

Mechanics

Linear Momentum of a Particle and a System of Particles, Momentum Conservation, Application of Momentum Principle, Angular momentum of a Particle and a System of Particles, Kepler's law of Planetary Motion, Laws of Universal Gravitation, Motion of Planets and Satellites.

Fundamentals of Physics (10th Ed) – Halliday, Resnick, Walker

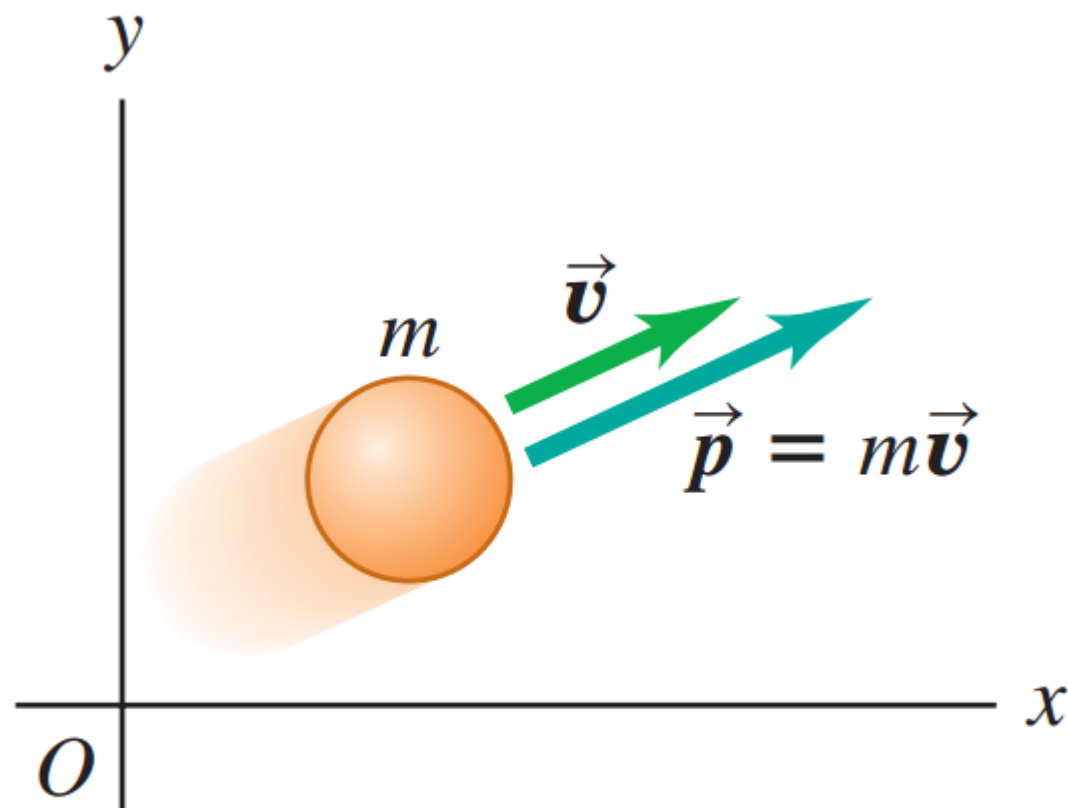
University Physics with Modern Physics (13th Ed) – Young, Freedman

Linear Momentum

$$p_x = mv_x$$

$$p_y = mv_y$$

$$p_z = mv_z$$



Momentum \vec{p} is a vector quantity

Newton's Second Law in Terms of Momentum

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt} (m\vec{v}) = m \frac{d\vec{v}}{dt} = m\vec{a}.$$

The Linear Momentum of a System of Particles

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n \\ &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n\end{aligned}$$

