Gravitation

Dr Mohammad Abdur Rashid

References

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Newton’s law of gravitation

Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

$$F_g = \frac{Gm_1m_2}{r^2} \qquad \text{(law of gravitation)}$$

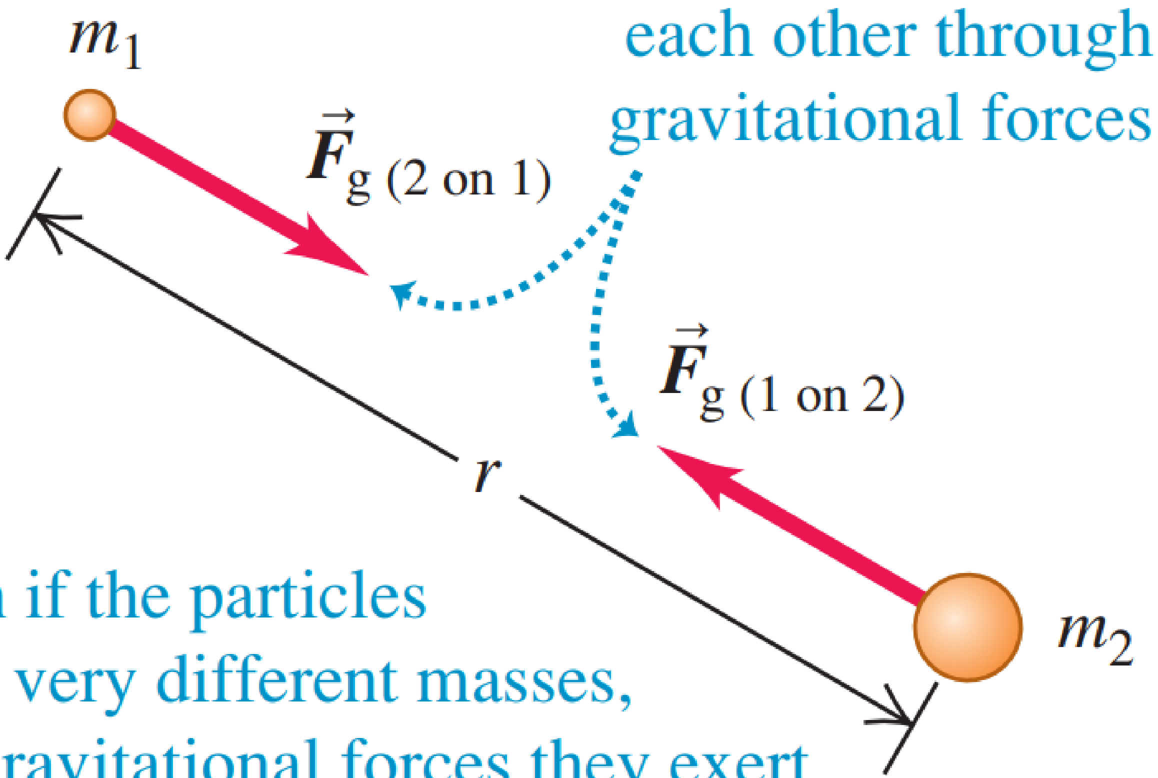
Caution

CAUTION **Don't confuse g and G** Because the symbols g and G are so similar, it's common to confuse the two very different gravitational quantities that these symbols represent. Lowercase g is the acceleration due to gravity, which relates the weight w of a body to its mass m: $w = mg$. The value of g is different at different locations on the earth's surface and on the surfaces of different planets. By contrast, capital G relates the gravitational force between any two bodies to their masses and the distance between them. We call G a *universal* constant because it has the same value for any two bodies, no matter where in space they are located. In the next section we'll see how the values of g and G are related.

Newton's law of gravitation

$$F_g = \frac{Gm_1 m_2}{r^2}$$

Any two particles attract each other through gravitational forces.



Even if the particles have very different masses, the gravitational forces they exert on each other are equal in strength:

