Periodic Motion

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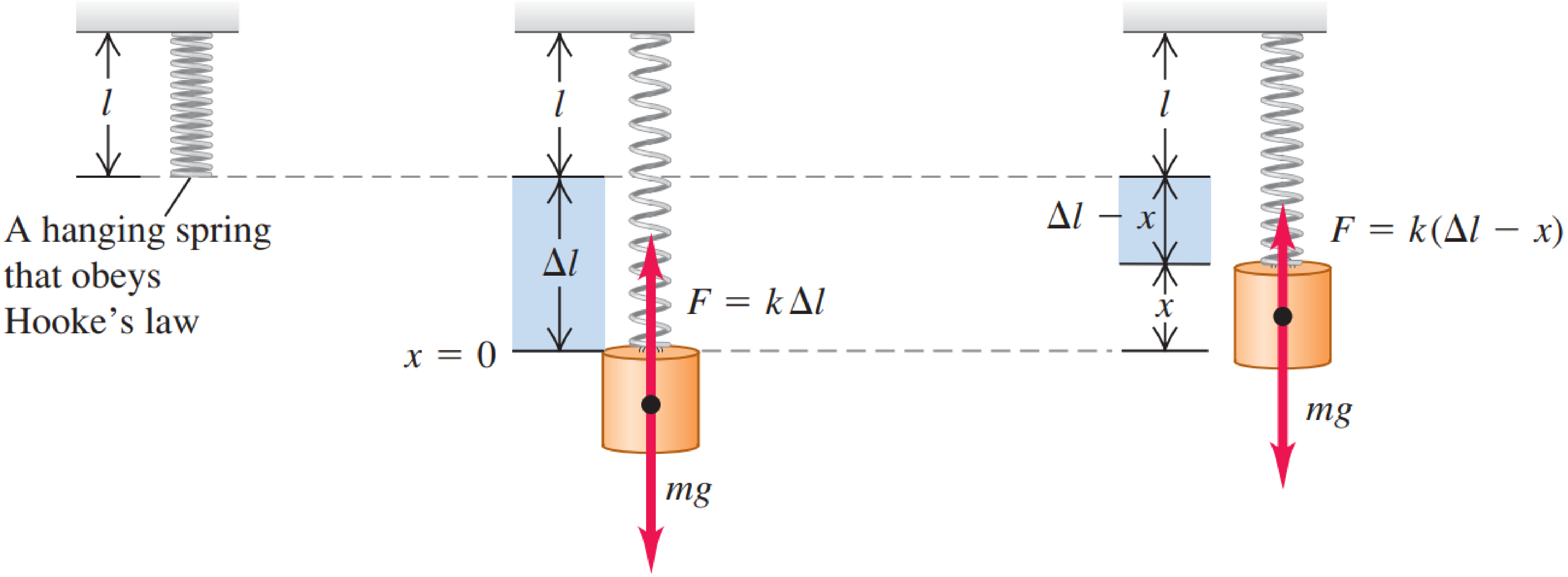
Vertical Simple Harmonic Motion

(a)

(b) A body is suspended from the spring. It is in equilibrium when the upward force exerted by the stretched spring equals the body's weight.

