LAKSHYA
JEE 2025

MATHEMATICS | **Lecture - 01**

CALCULUS

LIMIT, CONTINUITY & DIFFERENTIABILITY (LCD)

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Topics *to be covered*

1. Basics of Limits
2. Existence of Limits
3. One Sided Limit
3. Question Practice

LAST CLASS RECAP

Defining By "Feel" of Substitution:

$x = \tan\theta, \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$2\theta \in (-\pi, \pi)$

① $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}(\sin 2\theta)$

$= 2\theta, \quad 2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= \quad \quad 2\theta \in \left[\frac{\pi}{2}, \pi\right)$

QUESTION

If $\cos^{-1}\left(\frac{x^2}{3}\right)+\cos^{-1}\left(\sqrt{1-\frac{x^4}{9}}\right)+\cot^{-1}y=\frac{5\pi}{6}$, then (where [.] denotes G.I.F.)

(A) Maximum value of $[|xy|]$ is 1

(B) Minimum value of $[|xy|]$ is 1

(C) Maximum value of x^2+y^2 is $\frac{1}{3}$

(D) Maximum value of x^2+y^2 is $\frac{10}{3}$

$x\in[-\sqrt{3},\sqrt{3}]$

$y=\frac{1}{\sqrt{3}}$

$xy\in[-1,1]$

$x^2+y^2\big|_{max.}=3+\frac{1}{3}=\frac{10}{3}$

$[0,1]$

$\frac{x^2}{3}=t$

$\sin^{-1}t$

$LHS=\underbrace{\cos^{-1}t+\overbrace{\cos^{-1}\sqrt{1-t^2}}^{\sin^{-1}t}}_{\frac{\pi}{2}}+\cot^{-1}y=\frac{5\pi}{6}$

$\frac{\pi}{2}+\cot^{-1}y=\frac{5\pi}{6}$

$\cot^{-1}y=\frac{5\pi}{6}-\frac{\pi}{2}=\frac{5\pi-3\pi}{6}$

$\cot^{-1}y=\frac{\pi}{3}$

$y=\frac{1}{\sqrt{3}}$

$-1\le\frac{x^2}{3}\le1$

$0\le1-\frac{x^4}{9}\le1$

$-1\le-\frac{x^4}{9}\le0$

$-3\le x^2\le3$

$x\in[-\sqrt{3},\sqrt{3}]$

$1\ge\frac{x^4}{9}\ge0$

$\times\ominus$

$9\ge x^4\ge0$

$\Downarrow$

$x\in[-\sqrt{3},\sqrt{3}]$

CHALLENGER QUESTION

Let $g: R \to \left[\frac{\pi}{6}, \frac{\pi}{2}\right)$ is defined by $g(x) = \sin^{-1}\left(\frac{x^2-c}{1+x^2}\right)$. Then the possible values of 'c' for which g is surjective function, is

"onto"