$$F_{g\,(1\text{ on }2)} = F_{g\,(2\text{ on }1)}$$

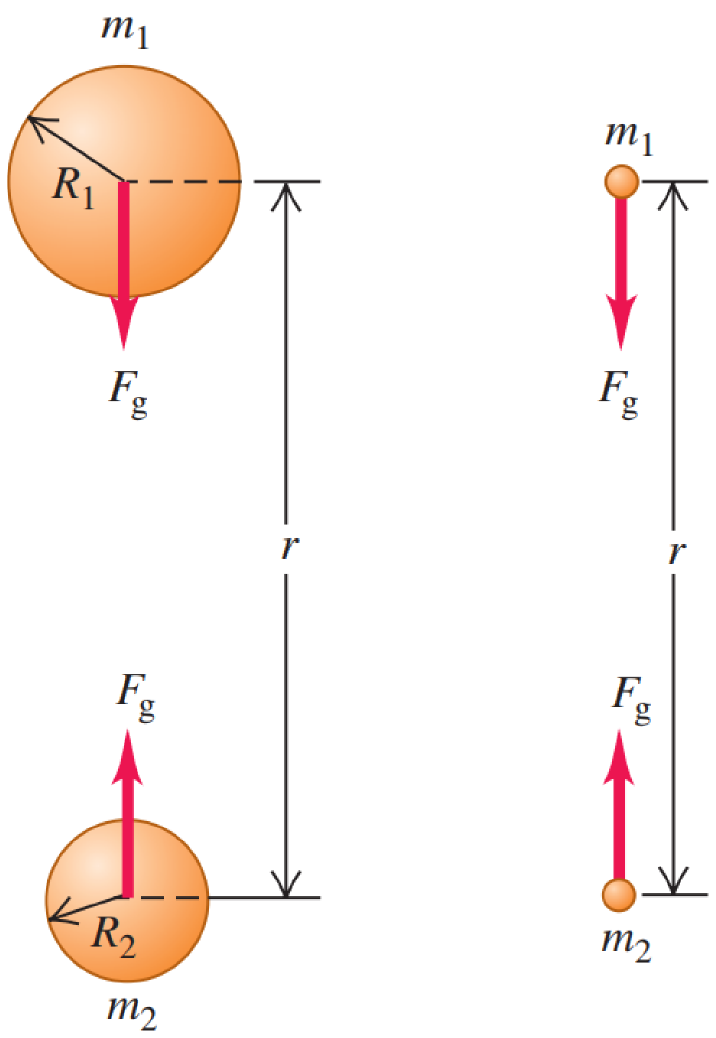
Newton's law of gravitation

$$F_g = \frac{G m_E m}{r^2}$$

$$G = 6.67428(67) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

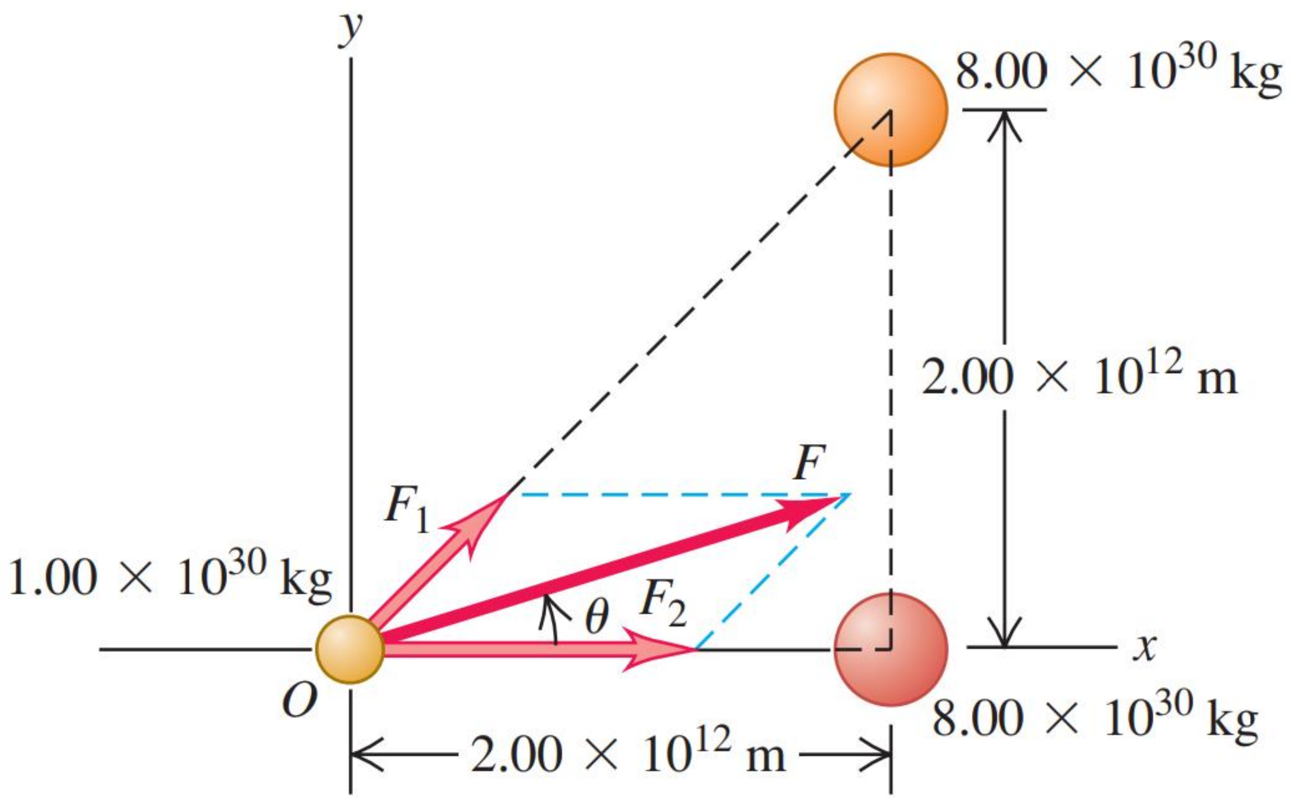
(a) The gravitational force between two spherically symmetric masses m_1 and m_2 ...

(b) ... is the same as if we concentrated all the mass of each sphere at the sphere's center.



Superposition of gravitational forces

Many stars belong to *systems* of two or more stars held together by their mutual gravitational attraction. Figure shows a three-star system at an instant when the stars are at the vertices of a right triangle. Find the total gravitational force exerted on the small star by the two large ones.



