Homework #3 solutions: MA 204

2.24 Let B be the event that the woman has breast cancer, + be the event of a positive mammogram, and - be the event of a negative mammogram. We are given $P(B) = 0.0238$, $P(+|B) = 0.85$, and $P(+|B^c) = 0.05$. By Bayes formula,

$$P(B|+) = \frac{P(+|B)P(B)}{P(+|B)P(B) + P(+|B^c)P(B^c)} = \frac{0.85 * 0.0238}{0.85 * 0.0238 + 0.05 * 0.9762} = 0.293$$

2.26 Let B be the event that the cab is blue, b be the event that the witness asserts the cab is blue. We are given $P(B) = 0.05$, $P(b|B) = 0.80$, and $P(b^c|B^c) = 0.8$. By Bayes formula,

$$P(B|b) = \frac{P(b|B)P(B)}{P(b|B)P(B) + P(b|B^c)P(B^c)} = \frac{0.8 * 0.05}{0.8 * 0.05 + 0.95 * 0.2} = 0.174$$

3.2 (a) $P(ABC) = (1/3)(1/4)(1/5) = 1/60$
(b) $P(AorBorC) = 1 - P(A^cB^cC^c) = 1 - (2/3)(3/4)(4/5) = 0.6$ (you can also use a different formula here)
(c) $P(AB|C) = P(AB) = 1/3 * 1/3 = 1/12$
(d) $P(B|AC) = P(B) = 1/4$
(e) P(At most one of the three events) =
P(0 events) + P(1 event) = $P(A^cB^cC^c)+P(A^cB^cC^c)+P(A^cBC^c)+P(A^cB^cC)$
$= (2/3)(3/4)(4/5) + (1/3)(3/4)(4/5) + (2/3)(1/4)(4/5) + (2/3)(3/4)(1/5) =$
$5/6$

3.4 (i) $P(ABC) = P(die1 = 3, die2 = 6) = \frac{1}{36}$, so
$P(A) * P(B) * P(C) = 1/2 * 1/2 * 1/4 = \frac{1}{36}$ so $P(ABC) = P(A)P(B)P(C)$
(ii) $P(A) = \frac{1}{2}$ but $P(A|B) = \frac{1}{3}$ so A, B are not independent
(iii) $P(A) = \frac{1}{2}$ but $P(A|C) = \frac{1}{4}$ so A, C are not independent
(iv) $P(B) = \frac{1}{2}$ but $P(B|C) = \frac{3}{4}$ so B, C are not independent

Carson Carson's misstep is that he assumed the probability of two people sharing any birthday is the same as the probability of two people sharing one specific given birthday.