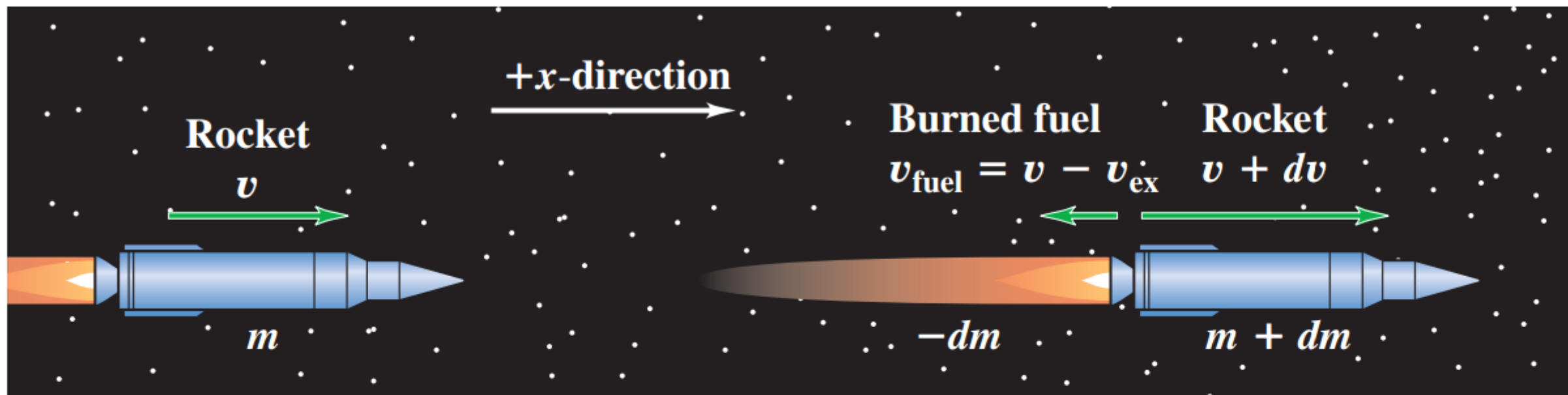
$$\vec{P} = M\vec{v}_{\text{com}}$$

Conservation of Linear Momentum

$$\left(\begin{array}{c} \text{total linear momentum} \\ \text{at some initial time } t_i \end{array} \right) = \left(\begin{array}{c} \text{total linear momentum} \\ \text{at some later time } t_f \end{array} \right)$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