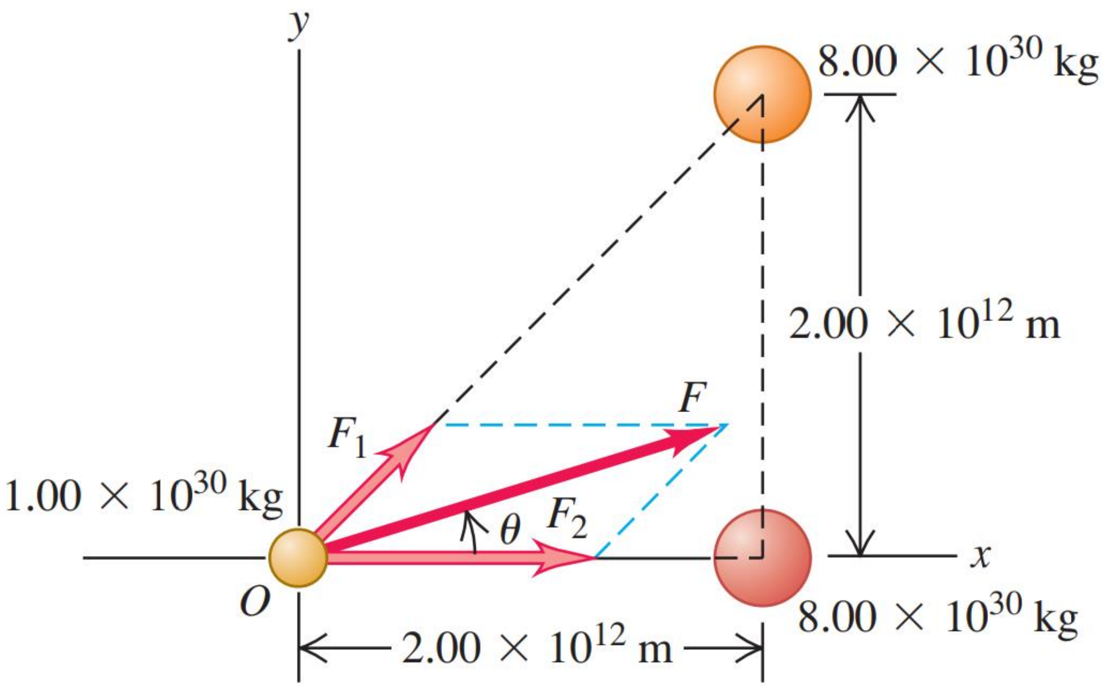
Superposition of gravitational forces

$$F_1 = \frac{\left[\begin{array}{c}(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}) \\ \times\,(8.00 \times 10^{30}\,\mathrm{kg})(1.00 \times 10^{30}\,\mathrm{kg})\end{array}\right]}{(2.00 \times 10^{12}\,\mathrm{m})^2 + (2.00 \times 10^{12}\,\mathrm{m})^2}$$

$$= 6.67 \times 10^{25}\,\mathrm{N}$$

$$F_2 = \frac{\left[\begin{array}{c}(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}) \\ \times\,(8.00 \times 10^{30}\,\mathrm{kg})(1.00 \times 10^{30}\,\mathrm{kg})\end{array}\right]}{(2.00 \times 10^{12}\,\mathrm{m})^2}$$

$$= 1.33 \times 10^{26}\,\mathrm{N}$$



Superposition of gravitational forces

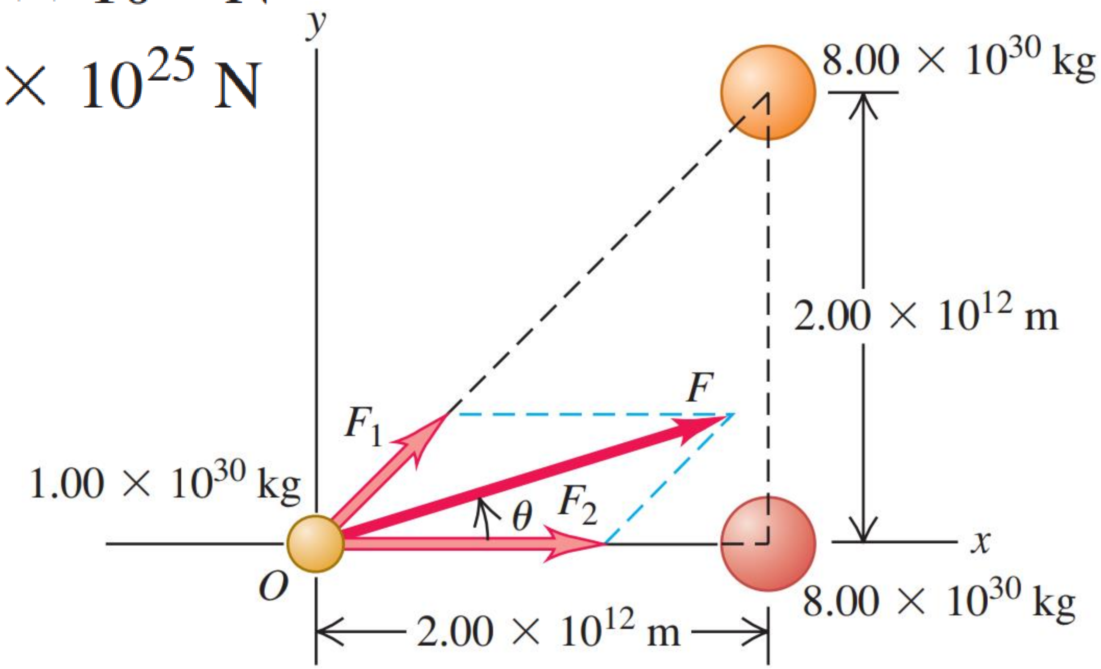
The x- and y-components of these forces are

$$F_{1x} = (6.67 \times 10^{25}\ \text{N})(\cos 45^\circ) = 4.72 \times 10^{25}\ \text{N}$$

$$F_{1y} = (6.67 \times 10^{25}\ \text{N})(\sin 45^\circ) = 4.72 \times 10^{25}\ \text{N}$$

$$F_{2x} = 1.33 \times 10^{26}\ \text{N}$$

$$F_{2y} = 0$$



Superposition of gravitational forces

The components of the total force $\vec{F}$ on the small star are

$$F_x = F_{1x} + F_{2x} = 1.81 \times 10^{26}\ \mathrm{N}$$
$$F_y = F_{1y} + F_{2y} = 4.72 \times 10^{25}\ \mathrm{N}$$

