# DIGITAL IMAGE PROCESSING (CS 402)

## Image restoration and reconstruction

By Dr. Rashmi Panda

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# Books

- Digital Image Processing (Authors: Rafael C. Gonzalez and Richard E. Woods)
- Image Processing, Analysis and Machine Vision (Authors: Milan Sonka, Vaclav Hlavac and Roger Boyle)
- Fundamentals of Digital Image Processing (Author: Anil K. Jain)

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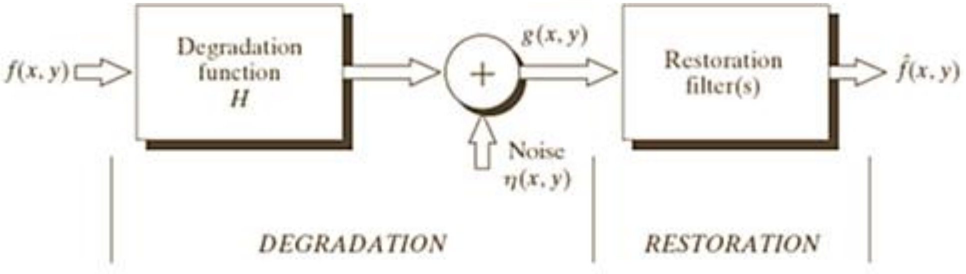
2) Restoration attempts to recover an image that has been degraded by using a prior knowledge of the degradation phenomenon. However, image enhancement techniques are generally heuristic.
3) Restoration process is oriented towards modelling the degradation and applying the inverse process in order to recover the original image. Where as in enhancement technique, image is manipulated to take the advantage of the psychophysical aspect of human visual system.

# A Model of Image degradation/Restoration Process



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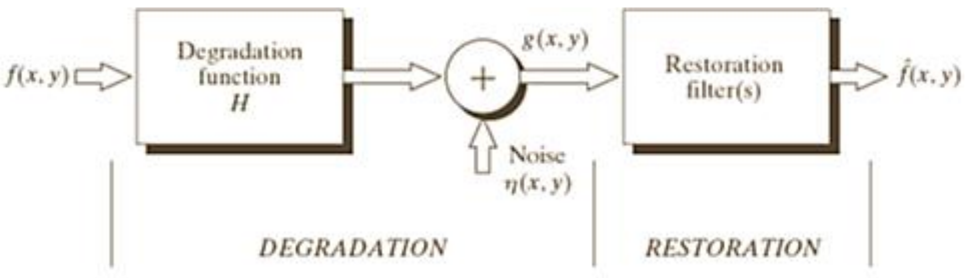


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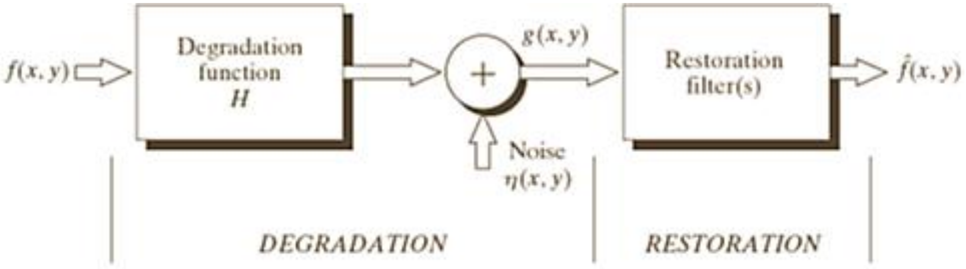


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- If H is linear and position invariant image then degraded image will be

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

- In frequency domain representation $G(u,v)=H(u,v)\times F(u,v)+N(u,v)$

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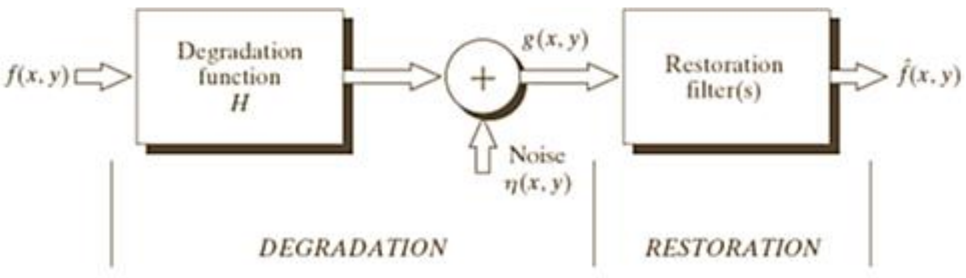


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- In frequency domain representation $G(u,v) = H(u,v) \times F(u,v) + N(u,v)$
- Restoration gives an estimate of the original image i.e. $\hat{f}(x,y)$
- The more we know about the $h(x,y)$ and $\eta(x,y)$, the closer is the estimate $\hat{f}(x,y)$.

# Image Restoration

- At present, we will consider the degradation process is *H* is the identity operator.

- We will deal with the degradation only in presence of noise.

$$g(x,y) = f(x,y) + \eta(x,y)$$

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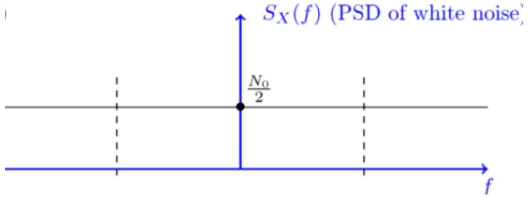

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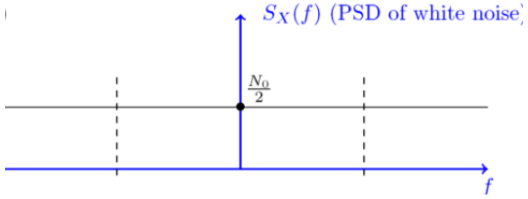


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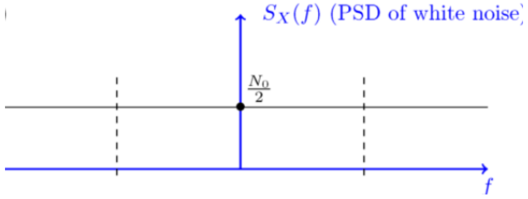


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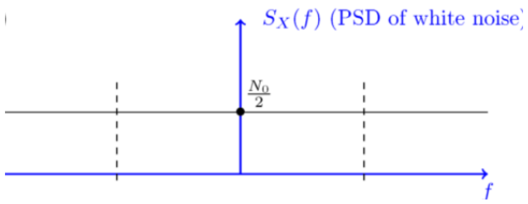