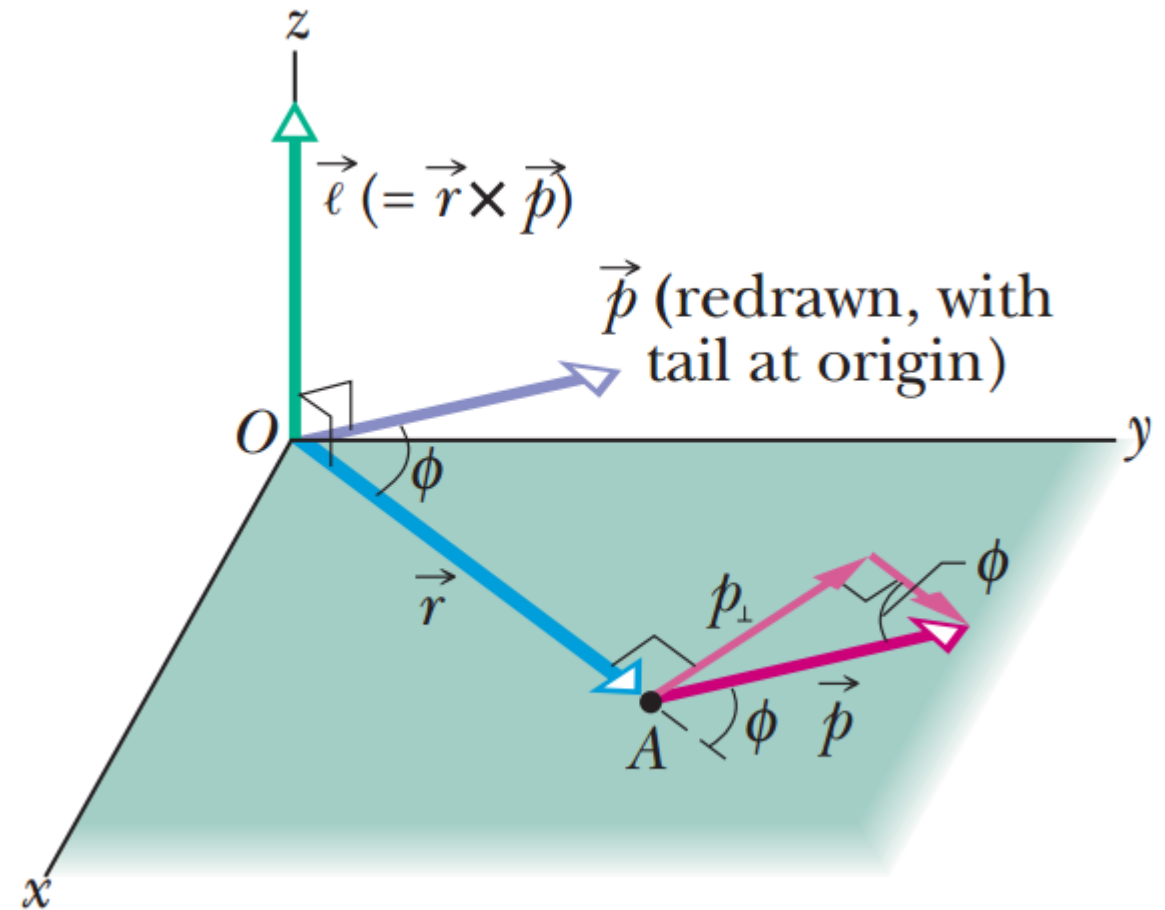


At time t , the rocket has mass m and x -component of velocity v .

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x -component of velocity $v + dv$. The burned fuel has x -component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