(A) $\left\{\frac{1}{2}\right\}$

(B) $\left(-1, -\frac{1}{2}\right]$

(C) $\left\{-\frac{1}{2}\right\}$

(D) $\left[-\frac{1}{2}, 1\right)$

$g(x) = \sin^{-1}\left(\underbrace{\quad}_{\downarrow}\right)$

$[-c, 1)$

$\frac{x^2-c}{1+x^2} = \frac{x^2+1-(c+1)}{x^2+1} = 1 - \frac{(c+1)}{x^2+1}$

$x^2 \in [0, \infty)$

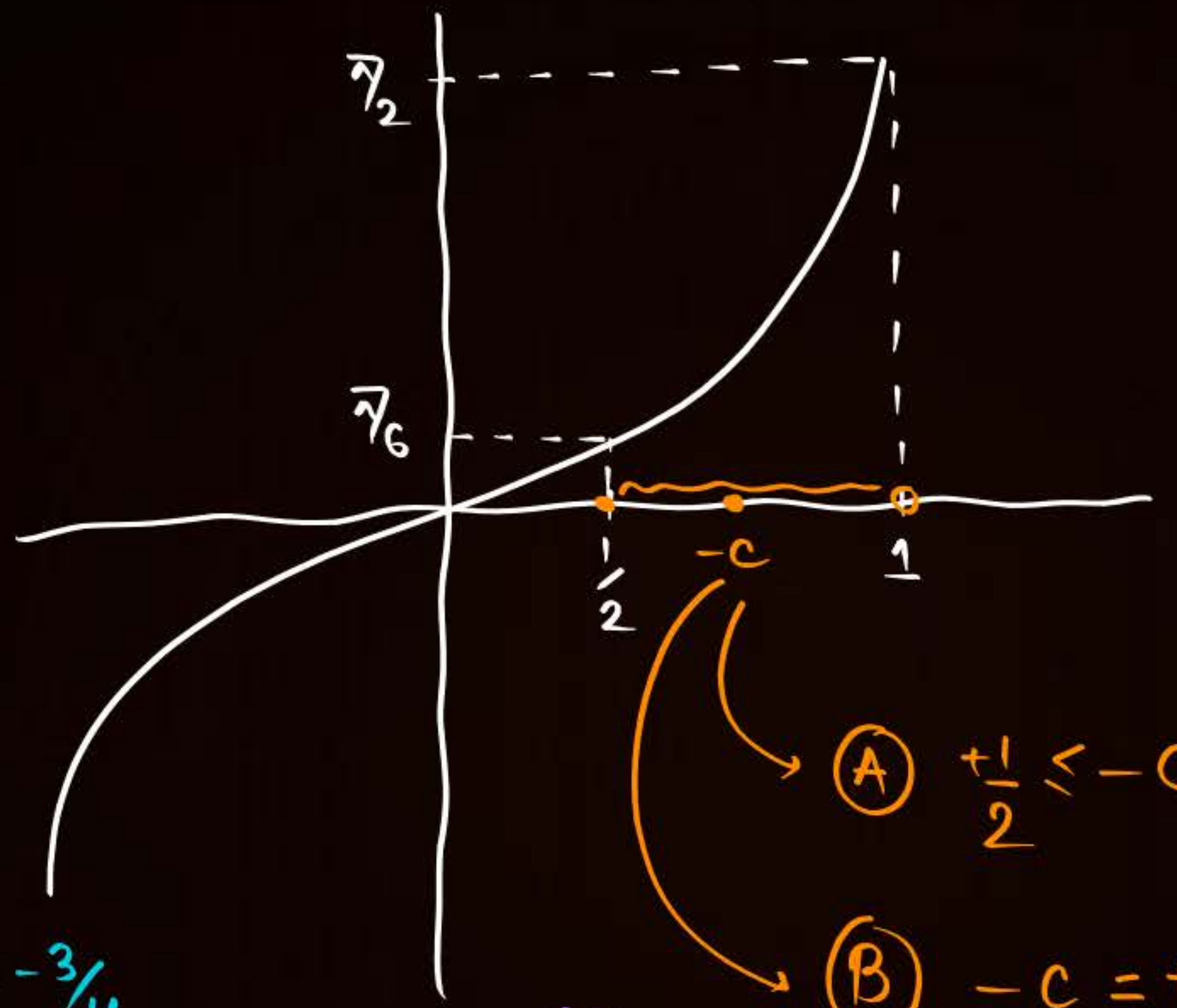
$x^2+1 \in [1, \infty)$

$\frac{1}{x^2+1} \in (0, 1]$

$c+1>0 \Rightarrow -\frac{c+1}{x^2+1} \in [-c-1, 0) \xrightarrow{+1} 1-\left(\frac{c+1}{x^2+1}\right) \in [-c, 1)$

$c+1<0 \Rightarrow -\frac{c+1}{x^2+1} \in (0, -c-1] \xrightarrow{+1} 1-\left(\frac{c+1}{x^2+1}\right) \in (1, -c]$ (Rejected!)

