Final Exam Review (mostly chapter 9): MA 204

Directions: Write clearly, show your work, and define your events and random variables where appropriate. You may use any material from the course. Outside material, consulting a friend or classmate, and looking up questions on the Internet is not allowed. The questions are arranged in no particular order.

1 Restaurant waiting time is exponentially distributed with a mean of 30. A total of 16 customers come through - what is the probability that their average weight time is less than 25 minutes?

Let $\lambda = 1/30$. We are interested in $S_{16}/16$, the average weight time of 16 students. Recall from the exponential distribution that $E[X] = SD[X] = 1/\lambda = 30$.

Thus, $P(S_{16}/16 < 25) = P(\frac{S_{16}/16-30}{30/sqrt(16)}) < \frac{25-30}{30/sqrt(16)} = P(Z < -2/3) =$ pnorm(-2/3) = 0.25

2 AP scores are awarded scores of 1, 2, 3, 4, or 5, assigned with probabilities 0.21, 0.21, 0.24, 0.20, and 0.14, respectively. 30 independent students are drawn from a sample of all students who take the test. What is the probability that they average higher than a 3?

Solution: Let $S_{30} = X_1 + X_2...X_{30}$ be the sum, where X_i is each student. Our interest is $P(S_{30}/30) > 3)$.

We know $E[X] = 1(0.21) + ... \ 5(0.14) = 2.844$ and $Var[X] = E[X^2] - (E[X])^2 = 1.84$ (Note: Solve for $E[X^2] = 1^2(0.21) + ...5^2(0.14) = 9.91$ first).

Next, $SD[X] = sqrt(1.84) = 1.36$

By the Central limit theorem, $P(S_{30}/30 > 3) = P(\frac{S_{30}/30-2.84}{1.36/sqrt(30)}) > \frac{3-2.84}{1.36/sqrt(30)} = P(Z > 0.63) = 0.26$.

3 On each of 365 days of the year, it rains with probability 0.1 independent of every other day. Let X be the number of days that it rains in a given year. We would like to estimate the probability that in a year's time it rains on at least 50 days.

- Describe the distribution of X
- What does Markov's inequality say about this probability
- What does Chebychev's inequality say about this probability
- What does the Central Limit Theorem say about this probability?
- Find the exact probability using *RStudio*.

Solution

- X is distributed binomial with n = 365 and probability = 0.1
- $P(X \geq 50) \leq 36.5/50$ (or 0.73)
- $P(X \geq 50) = P(X - 365 \geq 13.5) \leq 32.85/13.5^2$ (or 0.18)
- $P(X \geq 50) = P(Z \geq 2.36) = 0.009$
- 1-pbinom(49, 365, 0.1) = 0.014