

Statistical Inference Course Project 1

Overview

In this project I will investigate the exponential distribution in R and compare it with Central limit Theorem. The exponential distribution can be simulated in R with `rexp(n,lambda)`, where λ is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the Standard Deviation is also $1/\lambda$. I will set $\lambda=0.2$ for all the simulations. I will investigate the distribution of averages of 40 exponentials. Note that 1000 simulations will be required.

Simulations

```
# load necessary libraries
```

```
library(ggplot2)
```

```
set constants
```

```
lambda<-0.2 #lambda for rexp
```

```
n<-40 #number of exponentials
```

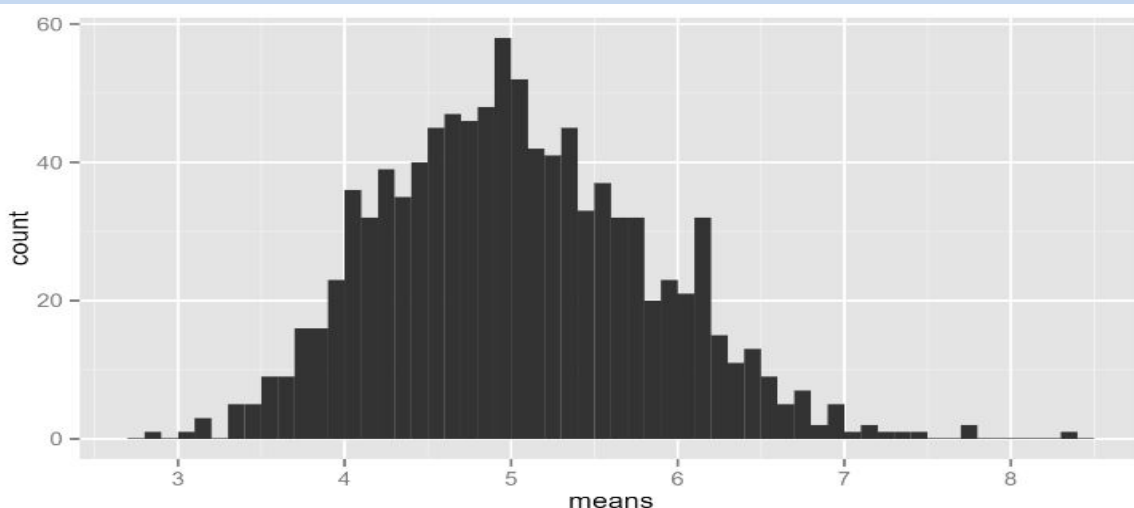
```
numberOfSimulations<-1000 #number of tests
```

```
# set seed to create reproducibility
```

```
set.seed(11081979)
```

```
# run the test resulting in n x numberOfSimulations matrix
```

```
exponentialDistributions <- matrix(data=rexp(n* numberOfSimulations, lambda), nrow=
numberOfSimulations)
exponentialDistributionsMeans <-
data.frame(means=apply(exponentialDistributions, 1 , mean) Sample
```



Sample Mean versus Theoretical Mean

The expected mean μ of an exponential distribution λ is

$$\mu = 1/\lambda$$

```
mu<- 1/lambda
```

```
mu
```

```
## [1] 5
```

Let \bar{X} be the average sample mean of 1000 simulations of 40 randomly sampled exponential distributions.

```
meanOfMeans <- mean(exponentialDistributionMeans$means)
```

```
meanOfMeans
```

```
## [1] 5.027126
```

As seen above the expected mean and the average sample mean are very close.

Sample Mean versus Theoretical Mean

The expected Standard Deviation σ of an exponential distribution of rate λ is

$$\sigma = 1/\lambda / \sqrt{n}$$

The e

```
sd <- 1/lambda/sqrt(n)
```

```
sd
```

```
## [1] 0.7905694
```

Then variance Var of the standard deviation σ is

$$\text{Var} = \sigma^2$$

```
Var <- sd^2
```

```
Var
```

```
##[1] 0.625
```

Let $\text{Var}\bar{X}$ be the variance of the average sample mean of 1000 simulations of 40 randomly sampled exponential distribution, and $\sigma_{\bar{X}}$ the corresponding standard deviation.

```
sd_x <- sd(exponentialDistributionMean$means)
```

```
sd_x
```

```
## [1] 0.8020334
```

```
Var_x <- var(exponentialDistributionMeans$means)
```

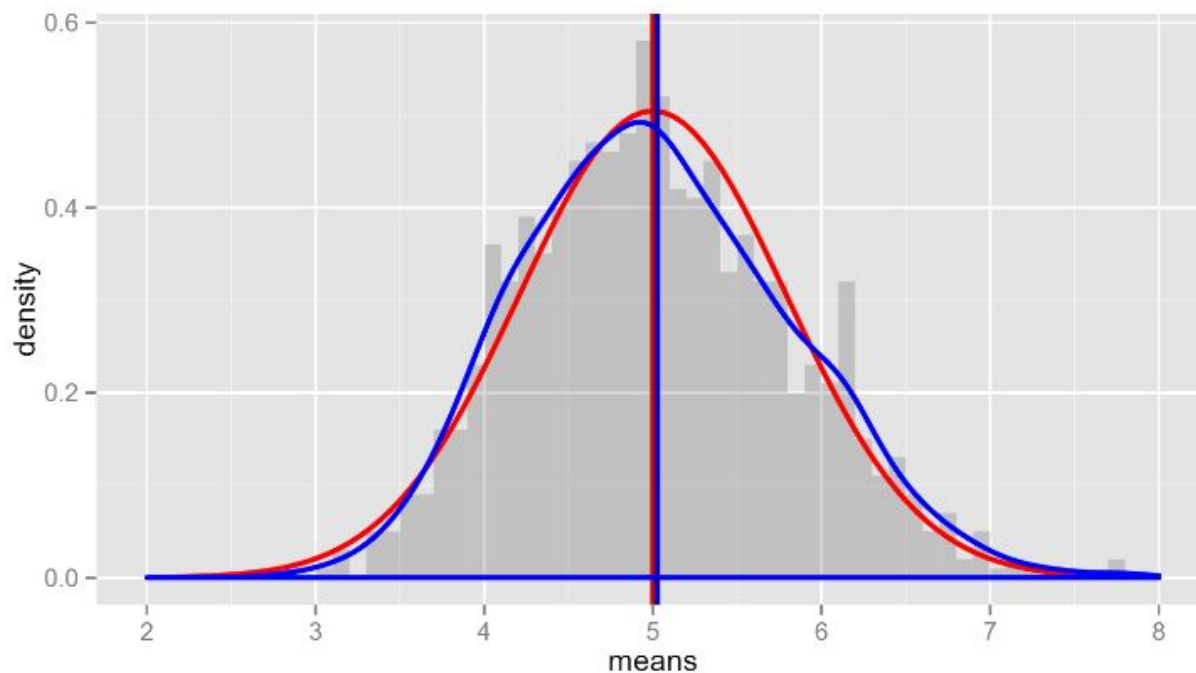
```
Var_x
```

```
## [1] 0.6432577
```

As evident above, the Standard deviations are very close. Since variance is the square of the standard deviations, minor differences will be enhanced, but are still pretty close.

Distribution

Comparing the population means and standard deviation with a normal distribution of the expected values . Added lines for calculated and expected means.



As you can see from the graph, the calculated distribution of means of random sampled exponential distributions , overlaps quite nice with the normal distribution with the expected values based on the given lambda.