$\frac{1}{2} = -c$



$y = \sin^{-1}(\;)$

$[-c, 1)$

(A) $+\frac{1}{2} \le -c < 1 \Rightarrow c \in \left(-1, -\frac{1}{2}\right] \rightarrow$ wrong

Check :-

$c = -\frac{3}{4}$

$-c = \frac{3}{4}$

(B) $-c = +\frac{1}{2}$

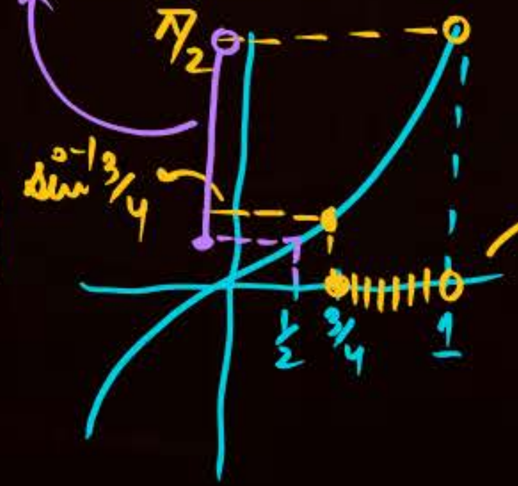
$c = -\frac{1}{2}$ ✓

$c = -3/4$

$y = \sin^{-1}(\;)$

$\left[\frac{3}{4}, 1\right)$

CD.



Range ∈ $\left[\sin^{-1}\frac{3}{4}, \frac{\pi}{2}\right)$

QUESTION

Solve the following equations :

(i) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = -\pi - 2\sin^{-1}x$

(ii) $3\tan^{-1}x = -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

(iii) $\cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1}x$

Q/BY!

Chat!

M-I :- by defining

$\Downarrow$

Solⁿ: $\Rightarrow x \in \left(-\infty, -\frac{1}{\sqrt{3}}\right)$

M-II :-

$\underbrace{3\tan^{-1}x + \pi}_{} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$-\frac{\pi}{2} < 3\tan^{-1}x + \pi < \frac{\pi}{2}$

$-\frac{3\pi}{2} < 3\tan^{-1}x < -\frac{\pi}{2}$

$-\frac{\pi}{2} < \tan^{-1}x < -\frac{\pi}{6} \Rightarrow \quad x < -\frac{1}{\sqrt{3}}$

QUESTION

Solve the following equations :

(i) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = -\pi - 2\sin^{-1}x$

(ii) $3\tan^{-1}x = -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

(iii) $\cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1}x$

$\sin^{-1}x = \theta$

M-I :- $\boxed{x = \sin\theta}$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$2\theta \in [-\pi, \pi]$

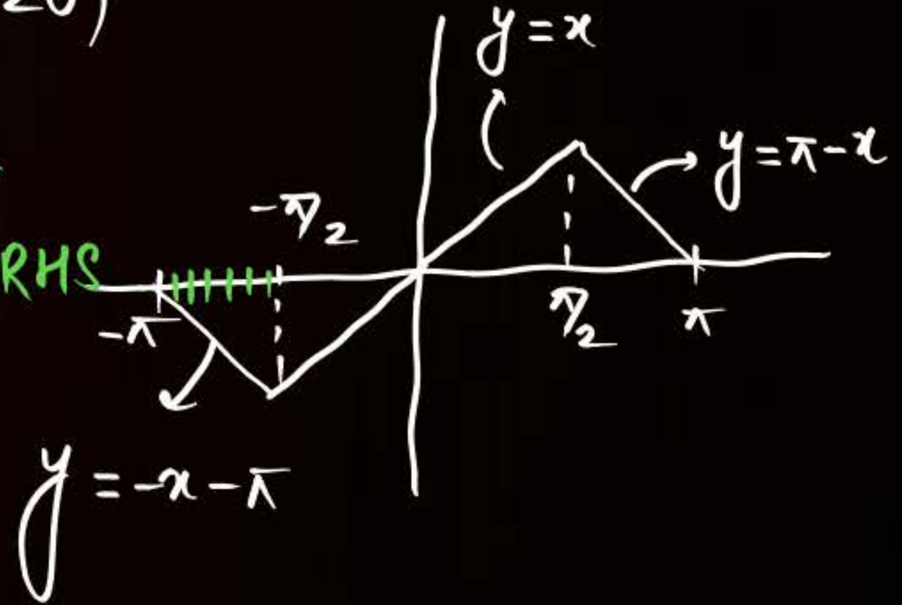
LHS $= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$= \boxed{-2\theta - \pi}$ = RHS