The magnitude of $\vec{F}$ and its angle θ (see Fig. 13.5) are

$$F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(1.81 \times 10^{26}\ \mathrm{N})^2 + (4.72 \times 10^{25}\ \mathrm{N})^2}$$
$$= 1.87 \times 10^{26}\ \mathrm{N}$$
$$\theta = \arctan\frac{F_y}{F_x} = \arctan\frac{4.72 \times 10^{25}\ \mathrm{N}}{1.81 \times 10^{26}\ \mathrm{N}} = 14.6^\circ$$

Weight

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe.

$$w = F_g = \frac{Gm_E m}{R_E^2} \qquad \text{(weight of a body of mass } m \text{ at the earth's surface)}$$

$$g = \frac{Gm_E}{R_E^2} \qquad \text{(acceleration due to gravity at the earth's surface)}$$

Gravity on Mars

A robotic lander with an earth weight of 3430 N is sent to Mars, which has radius $R_M = 3.40 \times 10^6$ m and mass $m_M = 6.42 \times 10^{23}$ kg (see Appendix F). Find the weight F_g of the lander on the Martian surface and the acceleration there due to gravity, g_M.

EXECUTE: The lander's earth weight is $w = mg$, so

$$m = \frac{w}{g} = \frac{3430\ \text{N}}{9.80\ \text{m/s}^2} = 350\ \text{kg}$$

Gravity on Mars

$$F_g = \frac{G m_M m}{R_M^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(6.42 \times 10^{23}\ \mathrm{kg})(350\ \mathrm{kg})}{(3.40 \times 10^6\ \mathrm{m})^2}$$

$$= 1.30 \times 10^3\ \mathrm{N}$$

The acceleration due to gravity on Mars is

$$g_M = \frac{F_g}{m} = \frac{1.30 \times 10^3\ \mathrm{N}}{350\ \mathrm{kg}} = 3.7\ \mathrm{m/s^2}$$

Gravitational Potential Energy

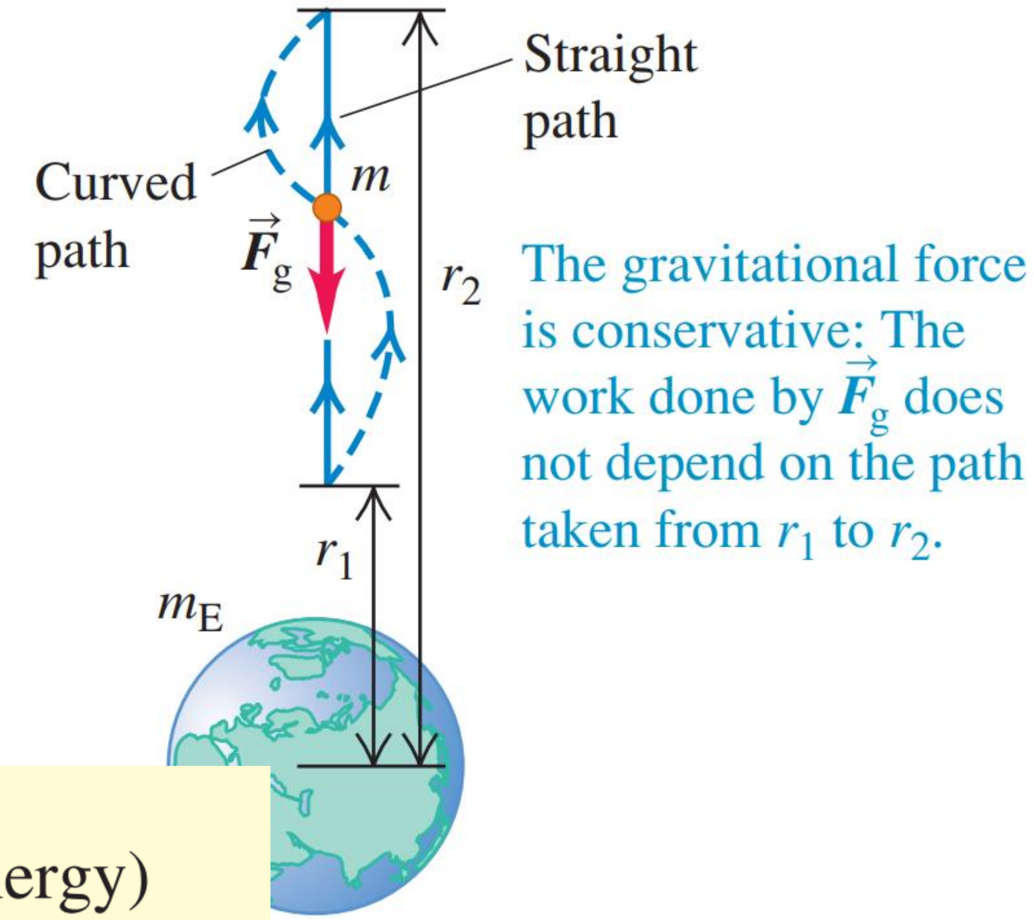
$$W_{\text{grav}} = \int_{r_1}^{r_2} F_r \, dr$$

$$F_r = -\frac{Gm_E m}{r^2}$$

$$W_{\text{grav}} = -Gm_E m \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{Gm_E m}{r_2} - \frac{Gm_E m}{r_1}$$

$$= U_1 - U_2$$

$$U = -\frac{Gm_E m}{r} \qquad \text{(gravitational potential energy)}$$



Gravitational potential energy

$$W_{\text{grav}} = Gm_{\text{E}}m\frac{r_1 - r_2}{r_1 r_2}$$

If the body stays close to the earth, then in the denominator we may replace r_1 and r_2 by R_{E}, the earth's radius, so