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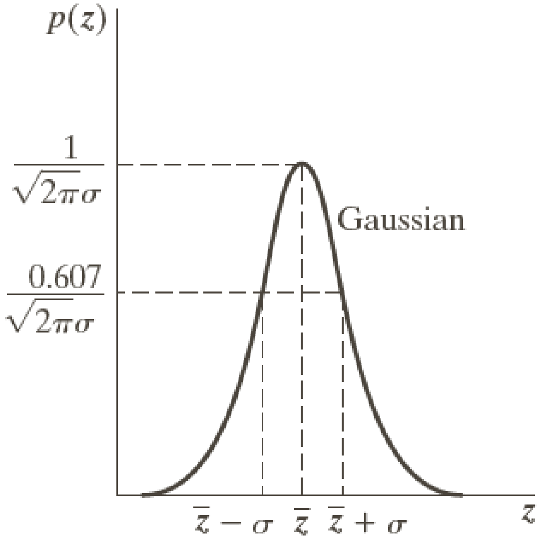


  - 70% of its values will be in the range $[(\mu-\sigma),(\mu+\sigma)]$
  - 95% of its values will be in the range $[(\mu-2\sigma),(\mu+2\sigma)]$

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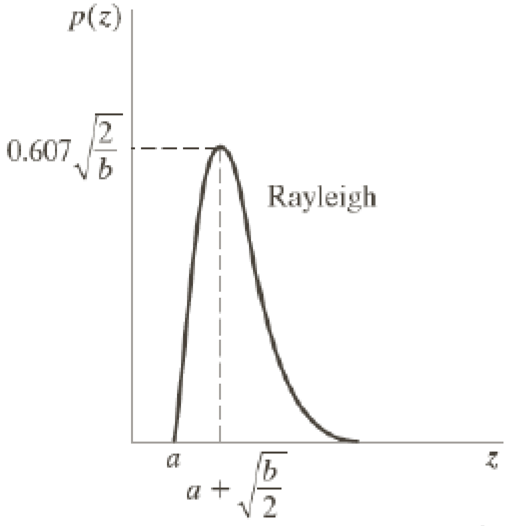

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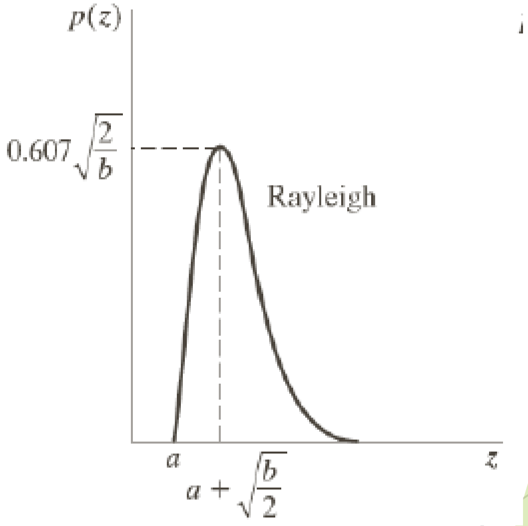


  - The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

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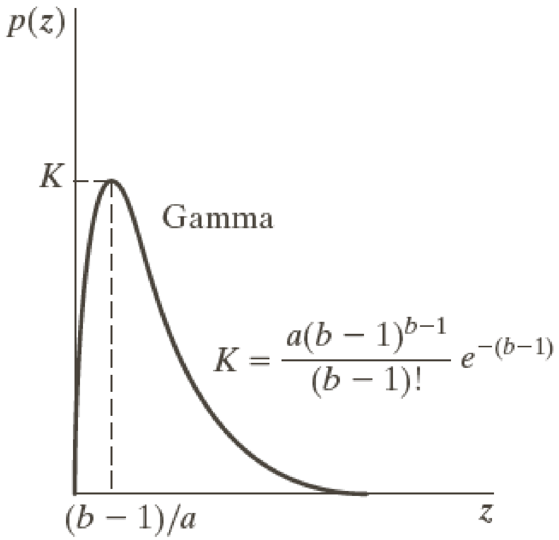

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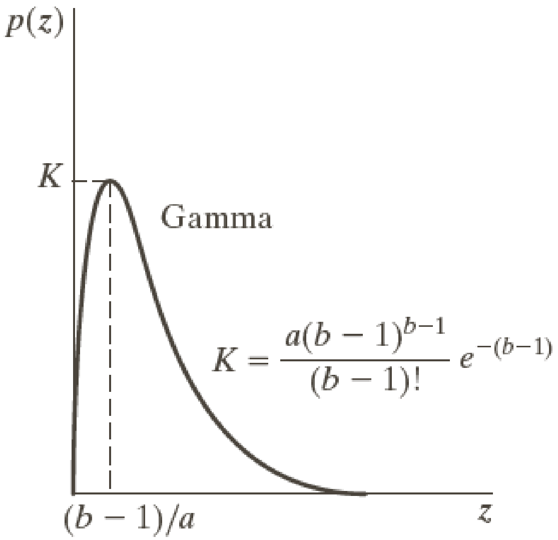

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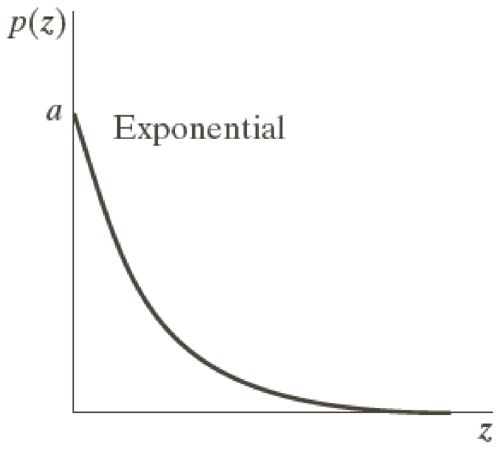

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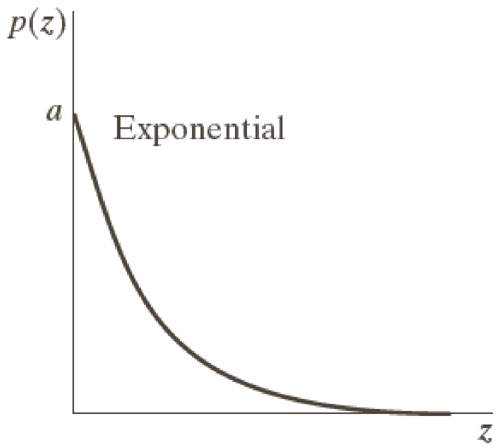


