

This question paper contains 8 printed pages.]

Your Roll No.....

No. of Question Paper : 2796

GC-4

Unique Paper Code : 32341401

Name of the Paper : Design and Analysis of Algorithms

Name of the Course : B.Sc. (H) Computer Science

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Question No. 1 of 35 marks is compulsory.

Attempt any **four** questions from Q. No. 2 to Q. No. 7.

(a) Arrange the following functions in the increasing order of their rate of growth : 2^{2^n} , 2^{n^2} , $n^2 \log(n)$, n^{2^n} (2)

(b) Consider the ternary search algorithm for searching an element in a given array: divide the array into three equal parts by taking two mid points, viz., left mid and right mid, If the search element is equal to the left mid, output left mid; if it is equal to the right mid, output the

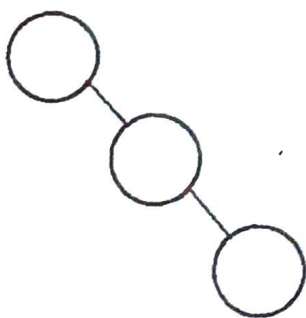
P.T.O.

right mid; if it is smaller than the left mid, perform recursive search in the leftmost partition; if it is greater than the right mid, perform a recursive search in the rightmost partition; else perform a recursive search in the middle partition. Thus, the algorithm performs comparisons in each iteration and recurses on one-third of the array. Write the recurrence relation for the running time of the algorithm and solve it.

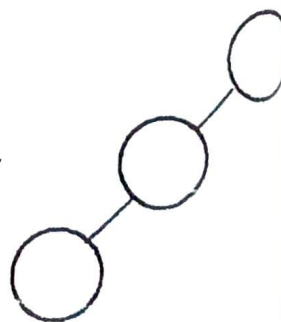
- (c) Consider an instance of the subset sum problem where bound $W = 6$, Items with weights $w_1 = 2$, $w_2 = 2$, $w_3 = 2$.

With the help of the above example argue that the memorized recursive algorithm solves lesser number of subproblems than the corresponding iterative algorithm.

- (d) Consider a linear chain of 3 nodes shown below :



or



Argue that such a chain cannot form a red-black tree. For each possible coloring of the three nodes, mention the property that is violated.

- (e) A shopkeeper has W marbles and n empty bottles. Let c_1, c_2, \dots, c_n respectively denote the number of marbles the bottles can contain. The shopkeeper wants to store the marbles in the bottles.
- (i) Describe a greedy algorithm which minimizes the number of bottles used.
 - (ii) How would you modify your algorithm if bottle i also has an associated cost price p_i and the goal is to minimize the total cost of the bottles used. (3+4)
- (f) Suppose an input to the bucket sort algorithm is not uniformly distributed. Then: (i) will the sort still give correct output? (ii) what will be the impact of relaxing this condition on the running time? Justify. (3)
- (g) Discuss the run time complexity of the naive string matching algorithm. (2)
- (h) Compare the space requirements of adjacency list and adjacency matrix representations of a graph having m edges and n vertices. (3)
- (i) Give an efficient algorithm to find the minimum element in a max-heap. Give the exact running time of the algorithm. (3)

- (j) Would you use BFS to find the shortest path between two nodes in a weighted graph with arbitrary edge weights? Justify your answer. (3)
2. (a) Give an efficient algorithm to check if a given undirected graph has a cycle. Discuss the time complexity of your algorithm. (5)
- (b) For each of the following operations does a Red Black Tree work faster than a Binary Search Tree? Elaborate your answer.
- (i) Search
- (ii) Postorder traversal (5)
3. (a) A priority queue is implemented in two different ways using a max heap and an array sorted in decreasing order of key values (higher key value indicates higher priority). Compare the time complexity of the following operations when performed on the two different implementations of the priority queue.
- (i) Finding the maximum element
- (ii) Deleting the maximum element
- (iii) Increase the priority of a certain element (6)

- (b) Suppose a graph G has two edge-disjoint spanning trees (two spanning trees that have no edges in common). Argue that in this graph G , every pair of nodes forms part of a cycle. (4)

- (a) Consider the following recursive algorithm to find an optimal schedule for weighted interval scheduling problem :

Compute_opt(j)

If $j = 0$ then

Return 0

Else

Return $\max(v_j + \text{Compute_opt}(p(j)), \text{Compute_opt}(j-1))$

- (i) Explain why does this algorithm take exponential time to run in the worst case.

- (ii) What changes should be made to the above algorithm to make it run in polynomial time. (6)

- (b) Suppose that an $n \times n$ array A consists of 1's and 0's such that in any row of A all the 1's come before any 0's in that row. Give an $O(n \lg n)$ algorithm for counting the number of 1's in A . (4)

P.T.O.

5. (a) Give an example graph having four nodes that does not have a topological ordering. (3)
- (b) Suppose a large array is maintained with the following policy: the list is initially sorted. When new elements are added, they are inserted at the end of the array and counted. Whenever the number of new elements reaches 10, the array is resorted and the counter is cleared. What strategy would be good to use for the resorting of the array? Why? (4)
- (c) We use Randomized-Select to select the minimum element of the array $A = \langle 3, 2, 9, 0, 7 \rangle$. Describe a sequence of partitions that would result in the worst case performance of the algorithm. (3)
6. (a) Which red-black tree properties may be violated when a node is deleted? (2)
- (b) Will Dijkstra's algorithm still give shortest path between two vertices if the edge weights are allowed to be negative. If yes, justify your answer with an argument. If no give an example. (4)
- (c) An instructor has given a test to her class of n students. The maximum marks for the test is 100. The instructor

decided not to give fractional marks while grading the test. The instructor wishes to sort the n integer scores in descending order. Design a linear time algorithm to perform this task. Also list all assumptions (if any) that you make to solve the problem. (4)

(a) Consider a k bit binary counter implemented using an array A such that $A[0]$ stores the lowest order bit and $A[k-1]$ stores the highest order bit. The only operation that can be performed on the counter is 'increment'. Using the aggregate method of analysis, determine the amortised cost per increment operation when a sequence of n increments is performed. (3)

(b) A certain input to the merge sort algorithm is such that the merging step always depicts the worst case behaviour. Determine the running time of the merge sort algorithm for this instance. (3)

(c) Consider the following algorithm that takes as input an array of n integers and an integer T . It finds whether there exist two elements in the array that sum up to T and returns 1 on success and 0 on failure. (4)

TargetSum (A, n, T)

 Heapsort (A, 1, n)

 for i = 1 to n

 flag = BinarySearch(A, i+1, n, |T-A[i]|)

 if (flag)

 return 1

 endif

 endfor

 return 0

TargetSum uses the following algorithms:

Heapsort (Array, First, Last)

BinarySearch(Array, First, last, element)

Analyze the worst case running time of TargetSum.