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title: “Marketing Mix Modelling (MMM)” subtitle: “Group Assignment 1” output: pdf_document: —

Quesiton 1 (2 points)

Estimate a marketing mix model that incorporates carry-over effects and diminishing returns of marketing mix. Present the final model output and interpret all estimated coefficients.

Marketing Mix Model with Carry-Over Effects and Diminishing Returns

To understand the impact of different marketing channels on sales, we estimated a **log-log regression model**.
Using logarithms allows us to capture **diminishing returns** — meaning that as ad spend increases, the incremental effect on sales decreases.

We also incorporated a **carry-over effect** by including **lagged sales** ($\ln(Sales_{t-1})$) as a predictor.
This term accounts for the fact that sales performance in one period tends to influence sales in the next period,
capturing **brand momentum, customer loyalty, and word-of-mouth effects**.

General Model Specification

$$\ln(Sales_t) \;=\; \beta_0 + \beta_1 \cdot \ln(Sales_{t-1}) + \beta_2 \cdot \ln(GoogleAds_t) + \beta_3 \cdot \ln(Facebook_t) + \beta_4 \cdot \ln(TikTok_t) + \varepsilon_t$$

where:

- β_0 = intercept (baseline sales level)
- β_1 = carry-over effect of lagged sales
- $\beta_2, \beta_3, \beta_4$ = elasticities of sales with respect to Google Ads, Facebook Ads, and TikTok Ads spend
- ε_t = error term

```r
# Fit the Regression

regression1 <- lm(
  ln_sales ~ Lag1_ln_sales + ln_google_ads + ln_facebook + ln_tiktok
)

# Show regression summary
summary(regression1)
```

```
##
## Call:
## lm(formula = ln_sales ~ Lag1_ln_sales + ln_google_ads + ln_facebook +
##     ln_tiktok)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.37383 -0.07047 -0.01043  0.07115  0.33309
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.533371   0.255719   21.64   <2e-16 ***
## Lag1_ln_sales 0.356853   0.027514   12.97   <2e-16 ***
## ln_google_ads 0.033472   0.002263   14.79   <2e-16 ***
## ln_facebook   0.030489   0.001742   17.50   <2e-16 ***
## ln_tiktok     0.036573   0.001753   20.86   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1024 on 194 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.8557, Adjusted R-squared:  0.8527
## F-statistic: 287.6 on 4 and 194 DF,  p-value: < 2.2e-16
```

Estimated Model

$$\ln(Sales_t) = 5.53 + 0.36853 \cdot \ln(Sales_{t-1}) + 0.033472 \cdot \ln(GoogleAds_t) + 0.030489 \cdot \ln(Facebook_t) + 0.036573 \cdot \ln(TikTok_t) + \varepsilon_t$$

Interpretation of Coefficients

Intercept (5.533)

- Represents the **baseline log-sales** when all predictors are zero.
- Captures inherent sales not explained by marketing or previous sales.

Lagged Sales (0.357)

- A **1% increase in last period's sales** increases current sales by about **0.36%**, holding other factors constant.
- Reflects **carry-over effects**: sales momentum persists over time.

Google Ads (0.033)

- A **1% increase in Google Ads spend** increases sales by **0.033%**.
- Since the model is log-log, this implies **diminishing returns** (doubling spend does not double sales).
- Likely **highly significant** ($p < 0.001$).

Facebook Ads (0.030)