  - The mean and variance of this density are given by

$$\bar{z} = 1/a$$
$$\sigma^2 = 1/a^2$$

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- **Uniform noise:**

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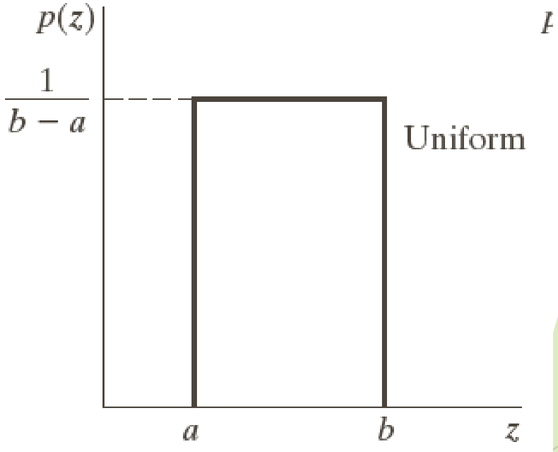
$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for a} \le z \le b \\ 0 & \text{otherwise} \end{cases}$$



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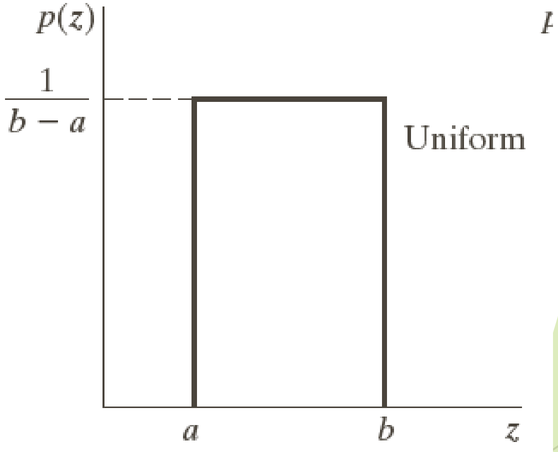
$$\sigma^2 = (b-a)^2/12$$



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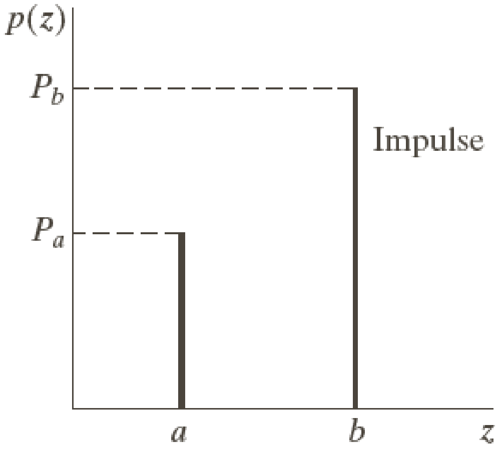

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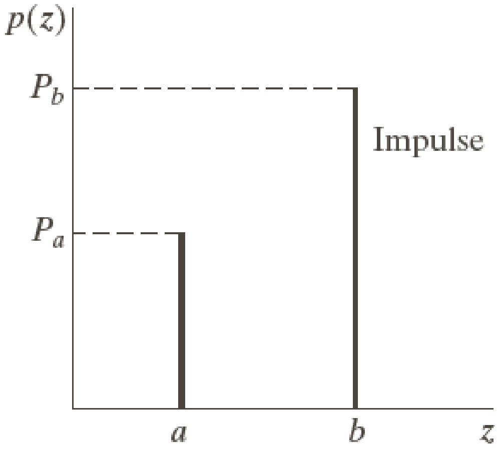

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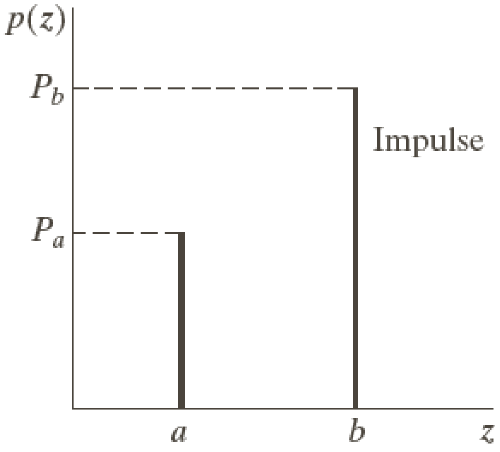
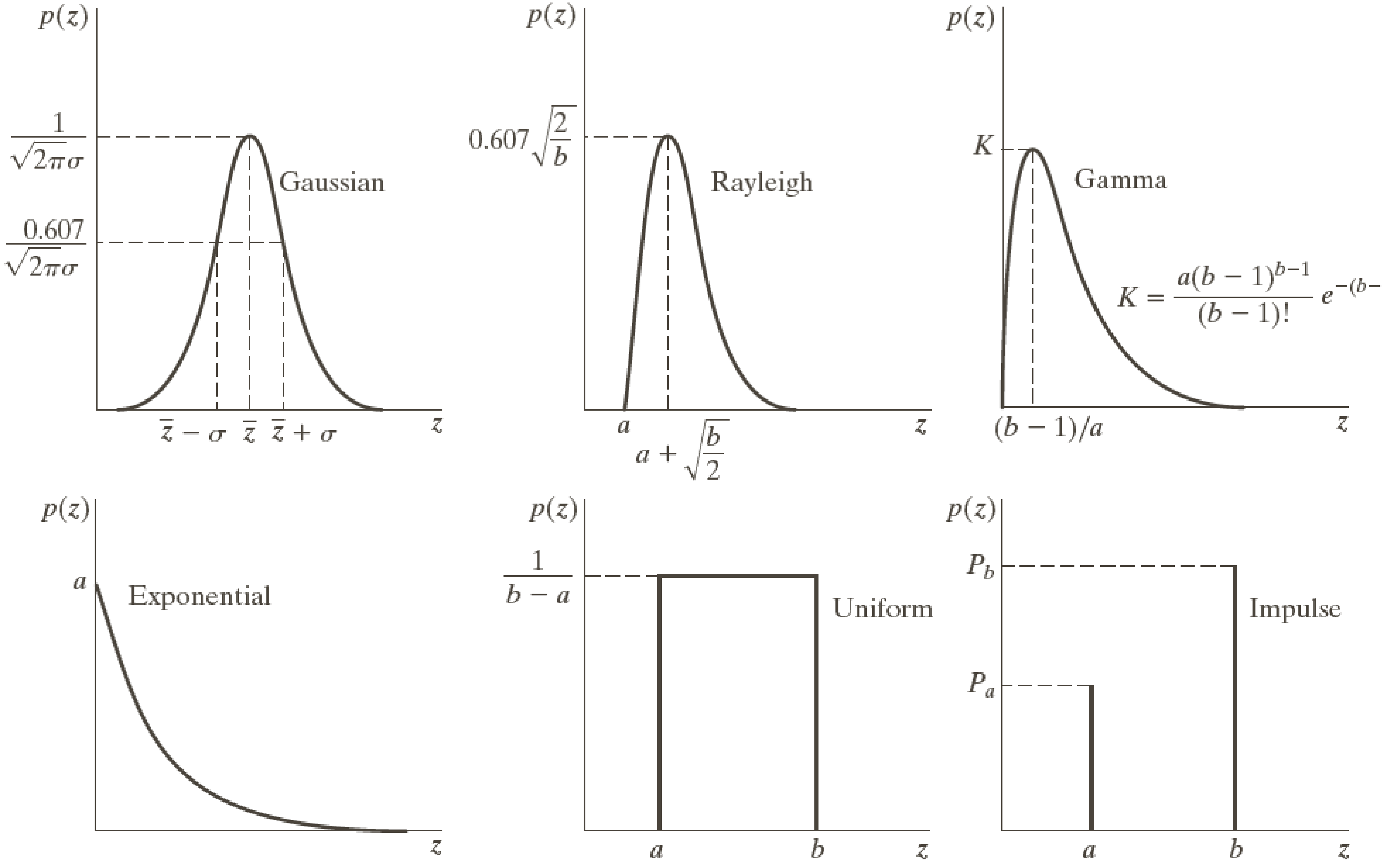


