

# Local Search and Optimization Problems

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### Outline

- Hill Climbing
- Simulated Annealing
- · Local Beam Search
- · Genetic Algorithms

### Reference

 Artificial Intelligence - A Modern Approach – Chapter 4 – Section 1

# Search Types

- · Backtracking state-space search
- Local Search and Optimization
- Constraint satisfaction search
- · Adversarial search

### Local Search and Optimization

- Previous searches: keep paths in memory and remember alternatives so search can backtrack. The solution is a path to a goal.
- · Path may be irrelevant if only the final configuration is needed
  - 8-queens
  - IC design
  - · Network optimization
  - Factory floor layout
  - · Job shop scheduling
  - · Automatic programming
  - · Cop planning etc.

### Local Search

- Use a single current state and move only to neighbours.
  - · No track of the paths
  - No track of the set of states that have been reached
- Not systematic 

   Might never explore a portion of the search space where the solution exists.
- · Advantages:
  - · Use little space
  - Can find reasonable solutions in large or infinite (continuous) state spaces for which the other algorithms are unsuitable.

# Optimization

- Local search is often suitable for optimization problems. Search for the best state by optimizing an objective function.
- F(x) where often x is a vector of continuous or discrete values.
- Begin with a complete configuration.
- · A successor of state S is S with a single element changed.
- Move from the current state to a successor state.
- Low memory requirements, because the search tree or graph is not maintained in memory (paths are not saved).

# Examples – 8 Queens

- Find an arrangement of 8 queens on a chess board such that no two queens are attacking each other (A queen attacks any piece in the same row, column, or diagonal.).
- · Start with some arrangement of the queens, one per column.
- X[j]: row of queen in column j
- Successors of a state: move one queen
- F(x): # pairs attacking each other





# Examples - Traveling Salesman Problem

- · Visit each city exactly once.
- · Start with some ordering of the cities.
- State representation order of the cities visited.
- Successor state: a change to the current ordering
- F(x): length of the route

### What is Local Search?

ocal Search algorithms operate on a single current state (or a small population of states) and iteratively nove to a neighboring state that is "better." They do not retain a search tree or remember paths. They have two leve advantages:

- Very Low Memory Use: They only need to keep track of the current state and its immediate neighbors, not the entire history of the search.
- Suitability for Large Problems: They can often find good solutions in large or infinite state space where systematic algorithms would fail due to memory constraints.
- The goal of local search is often optimization—finding the best possible state according to an object function.

### The Core Idea: Optimization

- ocal search is often suitable for optimization problems."
- specific measure, not just "a" valid solution.
- "Search for the best state by optimizing an objective function."
- The guide for the search is not a goal test ("is this the end?") but an objective function late call
  a cost function, loss function, or fitness function). This function assigns a value to every possible
  state, quantifying how "good" it is.

### 2. The Objective Function: F(x

- + F(x) where often x is a vector of continuous or discrete value
- This is a mathematical way to describe a solution or a configuration
- x is the state or solution candidate. It's a vector, meaning it's a list of parameters or decis that fully define the configuration.

### Examples:

- 8 Queens: x could be a vector of 8 numbers, where each number represents the row post
  of the queen in its column: x + [3, 1, 4, 2, 6, 5, 7, 6]. This is a vector of discrete
  values (integers from 1 to 8).
- Factory Layout: \*\* could be a list of all machine IDs, representing their assigned locations
  the factory floor.
- bettery\_capacity, body\_weight]. F(x) would then calculate the resulting fight to

### 1. Tegin with a complete configuration

- Unlike pathfinding which starts from a single start state and builds a solution, local search start with a full, but probably bad, solution.
- Example: For or queers, we contribute with an empty council, we start with all or queers are easy of the board, placed randomly. It will have many conflicts, but it's a complete configuration.
   A successor of state S is S with a simple element changed.
- This defines what a "neighbor" is. It's a new state generated by making a small, increment, change to the current state.
- x) and moving it to a different row in its same column. All other queens
- The algorithm evaluates the neighbors using the objective function: F(x) and selects one to become the new current state.
- In simple Hill-Climbing, it moves to the neighbor with the best value of F(x) (e.g., the low cost or highest fitness).

### Comparison to Tree Search Framework

- Chapter 3: start state is typically not a complete configuration. Chapter 4: all states are
- · Chapter 3: binary goal test;

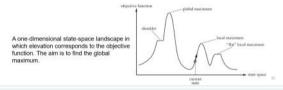
Chapter 4: no binary goal test, unless one can define one in terms of the objective function for the problem (e.g., no attacking pairs in 8-queens).