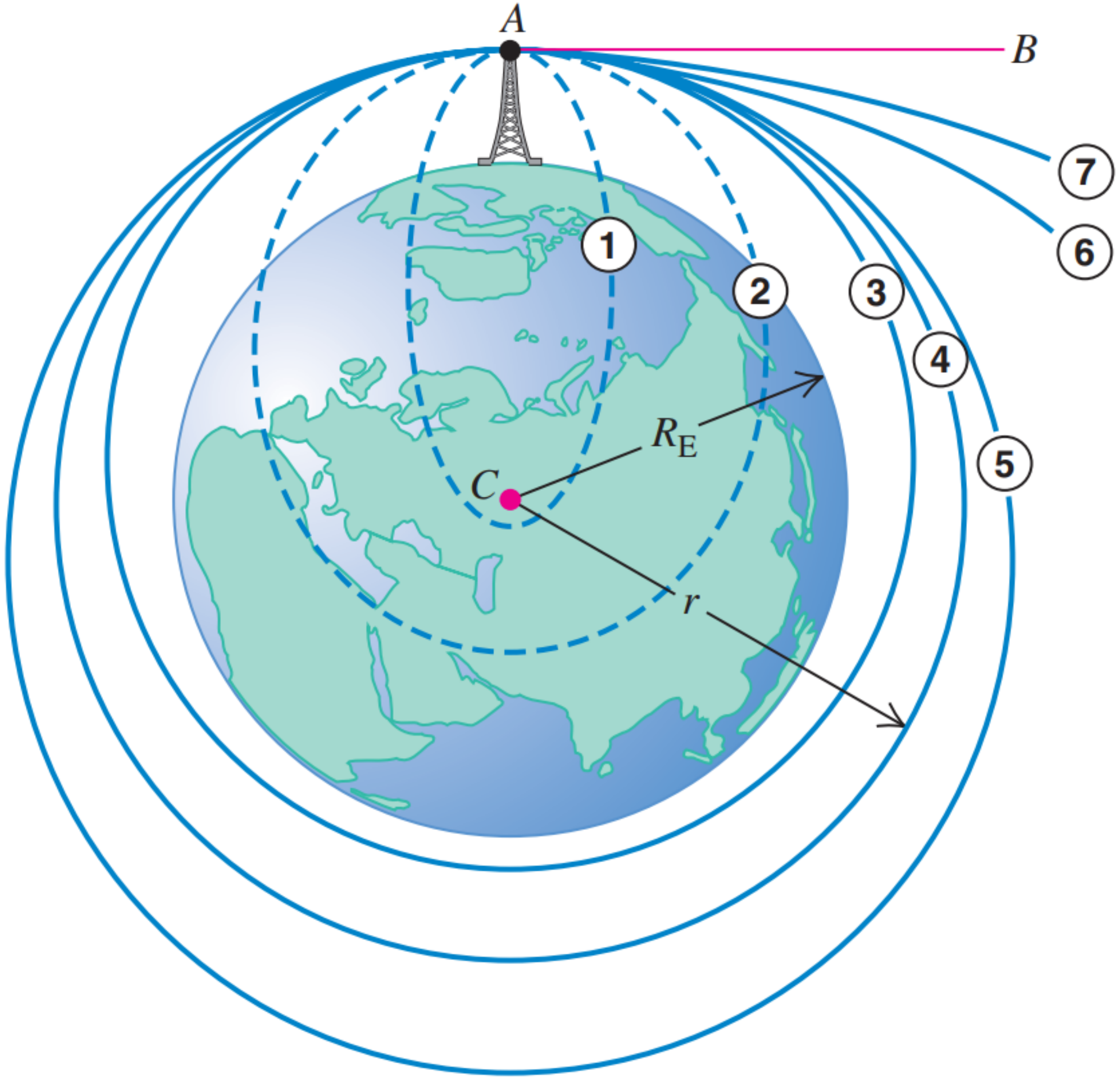
$$W_{\text{grav}} = Gm_{\text{E}}m\frac{r_1 - r_2}{R_{\text{E}}^2}$$