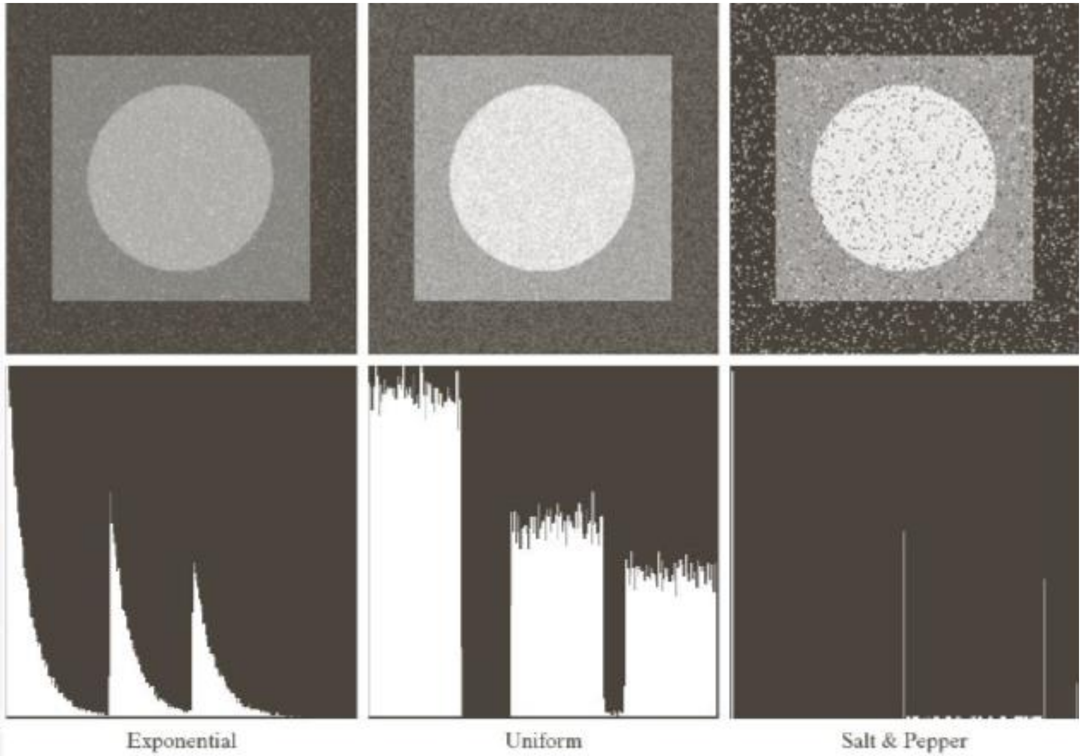

  - if $b>a$ , gray-level will appear as a light dot,

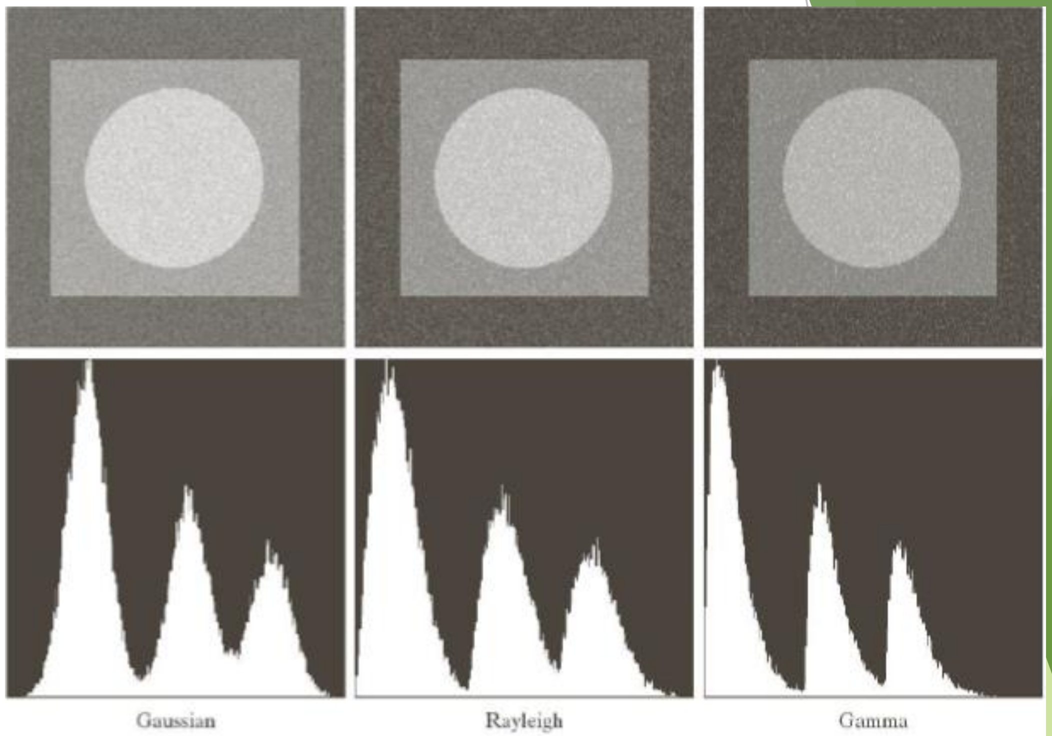
    while level will appear like a dark dot.
  - If either $Pa$ or $Pb$ is zero, the impulse noise is called unipolar