$= \pi - 2\theta$



when :- $-\pi \le 2\theta \le -\frac{\pi}{2}$ ⟶ $-\frac{\pi}{2} \le \theta \le -\frac{\pi}{4}$

$\Downarrow$

$-1 \le x \le -\frac{1}{\sqrt{2}}$

M-II :- Compare range.

LHS $= \sin^{-1}(\ldots) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

RHS ⇒ $-\frac{\pi}{2} \le -\pi - 2\sin^{-1}x \le \frac{\pi}{2}$

$\frac{\pi}{4} \le -\sin^{-1}x \le \frac{3\pi}{4}$

$\Downarrow$

$-\frac{\pi}{4} \ge \sin^{-1}x \ge -\frac{3\pi}{4}$

$-\frac{\pi}{4} \ge \sin^{-1}x \ge -\frac{\pi}{2}$

$-\frac{1}{\sqrt{2}} \ge x \ge -1$

Sol^n: $x \in \left[-1, -\frac{1}{\sqrt{2}}\right]$

Inf.

QUESTION

Which of the following functions represent identical graphs in $x-y$ plane $\forall x \in [-1, 1]$?

Multiple correct

Simplify!

HW.

(A) $f_1(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

(B) $f_2(x) = \frac{\pi}{4} - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\right)$

(C) $f_3(x) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$

(D) $f_4(x) = \frac{1}{2}\sin^{-1} x^2$

ACIDIC QUESTION

Consider a function

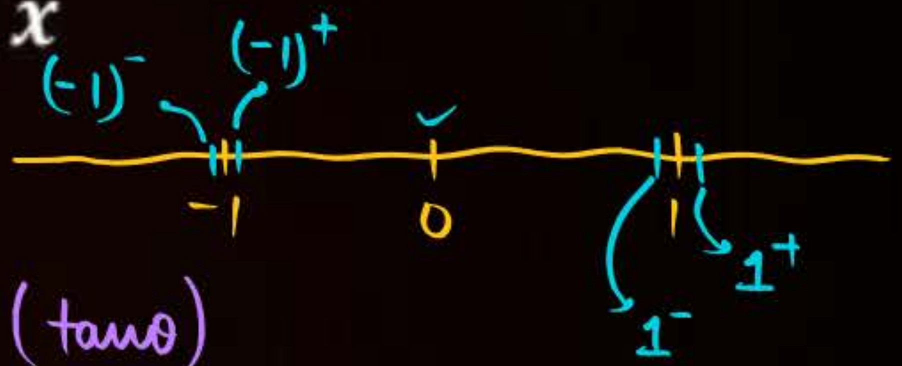
$f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) - a\tan^{-1}x$ where a is any real constant. Find the value of 'a' if $f(x) = 0$ for all x

$x < -1 \Rightarrow f(x) = -2\theta - \cancel{\pi} - \cancel{2\theta} + \cancel{2\theta} + \cancel{\pi} - a\theta$

$2\theta < -\frac{\pi}{2}$

$= -\theta(2+a) = 0 \rightarrow a = -2$.

