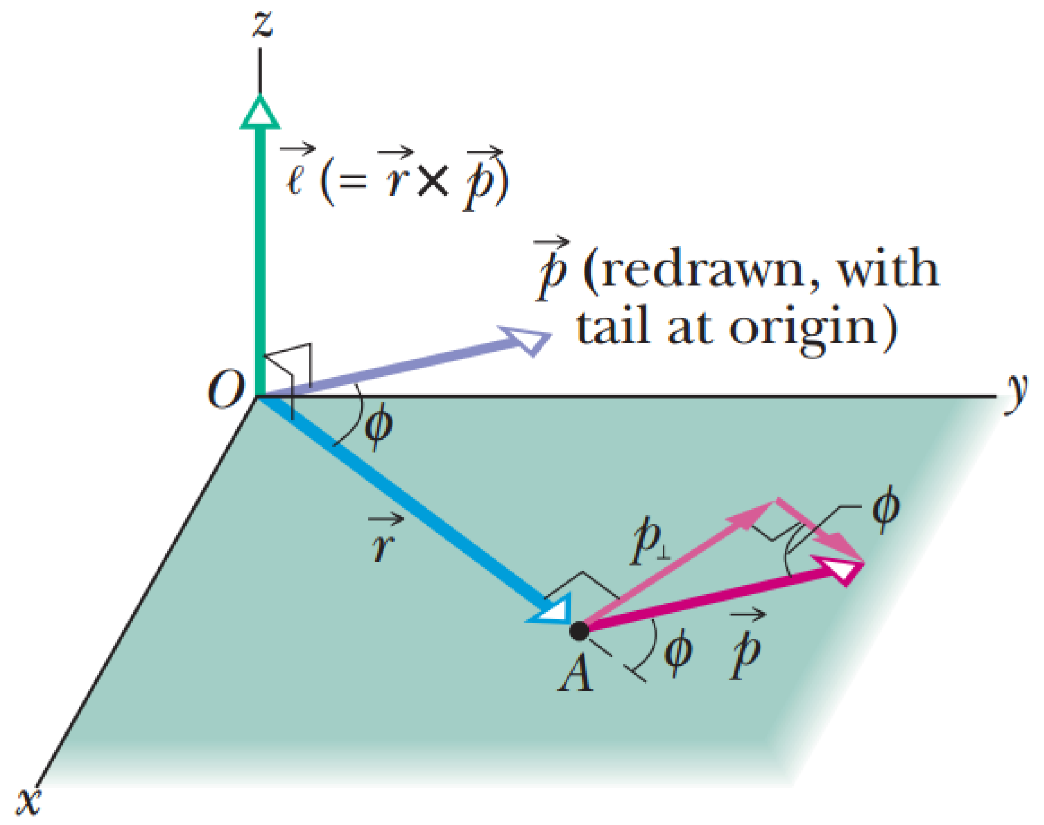
Angular momentum

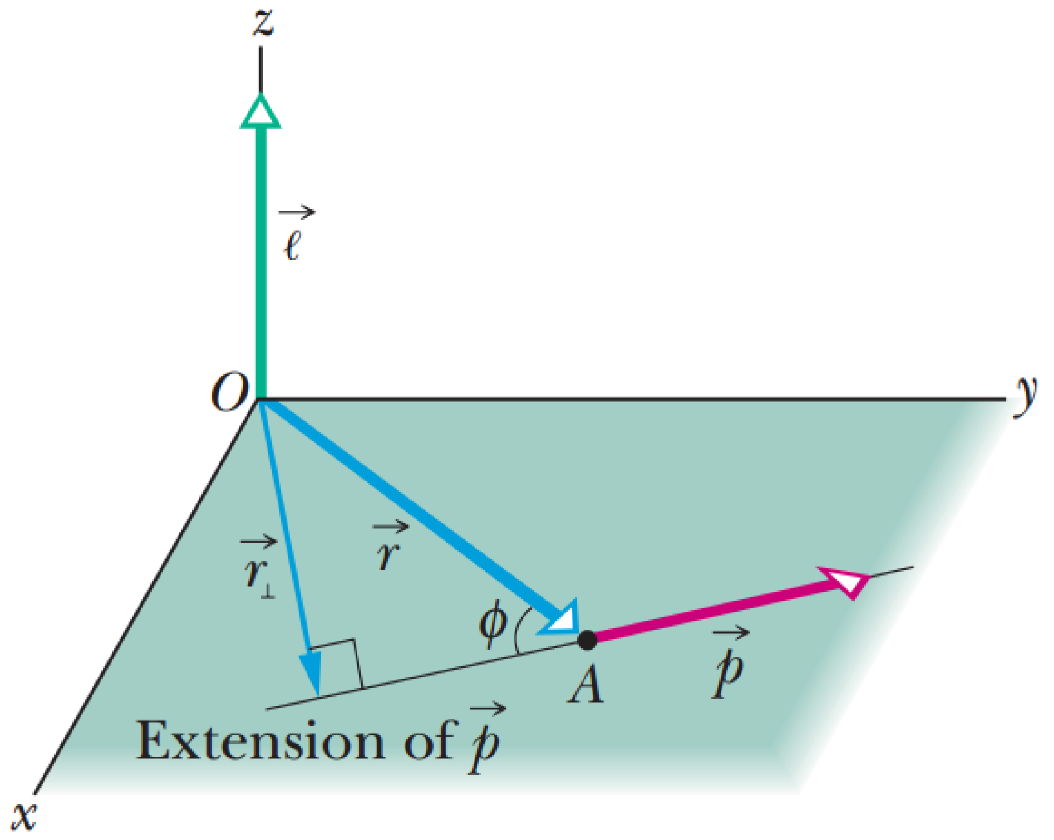




$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \qquad \text{(angular momentum of a particle)}$$

Angular momentum





Angular momentum

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Important. Note two features here: (1) angular momentum has meaning only with respect to a specified origin and (2) its direction is always perpendicular to the plane formed by the position and linear momentum vectors $\vec{r}$ and $\vec{p}$.

The units of angular momentum are $\mathrm{kg \cdot m^2/s}$.