$p(z)$
$\frac{1}{\sqrt{2\pi}\sigma}$
$\frac{0.607}{\sqrt{2\pi}\sigma}$
Gaussian
$\bar{z}-\sigma$ $\bar{z}$ $\bar{z}+\sigma$
$z$
$p(z)$
$0.607\sqrt{\frac{2}{b}}$
Rayleigh
$a$
$a+\sqrt{\frac{b}{2}}$
$z$
$p(z)$
$K$
Gamma
$K = \frac{a(b-1)^{b-1}}{(b-1)!}e^{-(b-}$
$(b-1)/a$
$z$
$p(z)$
$a$
Exponential
$z$
$p(z)$
$\frac{1}{b-a}$
Uniform
$a$ $b$ $z$
$p(z)$
$P_b$
$P_a$
Impulse
$a$ $b$ $z$

Original Image




Example of effect of different types of noises

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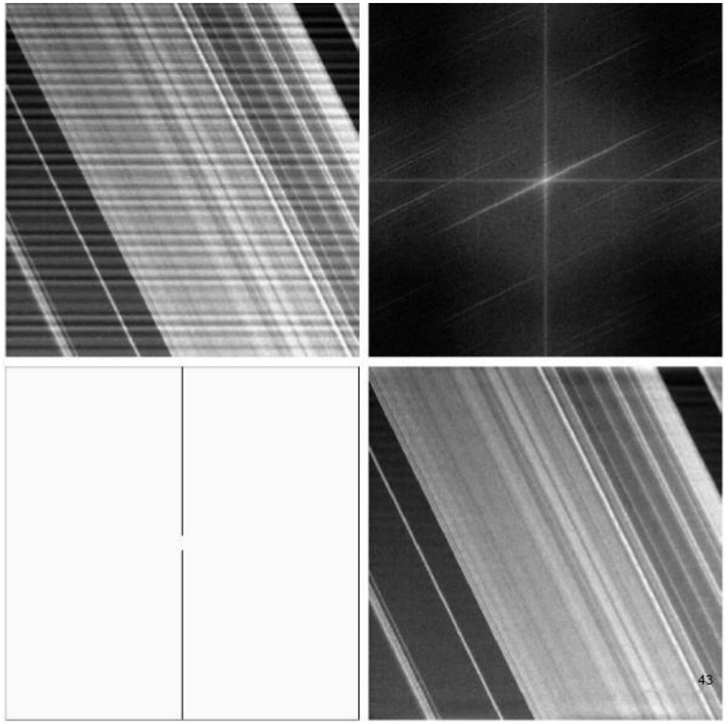

(a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

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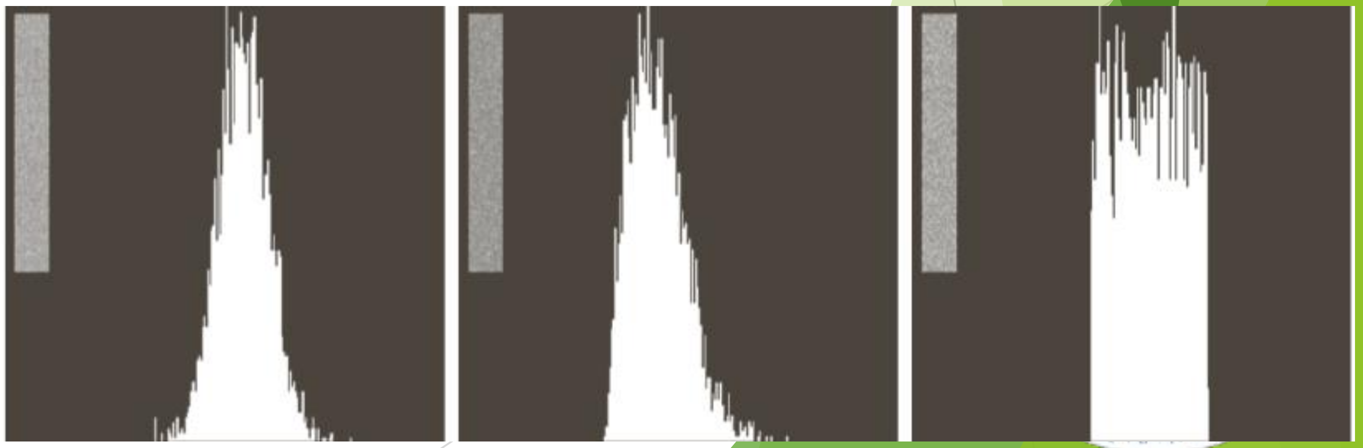

# Estimation of Noise Parameters

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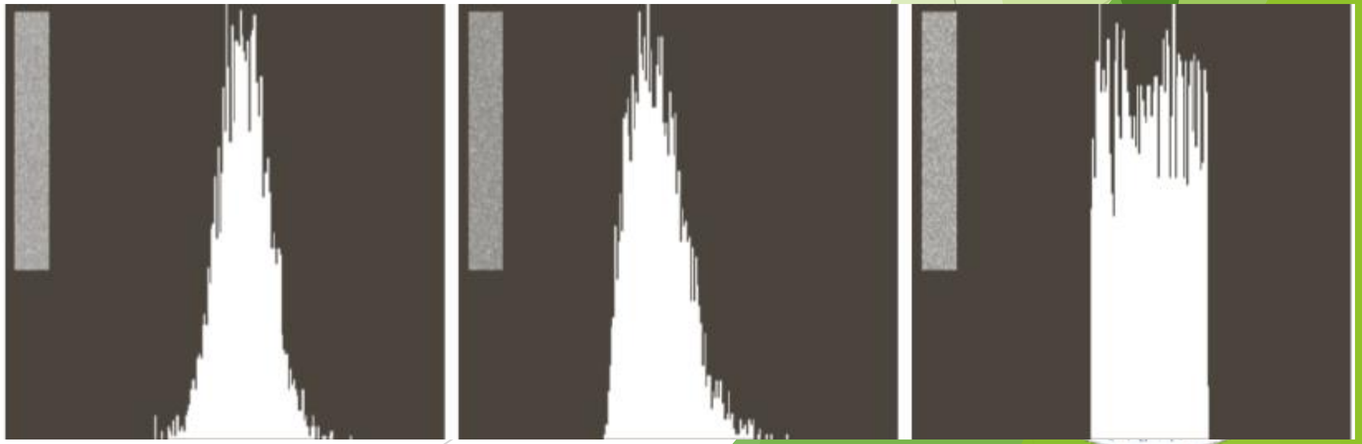
- The shape of the histogram identifies the closest PDF
- Consider a subimage denoted by *S* and let $P_s(z_i)$ , *i*= *0*, *1*, ..., *L*-*1*, denote the probability estimates of the intensities of the pixels in *S*. The mean and variance of the pixels in *S*:

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$



# Image restoration in presence of noise only-spatial filtering

- Noise model without degradation

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- Spatial filtering is a method of choice only when additive random noise is present.