$\boxed{x = \tan\theta}$ or $\boxed{\tan^{-1}x = \theta} \rightarrow \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



(A) 6

$f(x) = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) + \tan^{-1}(\tan 2\theta) - a\tan^{-1}(\tan\theta)$

(B) −6

$x > 1 \Rightarrow f(x) = \cancel{\pi} - \cancel{2\theta} + \cancel{2\theta} + 2\theta - \cancel{\pi} - a\theta = (2-a)\theta \Rightarrow f(x) = 0 \Rightarrow a = 2$

$\Downarrow$

$2\theta > \frac{\pi}{2}$

(C) 2

$0 < x < 1 \Rightarrow f(x) = 2\theta + 2\theta + 2\theta - a\theta = (6-a)\theta \Rightarrow f(x) = 0 \Rightarrow a = 6$

$\Downarrow$

$0 < 2\theta < \frac{\pi}{2}$

(D) −2

$-1 < x < 0 \Rightarrow f(x) = \cancel{2\theta} + (\cancel{-2\theta}) + 2\theta - a\theta = (2-a)\theta$

$\Downarrow$

$-\frac{\pi}{2} < 2\theta < 0$

CHALLENGER QUESTION

Let $f(x) = x^2 - 2ax + a - 2$ and $g(x) = \left[2 + \sin^{-1}\frac{2x}{1+x^2}\right]$. If the set of real values of 'a' for which $f(g(x)) < 0, \forall x \in R$ is (k_1, k_2), then find the value of $(10k_1 - 3k_2)$.

[Note : $[k]$ denotes greatest integer less than or equal to k.]

• HW

LIMIT, CONTINUITY & DIFFERENTIABILITY (LCD)

LIMITS

Easy

Majedaar!

Weightage → 1 Quesⁿ → JEE Mains → Jee-Adv.

BASICS OF LIMITS

NOTE-01:

Meaning of : $a^+ \equiv$ Just greater than a

$a^- \equiv$ Just smaller than a

'x' is exactly equals to 'a' $\equiv x = a$

'x' is approaching towards 'a' $\equiv x \to a$

from LHS $\quad x \to a^-$

from RHS $\quad x \to a^+$

$1.999\ldots9 \simeq 2^-$ $\quad$ $2^+ \simeq 2.0000\ldots1$

2

$-0.0000\ldots1 \simeq 0^-$ (-ve) $\quad$ $0^+ \simeq 0.0000\ldots1$ (+ve)

0

$-3.0000\ldots1 \simeq (-3)^-$ $\quad$ $(-3)^+ \simeq -2.999\ldots9$

-3

NOTE-02: What is limit or limiting value ?

Ex.: $\lim_{x \to 2}(x^2 + 2)$

as $x \to a \Rightarrow f(x) \to K$ (from both side)
$\quad$ ↳ any number

Then limiting value of $f(x)$ is said to exact 'K'.

NOTE-03: Need of limit

Ex.: If $f(x) = \frac{x^2-9}{x-3}$, then what is value of $f(x)$ is neighbourhood of $x = 3$?

$\boxed{x \in R - \{3\}}$

$\lim_{x \to 3^+}\left(\frac{x^2-9}{x-3}\right)$ $\left(\frac{\text{लगभग } 0}{\text{लगभग } 0} = \frac{0\uparrow}{0\uparrow}\right)$ $= \lim_{x \to 3^+} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 6$

Ex:-

$$\lim_{x\to 2}(x^2+2)$$

$\longrightarrow$ from LHS = LHL = $\lim_{x\to 2^-}(x^2+2) = 6$ $\longrightarrow$ limiting value.

$\longrightarrow$ from RHS = RHL = $\lim_{x\to 2^+}(x^2+2)$ = exact '6' $= 6$

$x \to 2^+$	$f(x) = x^2+2$
2.01	6.001
2.001	6.00001
2.00001	6.00000...1
2.0000..1	6.00......1

app. towards '6' from RHS.

$x \to 2^-$	$f(x) = x^2+2$
1.99	5.99
1.9999	5.9999
1.99999..	5.999999
⋮	⋮

app. towards '6' from LHS.

