Differential and Integral Calculus (DIC)

Note

□ Limit:

Evaluate:

$$\lim_{x \to -1} \frac{2x+2}{x+1}$$

$$\Rightarrow \lim_{x \to -1} \frac{2\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$\Rightarrow \lim_{x \to -1} (2)$$

$$= 2$$

$$f(x) = \frac{2x+2}{x+1}$$

$$= \frac{2(x+1)}{x+1}$$

$\therefore f(x) = 2$

or, $f(x) = 2 + 0 \cdot x$

যদি $x = -1$ বসালে হবে:

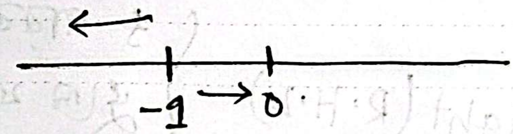
$f(x) = \infty$; $x = -1$

$\to (-1, \infty)$

Then we got:

$f(x) = 2$; when $x \neq 1$

i.e., x tensed to 1 means x is not equal to 1.



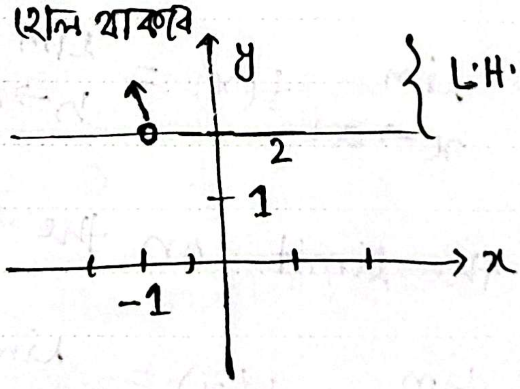
-1.0000001

-0.999999

$f(x) = \infty$; $x = -1$

$= 2$; $x \neq 1$

হোল থাকবে



$\{ L.H.L = R.H.L = f(-1) \}$

$ax + by = c$

$ax + by + cz = d \to$ তল (Plane) এর সমীকরণ।

Limit and Continuity:

$$\lim_{x\to 1}\frac{x^2-1}{x-1} \longrightarrow \frac{1-1}{1-1}=\frac{0}{0}=\infty$$

$$=\lim_{x\to 1}\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$=(x+1)$$

$$=(1+1)=2 \to 1.999,\ 1.999999$$

$$\to 2.00001,\ 2.001$$

A function will be continious or discontinuous when,

$$\lim_{x\to a}f(x)=\lim_{x\to a+}f(x)=\lim_{x\to a^-}f(x)$$

$\{f(x)\}$ (R.H.L) (L.H.L)

Working Method:

$$\lim_{x\to a}f(x)$$

i) To find the limit on the right (R.H.L)

$$\lim_{x\to a+}f(x)=\lim_{h\to 0}f(\underline{a+h})$$

f বারবার ভুলে যাবে

ii) To find the limit on the left (L.H.L)

$$\lim_{x\to a-}f(x)=\lim_{h\to 0}f(\underline{a-h})$$

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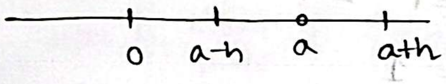
* Question arises why $h\to 0$?

$x=a+h$	$x=a-h$
$\Rightarrow a=a+h$	$\Rightarrow a=a-h$
$\therefore h=0$	$\Rightarrow 0=-h$
	$\therefore h=0$



Example:

If $f(x)=1+2x;\ -\frac{1}{2}\le x<0$

$=1-2x;\ 0\le x<\frac{1}{2}$

$=-1+2x;\ x\ge\frac{1}{2}$

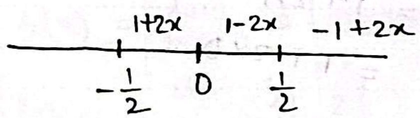
Then does the limit of the function

a) $\lim_{x\to 0}f(x)$ exist or not?

b) $\lim_{x\to\frac{1}{2}}f(x)$ exist or not?

Answer:

a)



To find the limit on the right (R.H.L) when $x>0$

then $f(x)=1-2x$.

R.H.L ⇒

$$\lim_{x \to 0+} f(x) = \lim_{h \to 0} f(0+h)$$

$$= \lim_{h \to 0} f(h)$$

$$= 1 - 2h$$

$$= 1 - 2 \times 0$$

$$= 1$$

To find the limit on the left (L.H.L) when $x < 0$
then $f(x) = 1 + 2x$

L.H.L ⇒

$$\lim_{x \to 0-} f(x) = \lim_{h \to 0} f(0-h)$$

$$= \lim_{h \to 0} f(-h)$$

$$= 1 - 2(-h)$$

$$= 1 + 2h$$

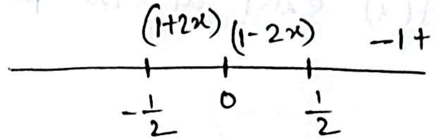
$$= 1 + 2 \times 0$$

$$= 1$$

As, R.H.L = L.H.L

∴ the function $f(x)$ exist at the point $x = 0$.