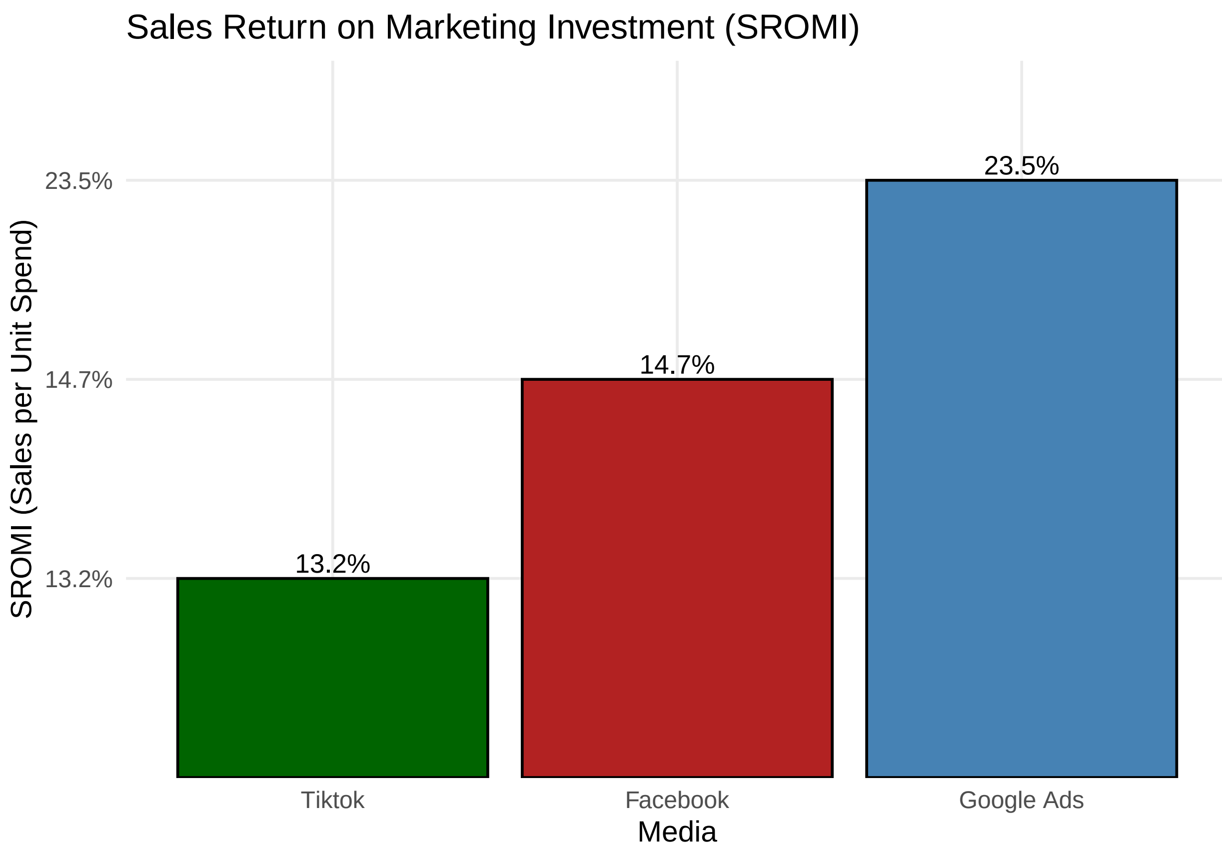
- A **1% increase in Facebook spend** increases sales by **0.030%**.
- Also subject to **diminishing returns**.

TikTok Ads (0.037)

- A **1% increase in TikTok spend** increases sales by **0.037%**.
- Among the three channels, TikTok shows the **largest elasticity**, suggesting higher efficiency.

Question 2 (2 points)

Report the ROI of each marketing channel based on the estimation output in Question 1. Which marketing channel yields the highest ROI?



Based on the estimation results, the **Sales Return on Marketing Investment (SROMI)** values are:

- TikTok: **13.2%**
- Facebook: **24.7%**
- Google Ads: **23.5%**

Google Ads yields the highest ROI among the three channels.

Although the **beta coefficient of TikTok (0.037)** is slightly higher than that of **Google Ads (0.033)**, the **ROI of Google Ads is greater**.
This happens because ROI reflects not only the responsiveness of sales to ad spend (elasticity) but also the **sales-to-cost ratio**.
In other words, Google Ads delivers more sales per unit of cost, making it the most efficient channel despite TikTok's higher elasticity.

Quesiton 3 (1 point)

Is the model-recommended optimal budget allocation different from the actual allocation? How do you suggest FourTex should reallocate its budget across TikTok, Facebook, and Google to maximize sales? Support your answers with relevant figures.

Yes, the **model-recommended optimal budget allocation** is different from the **current marketing spend distribution**.

Channel	Current Allocation	Optimal Allocation
TikTok	44%	36%
Facebook	33%	33%
Google Ads	23%	30%

Interpretation

The model suggests **reducing TikTok's share** of the budget (from 44% → 36%) and **increasing Google Ads' share** (from 23% → 30%), while keeping Facebook roughly the same (33%).