# Mean Filters

- Let, $S_{xy}$ presents the set of coordinates in a rectangle subimage window of size $m \times n$, centered at (x,y).

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## 1. Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- A mean filter smooths the local variation in an image and noise a result of blurring is also reduced.

# Mean Filters

2. Geometric mean filter

$$f(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

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$$f(x,y) = \left[ \prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

- Generally, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process

# Mean Filters

3. Harmonic mean filter

$$f(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

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$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- It works well for salt noise, but fails for pepper noise.
- It does well also with other types of noise like Gaussian noise.

# Mean Filters

## 4. Contraharmonic mean filter

$$f(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- Q is the order of the filter.

# Mean Filters

## 4. Contraharmonic mean filter

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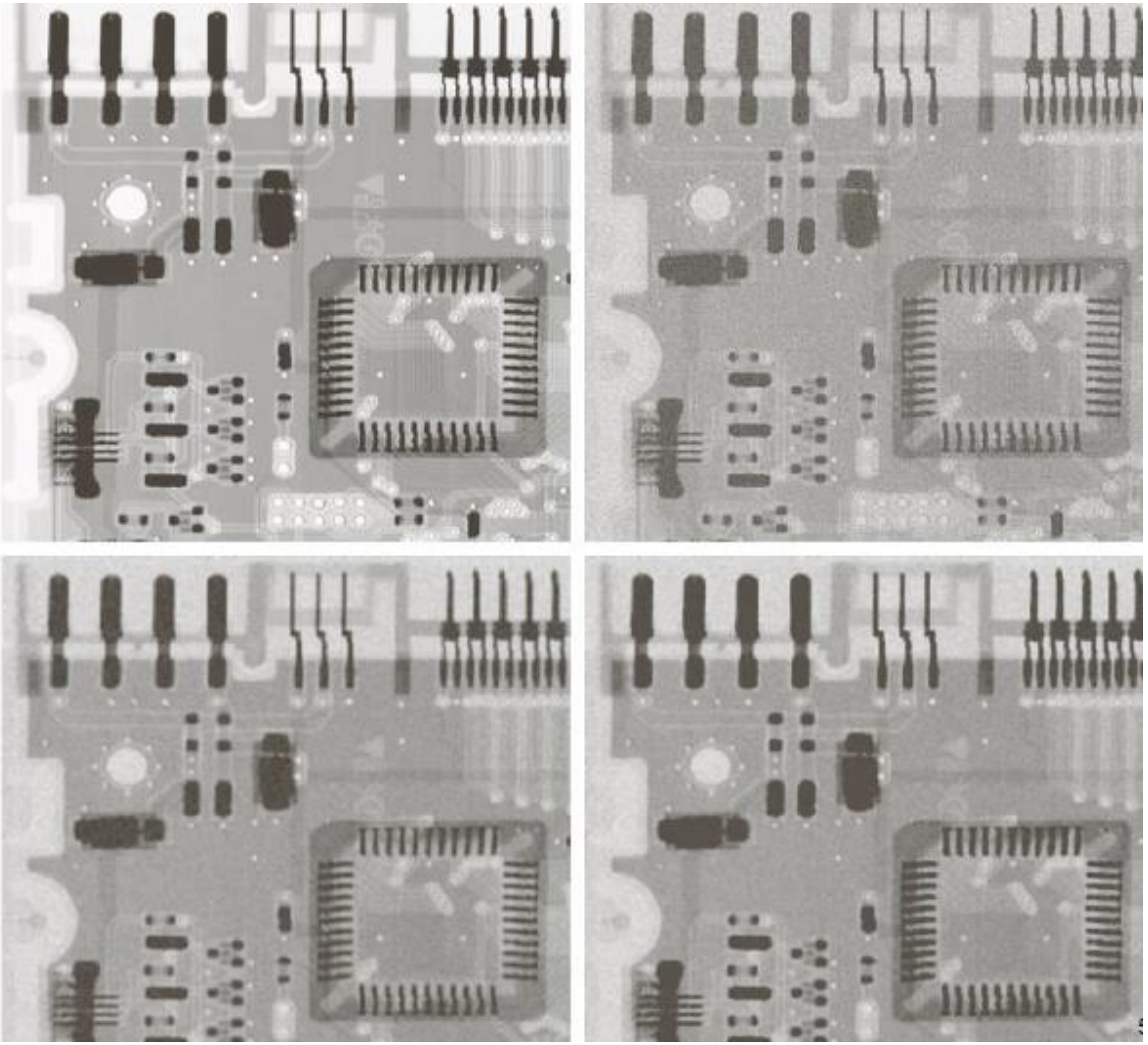
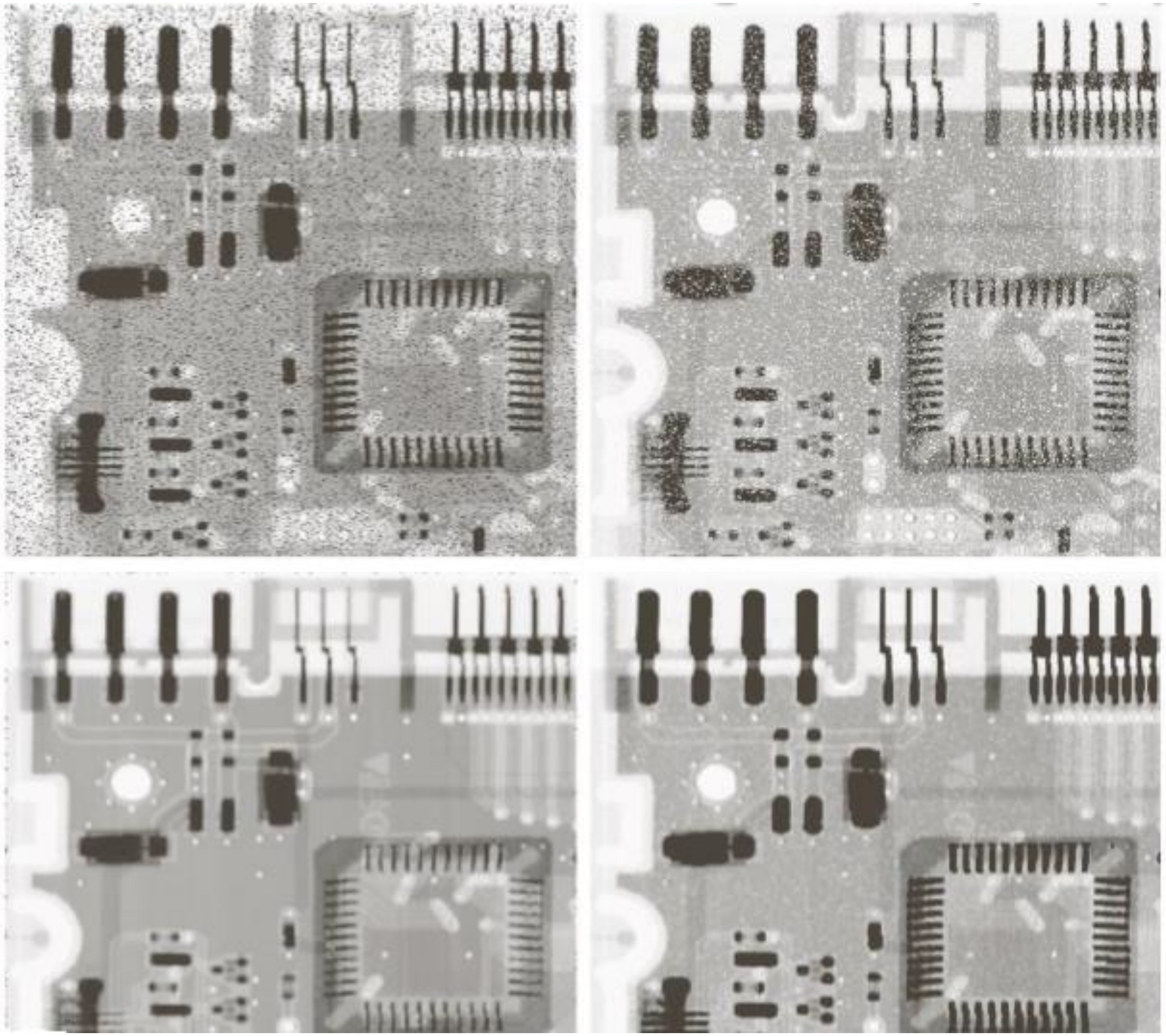
- Q is the order of the filter.
- It is well suited for reducing the effects of salt-and-pepper noise. Q>0 for pepper noise and Q<0 for salt noise.