b)



To find the limit on the right (R.H.L) when $x > \frac{1}{2}$
then $f(x) = -1 + 2x$

$$\lim_{x \to \frac{1}{2}+} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} + h\right)$$

ভুলে যায় বারবার
↓
caution!
caution!

$$= \lim_{h \to 0} \left\{-1 + 2\left(\frac{1}{2} + h\right)\right\}$$

$$= \lim_{h \to 0} \{-1 + 1 + 2h\}$$

$$= \lim_{h \to 0} \{2h\}$$

$$= 2 \times 0$$

$$= 0$$

To find the limit on the left (L.H.L) when $x < \frac{1}{2}$.
then $f(x) = 1 - 2x$

$$\lim_{x \to \frac{1}{2}-} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} - h\right)$$

$$= \lim_{h \to 0} \left\{1 - 2\left(\frac{1}{2} - h\right)\right\}$$

$$= \lim_{h \to 0} \{1 - 1 + 2h\}$$

$$= 0$$

$\therefore L\cdot H\cdot L = R\cdot H\cdot L$

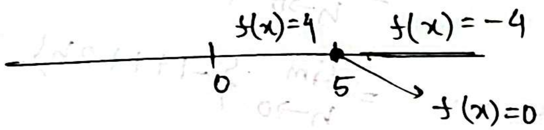
Then the function $f(x)$ exist at the point $x = \frac{1}{2}$,

Example:

$$f(x) = \begin{cases} 4 & ; x > 5 \\ 0 & ; x = 5 \\ -4 & ; x < 5 \end{cases}$$

Then does the limit of the function exist or not?

Answer:



To find the lim of the right $(R\cdot H\cdot L)$ when $x > 5$ then $f(x) = 4$,

$$\lim_{x\to 5+} f(x) = \lim_{h\to 0} f(5+h)$$

$$= \lim_{h\to 0} (4)$$

$$= 4$$

$$\left\{ \begin{array}{l} f(x) = 4 + \underline{0\cdot x} \\ \therefore f(1) = 4 \\ f(2) = 4 \end{array} \right.$$

To find the limit of the left $(L\cdot H\cdot L)$ when $x < 5$ then $f(x) = -4$

$$\lim_{x\to 5-} f(x) = \lim_{h\to 0} f(5-h)$$

$$= \lim_{h\to 0} \{-4\}$$

$$= -4,$$

$L\cdot H\cdot L \neq R\cdot H\cdot L$

Then the limit of the function $f(x)$ doesn't exist at the point $x = 5$,

Example:

$$f(x) = \begin{cases} 1 + 2x & ; -\frac{1}{2} \le x < 0 \\ 1 - 2x & ; 0 \le x < \frac{1}{2} \\ -1 + 2x & ; x \ge \frac{1}{2} \end{cases}$$

c) If the $\lim_{x\to 0} f(x)$ function continuous or discontinuous?

$\Rightarrow$ **R·H·L:**

when $x > 0$ then $f(x) = 1 - 2x$ {যে জায়গায় x সে জায়গায় h}

$$\lim_{x\to 0+} f(x) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} (1 - 2h)$$

$$= 1$$

L·H·L:

when $x<0$ the $f(x) = 1+2x$

$$\lim_{x\to 0-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} \{1+2(-h)\}$$

$$= 1$$

when $x=0$, then $f(x) = 1-2x$

$$\therefore f(0) = 1-2\times 0$$

$$= 1$$

$L\cdot H\cdot L = R\cdot H\cdot L = f(0)$

$\therefore$ The function is continuous at the point $x=0$।

কোনো বিন্দু $f(x)$ ফাংশনে exist করে কীনা দেখাতে
L·H·L = R·H·L দেখালে হয়.

কিন্তু continuous নাকি discontinuous দেখাতে
L·H·L = R·H·L = $f(x)$ দেখাতে হবে।

একটাও ≠ হলে discontinuous।

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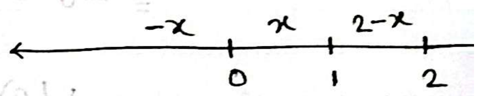
□ Checking Continuty | Discontinuty through mathematica and Graphyeally:

$$f(x) = -x \; ; \; x \leq 0$$
$$= x \; ; \; 0 < x \leq 1$$
$$= 1 \; ; \; x > 2$$

*) Analyze the point of discontinuty of the functio mathematically and graphycally.