Rate of change of angular momentum

$$\boxed{\vec{L} = \vec{r} \times m\vec{v}}$$

$$\vec{v} = d\vec{r}/dt$$

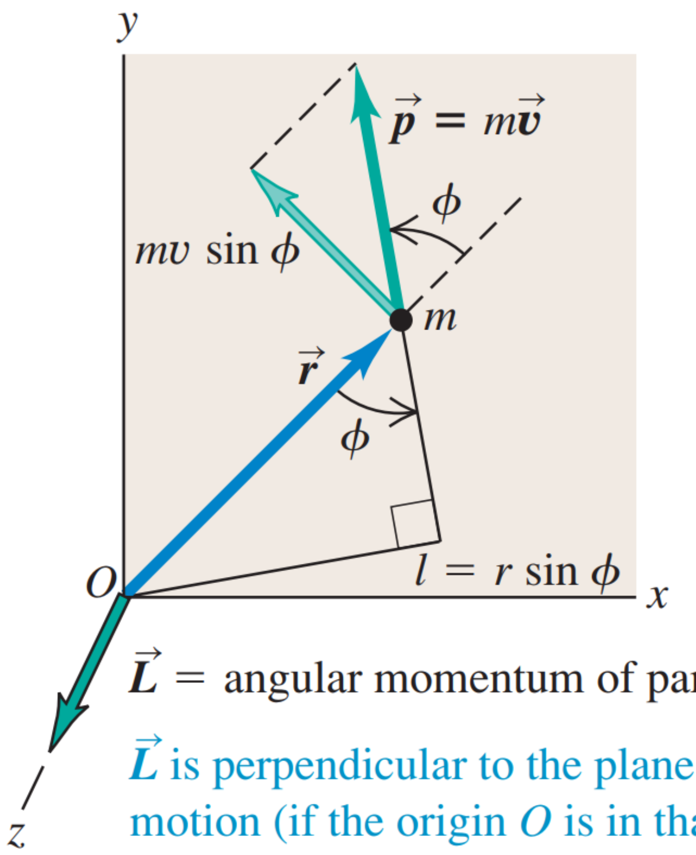
$$\frac{d\vec{L}}{dt} = \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times m\frac{d\vec{v}}{dt}\right) = (\vec{v} \times m\vec{v}) + (\vec{r} \times m\vec{a})$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}} \quad \text{(for a particle acted on by net force } \vec{F}\text{)}$$

The rate of change of angular momentum of a particle equals the torque of the net force acting on it.

