Computer Vision and Image Processing (EC 336)

Lecture 8: Frequency domain filtering

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Zero-Phase-Shift Filters

$$g(x,y)=\Im^{-1}\{H(u,v)F(u,v)\}$$

$$F(u,v)=R(u,v)+jI(u,v)$$

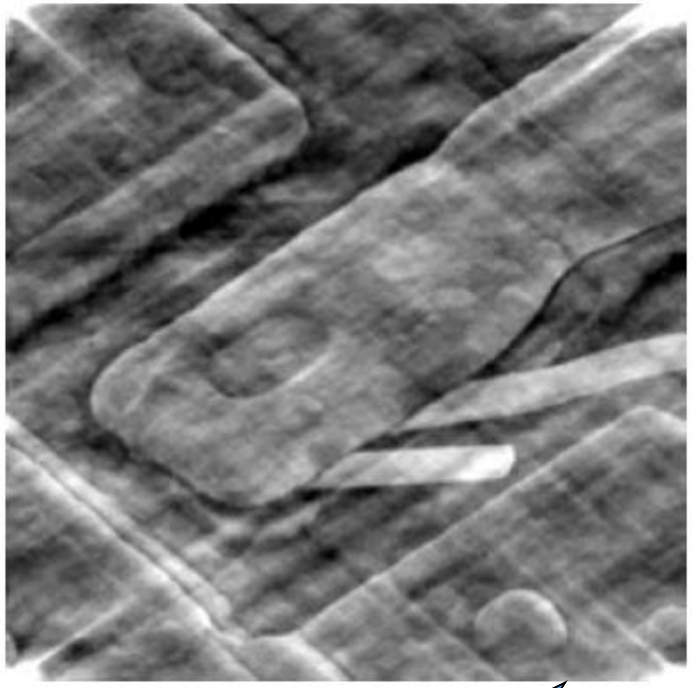
$$g(x,y)=\Im^{-1}\left[H(u,v)R(u,v)+jH(u,v)I(u,v)\right]$$

Filters affect the real and imaginary parts equally,

and thus no effect on the phase.

These filters are called **zero-phase-shift** filters

Examples: Nonzero-Phase-Shift Filters



a b

FIGURE 4.35
(a) Image resulting from multiplying by 0.5 the phase angle in Eq. (4.6-15) and then computing the IDFT. (b) The result of multiplying the phase by 0.25. The spectrum was not changed in either of the two cases.

Even s n the phase ang ramatic
(usuall effects on the t

Phase angle is multiplied by 0.5

Phase angle is multiplied by 0.25

Image Smoothing Using Filter Domain Filters: ILPF

Ideal Lowpass Filters (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

Image Smoothing Using Filter Domain Filters: ILPF

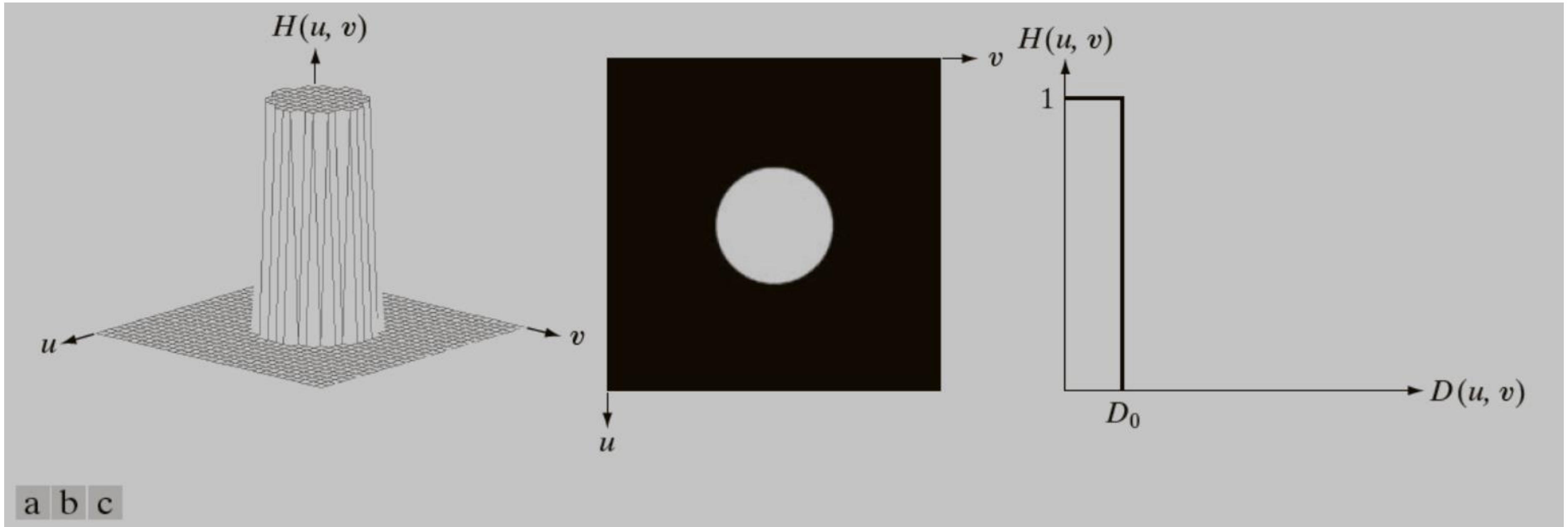
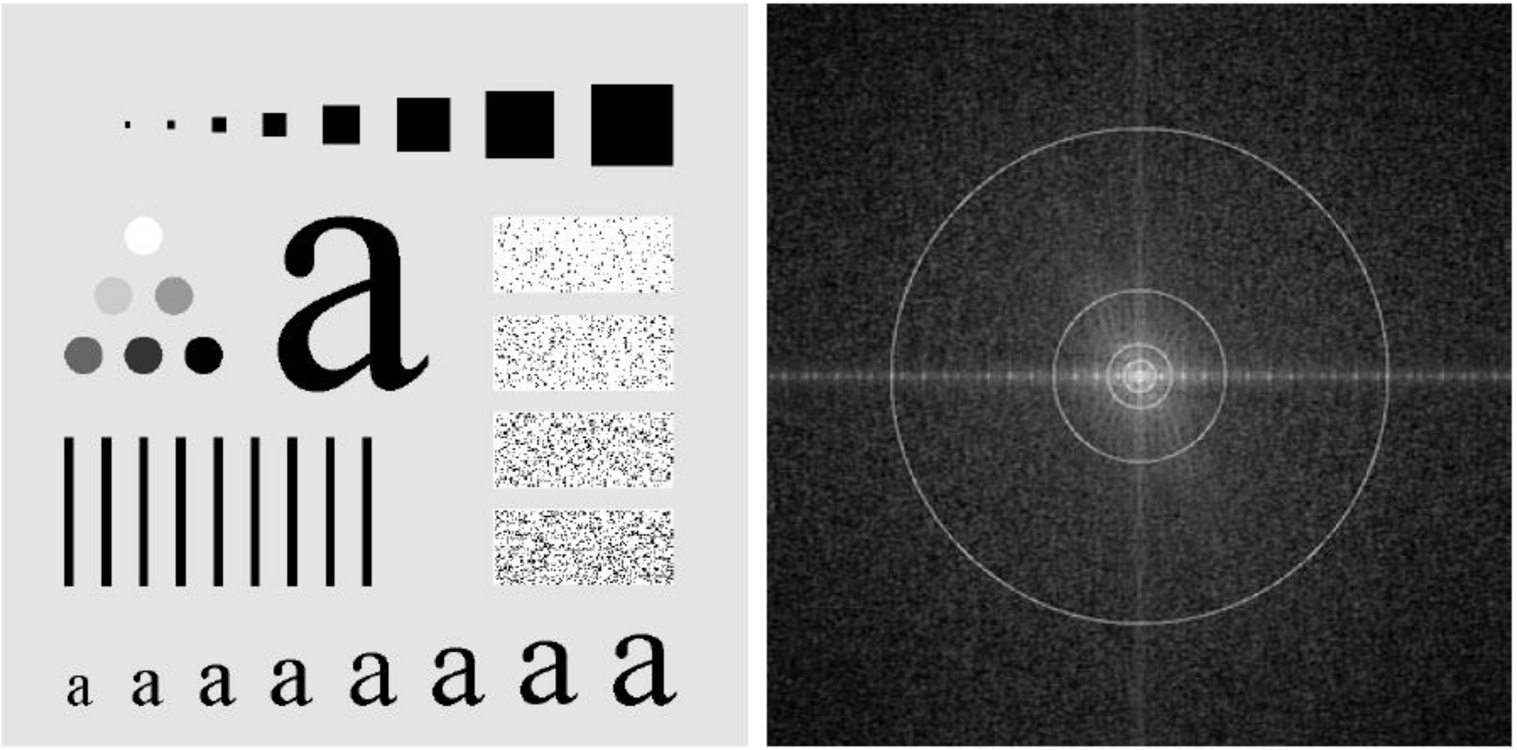