Answer:



Mathematically:

$x=0$ point:

L·H·L:

$$\lim_{x\to 0-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} -(-h)$$

$$= 0$$

R.H.L :

$$\lim_{x \to 0+} f(x) = \lim_{h \to 0} f(0+h)$$
$$= \lim_{h \to 0} f(h)$$
$$= \lim_{h \to 0} (h)$$
$$= 0$$

when, $x = 0$, $f(x) = -x$
$$= -0 = 0$$

$\therefore$ L.H.L = R.H.L = $f(0)$

The $f(x)$ function is continuous at the point $x=0$

$x = 1$ point :

L.H.L :

$$\lim_{x \to 1-} f(x) = \lim_{h \to 0} f(1-h)$$
$$= \lim_{h \to 0} (1-h)$$
$$= 1 - 0$$
$$= 1$$

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R.H.L :

$$\lim_{x \to 1+} f(x) = \lim_{h \to 0} f(1+h)$$
$$= \lim_{h \to 0} \{2 - (1+h)\}$$
$$= 2 - (1+0)$$
$$= 1$$

when, $x = 1$, $f(x) = x$
$$= 1$$

$\therefore$ L.H.L = R.H.L = $f(1)$

Then $f(x)$ function is continuous at the point

$x = 2$ point :

L.H.L :

$$\lim_{x \to 2-} f(x) = \lim_{h \to 0} f(\cancel{2+h}\ 2-h)$$
$$= \lim_{h \to 0} \{2 - (2-h)\}$$
$$= 2 - 2 + h$$
$$= 0$$

R.H.L :

$$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)$$

$$= \lim_{h \to 0} (1)$$

$\therefore$ L.H.L $\neq$ R.H.L

Then $f(x)$ function is discontinuous at the point $x=0$।

~~Graphycally :~~

~~i) $y = x^2$ $(x \leq 0)$~~

~~$x=0$~~
~~$x=-1$~~
~~$x=-2$~~
~~$x=-3$~~

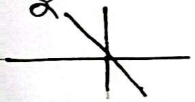
Graphycally :

i) $y = -x \to (x \leq 0)$

মূলবিন্দুগামী সরলরেখা

$x=0$
$x=-1$
$x=-2$
$x=-3$

কিন্তু এরকম হবে,



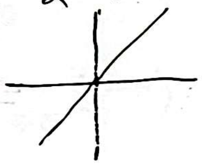
কারণ $-x$ আছে।

ii) $y = x$ $(0 < x \leq 1)$

মূলবিন্দুগামী সরলরেখা

$x=0$
$x=1$

কিন্তু এরকম হবে,



কারণ $y = x$ আছে

iii) $y = 2 - x$ $(1 \leq x \leq 2)$

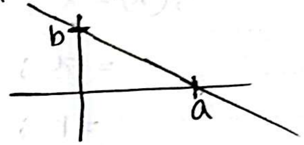
$\Rightarrow x + y = 2$

$\Rightarrow \frac{x}{2} + \frac{y}{2} = 1$

$x=1$
$x=2$

(উভয়অক্ষকে ছেদ বিন্দুগামী সরলরেখা)

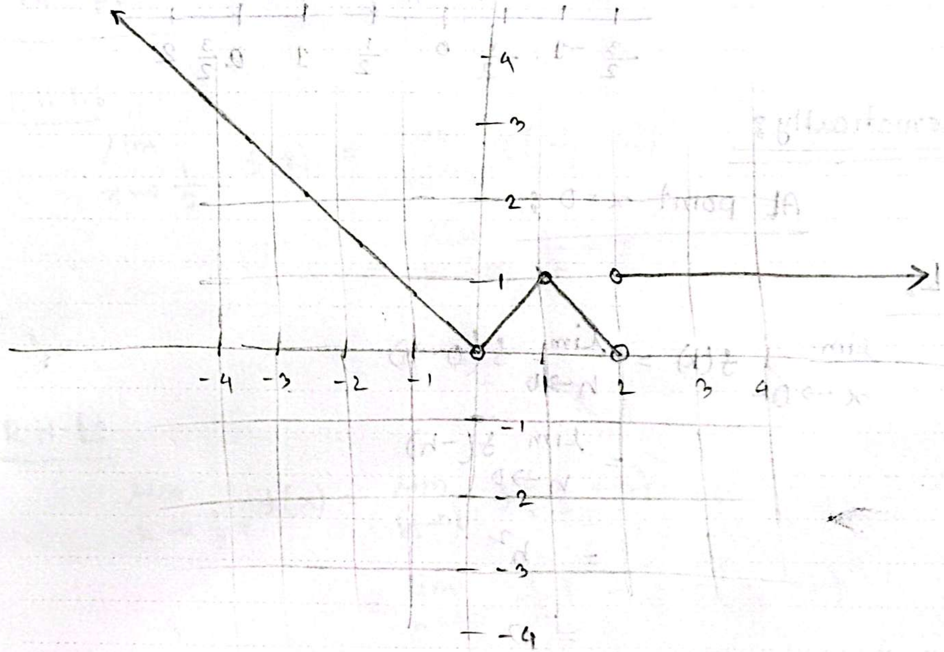
$$\frac{x}{a} + \frac{y}{b} = 1$$



iv) $y = 1$ $(x > 2)$

(x অক্ষের সমান্তরাল)