$$\boxed{g = Gm_{\text{E}}/R_{\text{E}}^2}$$

$$\boxed{W_{\text{grav}} = mg(r_1 - r_2)}$$

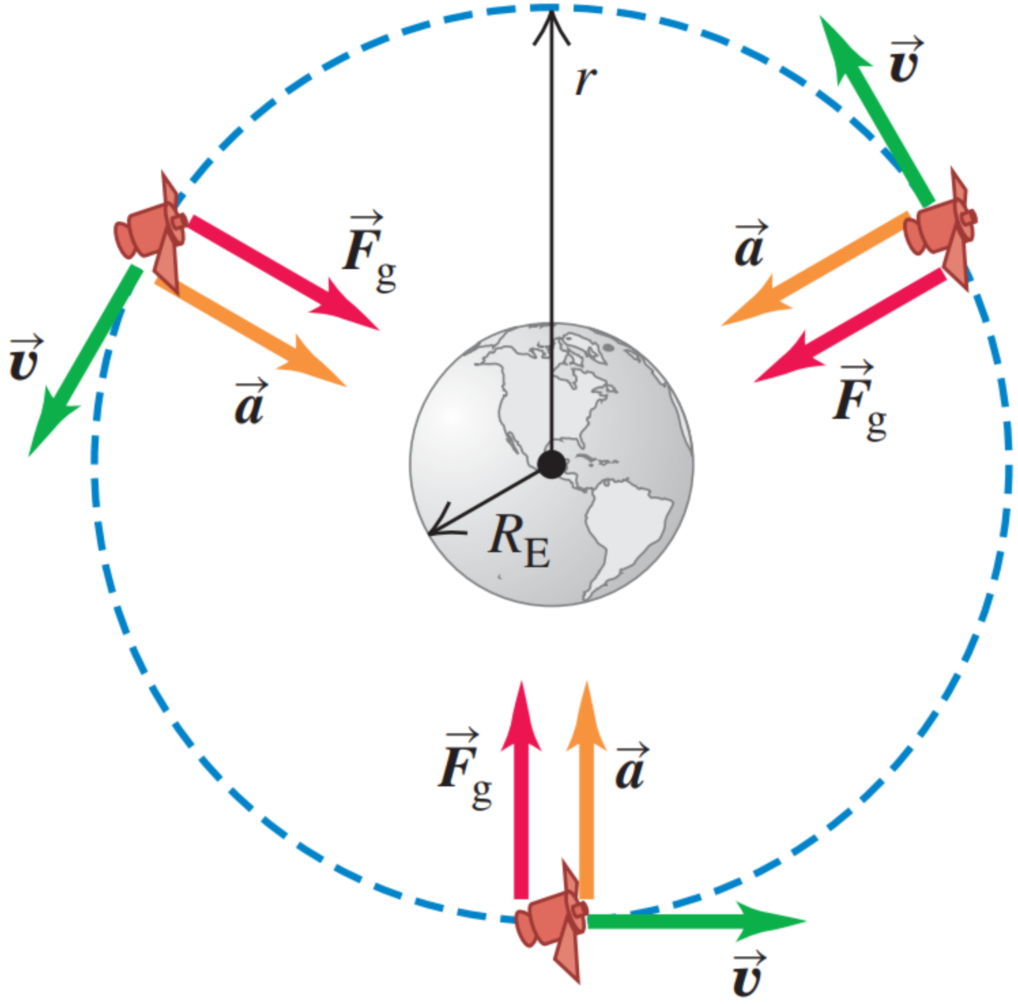
$$\boxed{U = mgy}$$

The motion of satellites



A projectile is launched from *A* toward *B*. Trajectories 1 through 7 show the effect of increasing initial speed.

Satellites: circular orbits



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Satellites: circular orbits

Let's see how to find the constant speed v of a satellite in a circular orbit. The radius of the orbit is r, measured from the *center* of the earth; the acceleration of the satellite has magnitude $a_{\mathrm{rad}} = v^2/r$ and is always directed toward the center of the circle. By the law of gravitation, the net force (gravitational force) on the satellite of mass m has magnitude $F_{\mathrm{g}} = Gm_{\mathrm{E}}m/r^2$ and is in the same direction as the acceleration. Newton's second law $(\sum\vec{F} = m\vec{a})$ then tells us that

$$\frac{Gm_{\mathrm{E}}m}{r^2} = \frac{mv^2}{r}$$

Solving this for v, we find

$$v = \sqrt{\frac{Gm_{\mathrm{E}}}{r}} \qquad \text{(circular orbit)} \qquad (13.10)$$

Satellites: circular orbits

We can derive a relationship between the radius r of a circular orbit and the period T, the time for one revolution. The speed v is the distance $2\pi r$ traveled in one revolution, divided by the period:

$$v = \frac{2\pi r}{T} \tag{13.11}$$

To get an expression for T, we solve Eq. (13.11) for T and substitute v from Eq. (13.10):

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_{\mathrm{E}}}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\mathrm{E}}}} \quad \text{(circular orbit)} \tag{13.12}$$

Satellites: circular orbits

Since the speed v in a circular orbit is determined by Eq. (13.10) for a given orbit radius r, the total mechanical energy $E = K + U$ is determined as well. Using Eqs. (13.9) and (13.10), we have

$$E = K + U = \frac{1}{2}mv^2 + \left(-\frac{Gm_E m}{r}\right) = \frac{1}{2}m\left(\frac{Gm_E}{r}\right) - \frac{Gm_E m}{r}$$

