Robotics Assignment 1

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Answer to Question 1

According to definition,
$P^n(S) = P(P^{n-1}(S))$

So, as $S = \phi = \{\}$,
$P(S) = \{\phi\}$

$P(P(S)) = \{\phi, \{\phi\}\}$

A powerset of set S has cardinality of 2^n where n is the cardinality of set S.
So cardinality $|P^n(\phi)| = 2^{2^{n-1}}$

Answer to Question 2

1) As G is a group, we know that it has at least one inverse. ——— (Property 3)
To prove, assume that G has 2 identity elements, e1 and e2.
Then, by the definition of an identity,
$e1 \cdot e2 = e1$, as e2 is an identity
Also,
$e1 \cdot e2 = e2$, as e1 is an identity
Hence,
e1=e1
Thus, group G has only 1 identity

2) Now, we know that each element $a \in G$ has an inverse.
To prove, suppose a has 2 inverses, i1 and i2.
Then, by property 4,
$a \cdot i1 = e$
$\therefore i1 = e/a$
Also,
$a \cdot i2 = e$
$\therefore i2 = e/a$
Hence,
i1=i2
Hence proved

Answer to Question 3

The objects a and c are convex, and the objects c and d is concave, as they are hollow.

Answer to Question 4

1) Looking at the graph for $cos\theta$,
$cos(-\pi/4) = cos(\pi/4) = 1/\sqrt{2}$
Similarly, many different θ values map to the same cos value. Hence, the function is not injective.
As we see from the graph,
$cos(-\pi/2) = 0$
$cos0 = 1$
$cos(\pi/2) = 0$
$\therefore$ Considering the map, every angle between $-\pi/2$ and 0 has a value between 0 and 1, and similarly, 0 to $\pi/2$ also has a value between 0 to 1.
Thus, the function is surjective, every codomain element has a domain element.
To make the function bijective, we can define it as,
$f : [0, \pi] \to [0, 1]$.
Now, every domain element has a unique co domain value.

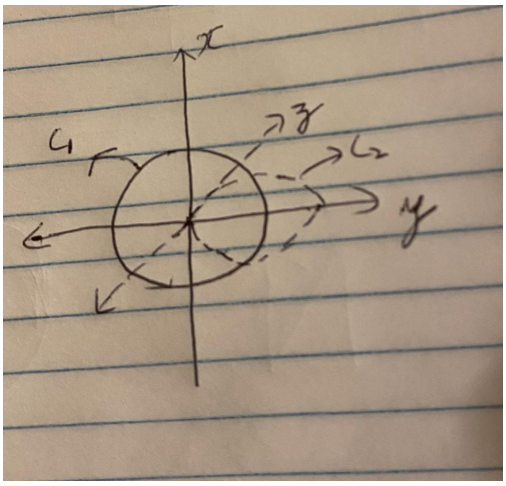
2) For the exponential function, every domain value has a single codomain value.
Let $f(x1) = f(x2)$
Hence, $e^x1 = e^x2$.
Hence, $e^{(x1-x2)} = 1$
$\therefore x1 - x2 = 0$
$\therefore x1 = x2$
Hence, each x has a unique $f(x)$.
Hence, the function is injective.
According to the exponential function, $e^0 = 1$, and after that the function is increasing,
But as x becomes negative, the function is asymptotic towards 0.
Hence, $f(x)$ does not take values below 0.
Hence the function is not surjective.
Hence, to make the function bijective, we can modify it as,
$f : R \to R+$.

Answer to Question 5

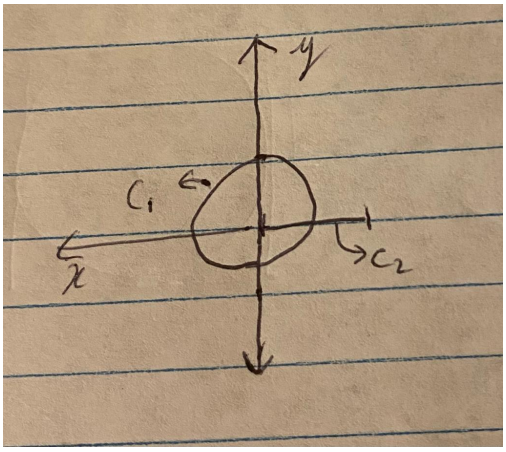
a) To find a continuous bijective function, we need to have one to one mapping between [-1,0] and $\mathbf{R}^-$ as well as [0,1] and $\mathbf{R}^+$.
We can use the fact that for numbers between (-∞,-1) and (1,∞), f(x)=1/x gives answers between [-1,1].
So, calculating, to include all n$\in \mathbf{R}$, we add 1 in the denominator.
Calculating, the final bijective map is,
$f(x) = x \div 1 - x^2$

b) To get the function, we have
The circle - $x^2 + y^2 = 1$ and the square - $||(x, y)||_\infty$.
Both intersect at (0,1),(1,0),(-1,0) and (0,-1).
If we draw a line through the origin, it would uniquely pass through a unique point on the circle as well as square.
Mapping points on square and circle using a parameter r.
For the circle, $r = \sqrt{x^2 + y^2}$, and for the square, $r = max(x, y)$.
So finding a ratio = $\dfrac{max(x, y)}{\sqrt{x^2 + y^2}}$.
This leads to our mapping of (x,y) from square to circle, which is,
$(x, y) \to \dfrac{max(x, y)}{\sqrt{x^2 + y^2}}(x, y)$
$\therefore (x, y) \to \dfrac{|x| + |y| + ||x| - |y||}{\sqrt{x^2 + y^2}}(x, y)$
The function is continuous over the circle and each value of x and y gives a unique value of the function, taking all the values in the range. Thus, this is a continuous bijective function.