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

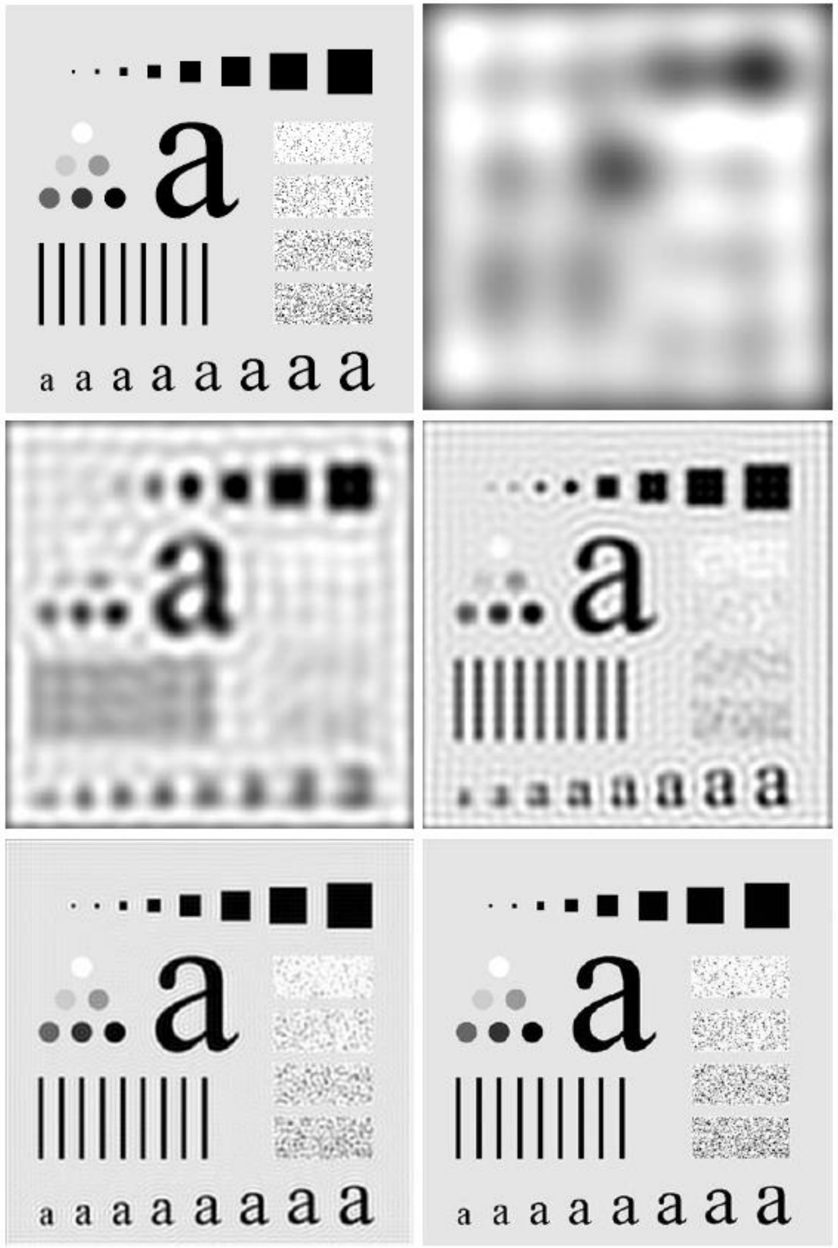
ILPF Filtering Example



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

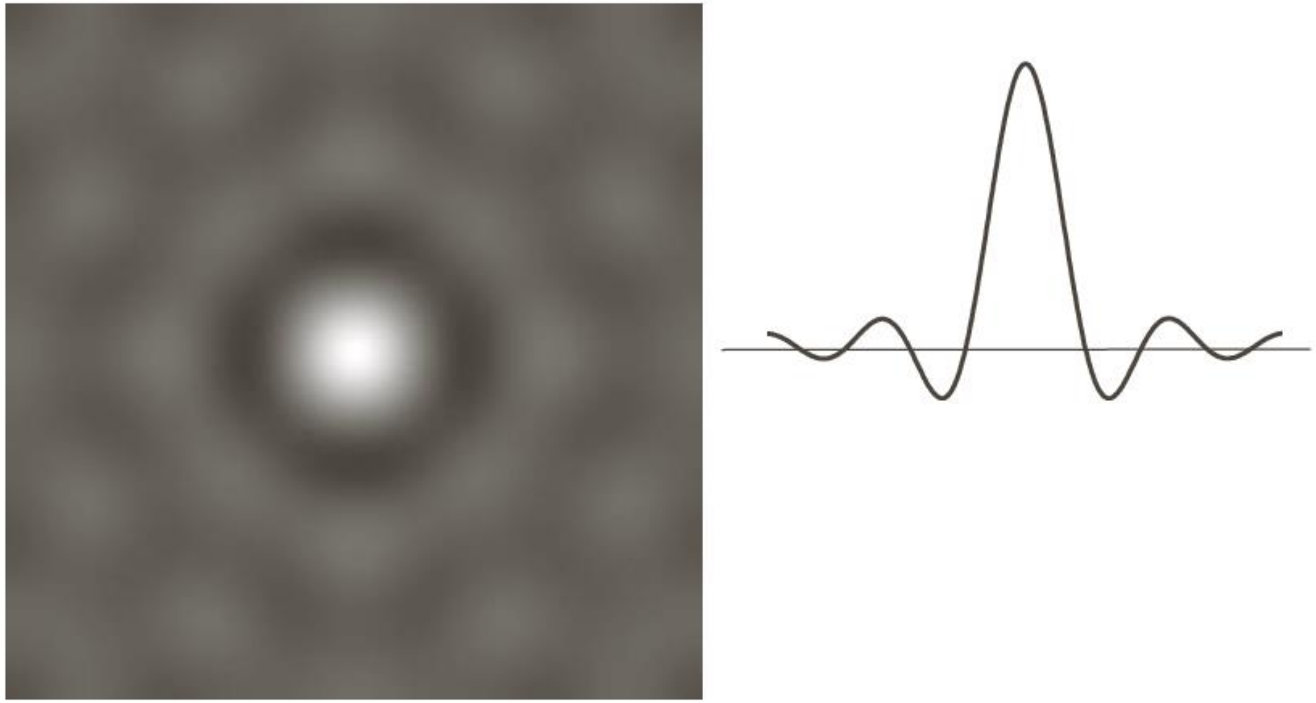
ILPF Filtering Example



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

The Spatial Representation of ILPF



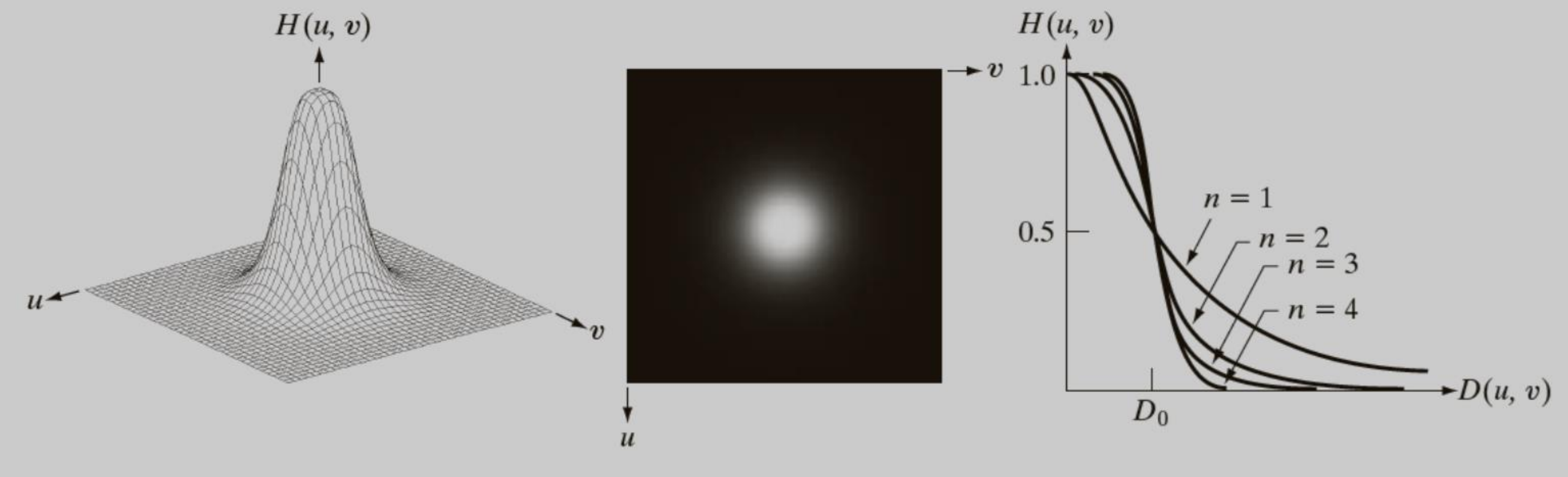
a b

FIGURE 4.43
(a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Image Smoothing Using Filter Domain Filters: BLPF