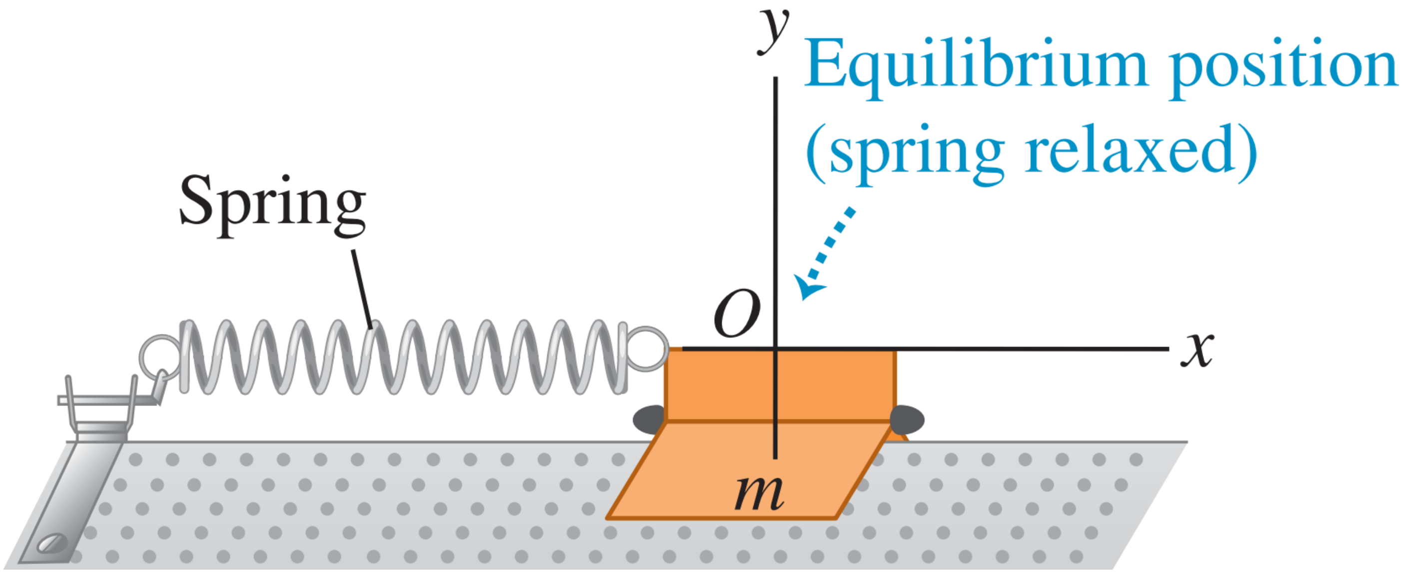
(c) If the body is displaced from equilibrium, the net force on the body is proportional to its displacement. The oscillations are SHM.