$$= -\frac{Gm_E m}{2r} \qquad \text{(circular orbit)} \qquad (13.13)$$

Example 13.6 A satellite orbit

Kepler's laws and the motion of planets

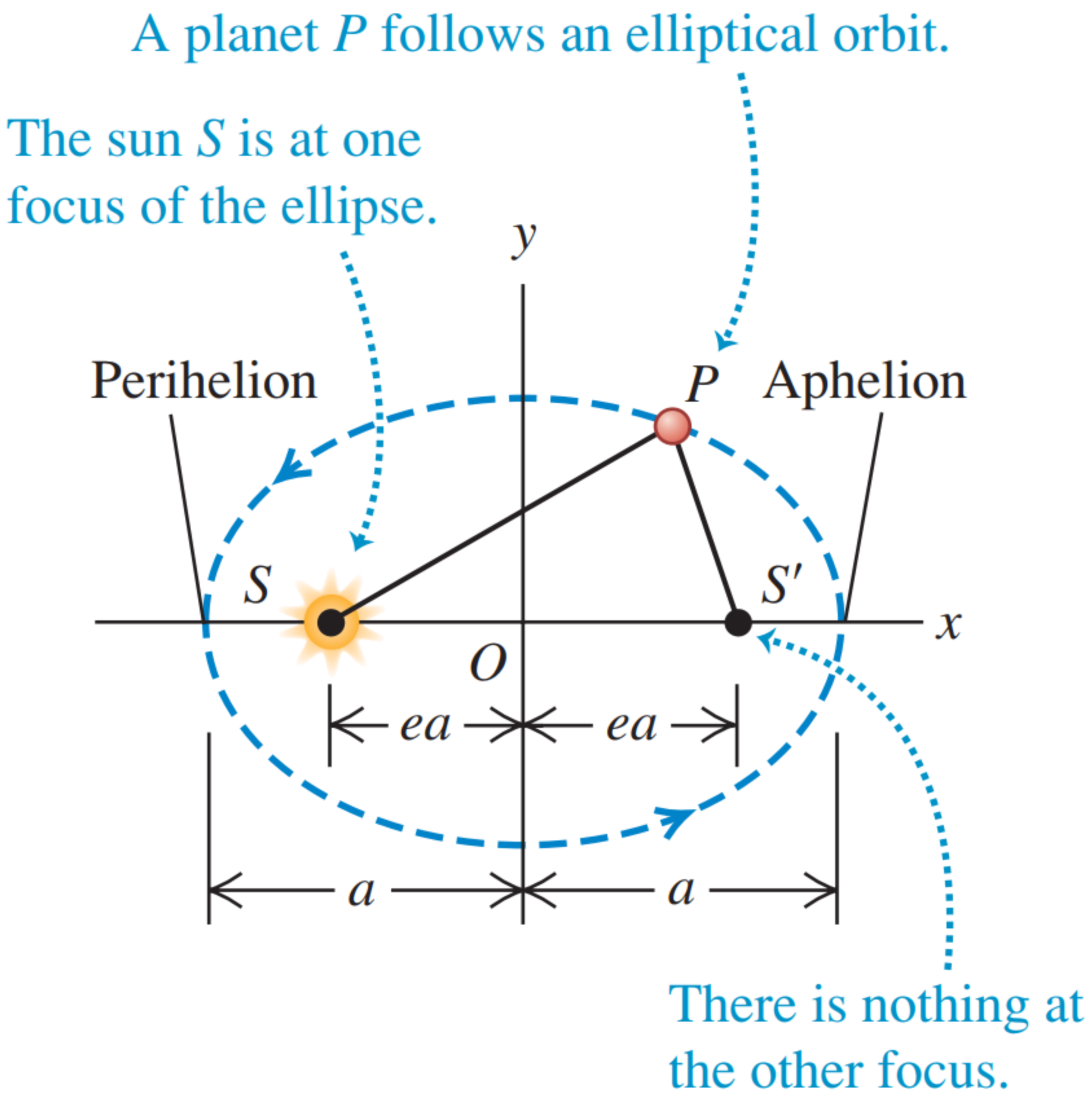
1. Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.

2. A line from the sun to a given planet sweeps out equal areas in equal times.

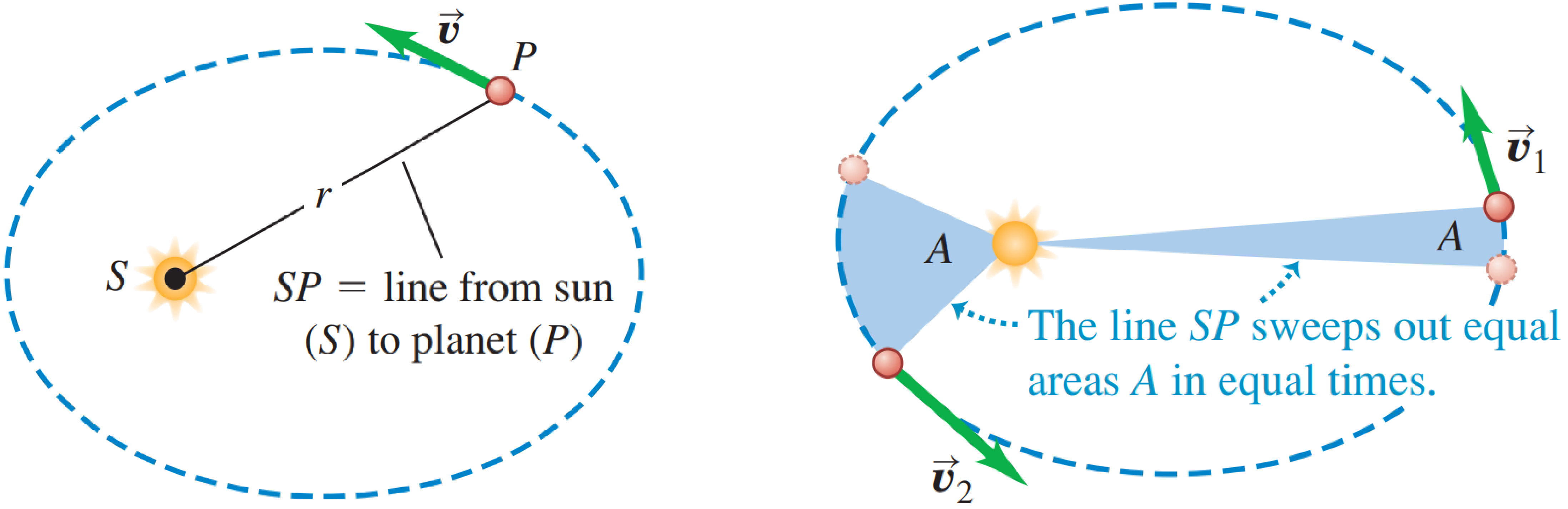
3. The periods of the planets are proportional to the powers of the major axis lengths of their orbits.