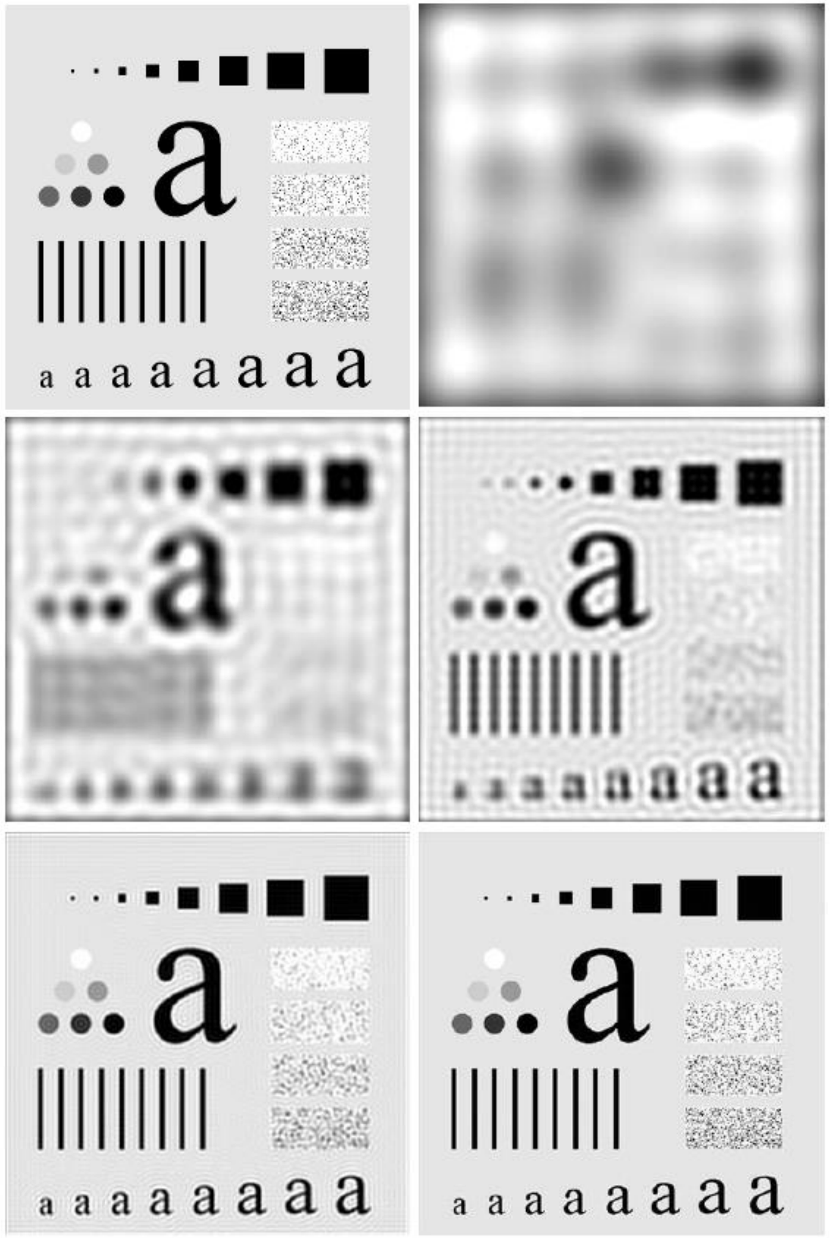
Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u,v) = \frac{1}{1 + \left[D(u,v) / D_0 \right]^{2n}}$$



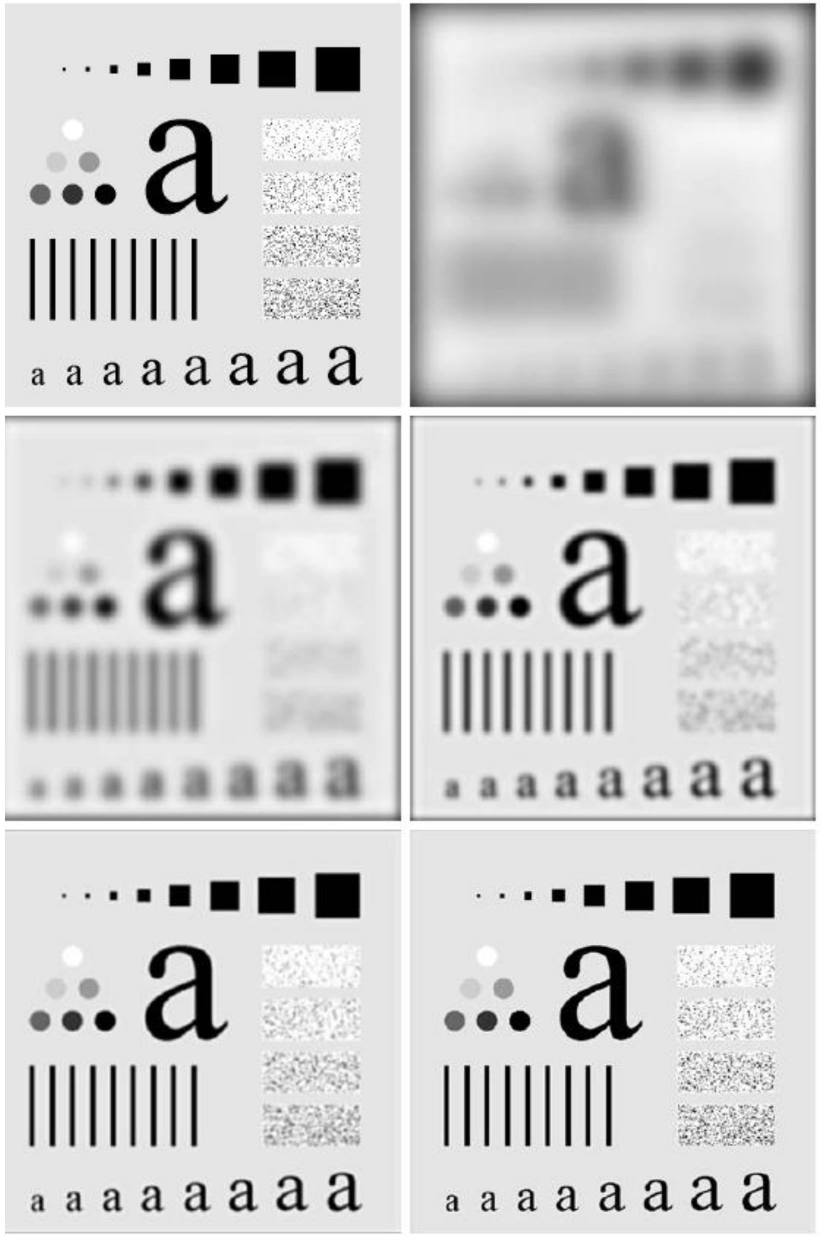
a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

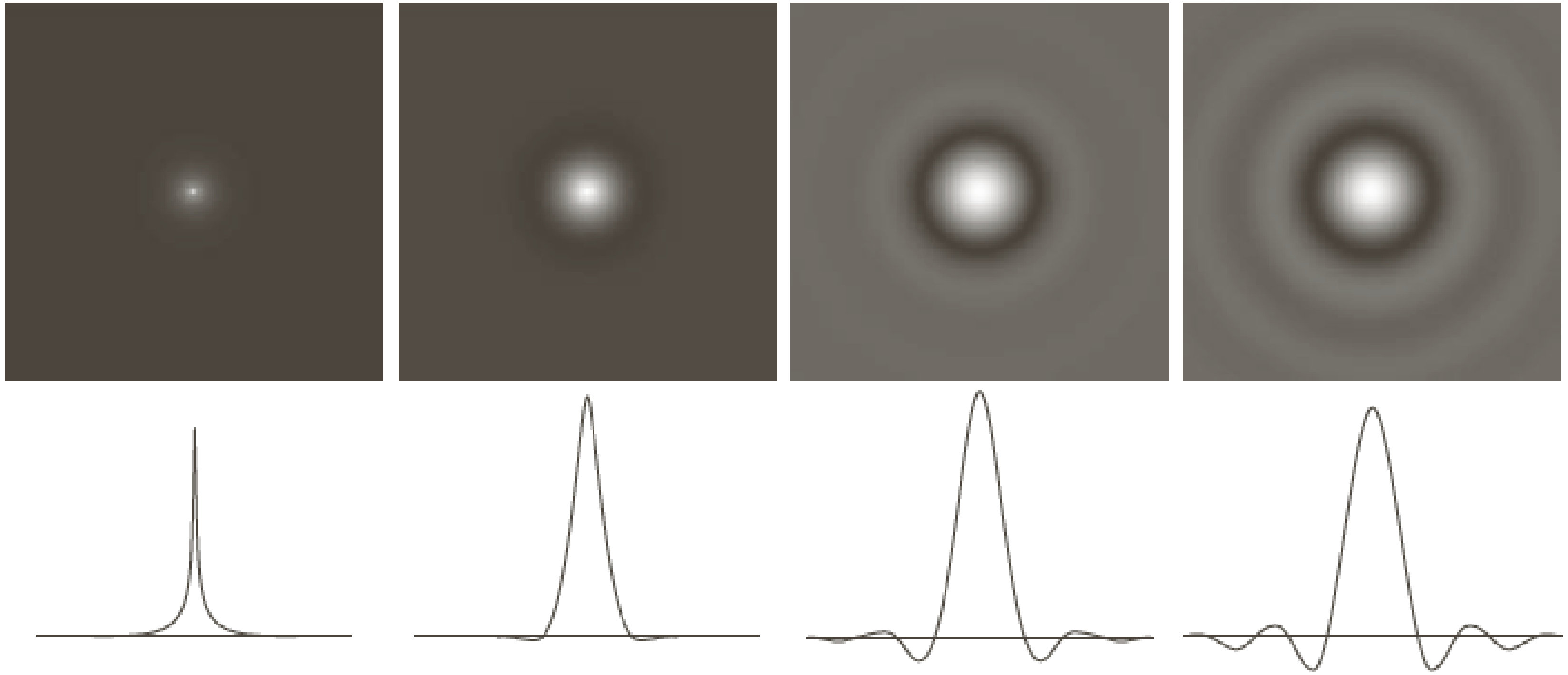
FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



b
d
f

GURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, ith cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

The Spatial Representation of BLPF



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000×1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Image Smoothing Using Filter Domain Filters: GLPF

