Test 1 Review: MA 204

Name _____

Directions: Write clearly, show your work, and define your events and random variables where appropriate. Caveat: The following sample questions are NOT exhaustive of the topics or questions which might be on the actual exam. You may use a calculator and a 3 x 5/4 x 6 index card (double sided).

1 Three people work independently at deciphering a message in code. The respective probabilities that they will decipher it are p, q, and r. What is the probability that the message will be deciphered?

2 Of all the applicants for a job it is felt that 80% are able to do the job and the rest are not. To aid in the selection process, an aptitude test is designed so that the capable applicant has probability 0.75 of passing the test, while an incapable applicant has probability 0.20 of passing the test. An applicant passes the test. What is the probability that the applicant will be able to do the work?

3 An airline knows that 10% of the people holding reservations on flights will probably not appear. The plane on one flight holds 120 people. If 125 reservations have been sold, find the probability that the airline will be able to accommodate everyone appearing for that flight. List using notation and identify the Rcode needed to find this probability.

4 There is a 70% chance that a tree is infected with either root rot or bark disease. The chance that it does not have bark disease is 0.4. Whether or not the tree has root rot is independent of whether it has bark disease. Find the probability that the tree has root rot

5 The probability of finding water when a well is dug in a certain locality is 75%. What is the probability that when 6 wells are dug, exactly 4 of them will have water?

6 Suppose X is a discrete uniform random variable so that $f(x) = \frac{1}{n}$ for $x = 1, 2, ..., n$. Find the mean and variance of X.

7 You toss a fair coin twice. A is the event that the first toss is a heads. B is the event that exactly 1 head is thrown. Are A and B independent events? Why?

8 At Pedagogy College there are 3 different sports teams, and each athlete at the college plays on exactly one sport. 20% of all athletes are soccer players, 30% of all athletes are field hockey players, and 50% of all athletes are synchronized swimmers. 50% of soccer players can run a 4 minute mile, 60% of field hockey players can run a 4 minute mile, and 30% of synchronized swimmers can run a 4 minute mile.

- What is the probability that an athlete at Pedagogy College can run a 4 minute mile?
- Given that an athlete at Pedagogy College cannot run a 4 minute mile what is the probability that they are a synchronized swimmer or a field hockey player?
- In this situation is being a synchronized swimmer independent of being a soccer player?

9 Identify a random variable and its distribution from the description. Define all relevant parameters. No calculations are necessary. For example, X = # of butterflies hatched from 10 caterpillars, X $\sim$ Binomial(10,0.6)

- Mufaro breaks into song on average 3 times an hour, assume independence.
- Sebastian buys 12 eggs, but he foolishly forgot to look to see if any of them were cracked before he bought them. The chance that an egg is cracked is 0.08. Egg cracks are independent.
- During a hockey game, Park Thomas is equally likely to score 0, 1, and 2 goals, and production from one game to the next is independent.

10 Let X equal the sum of two rolls of a 6-sided die. What is $P(X<5)$? And how about $P(X<3|X<5)$?

11 There are 500 students in a class, and each student has a 1% chance of being out sick. Assume that sickness is independent. What is the exact probability that 2 students will be out sick? What is the approximate probability that 2 students will be out sick? Make sure you distinguish which probability is exact and which is approximate.

12 A four-sided tetrahedron die is is labeled with the numbers 1, 2, 3, and 4. The die is rolled twice.

- Give the sample space for this experiment.
- Let X be the maximum of the two rolls. Find the distribution of X

13 Bob takes a multiple choice exam where each question has four possible answers. Bob knows the answer to half the questions. For one-fourth of the questions he will be able to eliminate two bad answers and reduce the correct answer to a fair coin flip. And for a fourth of the questions he will just guess. There are 32 questions on the exam. Find the expected number of questions which Bob gets right.

14 A DNA strand consists of a sequence of 8 letters from the set $\{A, C, G, T\}$. Assume that a DNA strand is created by a process that generates letters uniformly at random and independently of other letters, so that all strands are equally likely.

- What is the probability that a DNA strand has no A's?
- What is the probability that a DNA strand has 5 A's and 3 C's?
- What is the probability that a DNA strand has exactly 5 A's given that it has exactly 3 C's?

15 TRUE or FALSE (No explanations necessary.)

- If A and B are mutually exclusive, then A and B are independent.
- If A and B are independent events, then so are A and B^c

16 The joint distribution of X and Y is given by

		Y		
		1	2	3
	0	0.05	0.05	0.30
X	1	0.15	0.25	c

- Find c
- Find $P(X<Y)$
- Find the marginal distribution, mean, and variance of X
- Find E[Y]