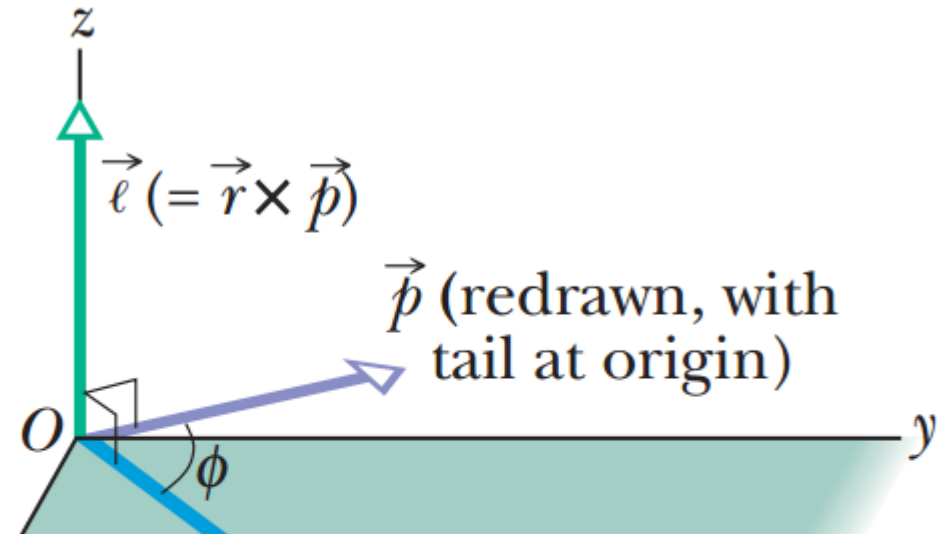
Angular Momentum

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$



$$\ell = r m v \sin \phi$$

Angular Momentum



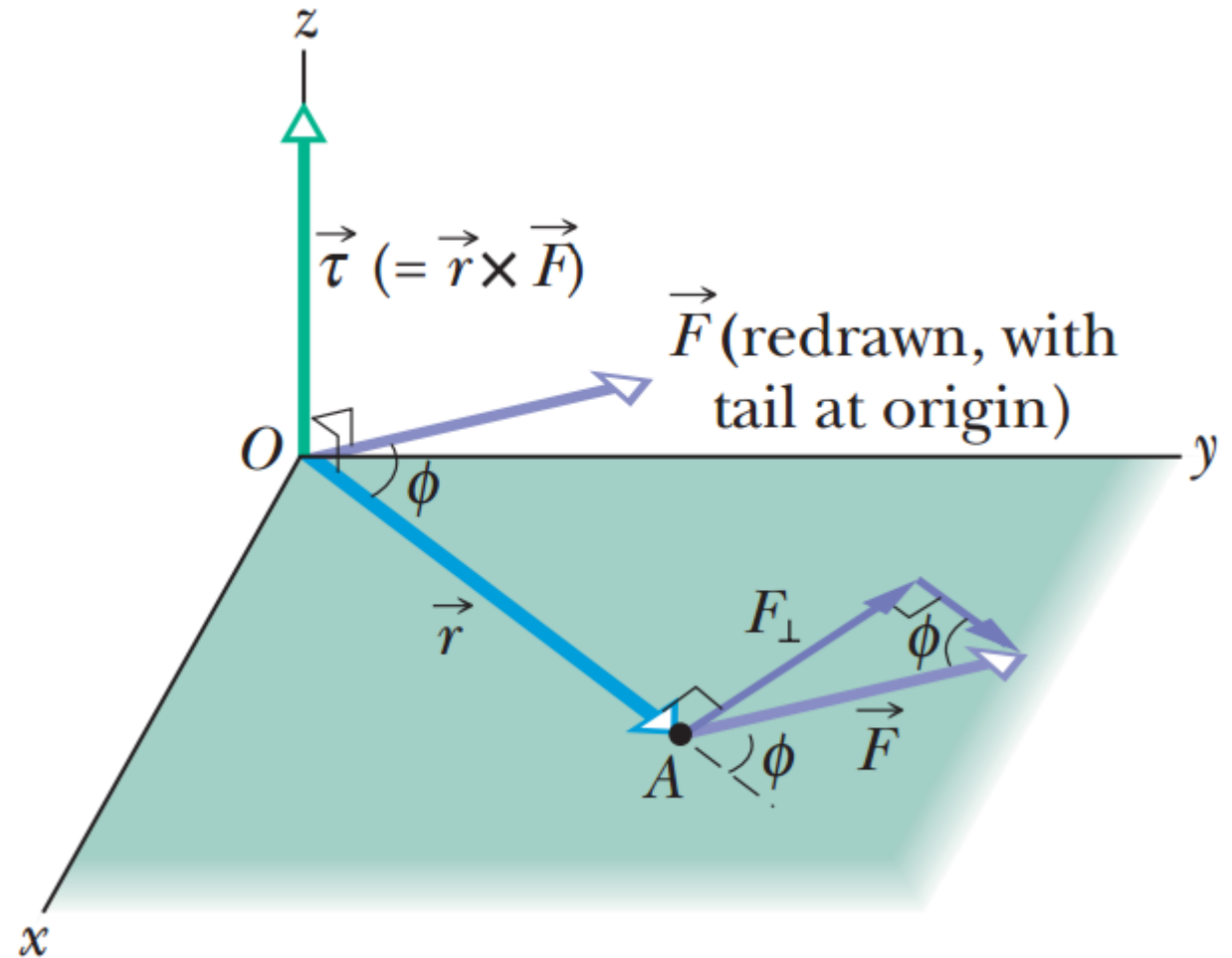
Important. Note two features here: (1) angular momentum has meaning only with respect to a specified origin and (2) its direction is always perpendicular to the plane formed by the position and linear momentum vectors \vec{r} and \vec{p} .

$$\ell = r m v \sin \phi$$

Newton's Second Law in Angular Form

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad (\text{single particle})$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt} \quad (\text{single particle})$$



Newton's Second Law in Angular Form

$$\vec{\ell} = m(\vec{r} \times \vec{v})$$

$$\frac{d\vec{\ell}}{dt} = m \left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v} \right) = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v})$$