BASICS OF LIMITS

NOTE-04:

While calculating limiting value of at $x = a$, hum sirf agal-bagal (neighborhood) mein value calculate karte hai.

At $x = a$, function ki kya value hai, function defined bhi hai ki nahi Hume use koi matlab nahi hai.

EXISTENCE OF LIMITS

$$\text{LHL at } (x = a) = \lim_{x\to a^-} f(\overset{a^-}{x}) = \lim_{h\to 0^+} f(\overbrace{a-h}^{a^-}) = \lim_{h\to 0^-} f(\overbrace{a+h}^{a^-})$$

$$\text{RHL at } (x = a) = \lim_{x\to a^+} f(\underset{a^+}{x}) = \lim_{h\to 0^+} f(\underbrace{a+h}_{a^+}) = \lim_{h\to 0^-} f(\underbrace{a-h}_{a^+})$$

At $(x = a)$ limit is said to existing iff:

$$\boxed{\text{LHL} = \text{RHL} = \text{finite}}$$

Note:

$$\boxed{\text{If LHL} = \text{RHL} = l \text{ (finite)} \Leftrightarrow \lim_{x\to a} f(x) = l}$$

ex:- $x = a$

	LHL	RHL	f(a)
DNE ←	1	2	3
DNE ←	-1	0	-1
L.E. ←	$\boxed{3}$	$\boxed{3}$	5
	-7	-7	-7
DNE ←	∞	0	nd
* DNE ←	∞	∞	nd
DNE ←	-∞	∞	nd

(#) $\lim_{x \to 2} f(x) = 3$

$$\left.\begin{array}{l} \lim_{x \to 2^-} f(x) = 3 \\ \lim_{x \to 2^+} f(x) = 3 \end{array}\right\}$$

QUESTION

(#) $[\cdot] \equiv$ GIF, $\{\cdot\} \equiv$ FPF

Calculate following limits

(i) $\lim_{x\to3}[x]$

RHL $= \lim_{x\to3^+}[x] = 3$

LHL $= \lim_{x\to3^-}[x] = 2$

DNE.

(iii) $\lim_{x\to1}(x^2+3)\operatorname{sgn}(x-1)$

LHL $= \lim_{x\to1^-}(x^2+3)\operatorname{sgn}(x-1) = 4(-1) = -4$

RHL $= \lim_{x\to1^+}(x^2+3)\operatorname{sgn}(x-1) = 4(1) = 4$

DNE.

(v) $\lim_{x\to1}\left(\frac{x^2-8x+7}{x^3-5x^2-11x+15}\right)$ $\to \frac{0\uparrow}{0\uparrow}$

$\lim_{x\to1}\frac{\cancel{(x-1)}(x-7)}{\cancel{(x-1)}(x^2-4x-15)} = \frac{-6}{(1-4-15)} = \frac{-6}{-18} = \frac{1}{3}$

(vii) $\lim_{x\to1}([x])^{\{x\}}$

LHL $= \lim_{x\to1^-}([x])^{\{x\}} = 0^1 = 0$

RHL $= \lim_{x\to1^+}([x])^{\{x\}} = 1^0 = 1$

DNE.

$\lim_{x\to1^+}\{x\} = 0$

$\lim_{x\to1^-}\{x\} = 1$

QUESTION

(ii) $\lim_{x\to 2}\{x\}$

$RHL = \lim_{x\to 2^+}\{x\} = 0$

$LHL = \lim_{x\to 2^-}\{x\} = 1$

DNE

(iv) $\lim_{x\to 2}\left(\frac{x^3-5x+3}{x^2+7}\right) = \frac{1}{11}$

(vi) $\lim_{x\to 7}\frac{|x-7|}{(x-7)}$

$LHL = \lim_{x\to 7^-}\frac{-\cancel{(x-7)}}{\cancel{(x-7)}} = -1$

$RHL = \lim_{x\to 7^+}\left(\frac{x-7}{x-7}\right) = 1$

DNE

Doubt

$\{x\} \in [0,1)$

$\lim_{x\to 2^-}\{x\} = 1$

limiting value.