- · Heuristic function: an estimate of the distance to the nearest goal
- Objective function: preference/quality measure how good is this state?
- · Chapter 3: saving paths

Chapter 4: start with a complete configuration and make modifications to improve it

### Visualization

- · States are laid out in a landscape.
- Elevation corresponds to the objective function value.
- · Move around the landscape to find the highest (or lowest) peak.
- · Only keep track of the current states and immediate neighbors.



### Local Search Algorithms

- · Strategies for choosing the state to visit next
  - Hill climbing
  - · Simulated annealing
- Then, an extension to multiple current states:
  - · Genetic algorithms

# Hill Climbing (Greedy Local Search)

- · Generate nearby successor states to the current state.
- Pick the best and replace the current state with that one.
- Loop

function HILL-CLIMBING(problem) returns a state that is a local maximum

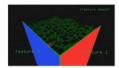
 $current \leftarrow \texttt{MAKE-NODE}(problem.\texttt{INITIAL-STATE})$ 

loop do

 $neighbor \leftarrow$  a highest-valued successor of current if neighbor. Value  $\leq$  current, Value then return current. State  $current \leftarrow neighbor$ 

### Hill Climbing (Greedy Local Search)

- · Generate nearby successor states to the current state.
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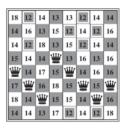


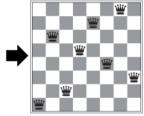
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### Hill Climbing Search Problems





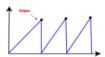
## Hill Climbing Search Problems

- · Here, we assume maximization rather than minimization.
- Local maximum: A local maximum is a peak that is higher than each of its neighboring states but lower than the global maximum.



### Hill Climbing Search Problems

- Ridges: It is a region that is higher than its neighbors but itself has a slope.
  - · It is a special kind of local maximum.
  - Results in a sequence of local maxima that is very difficult for greedy algorithms to navigate.
- Plateau: the evaluation function is flat, resulting in a random walk.





### Variants of Hill Climbing

### · Stochastic hill climbing

- Chooses at random from among the uphill moves; the probability of selection can vary with the steepness of the uphill move.
- This usually converges more slowly than steepest ascent, but in some state landscapes, it finds better solutions.

### · First-choice hill climbing

- Implements stochastic hill climbing by generating successors randomly until one is generated that is better than the current state.
- This is a good strategy when a state has many (e.g., thousands) of successors.

# Variants of Hill Climbing

### · Random-restart hill climbing

- Start different hill-climbing searches from random starting positions stopping when a goal is found.
- · Save the best result from any search so far.
- If all states have an equal probability of being generated, it is complete with a probability approaching 1 (a goal state will eventually be generated).
- Finding an optimal solution becomes the question of a sufficient number of restarts.
- · Surprisingly effective, if there aren't too many local maxima or plateau.

### 1. Stochastic Hill Climbin

Core idea: Don't always take the obsolute best move. Instead, choose randomly from all moves that improve the current state, with a bias towards better moves.

reprove the current state, with a list towards better moves.

ow it works: Instead of evaluating every single neighbor and pick creats. Stockwate: Hill Climbina:

identifies all neighbors that are an improvement (all "uptil instruer").
Assigns each of these good moves a probability. The steeper the uptill move (i.e., the bigger the improvement), the bigher its probability of being chosen.

2. First-Choice Hill Climbing

Core Idea: A specific, efficient way to implement Stochastic HRI Climbing when there are too many successors to evaluate fully.

How it works: This strategy is for when a state has a massive number of neighbors (e.g., thousands millions). Evaluating all of them to find the best one (steepest ascent) or even to build a probability distribution (stochastic) would be too computationally expensive.

I. Instead of generating all successors, it generates them one at a time, randomly

The moment it finds a successor that is better than the current state, it immediately moves to:
 It doesn't bother to check if there's an even better move available.

### Random-Restart Hill Climbing is a simple but powerful meta-algorithm

 Run a standard hill-dimbing search (e.g., steepest ascerd) from a randomly generated initial at 2. Stop the search when it converges to a solution (which could be a local maximum or the global maximum).

4. Repeat steps 1-3 many times, each time from a new, random starting po

### Simulated Annealing

- A hill-climbing algorithm that never makes "downhill" moves is always vulnerable to getting stuck in a local maximum.
- A purely random walk that moves to a successor state without concern for the value will eventually discover the global maximum but will be extremely inefficient.
- Combine the ideas: add some randomness to hill-climbing to allow the possibility of escape from a local optimum.
- Simulated annealing wanders around during the early parts of the search, hopefully toward a good general region of the state space
- Toward the end, the algorithm does a more focused search, making a few bad moves.