$$= m(\vec{r} \times \vec{a}) = \vec{r} \times m\vec{a}$$

$$\vec{v} \times \vec{v} = 0$$

$$= \vec{r} \times \vec{F}_{\text{net}} = \sum(\vec{r} \times \vec{F})$$

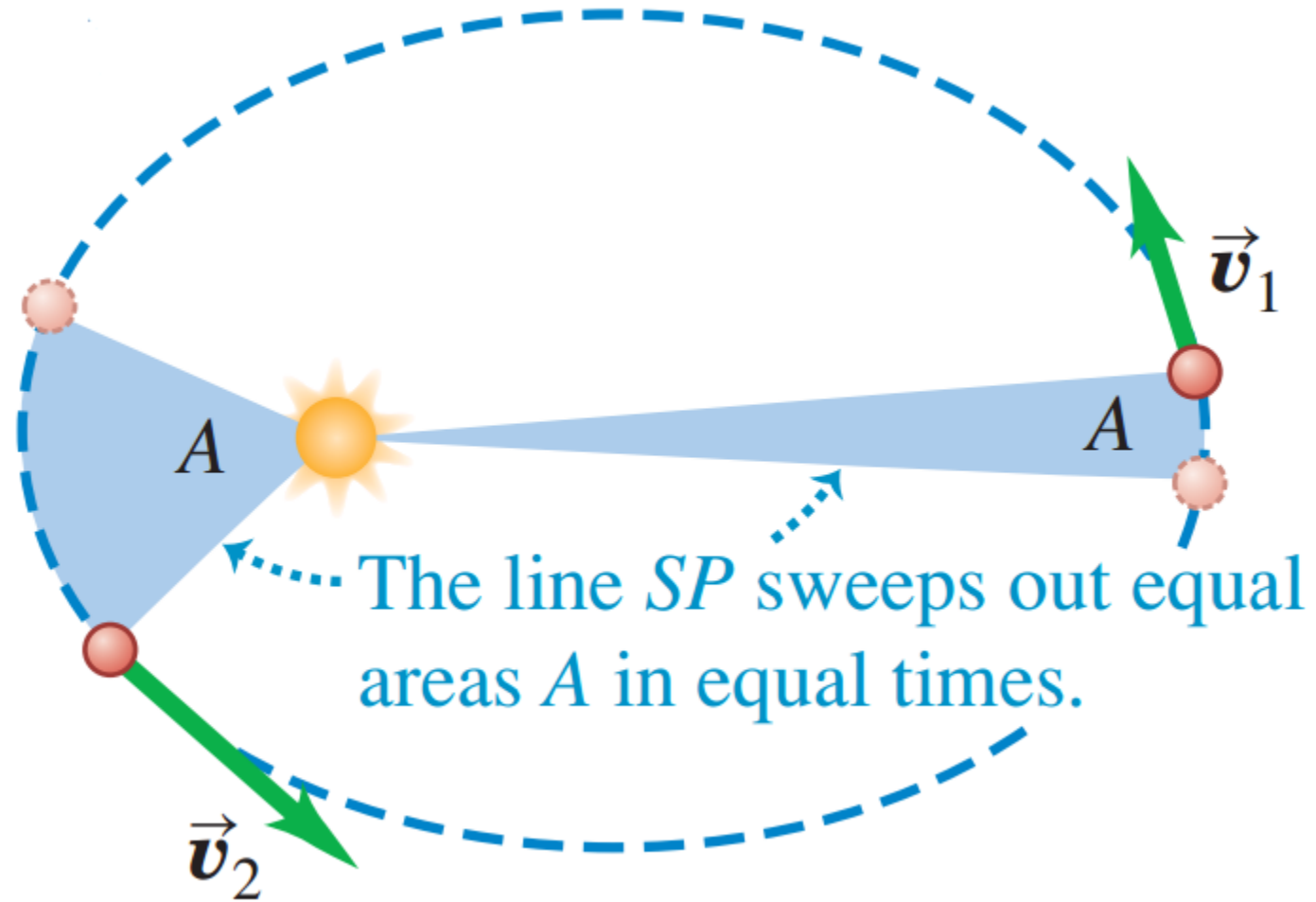
$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}$$

Planets and Satellites: Kepler's Laws

The law of orbits. All planets move in elliptical orbits with the Sun at one focus.

The law of areas. A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)

Planets and Satellites: Kepler's Laws



Planets and Satellites: Kepler's Laws

The law of periods. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r ,

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3 \quad (\text{law of periods}),$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis a is substituted for r .

Satellites: Orbits and Energy

The potential energy of the system is given by $U = -\frac{GMm}{r}$

Here r is the radius of the satellite's orbit,
 M and m are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit,
we write Newton's second law ($F = ma$) as

$$\frac{GMm}{r^2} = m \frac{v^2}{r},$$

Satellites: Orbits and Energy

The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$,

which shows us that for a satellite in a circular orbit, $K = -U/2$.