QUESTION [JEE (Adv.)-2018, Paper-1]

The number of real solutions of the equation

$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^{i}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^{i} - \sum_{i=1}^{\infty}(-x)^{i}\right)$ lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ________.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

HW

Homework

Re-attempt all the Questions of Lecture.

DPP

Module: Chapter – ITF

Exercise (Prarambh) : COMPLETE

Exercise (Prabal) : COMPLETE

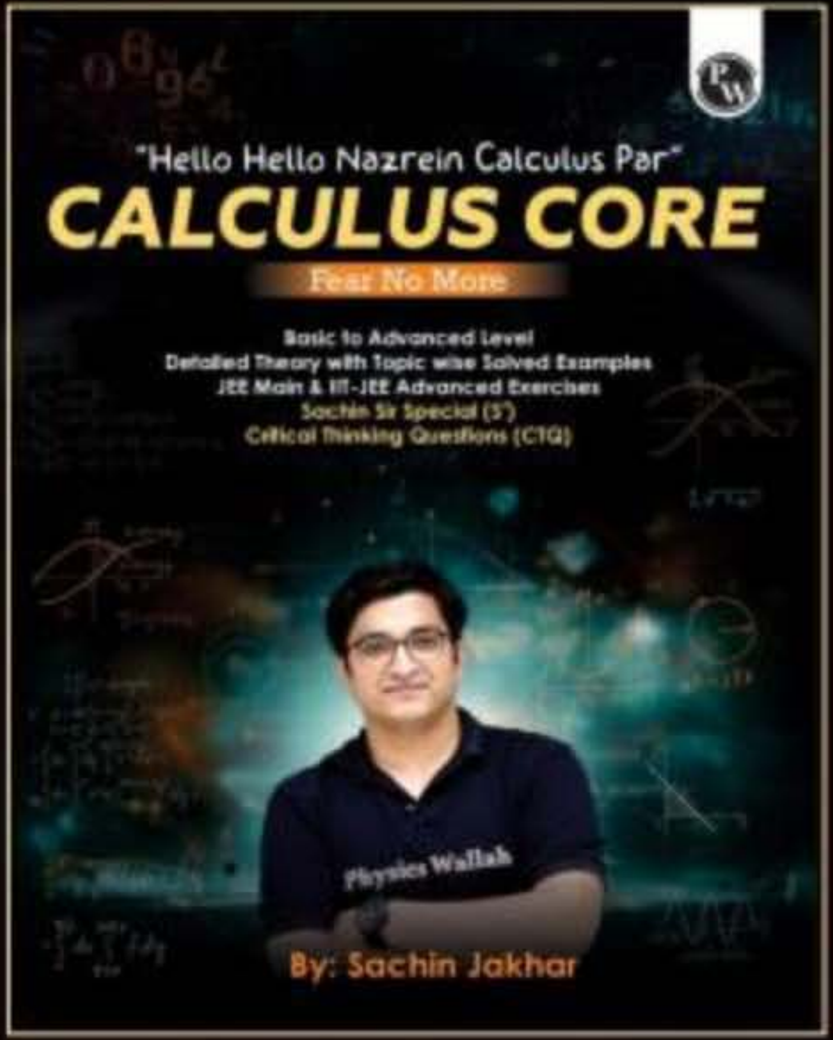
Exercise (Parikshit) : Ques: 1,5,7,9,11,23

NCERT:

INVERSE TRIGONOMETRIC FUNCTIONS

Exercise: 2.1 - COMPLETE

Exercise: 2.2 – Ques. 8 to 15



CALCULUS CORE

CHAPTER: ITF

DIBY : 3.3 COMPLETE (all)

JEE MAINS : COMPLETE

It's not about End Result,
It is all about JOURNEY

#futureIITians

THANK YOU