Test 1 Review Solutions: MA 204

Name ________________

Directions: Write clearly, show your work, and define your events and random variables where appropriate. Caveat: The following sample questions are NOT exhaustive of the topics or questions which might be on the actual exam. You may use a calculator and a 3 x 5/4 x 6 index card (double sided).

1 Three people work independently at deciphering a message in code. The respective probabilities that they will decipher it are p, q, and r. What is the probability that the message will be deciphered?

P(Message is Deciphered) = P(at least one coder deciphers) = 1-P(no successes) = 1-(1-p)(1-q)(1-r)

3 An airline knows that 10% of the people holding reservations on flights will probably not appear. The plane on one flight holds 120 people. If 125 reservations have been sold, find the probability that the airline will be able to accommodate everyone appearing for that flight. List using notation and identify the Rcode needed to find this probability.

Let X be the number of people that show up for a flight. X∼Binomial(125,0.90). We are interested in $P(X<121) = \sum_{k=0}^{120} \binom{125}{k} (0.90)^k (0.10)^{125-k}$**, which can be coded in** R **using 'pbinom(120,125,0.9)'**

5 The probability of finding water when a well is dug in a certain locality is 75%. What is the probability that when 6 wells are dug, exactly 4 of them will have water?

Let W **be the number of wells with water,** $W \sim Binomial(6, 0.75)$

$P(W = 4) = \binom{6}{4} (0.75)^4 (0.25)^2 = 0.297$

7 You toss a fair coin twice. A is the event that the first toss is a heads. B is the event that exactly 1 head is thrown. Are A and B independent events? Why?

Yes: $P(A) = 1/2$**,** $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.25}{0.50} = 0.50 = P(A)$**. Thus, because** $P(A|B) = P(A)$**,** A **and** B **are independent**

9 Identify a random variable and its distribution from the description. Define all relevant parameters. No calculations are necessary. For example, X = # of butterflies hatched from 10 caterpillars, X ∼ Binomial(10,0.6)

- Mufaro breaks into song on average 3 times an hour, assume independence.
- Sebastian buys 12 eggs, but he foolishly forgot to look to see if any of them were cracked before he bought them. The chance that an egg is cracked is 0.08. Egg cracks are independent.
- During a hockey game, Park Thomas is equally likely to score 0, 1, and 2 goals, and he cannot score anymore than 2 goals. Assume that production from one game to the next is independent.

Let M be the number of songs Mufaro breaks into during an hour. $M \sim Poisson(3)$
Let S be the number of egg cracks in 12 eggs. $S \sim Binomial(12, 0.08)$
Let P be the number of goals during a hockey game. $P \sim Uniform(0, 1, 2)$

11 There are 500 students in a class, and each student has a 1% chance of being out sick. Assume that sickness is independent. What is the exact probability that 2 students will be out sick? What is the approximate probability that 2 students will be out sick? Make sure you distinguish which probability is exact and which is approximate.

Let X be the number of students that are sick.
Exact: $X \sim Binomial(500, 0.01)$**,** $P(X = 2) = \binom{500}{2}(0.01)^2(0.98)^498 = 0.0836$
Appx: $X \sim Poisson(5)$**,** $P(X = 2) = \frac{e^{-5}5^2}{2} = 0.0842$

13 Bob takes a multiple choice exam where each question has four possible answers. Bob knows the answer to half the questions. For one-fourth of the questions he will be able to eliminate two bad answers and reduce the correct answer to a fair coin flip. And for a fourth of the questions he will just guess. There are 32 questions on the exam. Find the expected number of questions which Bob gets right.

We know Bob gets at least 16 questions correct each test (half the questions). He is expected to get half of the questions correct on which he has reduced to a coin flip (expected value, 4 questions right), and one-fourth of the questions on which he is guessing (expected value, 2 questions right). Thus, we expect him to get 22 questions correct

15 TRUE or FALSE (No explanations necessary.)

- If A and B are mutually exclusive, then A and B are independent.

- If A and B are independent events, then so are A and B^c

FALSE, TRUE

16 The joint distribution of X and Y is given by

		Y		
		1	2	3
	0	0.05	0.05	0.30
X	1	0.15	0.25	c

- Find c
- Find $P(X<Y)$
- Find the marginal distribution and mean of X
- Find E[Y]

$c = \mathbf{0.20}$
$P(X<Y) = \mathbf{0.85}$
$P(X = 0) = 0.40$**,** $P(X = 1) = 0.60$**,** $E[X] = 0.60$
$E[Y] = 1(0.20) + 2(0.30) + 3(0.50) = 2.3$