Gaussian Lowpass Filters (GLPF) in two dimensions is given

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0{}^2}$$

Image Smoothing Using Filter Domain Filters: GLPF

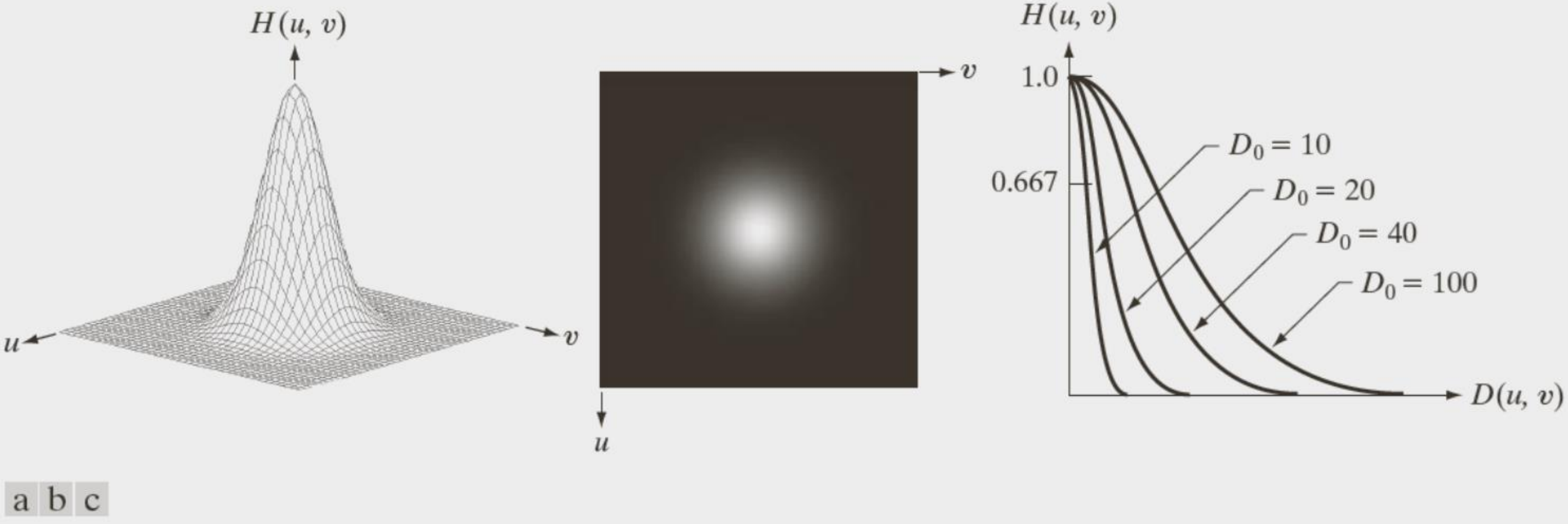
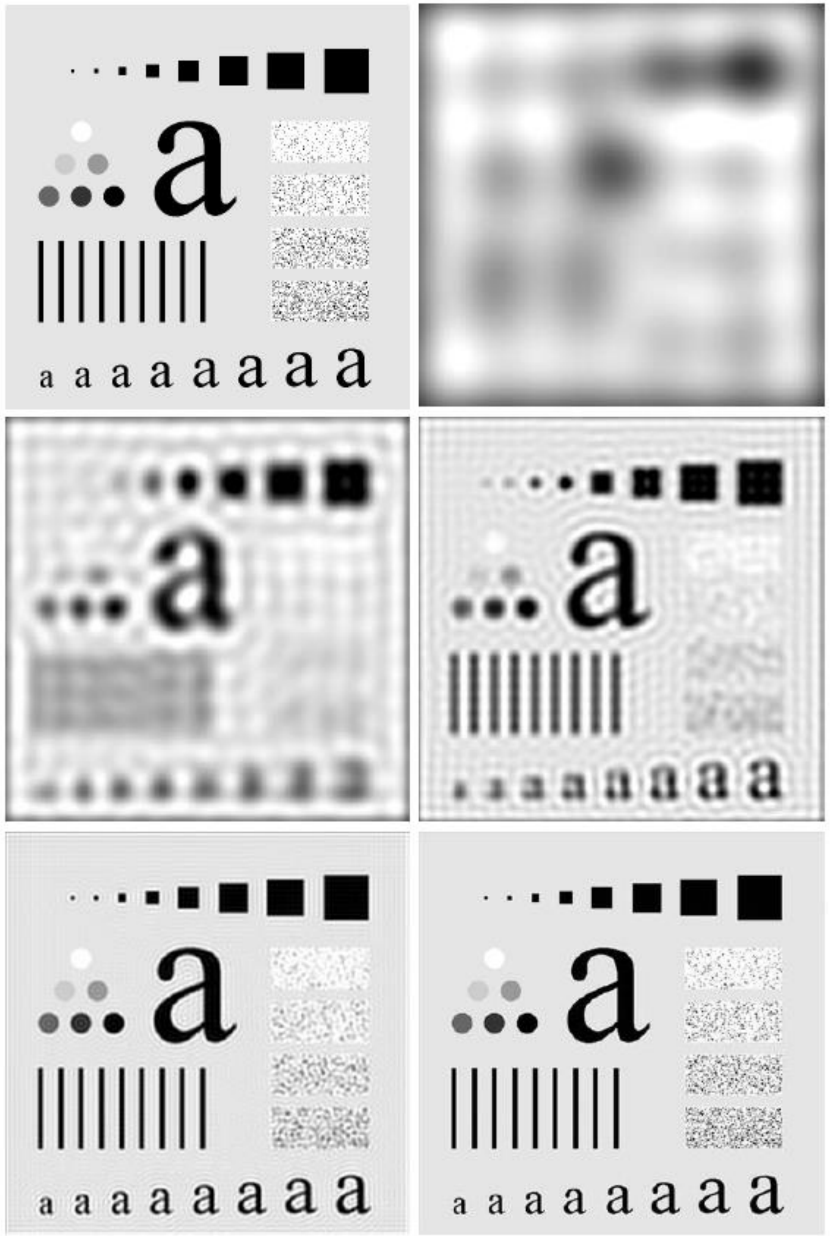
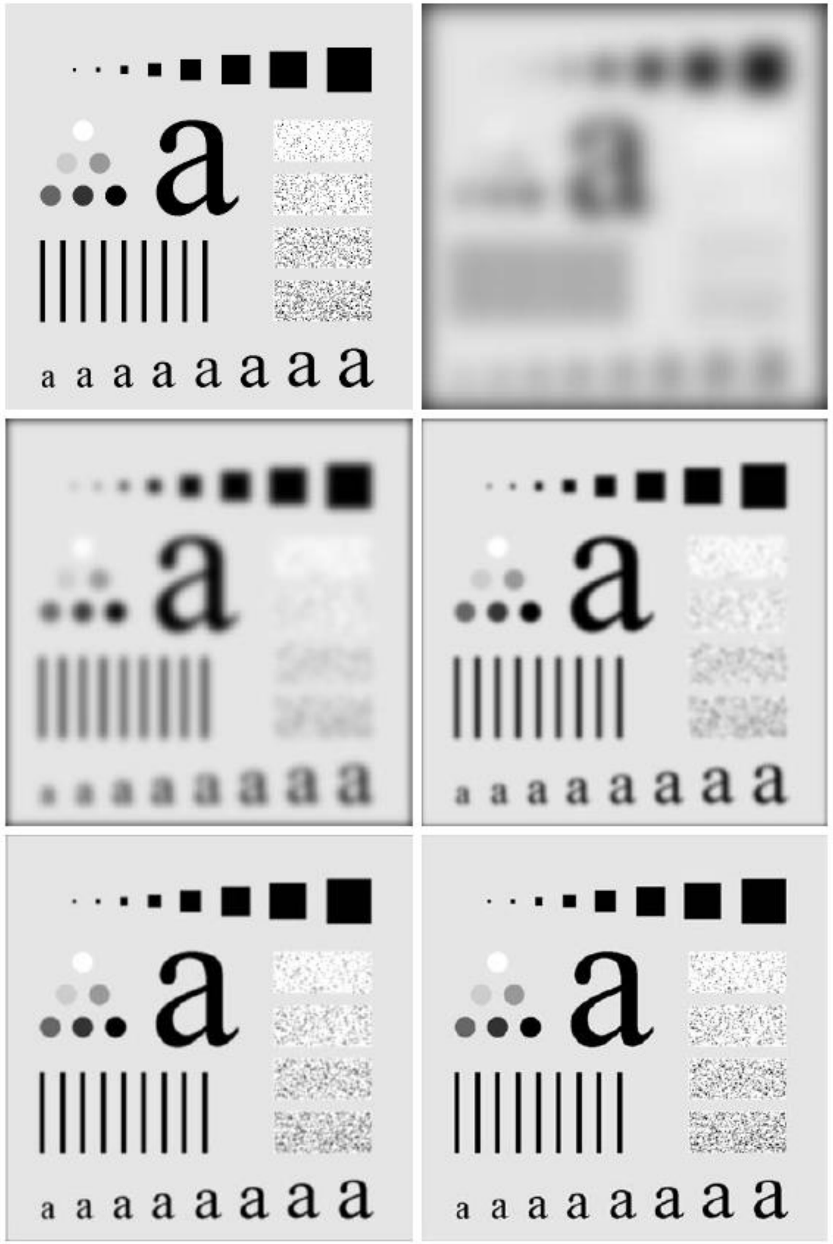


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.