a b
c d

(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size $3 \times 3$. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



| a | b |
|---|---|
| c | d |

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a $3 \times 3$ contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

# Spatial Filtering: Order-Statistic Filters

## 1. Median Filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

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- It provides less blurring than linear smoothing filter of same size.

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## 3. Min Filer

$$f(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

- Useful for finding the darkest pixel in the image. It reduces the salt noise.

# Spatial Filtering: Order-Statistic Filters

## 4. Midpoint Filter

$$f(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- This filter is useful randomly distributed noise.

# Spatial Filtering: Order-Statistic Filters

## 4. Midpoint Filter

$$f(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- This filter is useful randomly distributed noise.

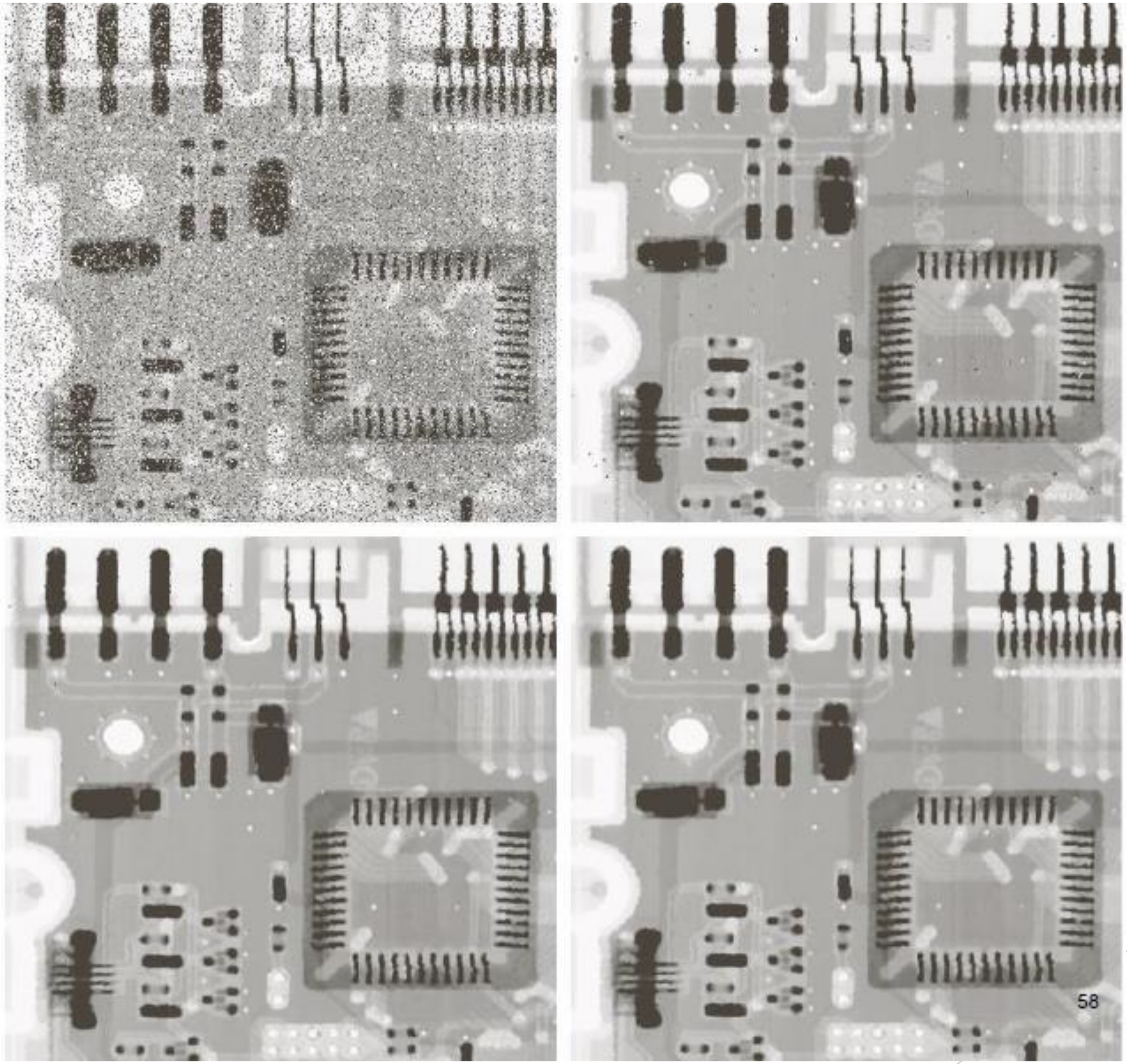
## 5. Alpha-trimmed Mean Filter

$$f(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}}\{g_r(s,t)\}$$

- The *d/2* lowest and *d/2* highest intensity values of *g(s,t)* are deleted in the image window.
- $g_r(s,t)$ denote the remaining *mn-d* pixels.
- This filter is useful for reducing multiple types of noise such as a combination of Salt-pepper noise and Gaussian noise.