$x=2$
$x=3$
$x=4$
$x=5$

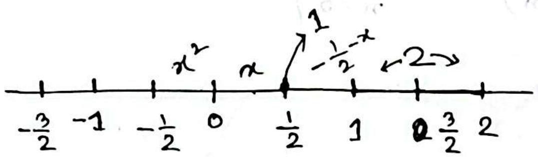


$\Rightarrow$ we see the graph, the function is continuous at the point $x=0$ and $x=1$ and discontinuous at the point $x=2$।

*) Analyze the point of discontinuty of the function mathematically and graphycally.

$$f(x) = x^2 \; ; \; x \le 0$$
$$= x \; ; \; 0 < x < \tfrac{1}{2}$$
$$= 1 \; ; \; x = \tfrac{1}{2}$$
$$= -\tfrac{1}{2} - x \; ; \; \tfrac{1}{2} < x < 1$$
$$= 2 \; ; \; 1 \le x < 2$$

Answer:



Mathematically:

At point $x=0$:

L.H.L:

$$\lim_{x\to 0-} f(x) = \lim_{h\to 0} f(0-h)$$
$$= \lim_{h\to 0} f(-h)$$
$$= h^2$$
$$= 0$$

R.H.L:

$$\lim_{x\to 0+} f(x) = \lim_{h\to 0} f(0+h)$$
$$= \lim_{h\to 0} (h)$$

$$= 0$$

when, $x=0$, $f(x) = x^2$

$$\therefore f(0) = 0^2$$
$$= 0$$

$\therefore$ L.H.S = R.H.S = $f(0)$

So, the function is continuous at the point, $x=0$

At point $x=\frac{1}{2}$:

L.H.L:

$$\lim_{x\to \frac{1}{2}-} f(x) = \lim_{h\to 0} f\left(\tfrac{1}{2}-h\right)$$
$$= \lim_{h\to 0} \tfrac{1}{2} - 0$$
$$= \tfrac{1}{2}$$

R.H.L:

$$\lim_{x\to \frac{1}{2}+} f(x) = \lim_{h\to 0} f\left(\tfrac{1}{2}+h\right)$$
$$= \lim_{h\to 0} \cancel{\tfrac{1}{2}} - \tfrac{1}{2} - \left(\tfrac{1}{2}+h\right)$$
$$= \lim_{h\to 0} -\tfrac{1}{2} - \tfrac{1}{2} - 0$$
$$= -1$$

$\therefore L.H.L \neq R.H.L$

So, the function is discontinuous at the point, $x=\frac{1}{2}$

At the point $x=1$:

L.H.L :

$$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0}\left\{-\frac{1}{2}-(1-h)\right\}$$

$$= \lim_{h\to 0}\left\{-\frac{1}{2}-1+h\right\}$$

$$= -\frac{1}{2}-1+0$$

$$= -\left(\frac{1}{2}+1\right)$$

$$= -\frac{1+2}{2}$$

$$= -\frac{3}{2}$$

R.H.L :

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0}(2)$$

$$= 2$$

$\therefore L.H.L \neq R.H.L$

So, the function is discontinuous at the point $x=1$ ।

Graphycally :

i) $y=x^2 \ (x\leq 0)$

$x=0$
$x=-1$
$x=-2$
$x=-3$

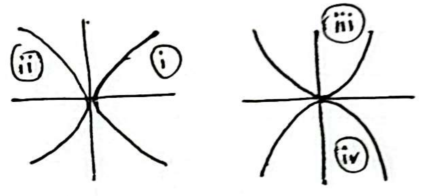
* দেখতে হবে, এটি (0,0) বিন্দু দিয়ে যায় কিনা, $x=0, y=0$ বসাতে হবে।
* (0,0) বিন্দুতে না গেলে, $x=0$ ও $y=0$ বসিয়ে x ও y এর মান নির্ণয় করতে হবে।

$$y=x^2$$
$$\Rightarrow 0=0^2$$
$$\Rightarrow 0=0$$

* Some parabolla Shit :

$y^2=4ax$ — (i) $\quad x^2=4ay$ — (iii)

$y^2=-4ax$ — (ii) $\quad x^2=-4ay$ — (iv)