Angular momentum

$$L = mvr\sin\phi = mvl$$



$\vec{L}$ = angular momentum of particle

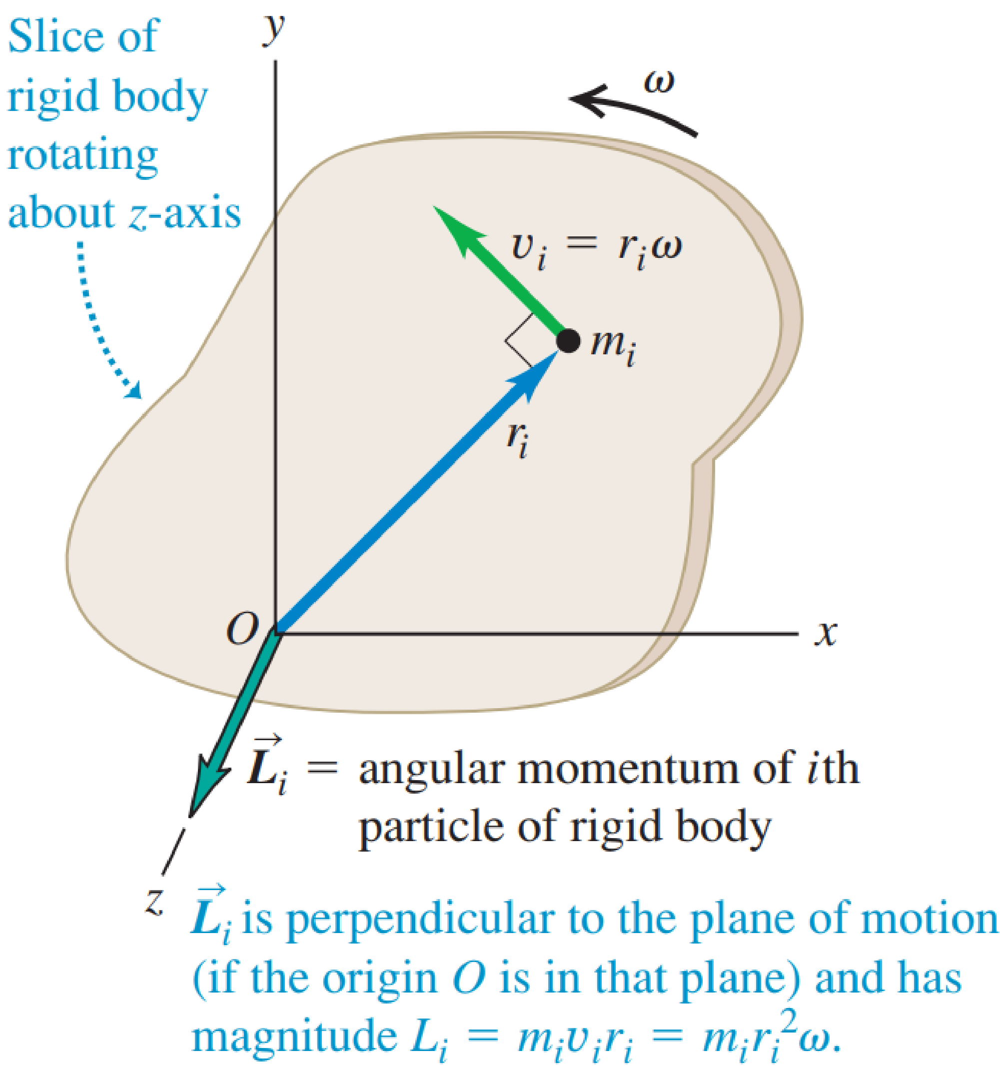
$\vec{L}$ is perpendicular to the plane of motion (if the origin O is in that plane) and has magnitude $L = mvl$.