Answer to Question 6



The space obtained by both circles forms a chain with one circle in xy and one in x-z plane. As we can see that try to compress C2 to x axis, it intersects with C1 which was not the case in original space. Thus X is not a manifold.



Answer to Question 7

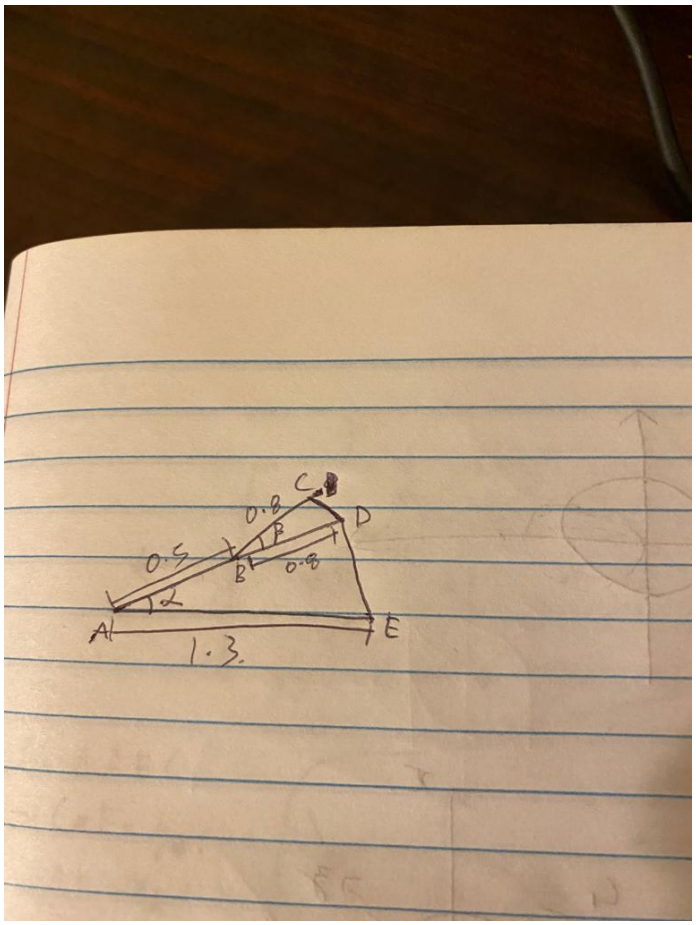


Figure 1: 50 Samples

Given that E_1 *is a* $10-$*bit encoder, thus there will be* 2^{10} *partitions and the angle of uncertainity* : $-$

$$\alpha = \frac{360}{2^{10}} \;=\; 0.3515625$$

Similarly E_2 is 15 bit encoder -

$$\beta = \frac{360}{2^{15}} \;=\; 0.0101$$

Therefore maximum uncertainity = CE= CD+DE

Using the law of cosine on Triangle ADE, DE =

$$\sqrt{(AE)^2 + (AD)^2 + 2(AE)(AD)Cos(\alpha)}$$

= 0.0079m

Similarly by Triangle BCD, we get CD=0.0001m

Therefore CE=0.008m

Answer to Question 8

For a n-sided die to get all sides:-

The probability of getting the first number $(p_1) = n/n = 1$
Number of tries to get the first number $= 1/p_1 = 1$

The probability of getting the second number $(p_2) = (n-1)/n$
Number of tries to get the second number $= 1/p_2 = n/(n-1)$

Similary, Number of tries to get the nth number = n

$$\begin{aligned} Total\ number\ of\ rolls &= (n/n) + (n/n-1) + (n/n-2) + ... + (n/1) \\ &= \Sigma_{i=0}^{n-1}(n/n-i) \qquad (harmonic\ series) \\ &\approx n\ ln(n) \qquad (For\ large\ n) \end{aligned} \tag{1}$$