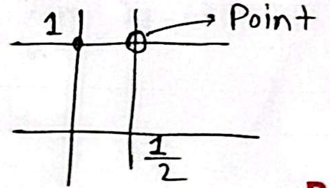
ii) $y=x \ (0<x<\frac{1}{2})$

মূলবিন্দুগামী

$x=0$
$x=\frac{1}{2}$

iii) $y=1 \ (x=\frac{1}{2})$

* দুইটির ছেদবিন্দু বসাবো। Just একটি বিন্দু create করবে



iv) $y = -\frac{1}{2} - x \quad \left(\frac{1}{2} < x < 1\right)$

$\Rightarrow x + y = -\frac{1}{2}$

$\Rightarrow \frac{x}{-\frac{1}{2}} + \frac{y}{-\frac{1}{2}} = 1$

$x = \frac{1}{2}$

$x = 1$

উভয়াক্ষকে ছেদানুসারী সরলরেখা

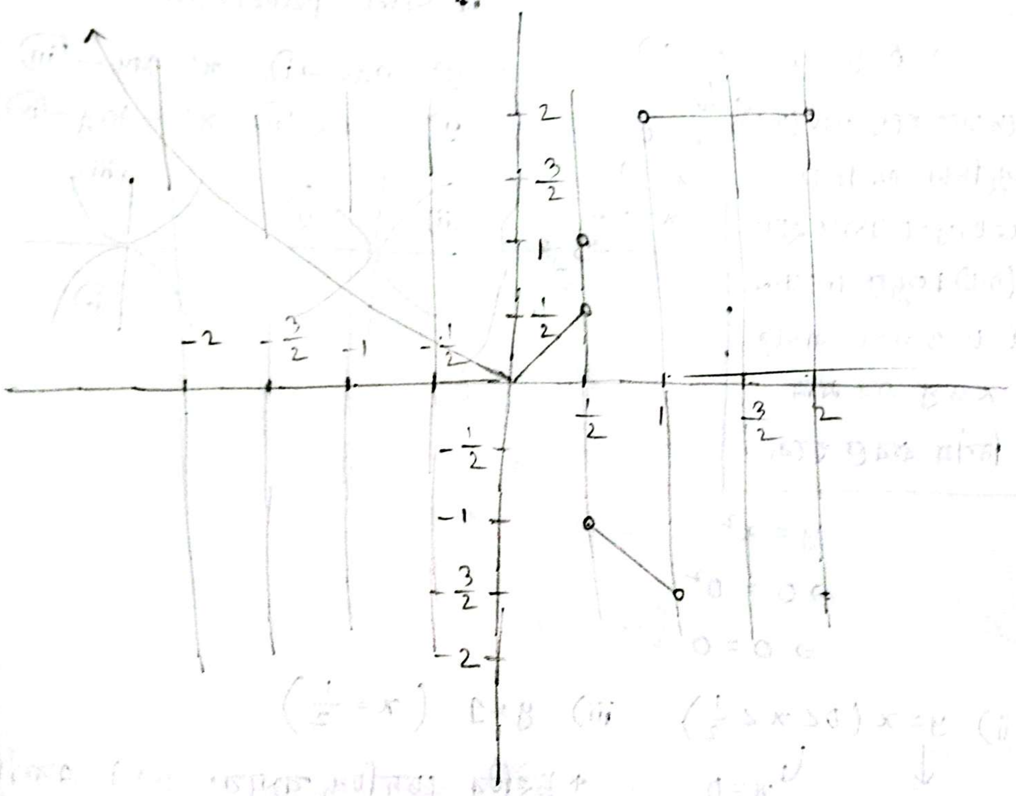
v) $y = 2 \quad (1 \le x < 2)$

x অক্ষের সমান্তরাল সরলরেখা

$x = 1$

$x = \frac{3}{2}$

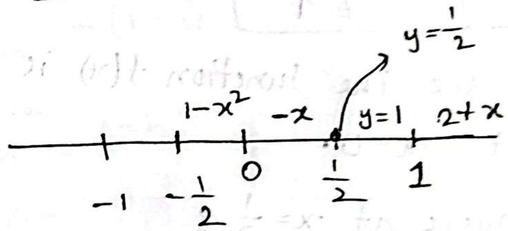
$x = 2$



*) Analyze the point of discontinuity of the function mathematically and graphycally.

$$f(x) = 1 - x^2 ; \quad x \le 0$$
$$= -x ; \quad 0 < x < \frac{1}{2}$$
$$= \frac{1}{2} ; \quad x = \frac{1}{2}$$
$$= 1 ; \quad \frac{1}{2} < x < 1$$
$$= 2 + x ; \quad x \ge 1$$

Answer:



Mathematically:

~~L.H.L:~~

Analysis at $x = 0$:

L.H.L:

$$\lim_{x \to 0-} f(x) = \lim_{h \to 0} f(0-h)$$
$$= \lim_{h \to 0} (-h)$$
$$= \lim_{h \to 0} \{1 - (-h)^2\}$$
$$= \lim_{h \to 0} \{1 - 0^2\}$$
$$= 1$$

R.H.L:

$$\lim_{x \to 0+} f(x) = \lim_{h \to 0} f(0+h)$$

$$= \lim_{h \to 0} (h)$$

$$= -h$$

$$= -0$$

$$= 0$$

And, $f(0) = 1 - 0^2$
$= 1$

$\therefore$ L.H.L $\neq$ R.H.L

We see the function $f(x)$ is discontinuous at the point $x=0$.