Angular momentum of a rigid body

$$\phi = 90^\circ$$

$$L_i = m_i(r_i\omega)\,r_i = m_i r_i^{\,2}\omega$$

$$L = \sum L_i = \left(\sum m_i r_i^{\,2}\right)\omega = I\omega$$

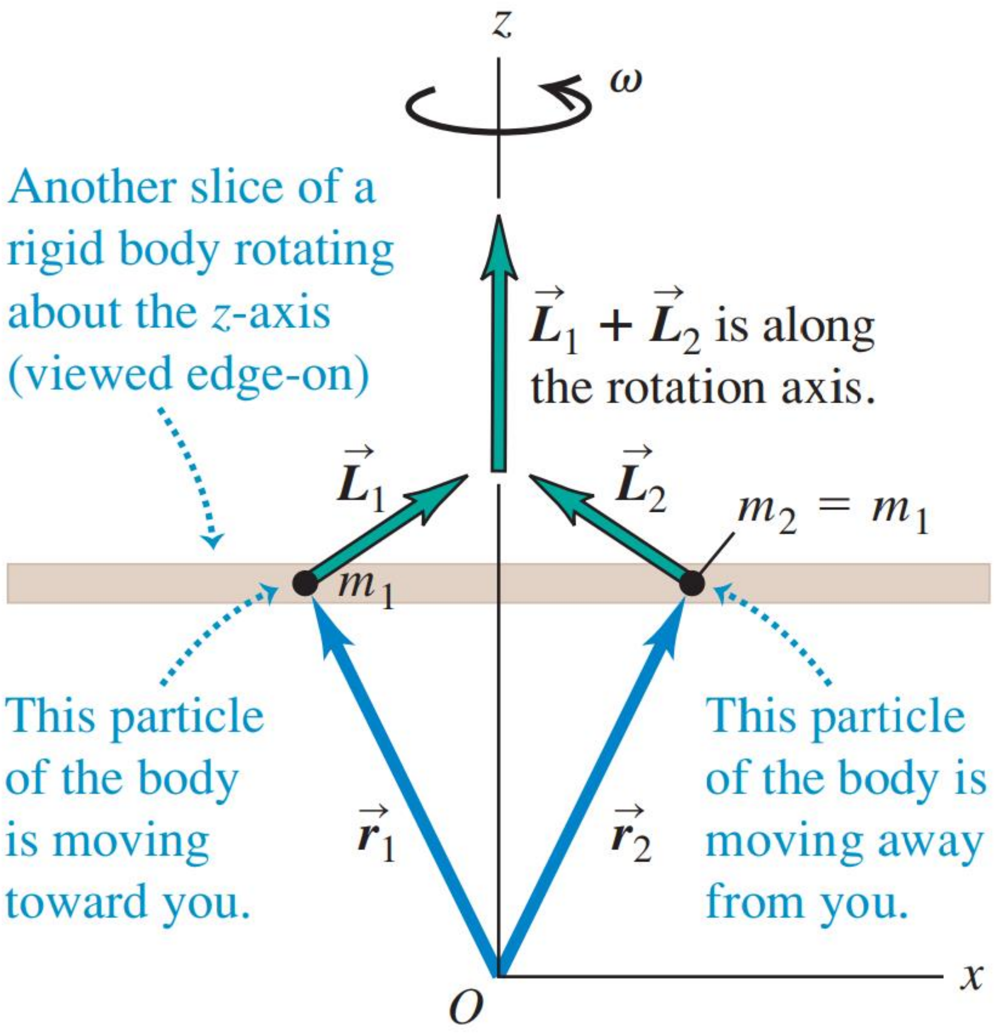


Angular momentum of a rigid body

$$\vec{L} = I\vec{\omega}$$

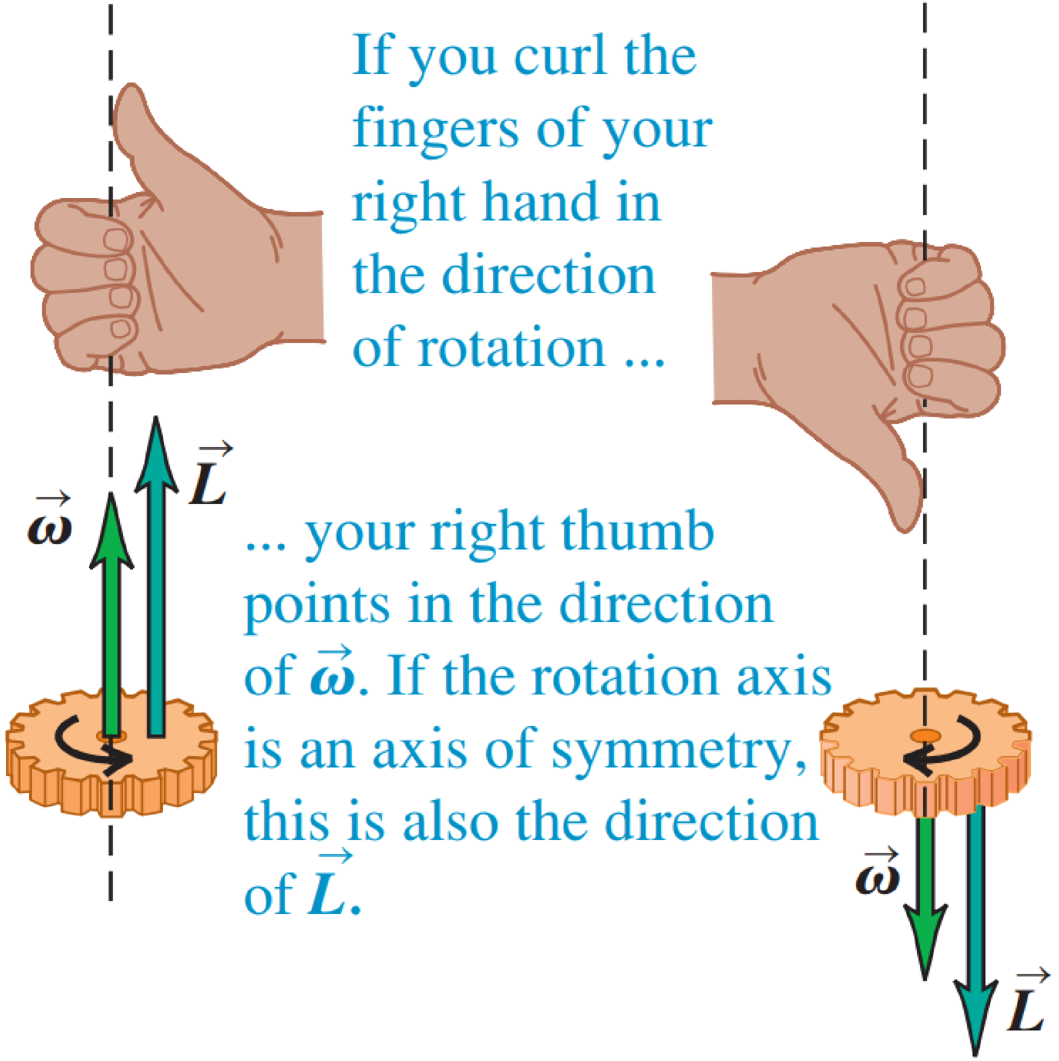
(for a rigid body rotating around a symmetry axis)

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt} \qquad \text{(for any system of particles)}$$



Angular momentum of a rigid body

$$\vec{L} = I\vec{\omega}$$



Angular momentum and torque

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

Angular momentum and torque

EXECUTE: (a) From Eq. (10.28),

$$L_z = I\omega_z = (2.5\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s}^3)t^2 = (100\ \text{kg}\cdot\text{m}^2/\text{s}^3)t^2$$

(We dropped the dimensionless quantity "rad" from the final expression.) At $t = 3.0\ \text{s}$, $L_z = 900\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) From Eq. (10.29),

$$\tau_z = \frac{dL_z}{dt} = (100\ \text{kg}\cdot\text{m}^2/\text{s}^3)(2t) = (200\ \text{kg}\cdot\text{m}^2/\text{s}^3)t$$

At $t = 3.0\ \text{s}$,

$$\tau_z = (200\ \text{kg}\cdot\text{m}^2/\text{s}^3)(3.0\ \text{s}) = 600\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 600\ \text{N}\cdot\text{m}$$