a b
c d
e f

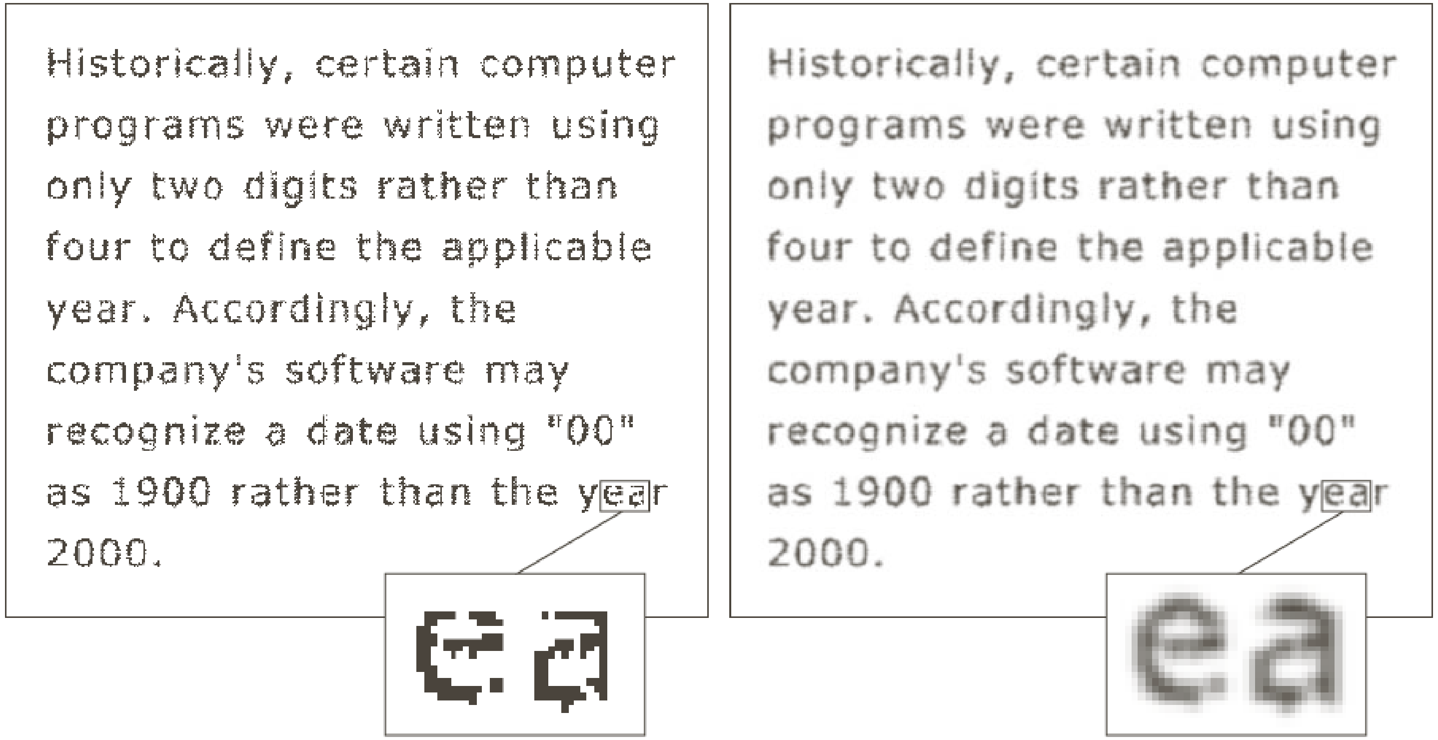
FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Examples of smoothing by GLPF (1)



a b

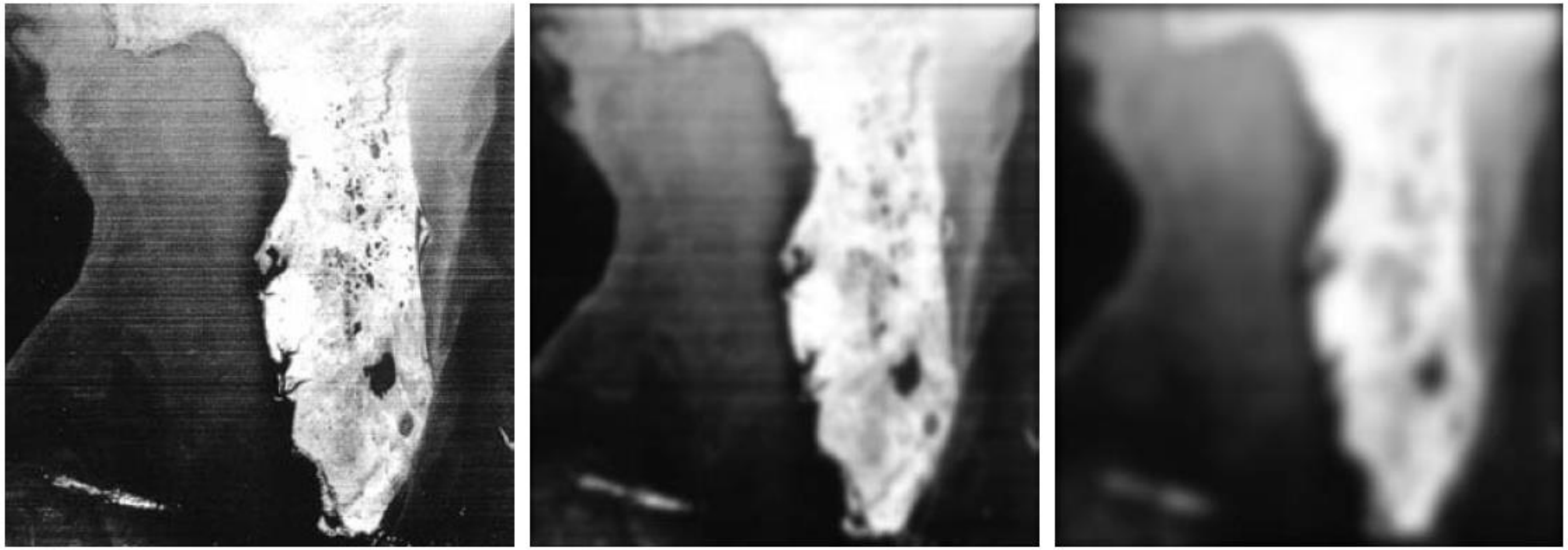
FIGURE 4.49
(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Examples of smoothing by GLPF (2)

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Examples of smoothing by GLPF (3)



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Image Sharpening Using Frequency Domain Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

