Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

The fact that both periodic dips in the host star's brightness and accompanying radial velocity shifts have been observed for GJ 8999 b indicates that the planet's orbit is aligned nearly edge-on relative to our line of sight. Therefore, the orbital inclination is close to 90 degrees.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

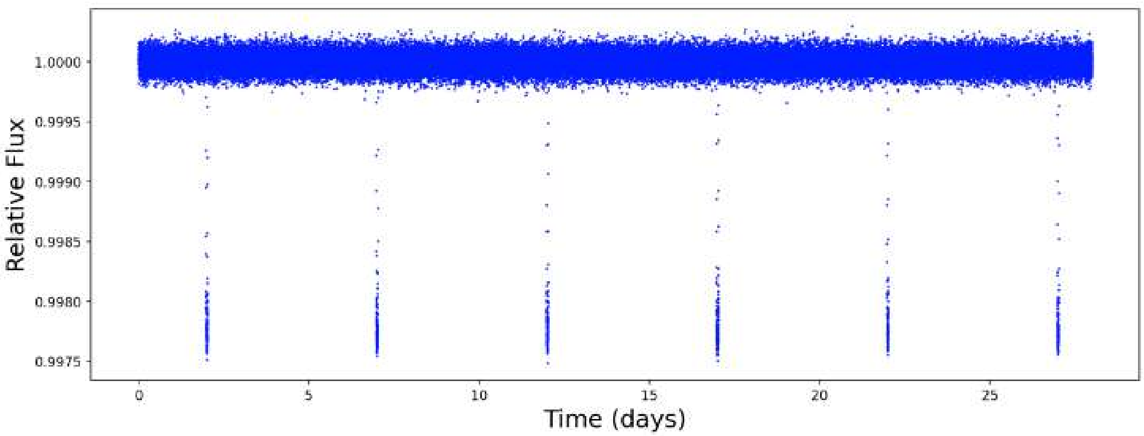

Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

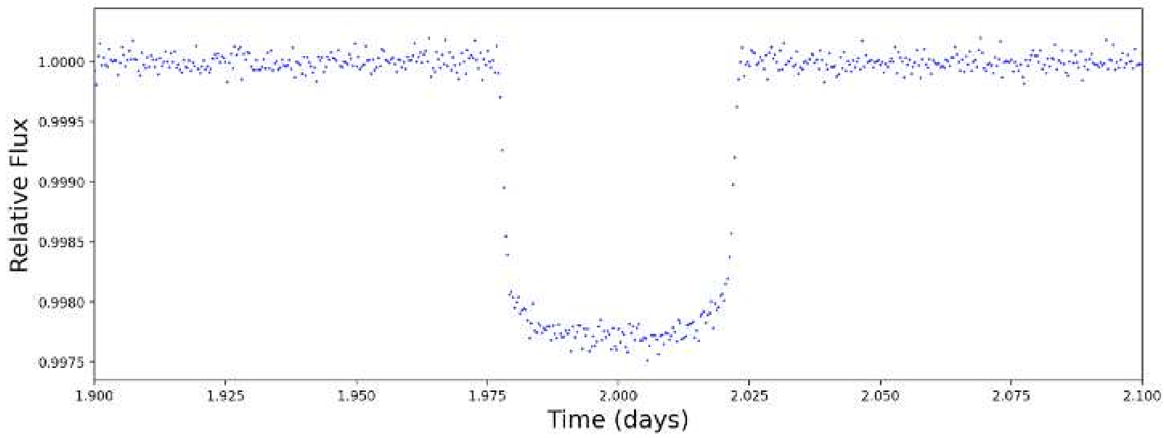

Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

Analysis of the stellar brightness over a 28-day window reveals six evenly spaced transits, indicating that each transit repeats approximately every five days. Thus, the exoplanet completes an orbit in roughly 5 days.

c) What is the radius of this planet?