### Simulated Annealing

- · Based on a metallurgical metaphor.
- · Annealing: harden metals and glass by heating them to a high temperature and then gradually cooling them.
- · Start with a temperature set very high and slowly reduce it.
- · Run hill-climbing with the twist that you can occasionally replace the current state with a worse state based on the current temperature and how much worse the new state is.
- · At the start, make lots of moves and then gradually slow down

# Simulated Annealing

- · Generate a random new neighbor from the current state.
- · If it's better take it.
- If it's worse, then take it with some probability proportional to the temperature and the delta between the new and old states.



https://upload.wikimedia.org/wikipedia/commons/d/d5/Hill\_Climbing\_with\_Simulated\_Annealing.glf

# Simulated Annealing

function Simulated-Annealing(start, schedule) current ← start for  $t \leftarrow 1$  to  $\infty$  do  $T \leftarrow schedule[t]$ if T=0 then return current  $\texttt{next} \, \leftarrow \, \texttt{a randomly selected successor of the current}$ ΔE ← Value[next] - Value[current] if  $\Delta E > 0$  then current  $\leftarrow$  next **else** current  $\leftarrow$  next only with probability  $e^{\Delta E/T}$ 

Simulated Annealing

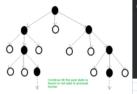
- $\bullet$  Probability of a move decreases with the amount  $\Delta E$  by which the evaluation is worsened.
- · A second parameter T is also used to determine the probability: high T allows worse moves, T close to zero results in few or no
- · Schedule input determines the value of T as a function of the completed cycles.



### Local Beam Search

something in between BFS and best first search

- Keep track of k states rather than just one, as in hill climbing
- Begins with k randomly generated states
- At each step, all successors of all k states are generated
- · If anyone is a goal, algorithm halts
- Otherwise, selects the best k successors from the complete list, and repeats



### Local Beam Search

- · Successors can become concentrated in a small part of state space
- · Stochastic beam search: choose k successors with the probability proportional to the successor's value.
  - Increase diversity

## Genetic Algorithms

- · Local beam search, but...
- A successor state is generated by combining two parent states.
- · Start with k randomly generated states (population).
- · A state is represented as a string over a finite alphabet (often a string of 0s and 1s).
- Evaluation function (fitness function). Higher = better.
- · Produce the next generation of states by selection, crossover,

# Genetic Algorithms Specific Mechanics

- · Representation of individuals
  - · Classic approach: an individual is a string over a finite alphabet, with each element in the string called a gene
  - · Usually, binary instead of AGTC as in real DNA
- Mixing number
- Selection strategy
  - Random
  - · Selection probability proportional to fitness
  - Selection is made with replacement to make a very fit individual reproduce several times

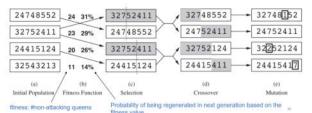
# Genetic Algorithms

- · Recombination procedure
  - Random pairing of selected individuals
  - · Random selection of cross-over points
- Mutation rate How often offspring have random mutations to their representation.
- · The makeup of next generation
  - · Forms with the newly generated offspring
  - Elitism Include a few top-scoring parents from the previous generation in addition to newly formed offspring.
  - Culling Discard individuals below a given threshold.

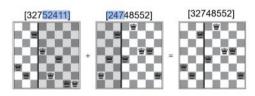
### Example - N-Queens

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• Put n gueens on an n × n board with no two queens on the same row, column, or diagonal

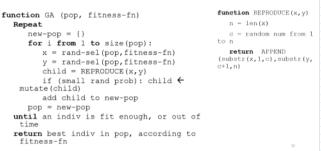


### Example - N-Queens



Has the effect of "jumping" to a completely different new part of the search space (quite non-local)

### Genetic Algorithms



# Comments on Genetic Algorithms

- Genetic Algorithm is a variant of "stochastic beam search".
- Positive points
  - · Random exploration can find solutions that local search can't (via crossover primarily).
  - · Appealing connection to human evolution.
- Negative points
  - Many "tunable" parameters
  - · Difficult to replicate performance from one problem to another
  - Lack of good empirical studies comparing to simpler methods
  - Useful on some (small?) problems, but no convincing evidence that GAs are generally better than hill-climbing w/random restarts.

domly changed with a small probability. This introduces new traits, sing:
set  $S_{ij} = S_{ij} = S_{ij} = S_{ij}$ so  $S_{ij} = S_{ij} = S_{ij}$ 

d mutation creates a new generation of individuals (e.g., 32768552

be fitter than the previous one because it was built from the best cycle repeats until it finds an individual with a perfect fitness score (a

example of such a solution that the algorithm could eventual