Reconstruction through MRI k-space Data

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Introduction

MRI is used to generate detailed and informative images of the **tissues and organs within the patient body** with the help of radio waves and magnetic field.

- MRI is a non-invasive scanning technique.
- It does not use ionizing radiations and hence it is preferred over Computed Tomography (CT).
- MRI produces higher resolution images of soft tissues compared to CT.

The raw data (k-space data) may contain some missing entries because of improper scanning or due to traveling in medium.

- It is very difficult to have a scanning device that can generate complete k-space data.
- The problem of reconstruction of an image from k-space is a challenging one.

Scope and Challenges

MRI uses the principle of nuclear magnetic resonance.

- Spatial information is provided by the spatial gradient of the applied magnetic field.
- The sampling trajectory in the k-space is determined by the time dependence of the applied magnetic field gradient.
- The computer system accepts the analog data from RF coils and performs analog to digital conversion of data.
    - The digitized version of the body part which will get stored in a temporary image space also called the k-space, the Fourier transformation of the image.
    - It represents the spatial position covered by frequency and its phase data generated by MRI scan.
    - K-space data is sent to the image processor where some reconstruction algorithm is applied to reconstruct an image.
    - Both phase and frequency data are produced in the form of matrix of complex numbers, whose central region has low frequency. This is generally referred as k-space data.

Modern MRI Techniques

Modern MRI gives the clear picture of temporal resolution of dynamic imaging and plays an important role in metabolic imaging.

- Functional MRI is used to explain the functional movement of body part with respect to the brain and to generate images of brain activity with the help of magnetization property.
- Diffusion MRI: It estimates the scattering of water molecules in the tissue. It was developed for capturing images that are continuously varying, e.g., brain motion, cardiac condition, etc.
- Real Time MRI: continuously monitors the objects in real time. FLASH (Fast Low Angle Shot) is a basic principle for measuring the k-space data that is obtained from scanning devices.
- Multinuclear Imaging: analyses the biological tissues which are having a tremendous amount of Hydrogen. It is used for determining nanolevel distances among the nerves.
- Interventional MRI: was developed for interventional radiology. It has no magnets but it has quasi static fields and strong magnetic radio frequency fields that are generated from the scanner.
- Susceptibility Weighted Imaging (SWI): uses echo constraints, pulse sequences for capturing images. It is applicable for detecting tumors, traumatic brain injury, stroke, etc.

K-Space Data

K-space data is related to image data by Fourier Transform. Gradient can be defined as vector in 3 dimensions.

The Larmor frequency (ω) represents the angular momentm of precession generated by magnetic moment of the protons and can be given as:

$$\omega = \gamma B \tag{1}$$

B is the strength of the static magnetic field, and γ is the gyromagnetic ratio. Due to the presence of main magnetic field (B_0) and gradient magnetic field (G), Larmor frequency can be calculated as:

$$\omega = \gamma(\vec{B_0} + \vec{G}\vec{r}) \tag{2}$$

$\vec{r}$ is the spatial location (coordinates of a point) in the image. In rotating frame, the main magnetic field vanishes, ω can be given as:

$$\omega(r, t) = \gamma(\vec{G}(t)\vec{r}) \tag{3}$$

K-Space Data

Phase, ϕ is the integral of angular frequency. Phase of the image at $t = 0$ can be given as $\phi(0) = 0$; At any time t, it is given by:

$$\phi(r, t) = \int_0^t \omega(r, \tau) d\tau \tag{4}$$

$$\phi(r, t) = \int_0^t G(\tau)\vec{r} d\tau \tag{5}$$

Let

$$k(t) = \gamma \int_0^t \vec{G}(\tau) d\tau \tag{6}$$

Now it can be written as:

$$\phi(r, t) = k(t) r \tag{7}$$

MRI scanner output can be represented as a function of time as

$$s(t) = \int M_{xy}(r, t) dr \tag{8}$$

M_{xy} is the vector sum of all transverse magnetization exists in the image at any point of time.