A system that can have periodic motion

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

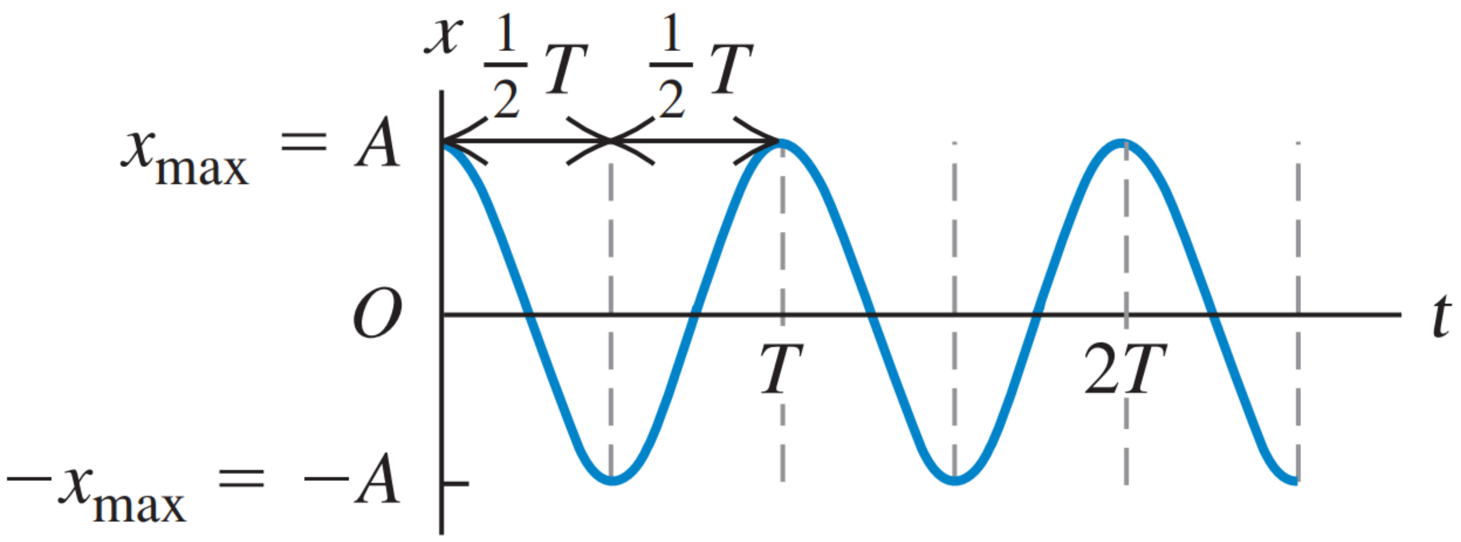
$$\omega = 2\pi f = \frac{2\pi}{T}$$



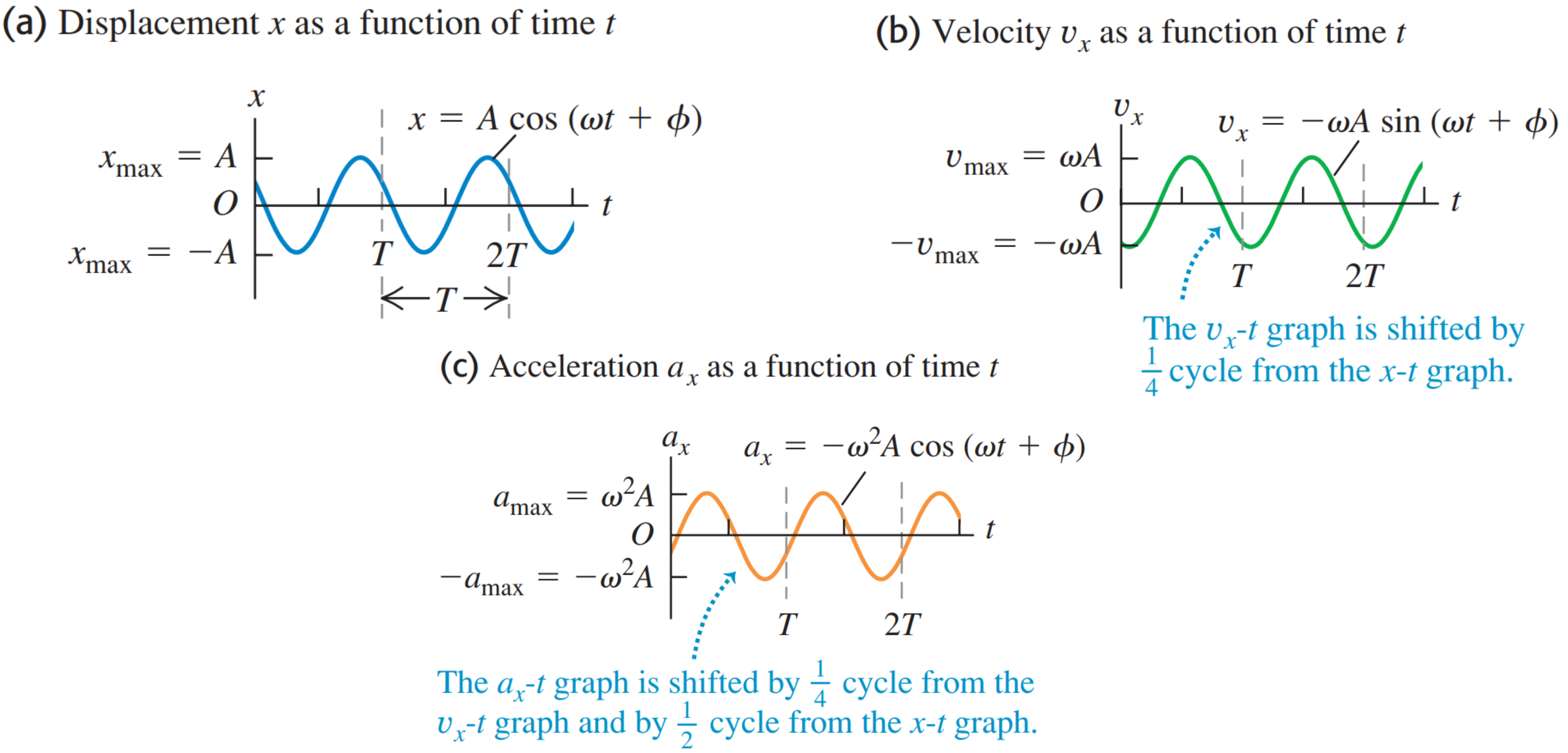
Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi) \qquad \text{(displacement in SHM)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(simple harmonic motion)}$$



Simple Harmonic Motion (SHM)