Analysis at $x = \frac{1}{2}$:

L.H.L:

$$\lim_{x \to \frac{1}{2}-} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} - h\right)$$

$$= \lim_{h \to 0} -\frac{1}{2} + h$$

$$= -\frac{1}{2}$$

R.H.L:

$$\lim_{x \to \frac{1}{2}+} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} + h\right)$$

$$= \lim_{h \to 0} (1)$$

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$$= 1$$

$\therefore$ R.H.L $\neq$ L.H.L

We see the function is discontinuous at the poin[t] $x = \frac{1}{2}$.

Analysis at $x = 1$:

L.H.L:

$$\lim_{x \to 1-} f(x) = \lim_{h \to 0} f(1-h)$$

$$= \lim_{h \to 0} 1-h$$

$$= \lim_{h \to 0} (1)$$

$$= 1$$

R.H.L:

$$\lim_{x \to 1+} f(x) = \lim_{h \to 0} (1+h)$$

$$= \lim_{h \to 0} 2+1+0$$

$$= 3$$

$\therefore$ L.H.L $\neq$ R.H.L

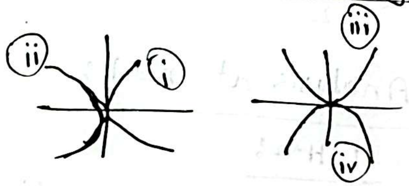
We see the function is discontinuous at the point $x=1$।

Graphycally:

i) $y=1-x^2$ $(x\le 0)$ → $x=0$, $x=-1$, $x=-2$, $x=-3$

$y^2=4ax$ — (i) $x^2=4ay$ — (iii)

$y^2=-4ax$ — (ii) $x^2=-4ay$ — (iv)



$(x,y)=(0,0)$ বসিয়ে:

$y=0$

$1-x^2=0$

$\Rightarrow -x^2=-1$

$\Rightarrow x^2=1$

$\therefore x=\pm 1$

প্রথমে এটি ৪টির মধ্যে কোন প্যারাবোলার ফরম্যাট পারবে তা দেখাতে হবে।

$y=1-x^2$

$\Rightarrow x^2=-y+1$

$\Rightarrow x^2=-(y-1)$ → (iv) নম্বর টির সাথে মেলে

অতএব, বিন্দু হলো:

$y=0$ বসিয়ে → $x=\pm 1$ পাই.

$y=0$ হলে $(1,0)$ $(-1,0)$

আবার, $x=0$ বসিয়ে → $y=1-0^2$

$\Rightarrow y=1$

$x=0$ হলে, $(0,1)$

ii) $y=-x$ $(0<x<\frac{1}{2})$ → $x=0$, $x=\frac{1}{2}$

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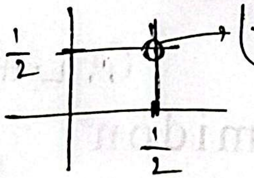
iii) $y=\frac{1}{2}$ $(x=\frac{1}{2})$



$x=1$ $x=2$

$x=\frac{3}{2}$ $x=\frac{5}{2}$

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iv) $y=1$ $(\frac{1}{2}<x<1)$ → $x=\frac{1}{2}$, $x=1$