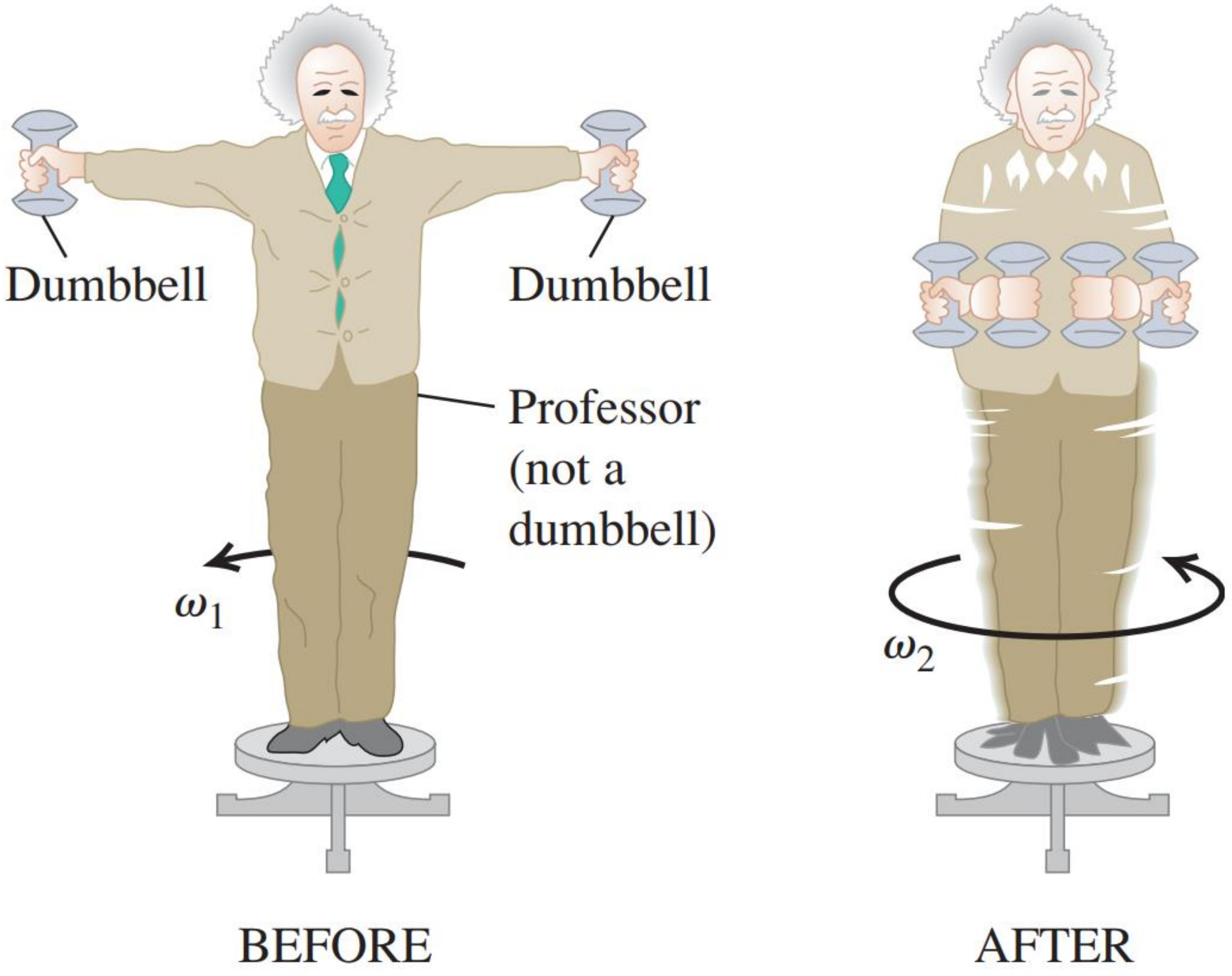
Conservation of angular momentum

When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved).

$$\frac{d\vec{\boldsymbol{L}}}{dt} = \mathbf{0} \qquad \text{(zero net external torque)}$$

Anyone can be a ballerina

$$\vec{L} = I\vec{\omega}$$



Lets watch this video

8.01x - Lect 20 - Angular Momentum- Torques- Conservation of Angular Momentum

http://www.youtube.com/watch?v=sNaaL19opxw

Readings

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Chapter 10: Dynamics of Rotational Motion