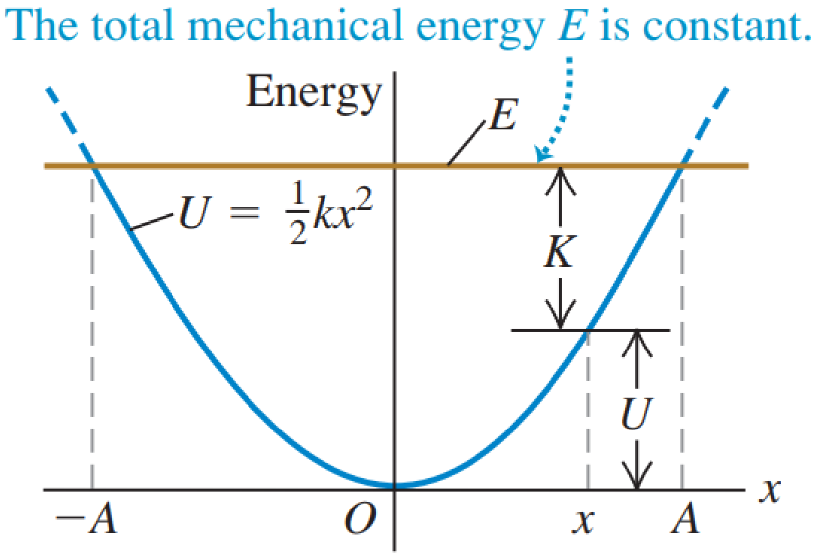
Energy in Simple Harmonic Motion

$$E = \tfrac{1}{2}m{v_x}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \qquad \text{(total mechanical energy in SHM)}$$

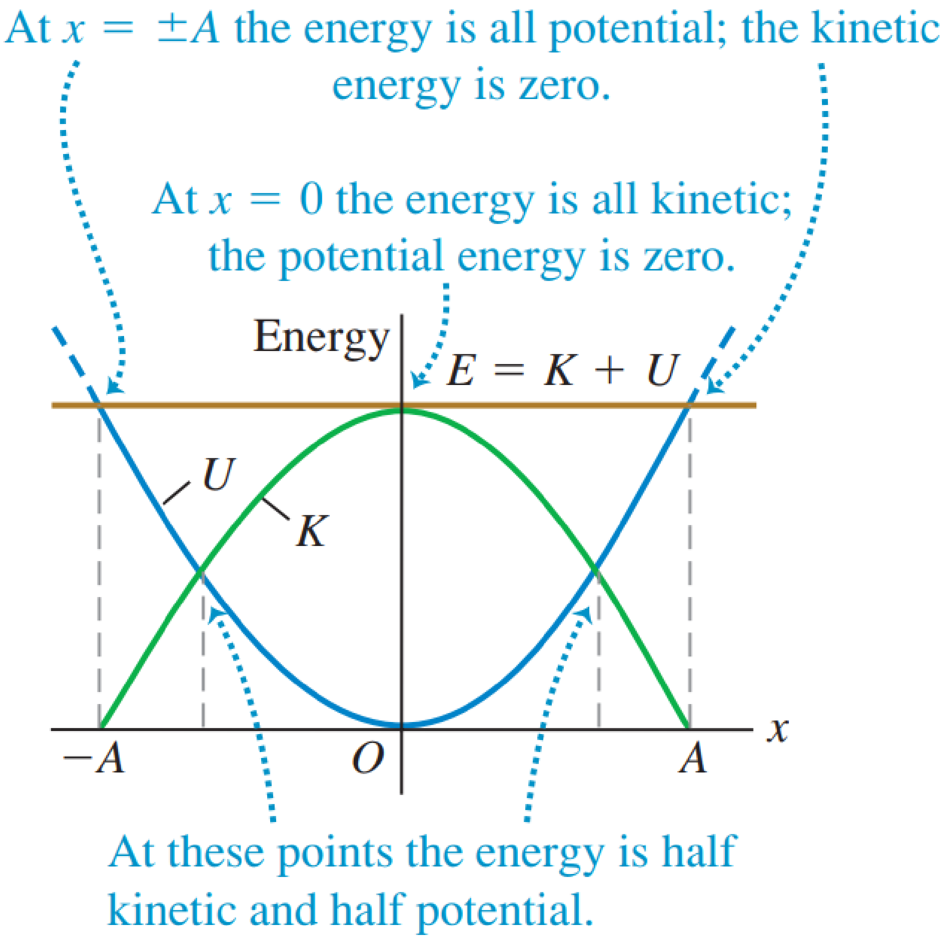
$$\begin{aligned} E &= \tfrac{1}{2}m{v_x}^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}m[-\omega A \sin(\omega t + \phi)]^2 + \tfrac{1}{2}k[A\cos(\omega t + \phi)]^2 \\ &= \tfrac{1}{2}kA^2 \sin^2(\omega t + \phi) + \tfrac{1}{2}kA^2 \cos^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2 \end{aligned}$$