x এর সমান্তরাল

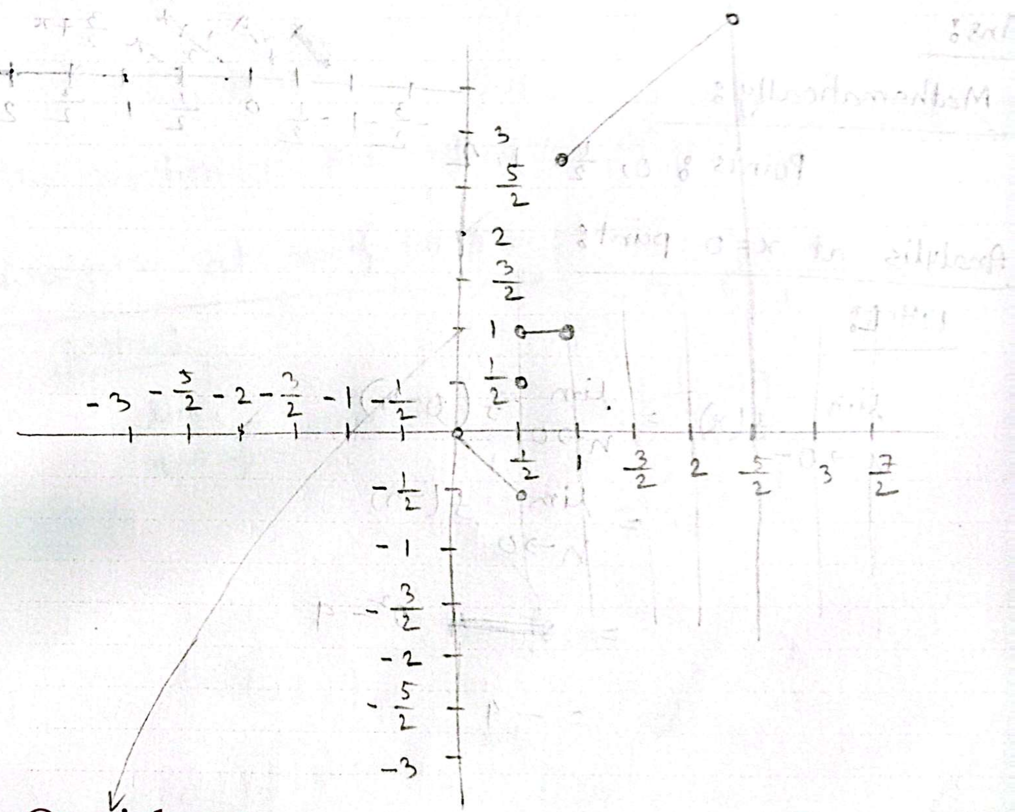
v) $y=2+x$ $(x\ge 1)$

$\Rightarrow y-x=2$

$\Rightarrow \frac{y}{2}-\frac{x}{2}=1$

$\Rightarrow \frac{x}{-2}+\frac{y}{2}=1$

উভয় অক্ষকে $(-2,2)$ বিন্দুতে ছেদ করবে এমন সমীকরণ

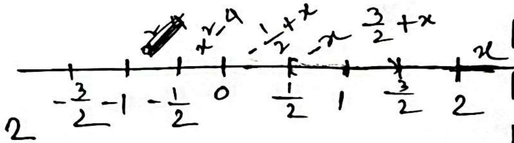


*) Analyze the point of discontinuty of the function mathematically and graphycally.

$$f(x) = x^2-4 \; ; \; x \le 0$$
$$= -\frac{1}{2} + x \; ; \; 0 < x < \frac{1}{2}$$
$$= -x \; ; \; \frac{1}{2} < x \le 1$$
$$= -\frac{3}{2} + x \; ; \; 1 < x < 2$$
$$= x \; ; \; x \ge 2$$

Ans:

Mathematically:



Points: $0, \frac{1}{2}, 1, 2$

Analylis at $x=0$ point:

L.H.L:

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$$
$$= \lim_{h \to 0} f(-h)$$
$$= 0^2 - 4$$
$$= -4$$

R.H.L:

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$$
$$= \lim_{h \to 0} f(h)$$
$$= -\frac{1}{2} + 0$$
$$= -\frac{1}{2}$$

$\therefore$ L.H.L $\neq$ R.H.L

The function is discontinuous at the point $x=0$.

Analysis at $x=\frac{1}{2}$ point:

L.H.L:

$$\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{1}{2}-h\right)$$
$$= \lim_{h \to 0} -\frac{1}{2} + \frac{1}{2} - h$$
$$= \lim_{h \to 0} -0$$
$$= 0$$

$\therefore R\cdot H\cdot L \neq L\cdot H\cdot L$

The function is discontinuous at the point $x=2$।

Graphycally :

i) $y = x^2 - 4 \quad (x \le 0)$

$x=0$

$x = -\frac{1}{2}$

$\Rightarrow x^2 = y+4$

(iii) নং বিবেচনা $x = -\frac{3}{2}$

$x = -2$

$(x,y)=(0,0)$ ধরলে

$L\cdot H\cdot S = R\cdot H\cdot S$ মিলে না।

তাই,

$x=0$ ধরে,

$0^2 = y+4$

$\Rightarrow y = -4$।

বিন্দু $(0,-4)$

$y=0$ ধরে,

$x^2 = 0+4$

$\Rightarrow x^2 = 4$

$\therefore x = \pm 2$

$(2,0)$ $(-2,0)$

ii) $y = -\frac{1}{2} + x \quad (0 < x < \frac{1}{2})$

$x=0$

$x=\frac{1}{2}$

$\Rightarrow y - x = -\frac{1}{2}$

$\Rightarrow \frac{y}{-\frac{1}{2}} - \frac{x}{-\frac{1}{2}} = 1$

$\Rightarrow \frac{x}{\frac{1}{2}} + \frac{y}{-\frac{1}{2}} = 1$

iii) $y = -x \quad (\frac{1}{2} < x \le 1)$

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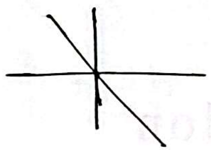