The total mechanical energy of the orbiting satellite is

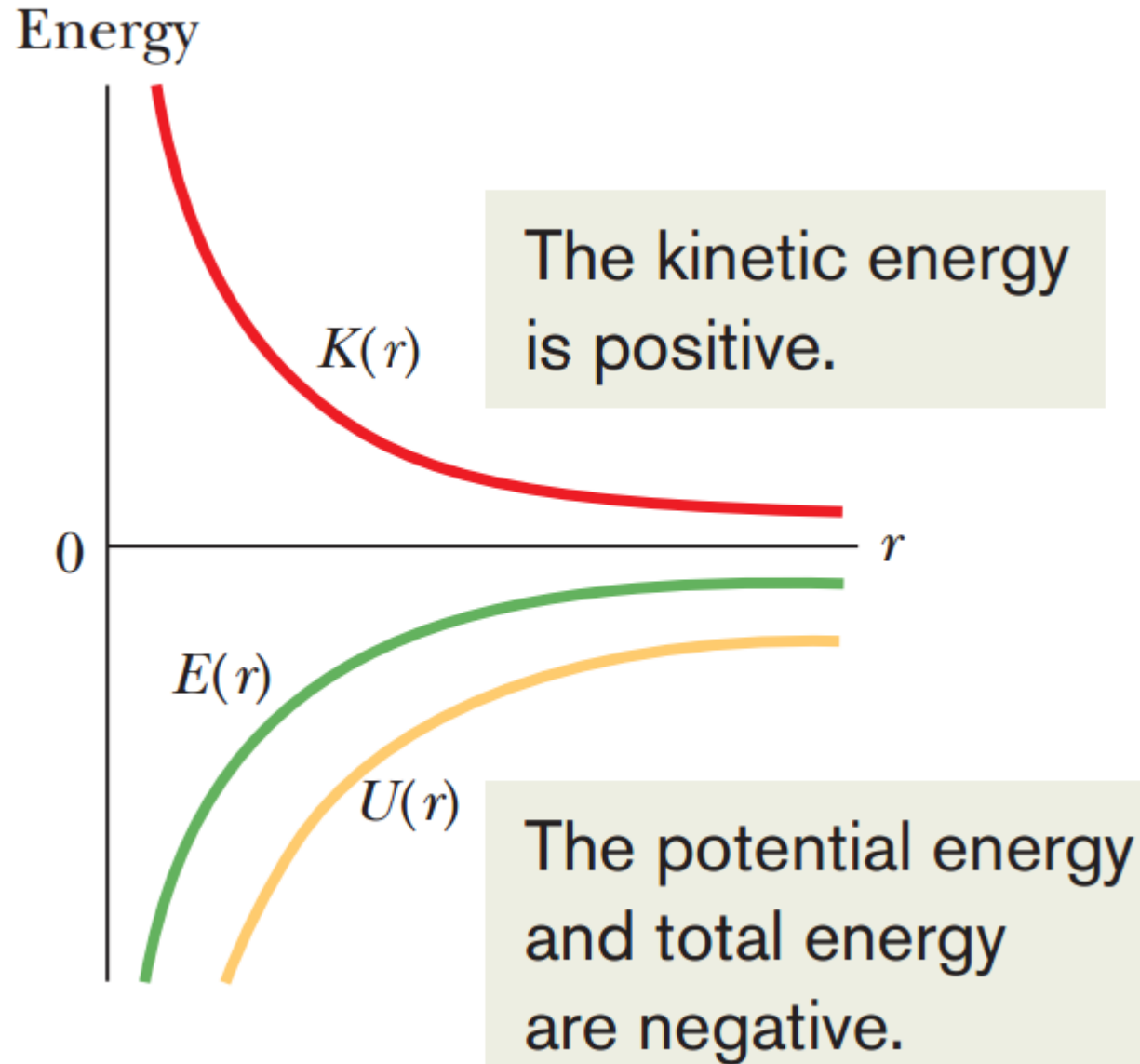
$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \quad (\text{circular orbit}).$$

$$E = -K$$

Satellites: Orbits and Energy

This is a plot of a satellite's energies versus orbit radius.



Satellites: Orbits and Energy

For a satellite in an elliptical orbit of semimajor axis a , we can substitute a for r to find the mechanical energy:

$$E = -\frac{GMm}{2a} \quad (\text{elliptical orbit}).$$

Satellites: Orbits and Energy

A playful astronaut releases a bowling ball, of mass $m = 7.20$ kg, into circular orbit about Earth at an altitude h of 350 km.

(a) What is the mechanical energy E of the ball in its orbit?

Satellites: Orbits and Energy

Calculations: The orbital radius must be

$$r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},$$

in which R is the radius of Earth. Then, from Eq. 13-40 with Earth mass $M = 5.98 \times 10^{24} \text{ kg}$, the mechanical energy is

$$\begin{aligned} E &= -\frac{GMm}{2r} \\ &= -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.20 \text{ kg})}{(2)(6.72 \times 10^6 \text{ m})} \\ &= -2.14 \times 10^8 \text{ J} = -214 \text{ MJ.} \end{aligned} \quad (\text{Answer})$$

Satellites: Orbits and Energy

Calculate the total energy of a five metric ton telecommunications satellite circulating in a geostationary orbit.

The mass of earth is 5.98×10^{24} kg.

1 metric ton = 1000 kg.

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

$$E = -\frac{GMm}{2r}$$

$$E = -23.60 \text{ GJ}$$