K-Space Data Generation

Most of the k-space data reside in the lower frequency region. The higher frequency regions are generally sparse in k-space, and most of the information of an image is present near the central region of k-space.
The Fourier Transform of a real signal is a conjugate symmetry. Due to this reason, the k-space data also follows conjugate symmetry. **The high spatial frequencies correspond to the region of an image where the rate of change in intensity is more. The low spatial frequency corresponds to the region of an image where the rate of change in intensity is less.**

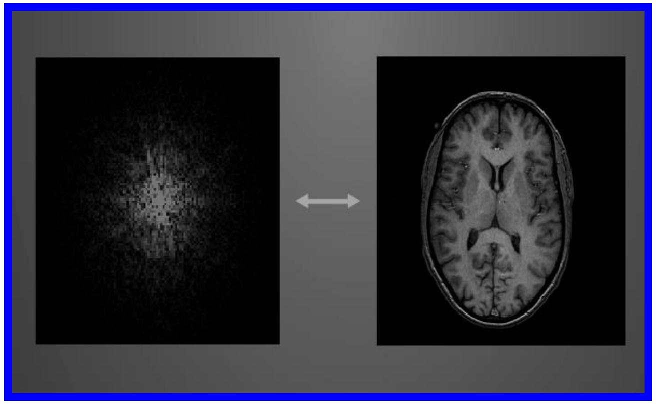

Figure: Inverse Fourier Transform of an k-space

k-space Data and Partial k-space

- K-space data can be referred as Full k-space data when there are no missing entries present in k-space.
- It is practically difficult to obtain the full k-space.
- Ideally, the Fourier Transform of an image is equivalent to the full k-space data.
- The symmetric nature of k-space enables rebuilding the missing k-space data to a greater extent.

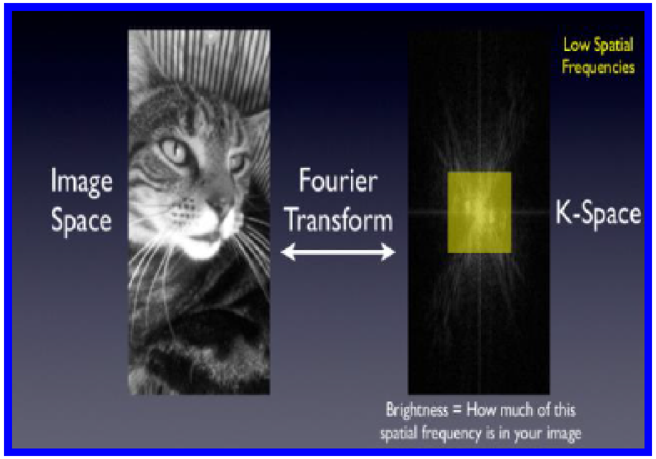

Figure: Fourier transform of an image gives full k-space data

Fourier Transform and k-space Data

- k-space Imaging Sequence
  - The imaging sequence consists of four phases: data collection, preprocessing, reconstruction, and postprocessing
  - The collected k-space is analog in nature
  - In preprocessing phase, it converts the analog data to digital form.
  - Meaningful image is done in the reconstruction phase.
  - In the postprocessing phase, it is needed for removing noise in the reconstructed image.
- Phase Correction
  - Acquisition of the complete k-space is not feasible.
  - Error varies when processing occurs from sub section to sub section that should be eliminated for constructing proper image.
- Gibbs Ringing Artifact
  - Gibb's artifacts are unwanted signals which usually appear near the edges.
  - It usually occurs because of the low pass filter. Here in k-space reconstruction it occurs due to the variation in phase.
  - This artifact gives noise near the edge in an image because there is a transformation happen when it shifts from low spatial frequency to high spatial frequency or vice versa.
  - These artifacts can be prevented by using the concept of convolution as a function of smoothing.

Classical Algorithms for MRI Image Reconstruction

Image reconstruction is the process of converting k-space data to an image.

- