Homework #2 solutions: MA 204

2.6

- A beats B in 20 of 36 opportunities:

$$(5/2), (5/2), (5,2), (5,2), (3,2), (3,2), (3,2), (3,2), (7/2), (7/2),$$
$$(7,2), (7,2), (5/4), (5/4), (5,4), (5,4), (7,4), (7,4), (7,4), (7,4)$$

- B beats C in 20 of 36 opportunities:

$$(2/1), (2/1), (2,1), (2,1), (4,1), (4,1), (4,1), (4,1), (9/1), (9/1),$$
$$(9,1), (9,1), (9/6), (9/6), (9,6), (9,6), (9/8), (9/8), (9,8), (9,8)$$

- C beats A in 20 of 36 opportunities:

$$(8/3), (8/3), (8/3), (8/3), (8/5), (8/5), (8/5), (8/5), (8/7), (8/7),$$
$$(8/7), (8/7), (6/3), (6/3), (6/3), (6/3), (6/5), (6/5), (6/5), (6/5)$$

$$\therefore P(A > B) = P(B > C) = P(C > A) = \tfrac{20}{36}$$

2.10

$$\begin{aligned} P(W) &= P(W|\text{first ball W})P(\text{first ball W}) + P(W|\text{first ball R})P(\text{first ball R}) \\ &= 2/5 * 1/3 + 1/5 * 2/3 = 4/15 \end{aligned}$$

2.14 We want $P(B) = 1 - \prod_{i=1}^{k-1}(1 - \frac{i}{687})$ to be greater than 0.50.

For k = 31, P(B) = 0.497; for k = 32, P(B) =0.52. Thus, 32 works, although I'll accept 31 if you showed accurate work.

2.16 (a) Let H be the event that the selected card is a heart. Let M be the event that the missing card is a heart. Thus, $P(H) =$

$P(H) = P(H|M)P(M)+P(H|M^c)P(M^c) = (12/51)(1/4)+(13/51)(3/4) = \frac{1}{4}$

(b) The selected card is equally likely to be one of the four suits. Thus, $P(H) = 0.25$