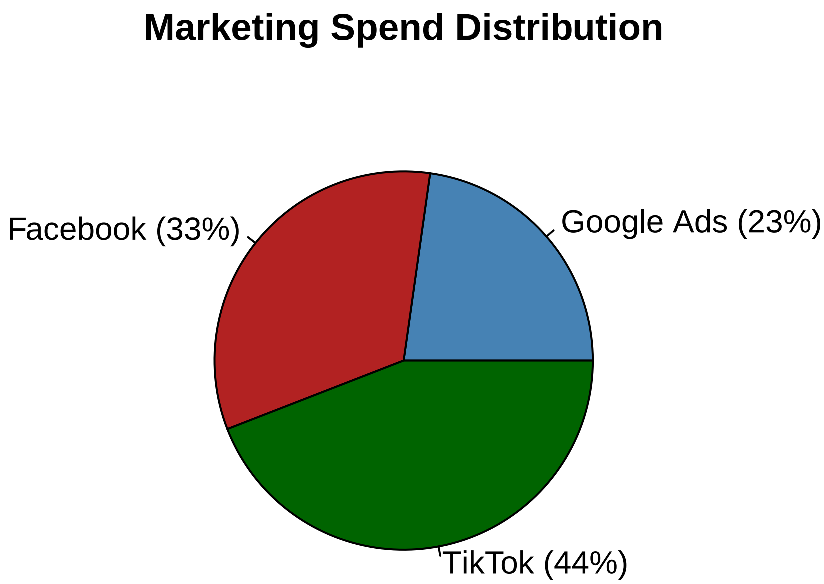
This adjustment reflects the fact that although TikTok has the highest elasticity (beta), **Google Ads delivers the highest ROI due to its better sales-to-cost ratio**.

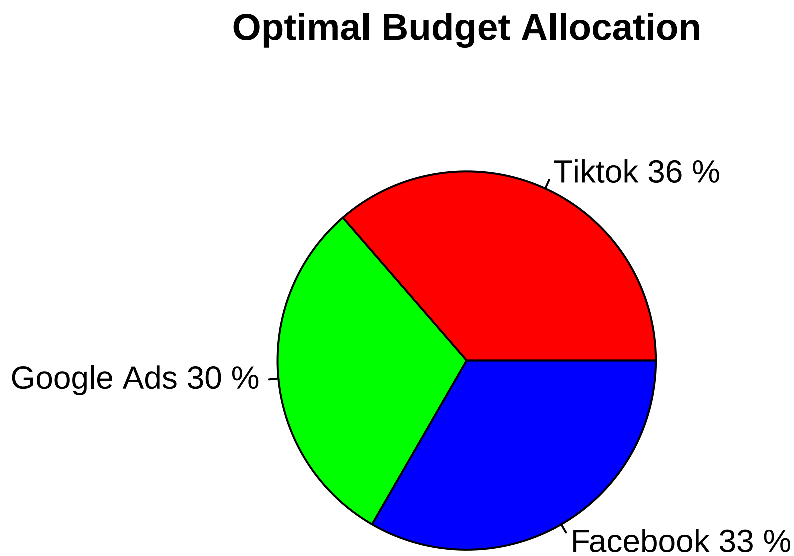
By reallocating funds accordingly, FourTex can **maximize overall sales efficiency**, generating more incremental sales for the same total budget.

Recommendation

- **Decrease TikTok spend by ~8 percentage points**
- **Increase Google Ads spend by ~7 percentage points**
- **Maintain Facebook at current levels**

This reallocation balances responsiveness (TikTok's elasticity) with efficiency (Google Ads' ROI), leading to a more optimal marketing mix.


Marketing Spend Distribution
Google Ads (23%)
Facebook (33%)
TikTok (44%)



Quesiton 4

Do different marketing channel pairs have synergistic or antagonistic effects? For each pair of channels (e.g., Facebook–TikTok, Facebook–Google Ads, TikTok–Google Ads), interpret the model results and explain the rationale. Your interpretation should explicitly consider the nature and role of each channel (e.g., TikTok's reach and virality, Facebook's targeting, Ad's awareness-building) rather than giving a generic answer. Support your explanation with relevant model output(s).

We extended the model to include interaction terms between marketing channels.