$x = \frac{1}{2}$

$x = 1$

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iv) $y = \frac{3}{2} + x \quad (1 < x < 2)$

$x=1$

$x = \frac{3}{2}$

$x = 2$

$\Rightarrow y - x = \frac{3}{2}$

$\Rightarrow \frac{y}{3/2} - \frac{x}{\frac{3}{2}} = 1$

$\Rightarrow \frac{x}{-\frac{3}{2}} + \frac{y}{\frac{3}{2}} = 1$

v) $y = x \quad (x \ge 2)$

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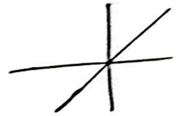
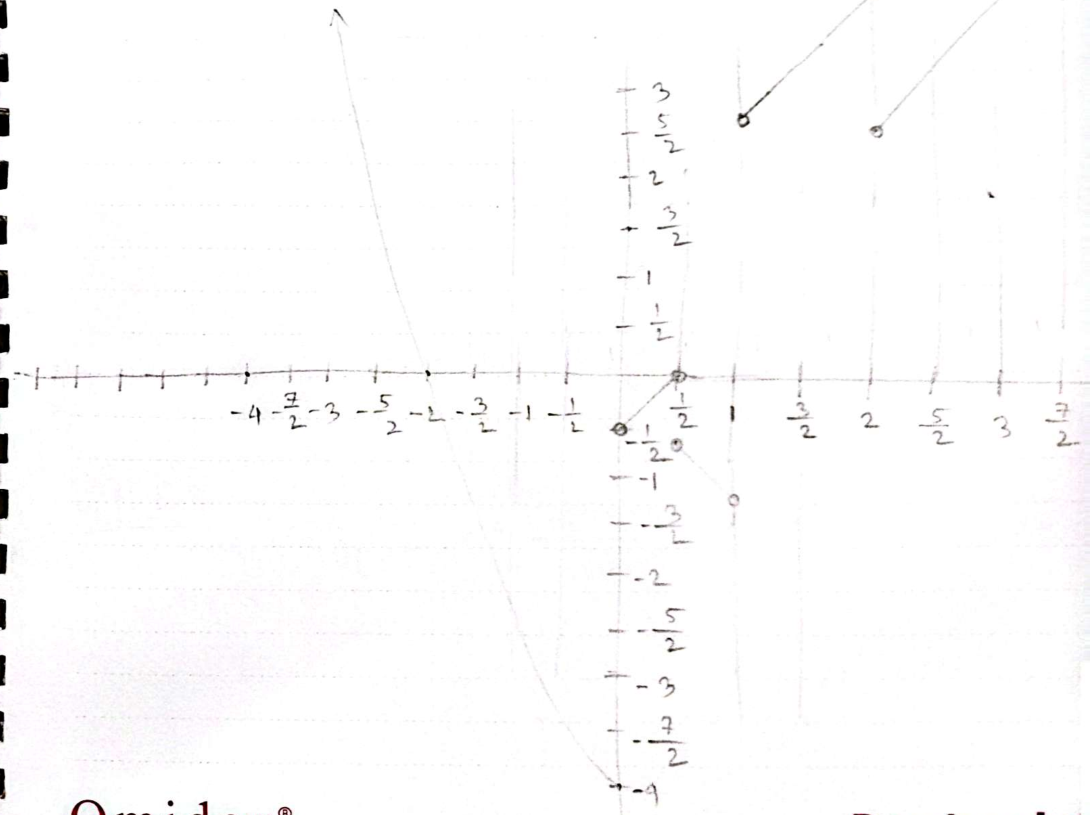


$x=2$

$x=\frac{5}{2}$

$x=3$

$x=\frac{7}{2}$



R.H.L:

$$\lim_{x \to \frac{1}{2}^+} f(x) = \lim_{h \to 0} f\left(\frac{1}{2}+h\right)$$

$$= \lim_{h \to 0} -\frac{1}{2} - h$$

$$= -\frac{1}{2}$$

$\therefore$ L.H.L $\neq$ R.H.L

The function is discontinuous at the point $x = \frac{1}{2}$

Analysis at point $x = 1$:

L.H.L:

$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$$

$$= \lim_{h \to 0} -1 + h$$

$$= -1 + 0$$

$$= -1$$

R.H.L:

$$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$$

$$= \lim_{h \to 0} \frac{3}{2} + 1 + h$$

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$$= \frac{3+2}{2}$$

$$= \frac{5}{2}$$

$\therefore$ L.H.L $\neq$ R.H.L

The function is discontinuous at the point $x=$

Analysis at the point $x = 2$:

L.H.L:

$$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h)$$

$$= \lim_{h \to 0} \frac{3}{2} + 2 - h$$

$$= \frac{3+4}{2}$$

$$= \frac{7}{2}$$

R.H.L:

$$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)$$

$$= \lim_{h \to 0} 2 + h$$

$$= 2$$