Energy in Simple Harmonic Motion

(a) The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x



(b) The same graph as in **(a)**, showing kinetic energy K as well



The Principle of Superposition for Waves

Suppose that two waves travel simultaneously along the same stretched string. Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

This summation of displacements along the string means that

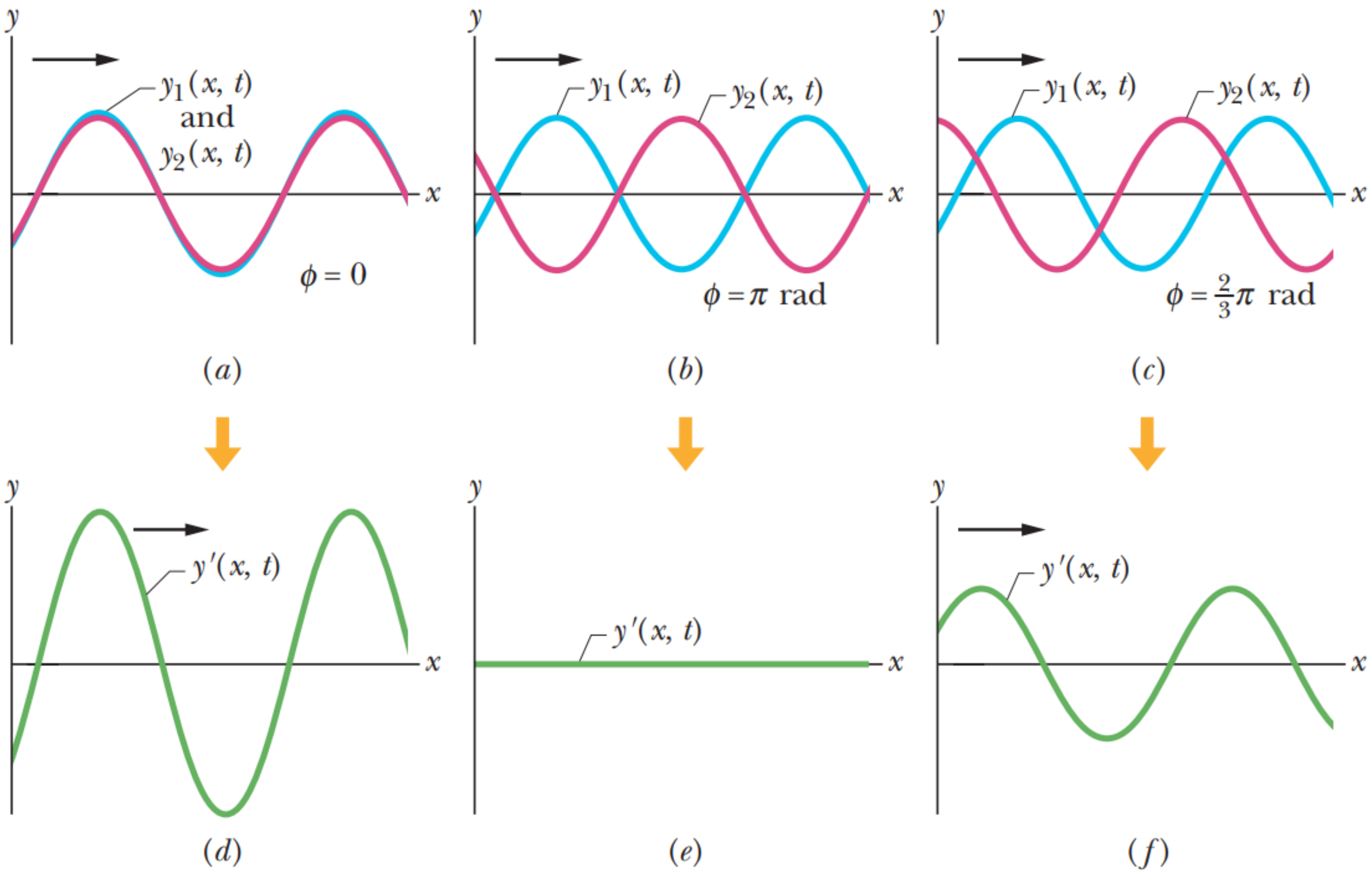
Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