Kepler's First Law

Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.



Kepler’s Second Law



Kepler's Third Law

The law of periods. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad \text{(law of periods),}$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor ax

$$\frac{GMm}{r^2} = (m)(\omega^2 r)$$

Satellites: Orbits and Energy

The potential energy of the system is given by $$U = -\frac{GMm}{r}$$

Here r is the radius of the satellite's orbit,
M and m are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit, we write Newton's second law ($F = ma$) as

$$\frac{GMm}{r^2} = m\frac{v^2}{r},$$

Satellites: Orbits and Energy

The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$,

which shows us that for a satellite in a circular orbit, *K = -U/2.*

The total mechanical energy of the orbiting satellite is

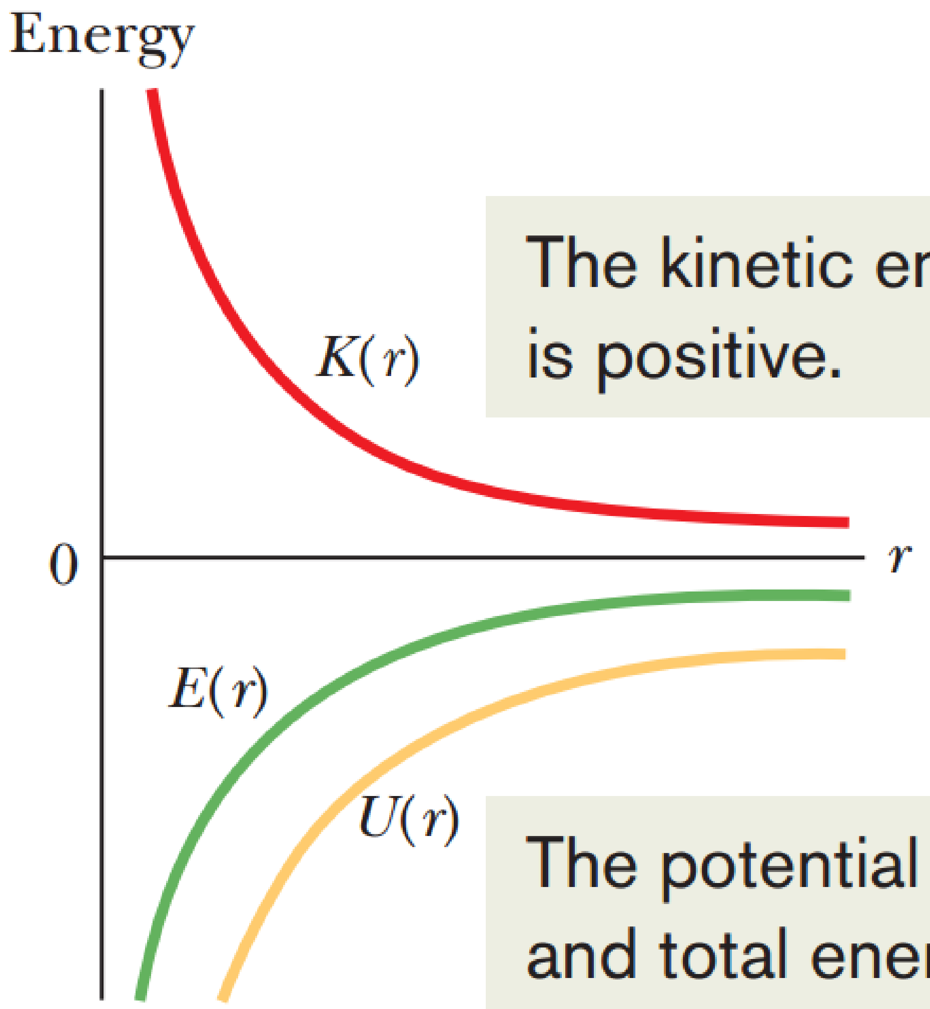
$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$\boxed{E = -K}$$

$$E = -\frac{GMm}{2r} \quad \text{(circular orbit).}$$

Satellites: Orbits and Energy

This is a plot of a satellite's energies versus orbit radius.



The kinetic energy is positive.

The potential energy and total energy are negative.

Satellites: Orbits and Energy

For a satellite in an elliptical orbit of semimajor axis *a*, we can substitute *a* for *r* to find the mechanical energy:

$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit).}$$

Satellites: Orbits and Energy

A playful astronaut releases a bowling ball, of mass m = 7.20 kg, into circular orbit about Earth at an altitude h of 350 km.

(a) What is the mechanical energy E of the ball in its orbit?

Satellites: Orbits and Energy

Calculations: The orbital radius must be

$$r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},$$

in which R is the radius of Earth. Then, from Eq. 13-40 with Earth mass $M = 5.98 \times 10^{24}$ kg, the mechanical energy is

$$\begin{aligned} E &= -\frac{GMm}{2r} \\ &= -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.20 \text{ kg})}{(2)(6.72 \times 10^6 \text{ m})} \\ &= -2.14 \times 10^8 \text{ J} = -214 \text{ MJ}. \qquad \text{(Answer)} \end{aligned}$$

Satellites: Orbits and Energy

Calculate the total energy of a five metric ton telecommunications satellite circulating in a geostationary orbit.

The mass of earth is 5.98×10^{24} kg.

1 metric ton = 1000 kg.

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\boxed{E = -\frac{GMm}{2r}}$$

E = -23.60 GJ

Readings

University Physics with Modern Physics

– Hugh D. Young, Roger A. Freedman

Chapter 13: Gravitation