The transit depth tells us how much of the star's light is blocked when the planet passes in front. This is given by:

$$\Delta F = \left(\frac{R_p}{R_*}\right)^2$$

From the light curve, the star's brightness drops from 1.0000 to 0.9975 during transit. So,

$$\Delta F = 1.0000 - 0.9975 = 0.0025$$

This means:

$$\frac{R_p}{R_*} = \sqrt{0.0025} = 0.05$$

So, the planet is about 5% the size of its star.

Given the star's radius is 0.2 times the radius of our Sun:

$$R_p = 0.05 \times 0.2R_\odot = 0.01R_\odot$$

And since the Sun is about 109 times wider than the Earth:

$$R_p = 0.01 \times 109R_\oplus \approx 1.09R_\oplus$$

So, the planet GJ 8999 b is **almost the same size as Earth**.

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

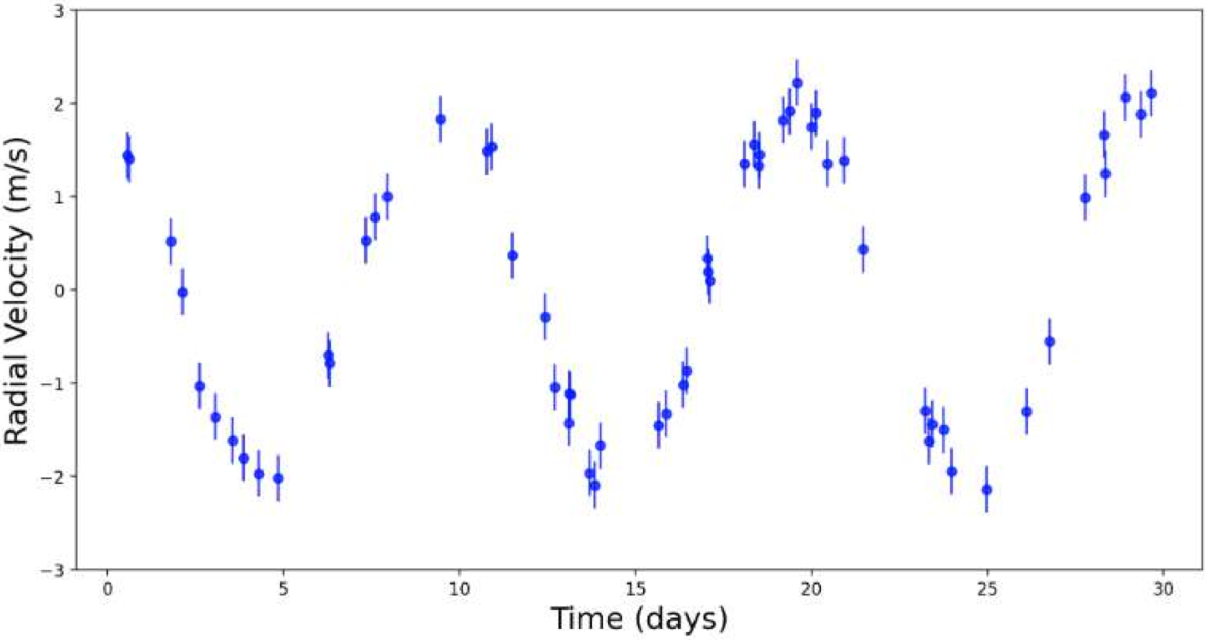

Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

The radial velocity data shows periodic shifts reaching about ±2 m/s, yielding a semi-amplitude of 2 m/s for the star’s motion induced by the planet.

e) What is the mass of this planet?

To determine the planet's mass, we use the radial velocity formula:

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{M_*^{2/3}}$$

Assuming the planet's orbit is circular (so $e = 0$), and that the planet's mass is much smaller than the star's mass, the equation simplifies as above.