This is another example of the **principle of superposition,** which says that when several effects occur simultaneously, their net effect is the sum of the individual effects. (We should be thankful that only a simple sum is needed. If two effects somehow amplified each other, the resulting nonlinear world would be very difficult to manage and understand.)

Interference of Waves

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi)$$



Interference of Waves

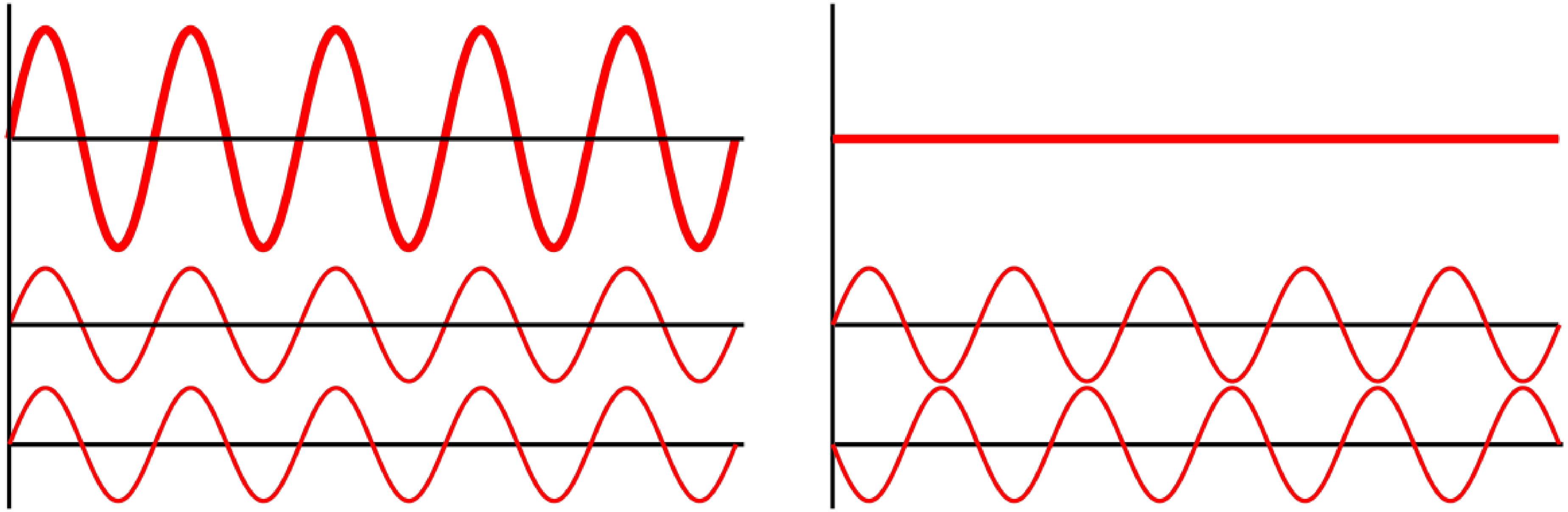
$$y'(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$

