[This question paper contains 12 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1235 **F**

Unique Paper Code : 2342011203

Name of the Paper : Probability for Computing

Name of the Course : **B.Sc. (H) Computer Science**

Semester / Type : II / DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **First** question is compulsory and attempt any **four** questions from remaining.

3. Part of the questions to be attempted together.

4. Attempt all questions from **Section A.**

5. Attempt any four questions from **Section B.**

6. Use of non-programmable scientific calculator is allowed.

SECTION A

1. (a) State the Central Limit Theorem. (2)

(b) A box contains six red, four orange, and two blue balls. Two balls are randomly selected. What is the sample space of this experiment? Let X represent the number of orange balls selected. What are the possible values of X? Calculate $P\{X = 0\}$. (5)

(c) (i) For any two random variables X and Y, prove that :

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

(ii) Calculate the expected sum obtained when three fair dice are rolled. (4+3)

(d) Coming home from work, Neha always encounters traffic signal. The probability that she makes it through a traffic signal is 0.2. How many traffic signals can she expect to hit before making it through one? What is the probability of the third traffic light being the first one that is green? (5)

(e) Assume that each child who is born is equally likely to be a boy or a girl. If a family has two children, what is the probability that both are girls given that :

(i) the eldest is a girl?

(ii) at least one is a girl? (2+2)

(f) (i) When are two states of a Markov chain said to communicate with each other?

(ii) For the given transition probability matrix of a four-state Markov chain with states 0,1,2, and 3, answer the following :

$$P=\begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

(a) Which state is an absorbing state?

(b) Do states 0 and 2 communicate?

(c) Do states 0 and 1 communicate? (2+3)

(g) Name and define one technique to generate pseudorandom numbers. (2)

SECTION B

4. (a) Suppose the joint density of two random variables X and Y is given by :

$$f(x,y) = \begin{cases} 6xy(2 - x - y), & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Compute the conditional expectation $E[X | Y = y]$, where $0 < y < 1$. (7)

(b) Calculate $E[X]$ for a Poisson distribution with parameter λ. (3)

(c) Consider two bags. The first contains two white and seven black balls, and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first bag or the second bag depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected? (5)

3. (a) Prove that for all discrete random variables X and Y: $E[X] = E[E[X|Y]]$. (5)

(b) Suppose that we toss two coins. What is the sample space for this experiment? What is the probability that either the first or the second coin falls heads? (3)

(c) Derive the expectation of a uniform random variable with interval [a,b]. (3)

(d) A manufacturer produces medicine bottles out of which 0.1% are defective. Bottles are contained in a box containing 500 bottles. A drug company buys 100 boxes. Using Poisson distribution, find out how many boxes will contain no defective bottles. (4)

(a) Explain the n-step transition probabilities of a Markov chain using Chapman Kolmogorov equations. (6)

(b) A company pays dividends on a monthly basis when it is earning profits, and suspends the dividend payments in unprofitable times. Suppose that after a dividend has been paid in the current month, the dividend is paid in the next month with probability 0.9, while after a dividend is suspended the next one will be suspended with probability 0.6.

(i) What is the one-step transitional probability matrix for the above problem?

(ii) What will be the probability that dividend is paid in March 2023, given dividend is suspended in January 2023? (4+5)

(a) Let X denote the number of hours you spend in lab doing programming during a randomly selected college day. The probability that X can take on x values has the following form, where k is some unknown constant :

$$P(X=x) = \begin{cases} 0.1, & if\ x = 0 \\ kx, & if\ x = 1\ or\ x = 2 \\ k(5-x), & if\ x = 3\ or\ x = 4 \\ 0, & otherwise \end{cases}$$

(i) Find value of k.

(ii) What is the probability that you spend time on programming in lab for at least 3 hours? (3+2)

(b) Let c be a constant. For a continuous random variable X, show the following :

(i) $Var(cX) = c^2 Var(X)$

(ii) $Var(c + X) = Var(X)$ (3+3)

(c) Ram and Shyam go target shooting together. Both shoot at a target at the same time. Suppose Ram hits the target with probability 0.7, whereas

Shyam, independently, hits the target with probability 0.4.

(i) Given that exactly one shot hit the target, what is the probability that it was Shyam's shot?

(ii) Given that the target is hit, what is the probability that Shyam hit it? (2+2)

6. (a) Consider the Markov chain consisting of the three states 0, 1, 2 and having transition probability matrix P given as :

$$P=\begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{Vmatrix}$$

List the classes of the above Markov chain. Verify whether this Markov chain is irreducible or not. (4)

(b) Suppose that 5 percent of men and 0.25 percent of women are colour-blind. A colour-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. (4)

(c) Consider two random variables X and Y. Their joint probability mass function is defined as :

$$p(-1,1)=1/3$$

$$p(0,1)=1/3$$

$$p(1,1)=1/3$$

(i) Compute the conditional probability mass function of X given that Y = 1 i.e., $p_{X|Y}(x|1)$.

(ii) Compute probability mass function of X i.e., $p_X(x)$. (4+3)

7. (a) State and prove Chebyshev's Inequality. (5)

(b) A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safely? (5)

(c) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Transform this process into a Markov chain. How many states will it have after transformation? Provide the transition probability matrix for the same. (5)