For 10 sided die, number of rolls = 10/10 + 10/9 + ...+10/1 = 29.28

Answer to Question 9

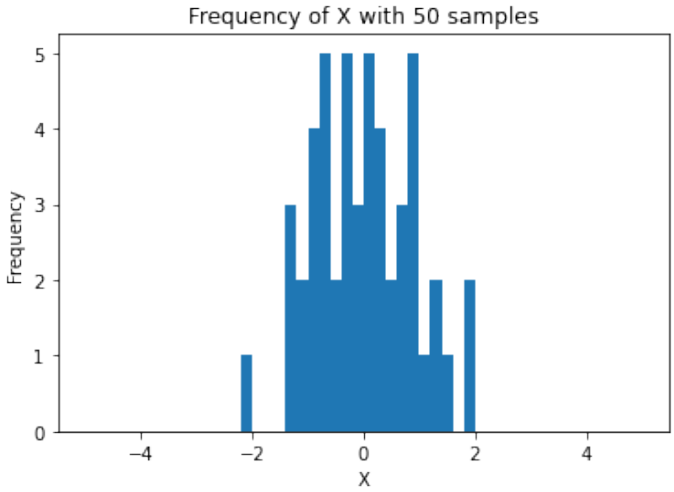


Figure 2: 50 Samples

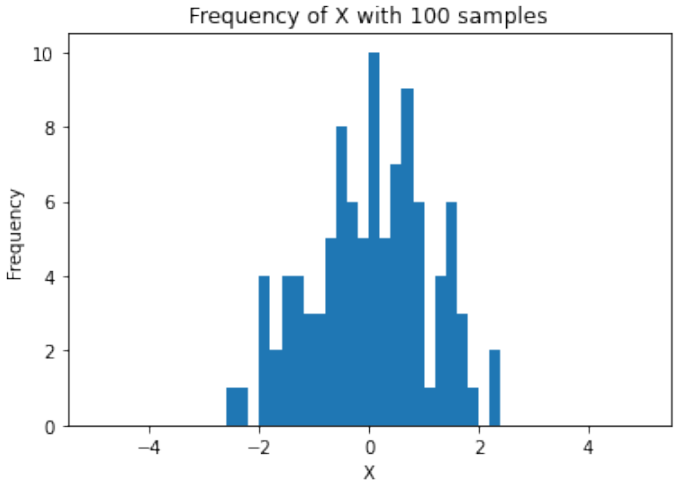


Figure 3: 100 Samples

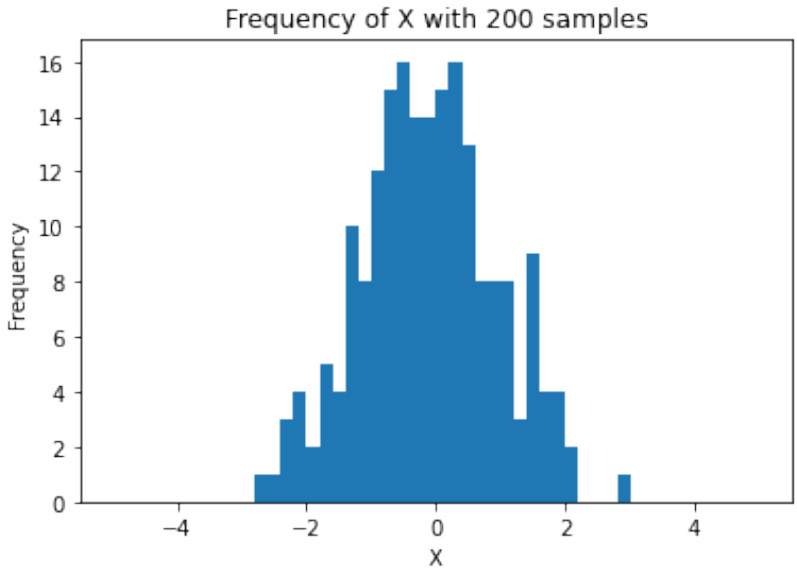


Figure 4: 200 Samples

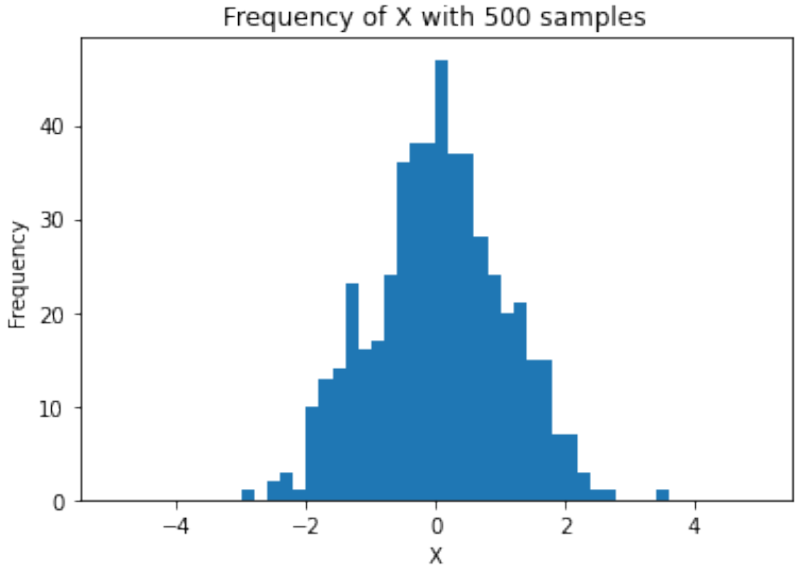


Figure 5: 500 Samples