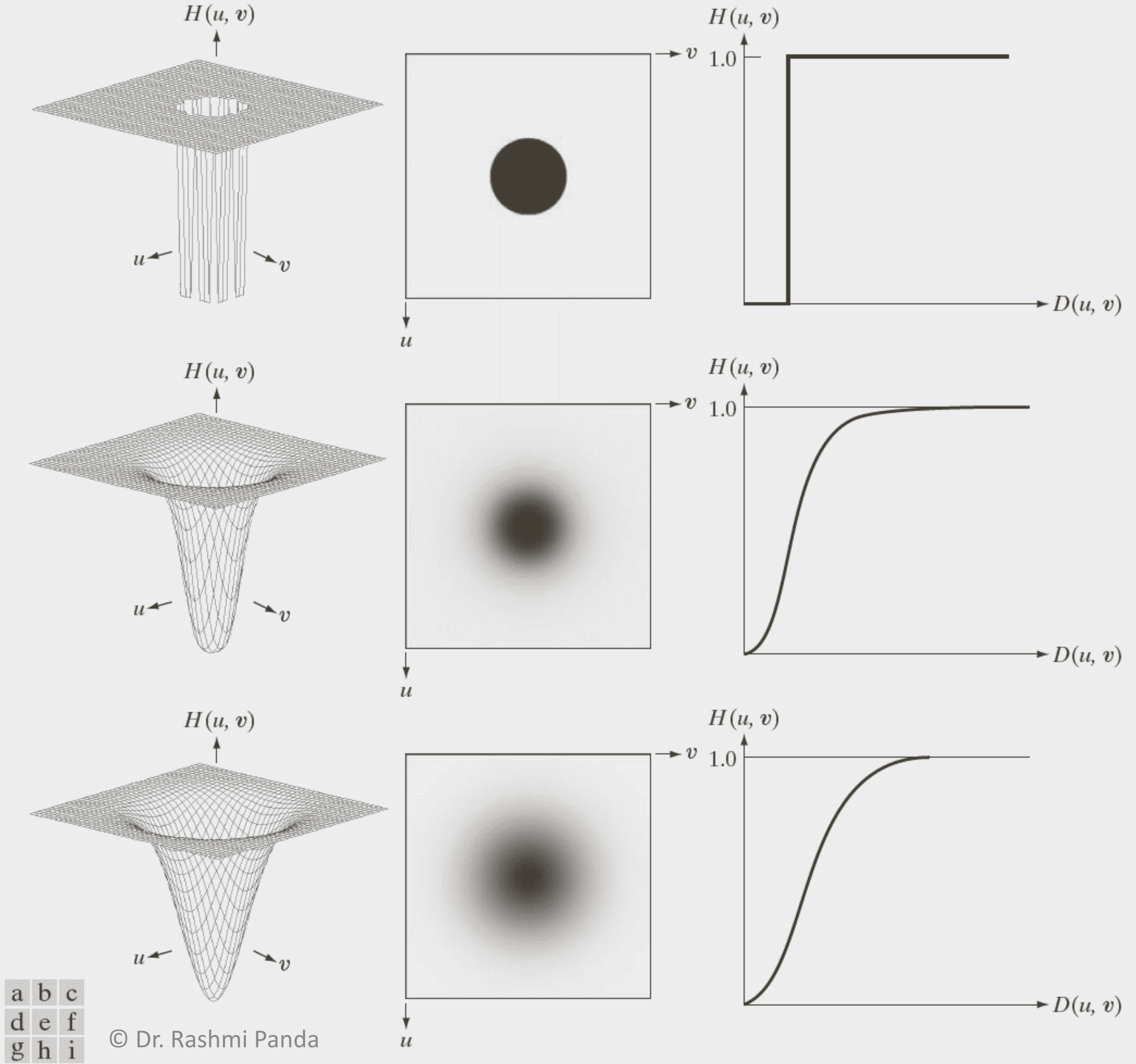


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

The Spatial Representation of Highpass Filters

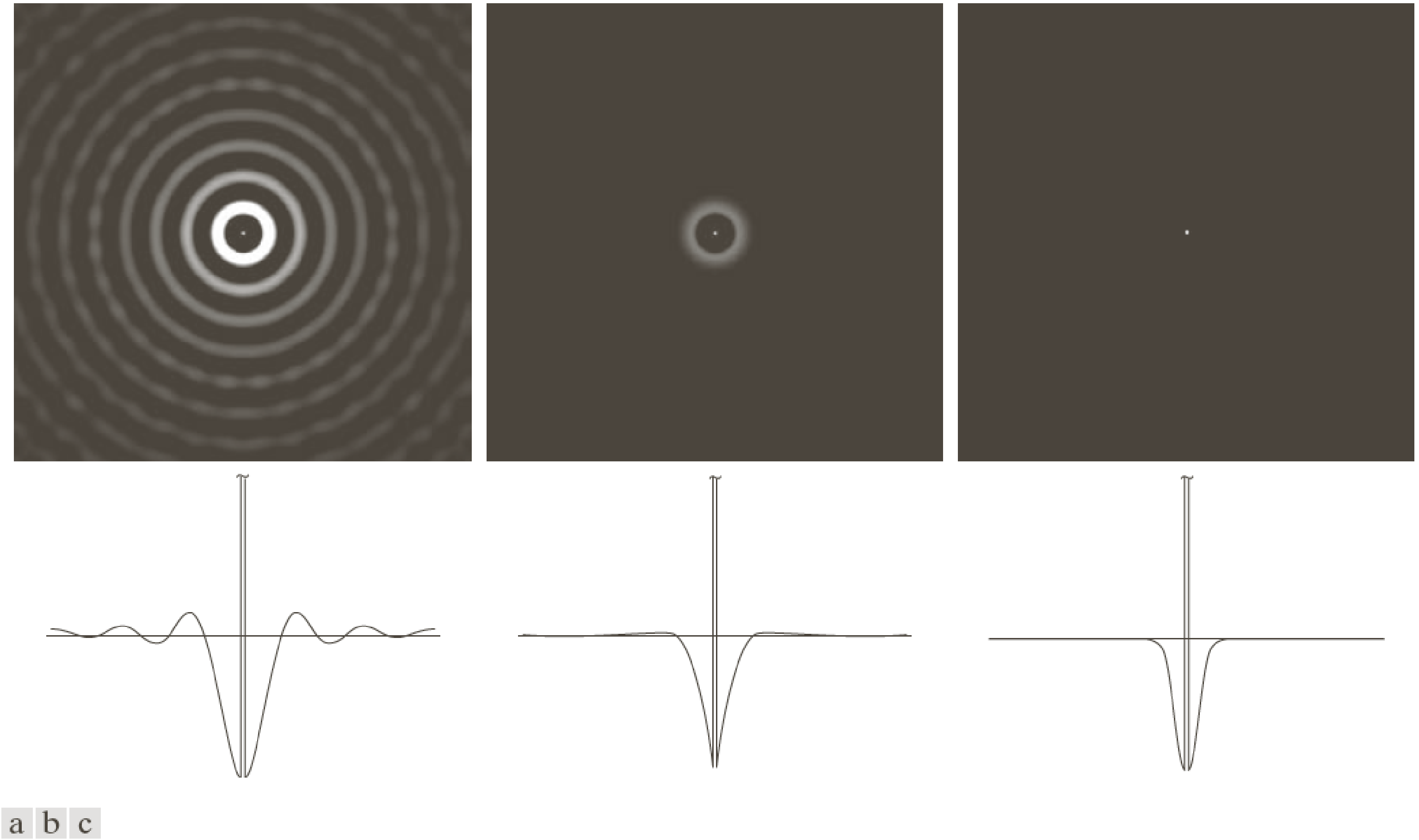


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Filtering Results by IHPF

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

Filtering Results by BHPF

a b c

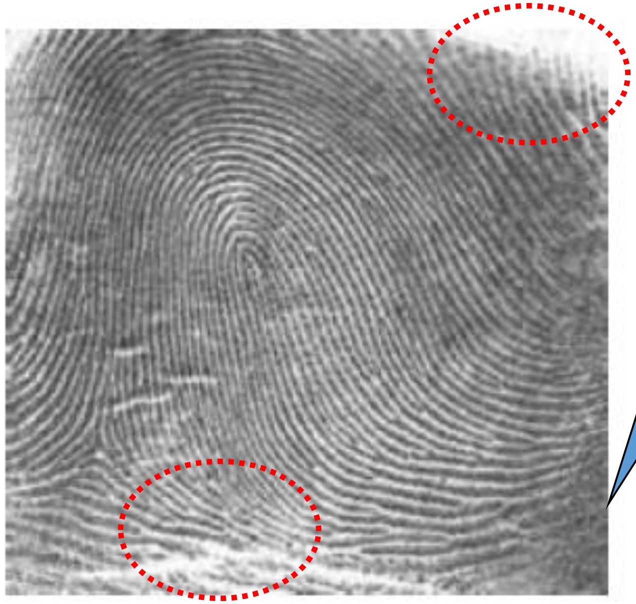
FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Filtering Results by GHPF

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Using Highpass Filtering and Threshold for Image Enhancement



BHPF
(order 4 with a cutoff frequency 50)

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

The Laplacian in the Frequency Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

$$H(u,v) = -4\pi^2\left[(u - P/2)^2 + (v - Q/2)^2)\right]$$

$$= -4\pi^2 D^2(u,v)$$

With respect to center

The Laplacian image

$$\nabla^2 f(x,y) = \Im^{-1}\left\{H(u,v)F(u,v)\right\}$$

Enhancement is obtained

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y) \qquad c = \text{-}1$$

The Laplacian in the Frequency Domain

The enhanced image

$$
\begin{aligned}
g(x,y) &= \mathfrak{I}^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \mathfrak{I}^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \mathfrak{I}^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$

The Laplacian in the Frequency Domain

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Unsharp Masking, Highboost Filtering and High-Frequency-Emphasis Fitering

$$g_{mask}(x, y) = f(x, y) - f_{LP}(x, y)$$

$$f_{LP}(x, y) = \Im^{-1}\left[H_{LP}(u, v)F(u, v)\right]$$

Unsharp masking and highboost filtering

$$g(x, y) = f(x, y) + k * g_{mask}(x, y)$$

$$\begin{aligned} g(x, y) &= \Im^{-1}\left\{\left[1 + k * \left[1 - H_{LP}(u, v)\right]\right] F(u, v)\right\} \\ &= \Im^{-1}\left\{\left[1 + k * H_{HP}(u, v)\right] F(u, v)\right\} \end{aligned}$$

Unsharp Masking, Highboost Filtering and High-Frequency-Emphasis Fitering

$$g(x,y) = \Im^{-1}\left\{\left[k_1 + k_2 * H_{HP}(u,v)\right]F(u,v)\right\}$$

$$k_1 \geq 0 \text{ and } k_2 \geq 0$$

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Gaussian Filter
D_0=40

High-Frequency-Emphasis Filtering
Gaussian Filter
K1=0.5, k2=0.75

a	b
c	d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$$\Im\left[f(x,y)\right] \underset{=}{\neq} \Im\left[i(x,y)\right]\Im\left[r(x,y)\right]$$

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\Im\{z(x,y)\} = \Im\{\ln f(x,y)\} = \Im\{\ln i(x,y)\} + \Im\{\ln r(x,y)\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

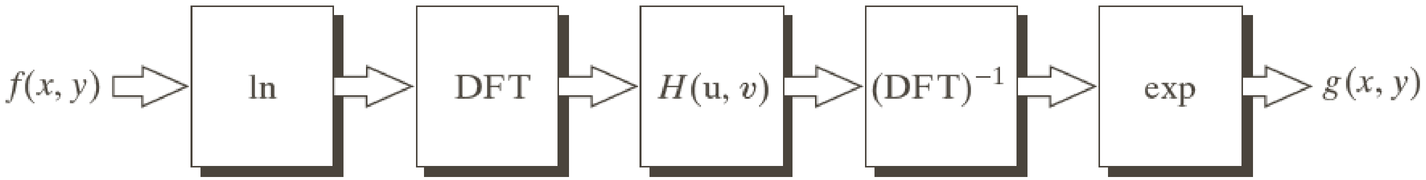


FIGURE 4.60 Summary of steps in homomorphic filtering.

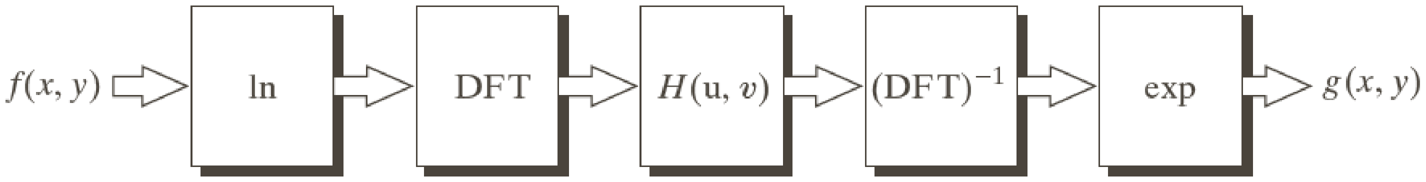
Homomorphic Filtering

$$S(u,v) = H(u,v)Z(u,v)$$
$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

$$s(x,y) = \Im^{-1}\{S(u,v)\}$$
$$= \Im^{-1}\{H(u,v)F_i(u,v) + H(u,v)F_r(u,v)\}$$
$$= \Im^{-1}\{H(u,v)F_i(u,v)\} + \Im^{-1}\{H(u,v)F_r(u,v)\}$$
$$= i'(x,y) + r'(x,y)$$

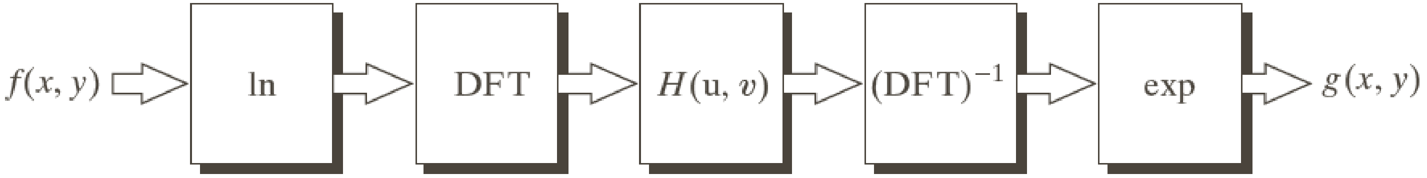
$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}e^{r'(x,y)} = i_0(x,y)r_0(x,y)$$

FIGURE 4.60
Summary of steps in homomorphic filtering.