If we solve for the planet's mass (M_p), the equation becomes:

$$M_p \sin i = K \left(\frac{P}{2\pi G}\right)^{1/3} M_*^{2/3}$$

Now, let's plug in the given values:

- $K = 2$ m/s
- $P = 5$ days $= 432,000$ s
- $M_* = 0.2\, M_\odot = 3.978 \times 10^{29}$ kg
- $G = 6.674 \times 10^{-11}$ m³/kg·s²
- $i = 90^\circ$, so $\sin i = 1$

4% ∨ Carrying out the calculation, we get: 5 o

$$M_p \approx 1.174 \times 10^{25}\ \mathrm{kg}$$

This value is approximately **1.97 times the mass of Earth**:

$$M_p \approx 1.97\, M_\oplus$$

Therefore, GJ 8999 b is nearly twice as massive as our own planet.

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

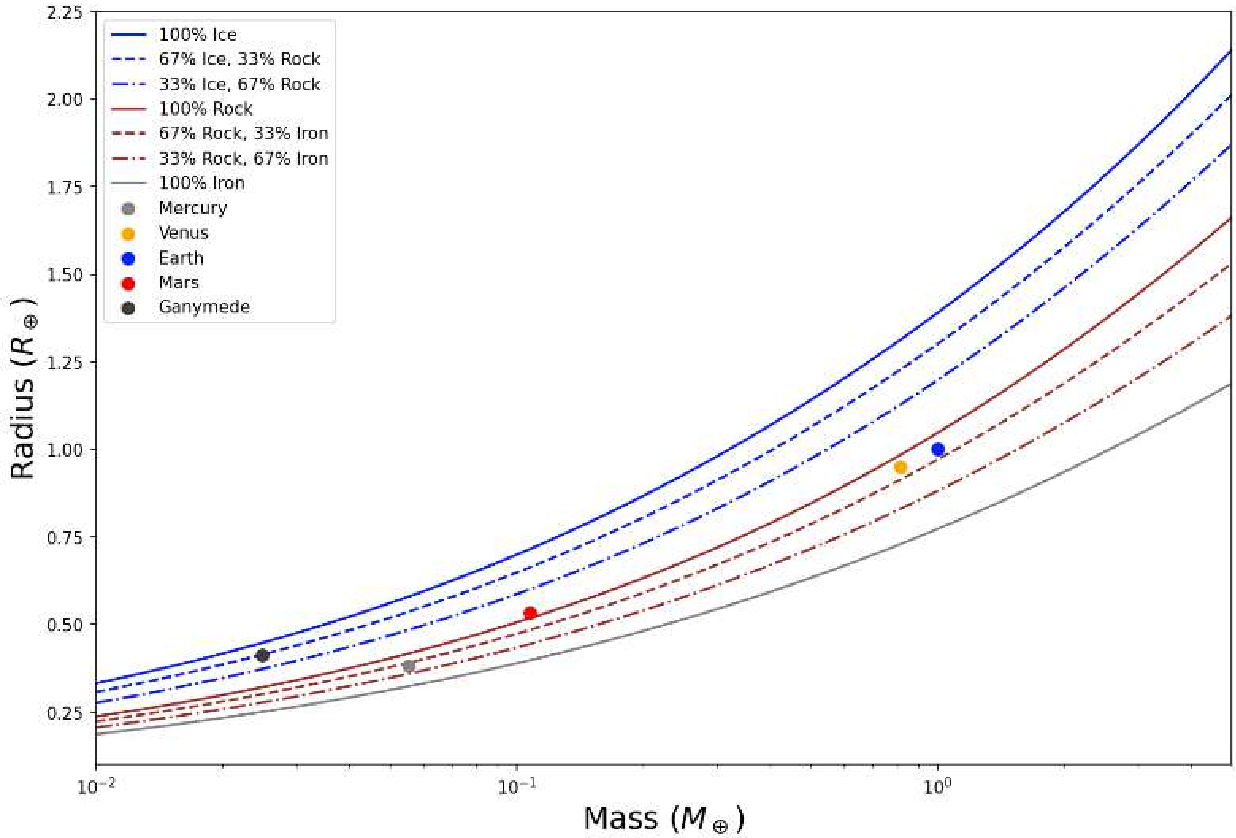

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

Its high density places indicates near the "67% iron, 33% rock" curve on mass-radius plots, implying that the planet likely has a higher fraction of iron in its interior compared to Earth, but not as high as Mercury.