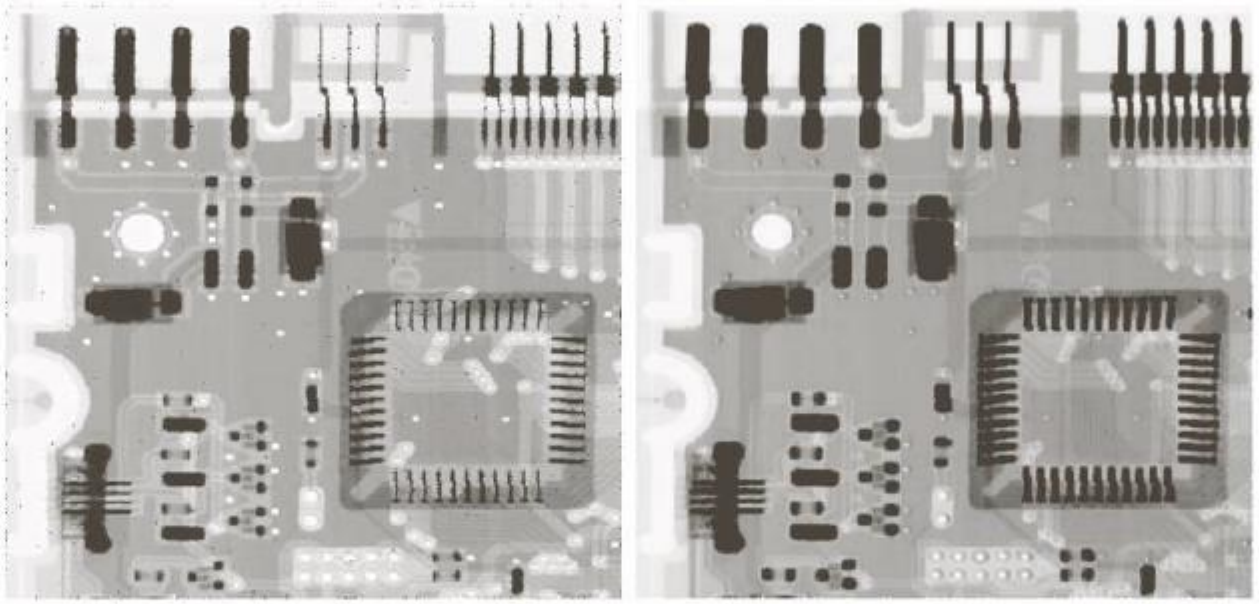
| a | b |
|---|---|
| c | d |

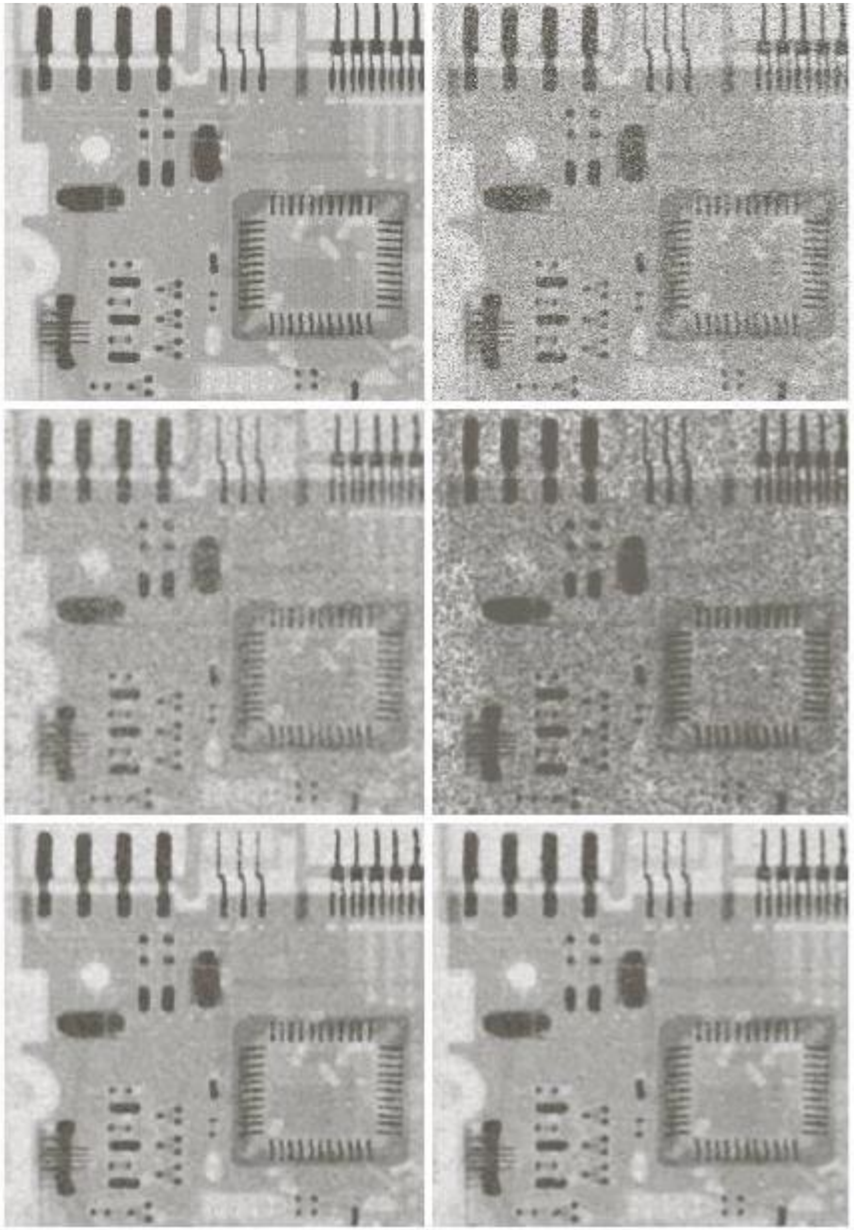
**FIGURE 5.10**
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size $3 \times 3$.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



a b

(a) Result of filtering Fig. 5.8(a) with a max filter of size $3 \times 3$. (b) Result of filtering 5.8(b) with a min filter of the same size.





| a | b |
|---|---|
| c | d |
| e | f |

**FIGURE 5.12** (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a $5 \times 5$; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Thank you !