Homomorphic Filtering

FIGURE 4.60 Summary of steps in homomorphic filtering.



The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly

These characteristics lead to associating the low frequencies of the Fourier transform of the algorithm of an image with illumination the high frequencies with reflectance.

Homomorphic Filtering

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left[D^2(u,v)/D_0^2\right]}\right] + \gamma_L$$

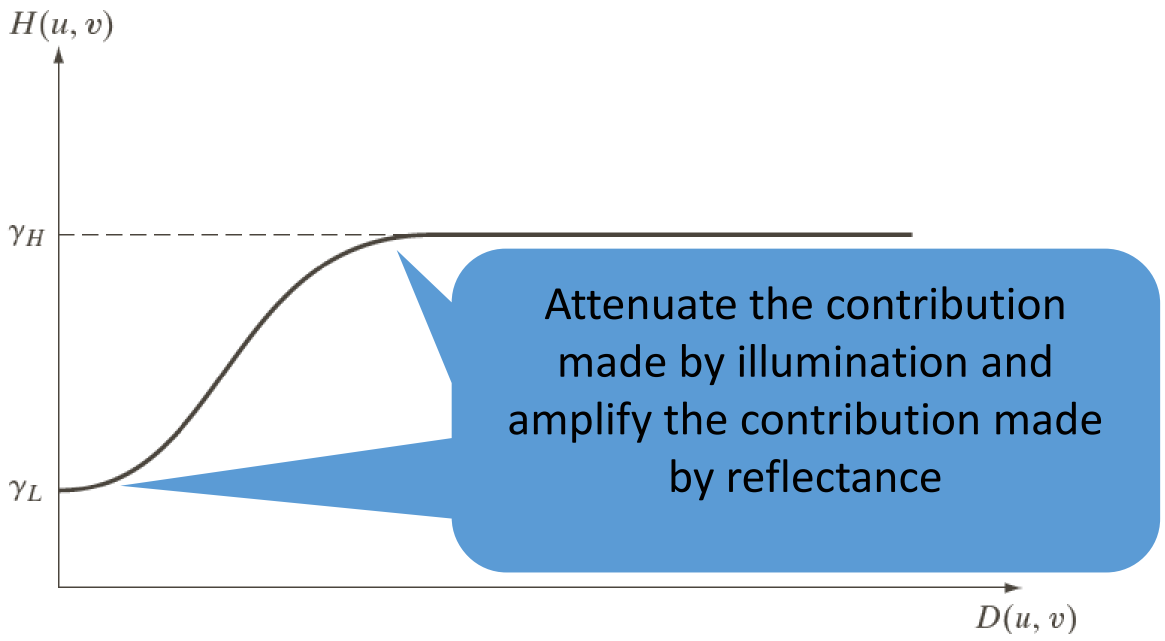


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

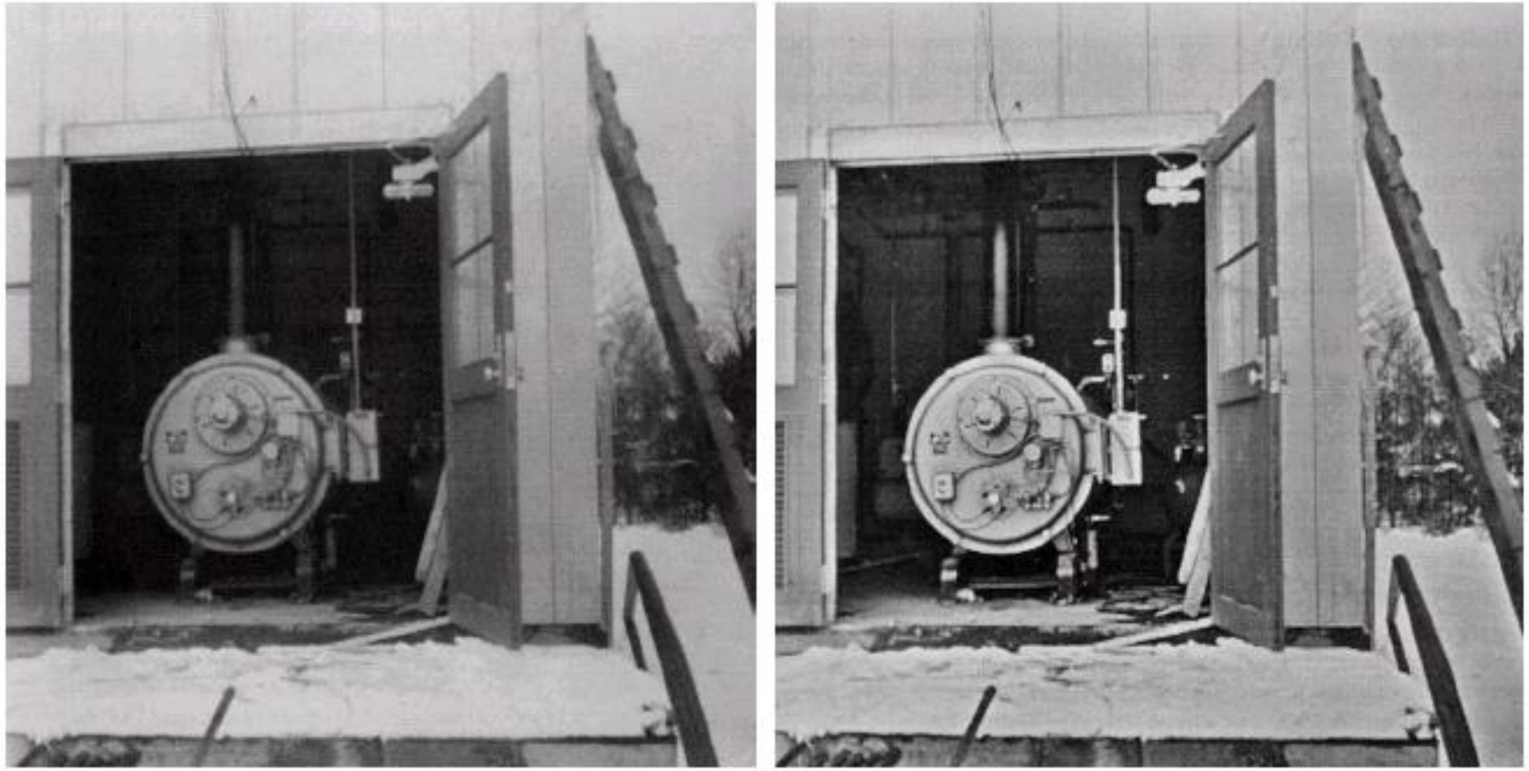
Homomorphic Filtering

$$\gamma_L = 0.25$$
$$\gamma_H = 2$$
$$c = 1$$
$$D_0 = 80$$

a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

Homomorphic Filtering



a b

FIGURE
(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

Selective Filtering

Non-Selective Filters:
operate over the entire frequency rectangle

Selective Filters
operate over some part, not entire frequency rectangle
- **bandreject or bandpass**: process specific bands
- **notch filters**: process small regions of the frequency rectangle

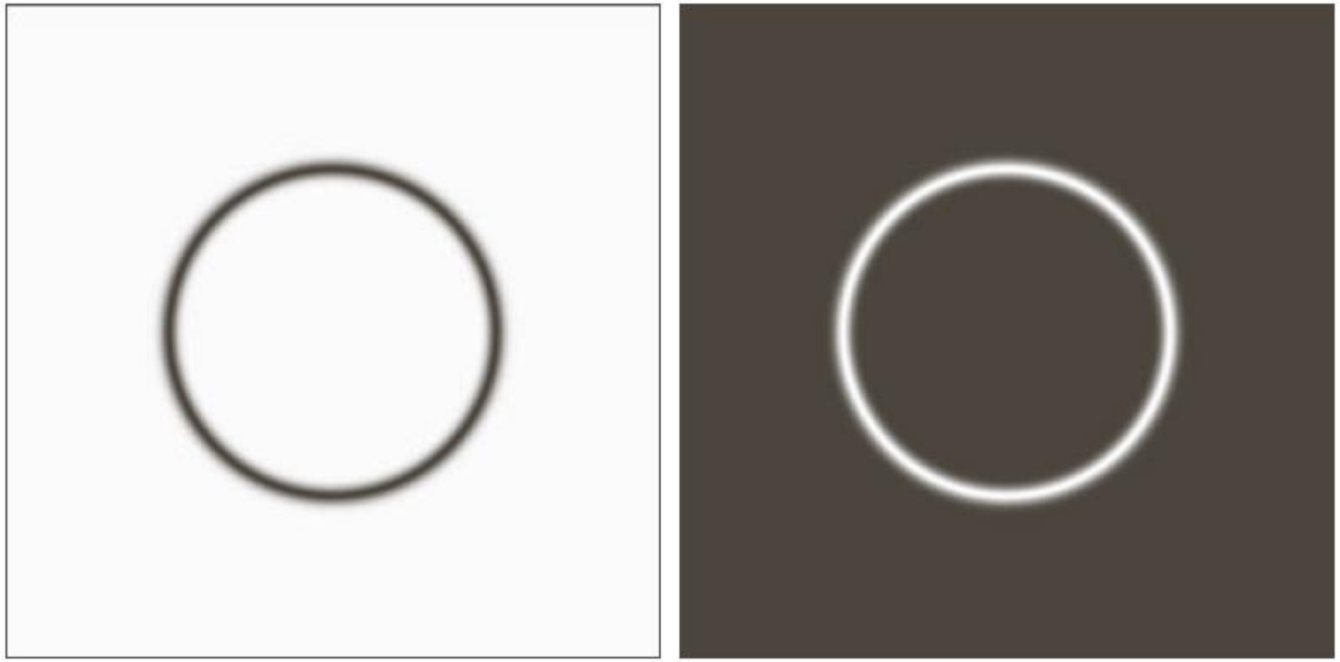
Selective Filtering:
Bandreject and Bandpass Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Selective Filtering:
Bandreject and Bandpass Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Selective Filtering: Notch Filters