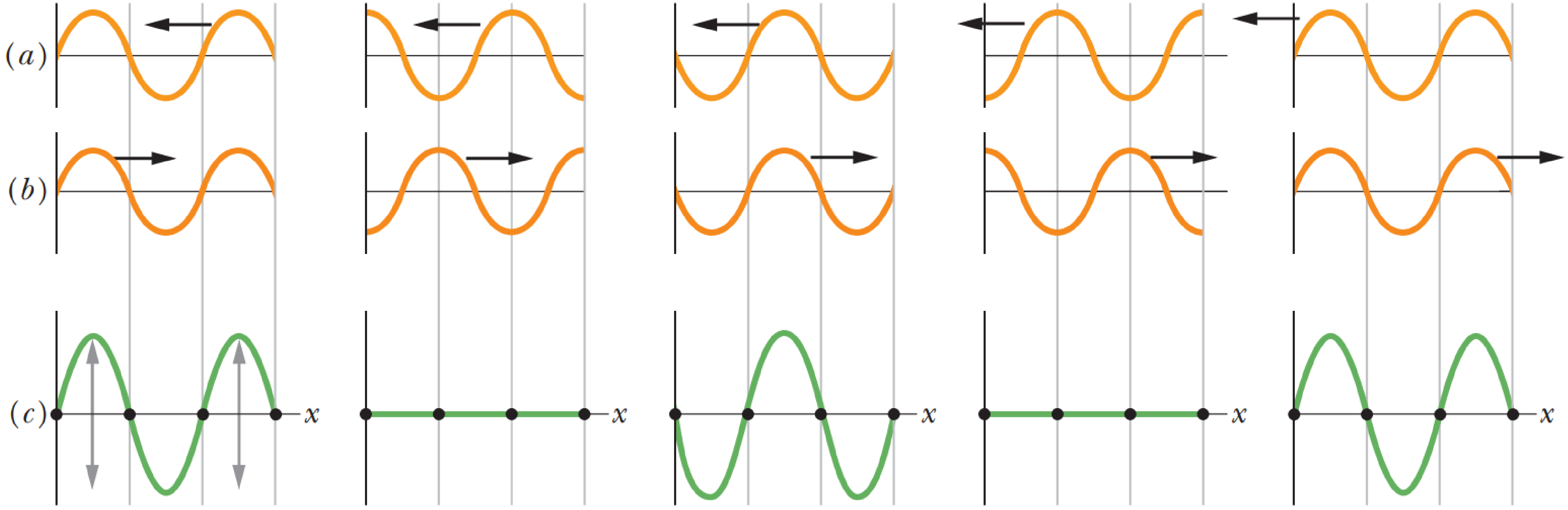
Interference of Waves

If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.



Standing Waves

As the waves move through each other, some points never move and some move the most.



$t = 0$ $t = \frac{1}{4}T$ $t = \frac{1}{2}T$ $t = \frac{3}{4}T$ $t = T$

Standing Waves

If two sinusoidal waves of the same amplitude and wavelength travel in *opposite* directions along a stretched string, their interference with each other produces a standing wave.

To analyze a standing wave, we represent the two waves with the equations

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

and

$$y_2(x, t) = y_m \sin(kx + \omega t).$$

The principle of superposition gives, for the combined wave,

$$y'(x, t) = y_1(x, t) + y_2(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

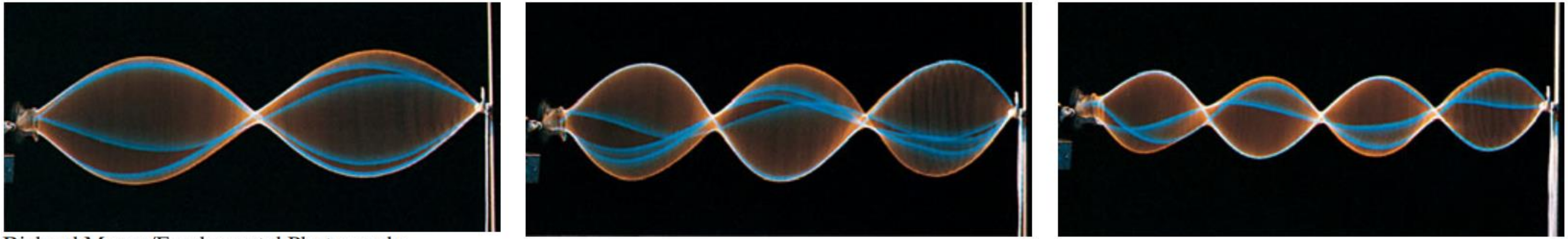
Applying the trigonometric relation

$$y'(x, t) = [2y_m \sin kx] \cos \omega t.$$

Standing Waves

$$x = n\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(nodes)}$$

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)}$$



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