Directions: Write clearly, show your work, and define your events and random variables where appropriate. You may use a calculator and a index card (double sided). For any probabilities involving the Normal distribution, you can leave answers in terms of R code.

Test 2 review: MA 204

1 You have a bag containing 7 blue marbles and 3 red marbles, and draw from the bag with replacement. Find the probability that it takes exactly 5 trials for you to draw your first two red marbles?

2 Tina is going to ask Jimmy out on a date until he says yes. Each time Tina asks Jimmy out on a date, there is a 0.18 chance that Jimmy says yes, and we can assume Jimmy's responses are independent.

- Find the probability that Jimmy agrees to go out with Tina on the 3rd try
- Find the probability that it takes Tina more than three tries until Jimmy says yes.
- Find the expected number of times that Tina must ask Jimmy until she receives a yes.

3 A continuous random variable X has a density proportional to x^{-3} for $x > 1$.

- Find the proportionality constant
- Find $P(2<X<3)$

4 It is known that $X \sim N(1, \sigma^2 = 1)$, $Y \sim N(2, 3)$ and $Z \sim N(-4, 5)$

- Describe the distribution of $X + Y - 2Z$ and its parameters
- Find the probability that $X + Y - 2Z$ is negative

5 The length of time until an chicken and platypus eggs hatch can be accurately described with by exponential distributions with means 2 hours and 3 hours, respectively. Let C be the number of hours until a chicken egg hatches and let P be the number of hours until a platypus egg hatches. Suppose C is independent of P. Find

- F(C)
- $P(C>4)$
- $P(C>7|C>3)$.
- The joint density function of C and P.

6 Leonard wants to guess Benjamin's height. Let X = Leonard's guess in inches, such that $X \sim Normal(63, \sigma^2 = 9)$.

- Using the 68-95-100 rule, estimate the probability that Leonard predicts a height of 60 inches or lower.
- Leonard makes 4 independent guesses, X_1, X_2, X_3 and X_4. Describe the distribution of $\frac{X_1+X_2+X_3+X_4}{4}$.

- Benjamin's true height is 64 inches. Describe why $P(X = 64) = 0$, and provide a more reasonable probability for Benjamin and Leonard to calculate to determine if Leonard accurately guessed Benjamin's height.

7 Random variables X and Y have joint probability density function $f(x, y) = 12x^2, 0<x<y<1$.

- Find the marginal distribution of X
- Find $E[X]$

8 Suppose that random variables X and Y are independent with $E[X] = 2$, $Var[X] = 3$, $E[Y] = -3$, $Var[Y] = 4$.

- Find $Var[4 - 5X]$
- Find $E[X^2]$
- Find $Var[2X - Y]$
- Find $Cov[X, Y]$

9 David and Emily arrive at the Rec Center at times uniformly distributed between noon and 2 pm independently of each other. What's the probability that they will get there within 20 minutes of each other?

10 Refer to Question 5. Find $P(C > P)$.

11 Find all the inflection points of a normal density curve. Show how this information can be used to draw a normal curve given the mean and variance