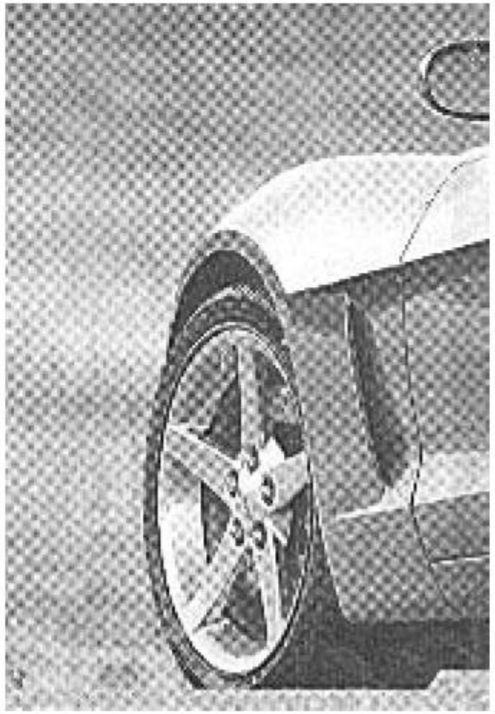
A Butterworth notch reject filter of order n

$$H_{NR}(u,v) = \prod_{k=1}^{3} \left[\frac{1}{1 + \left[D_{0k} / D_k(u,v) \right]^{2n}} \right] \left[\frac{1}{1 + \left[D_{0k} / D_{-k}(u,v) \right]^{2n}} \right]$$

$$D_k(u,v) = \left[(u - M/2 - u_k)^2 + (v - N/2 - v_k)^2 \right]^{1/2}$$

$$D_{-k}(u,v) = \left[(u - M/2 + u_k)^2 + (v - N/2 + v_k)^2 \right]^{1/2}$$

Examples: Notch Filters (1)



a	b
c	d

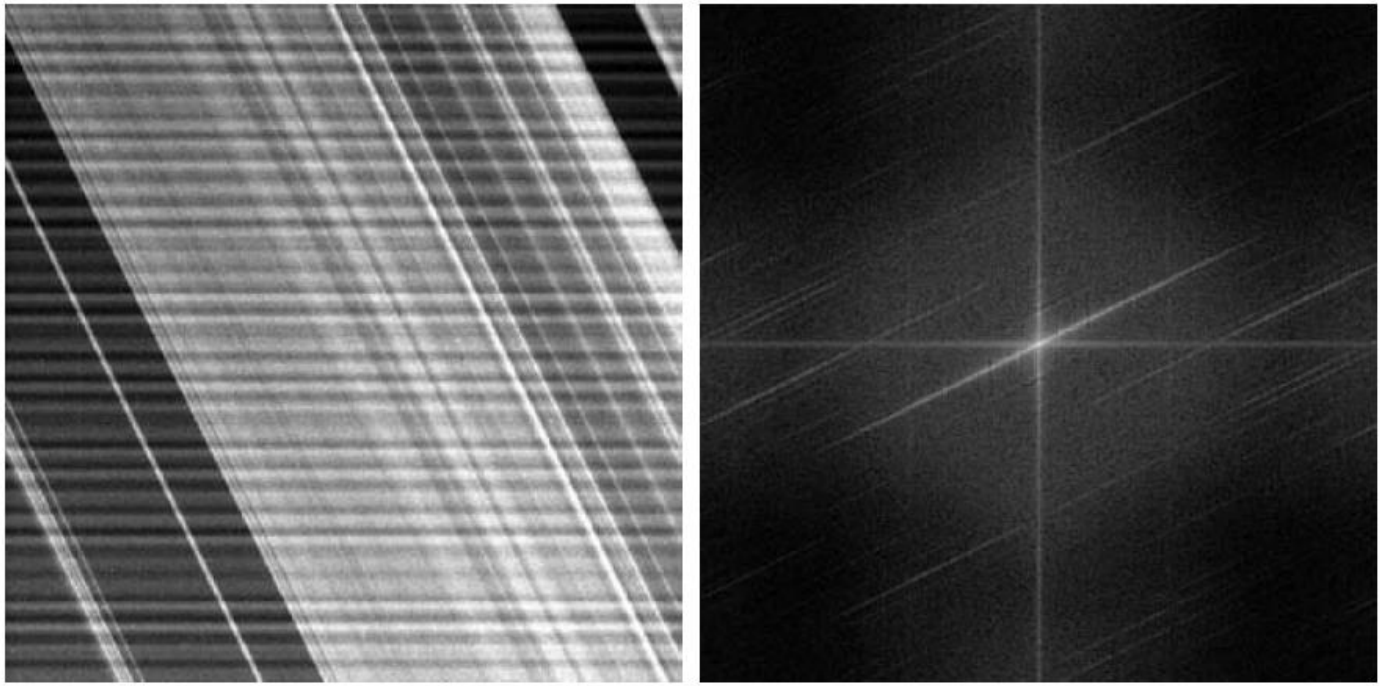
FIGURE 4.64
(a) Sampled newspaper image showing a moiré pattern.
(b) Spectrum.
(c) Butterworth notch reject filter multiplied by the Fourier transform.
(d) Filtered image.

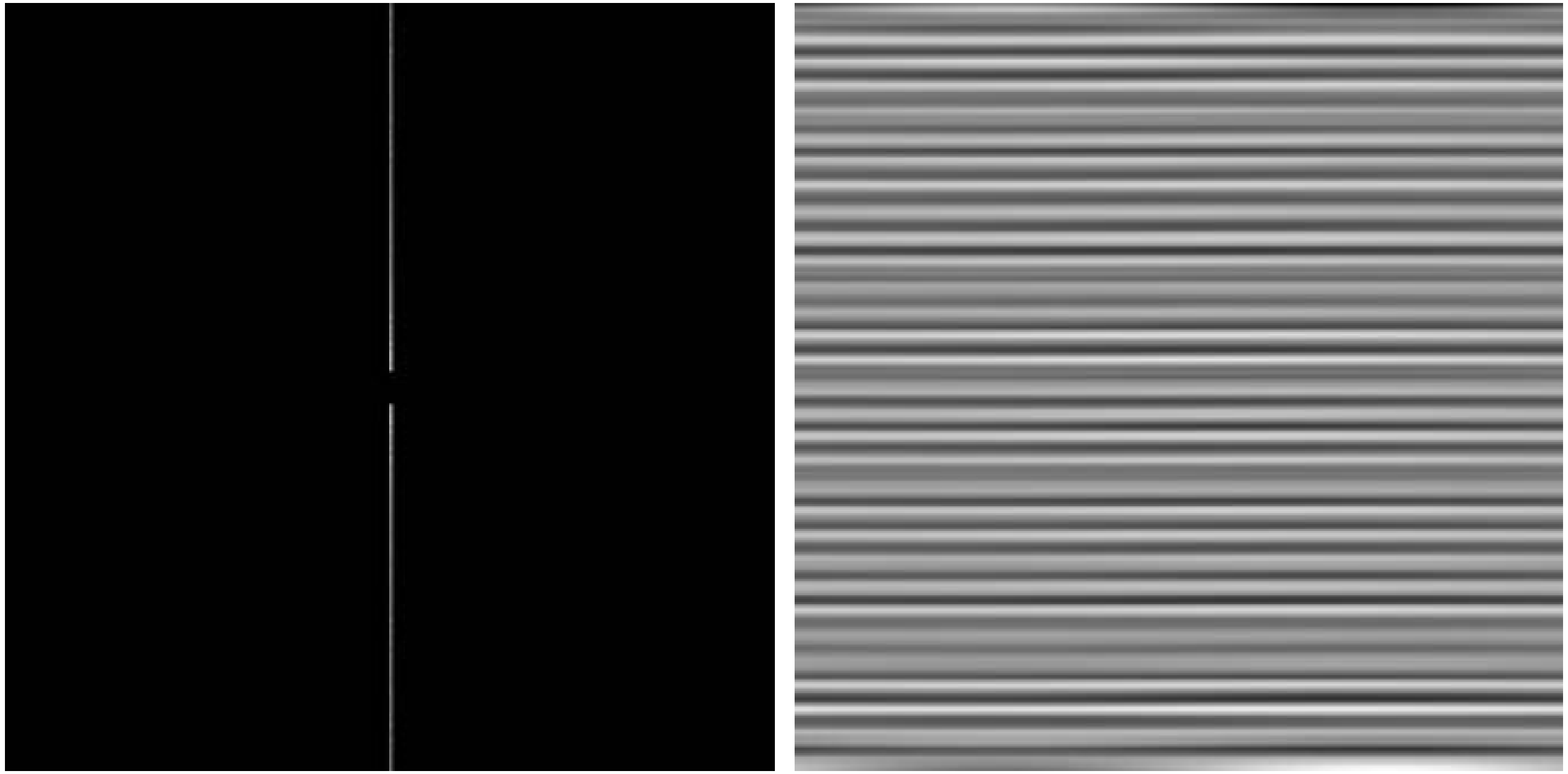
A Butterworth notch reject filter $D_0=3$ and n=4 for all notch pairs

Examples: Notch Filters (2)

a	b
c	d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)





a b

FIGURE 4.66
(a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a).
(b) Spatial pattern obtained by computing the IDFT of (a).

Thank you!