$$\ln(Sales_t) = \beta_0 + \beta_1 \cdot \ln(Sales_{t-1}) + \beta_2 \cdot \ln(GoogleAds_t) + \beta_3 \cdot \ln(Facebook_t) + \beta_4 \cdot \ln(TikTok_t) + \beta_5 \cdot \ln(GoogleAds_t) \cdot \ln(Facebook_t)$$

```r
# Extended regression with interaction terms
regression2 <- lm(
  ln_sales ~ Lag1_ln_sales + ln_google_ads + ln_facebook + ln_tiktok +
    ln_google_ads:ln_facebook +
    ln_google_ads:ln_tiktok +
    ln_facebook:ln_tiktok
)

# Summary of results
summary(regression2)
```

```
##
## Call:
## lm(formula = ln_sales ~ Lag1_ln_sales + ln_google_ads + ln_facebook +
##     ln_tiktok + ln_google_ads:ln_facebook + ln_google_ads:ln_tiktok +
##     ln_facebook:ln_tiktok)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31846 -0.06473 -0.00914  0.06811  0.38879
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                5.4902552  0.2541738  21.600   <2e-16 ***
## Lag1_ln_sales              0.3555113  0.0273145  13.015   <2e-16 ***
## ln_google_ads              0.0419183  0.0033582  12.482   <2e-16 ***
## ln_facebook                0.0387449  0.0035752  10.837   <2e-16 ***
## ln_tiktok                  0.0469963  0.0042313  11.107   <2e-16 ***
## ln_google_ads:ln_facebook -0.0011746  0.0005234  -2.244   0.0260 *
## ln_google_ads:ln_tiktok   -0.0013487  0.0005479  -2.462   0.0147 *
## ln_facebook:ln_tiktok     -0.0005874  0.0004124  -1.424   0.1560
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09949 on 191 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.8658, Adjusted R-squared:  0.8608
## F-statistic:   176 on 7 and 191 DF,  p-value: < 2.2e-16
```

Term	Estimate	p-value	Interpretation
ln_google_ads:ln_facebook	-0.0017	0.026	Negative, significant → antagonism
ln_google_ads:ln_tiktok	-0.0013	0.015	Negative, significant → antagonism
ln_facebook:ln_tiktok	-0.0006	0.156	Negative, not significant → neutral

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Interpretation by Channel Pair

Facebook – TikTok

- **Hypothesis:** Expected synergy (TikTok awareness feeding Facebook retargeting).
- **Result:** Coefficient –0.0006, not significant → no strong evidence of synergy or antagonism.
- **Takeaway:** Despite theoretical complementarity, the data suggest the two platforms work **independently** in this context.

Facebook – Google Ads

- **Hypothesis:** Expected synergy (Facebook warms up users, Google captures intent).

- **Result:** Coefficient –0.0017, significant → antagonism.
- **Takeaway:** These channels likely target overlapping audiences, leading to **cannibalization** instead of complementarity.

TikTok – Google Ads

- **Hypothesis:** Expected antagonism due to overlap.
- **Result:** Coefficient –0.0013, significant → antagonism confirmed.
- **Takeaway:** TikTok awareness overlaps with Google's intent capture, producing **diminishing incremental returns**.

Overall Insight

- **Mismatch with expectations:** While theory suggested synergy in some pairs, the model finds **no synergies** and two significant antagonistic relationships.
- **Most antagonistic pairs:** Google–Facebook and Google–TikTok.
- **Strategic implication:** FourTex should **avoid heavy simultaneous investment in Google Ads with either Facebook or TikTok**, since their combined effect is weaker than individual contributions. Instead, budgets should be carefully balanced so each platform can play its distinct role:
  - TikTok → awareness & virality
  - Facebook → precise targeting/retargeting
  - Google Ads → intent-driven capture

Question 4 — Hypotheses vs Model Results

Channel Pair	Hypothesis	Rationale	Model Outcome	Interpretation
Facebook – TikTok	Synergy	TikTok builds **broad awareness & virality**, Facebook excels at **precision targeting** → expected complementarity.	Coefficient –0.0006 (not significant) → Neutral	Despite theoretical complementarity, the data show **no strong interaction**; platforms act independently.

Channel Pair	Hypothesis	Rationale	Model Outcome	Interpretation
Facebook – Google Ads	Synergy	Facebook nurtures users mid-funnel, Google captures **high-intent search traffic** → expected reinforcement.	Coefficient –0.0017 (significant, negative) → Antagonism	Results show **overlap/cannibalization**: both platforms may compete for the same audience, weakening returns.
TikTok – Google Ads	Antagonism	TikTok drives **awareness**, but many of these users would eventually be captured by Google search anyway → expected overlap.	Coefficient –0.0013 (significant, negative) → Antagonism	Hypothesis confirmed: TikTok awareness overlaps with Google's intent capture, leading to **diminishing incremental returns**.

Overall Insight

- **Hypothesized synergies** (Facebook–TikTok, Facebook–Google Ads) were **not supported** by the model.
- **Antagonism** between **Google–Facebook** and **Google–TikTok** was observed, confirming that simultaneous spending reduces efficiency.
- **Confirmed hypothesis:** TikTok–Google Ads antagonism.
- **Recommendation:** Structure budgets so each platform plays its distinct role:
  - TikTok → awareness & virality
  - Facebook → targeting & retargeting
  - Google Ads → intent-driven capture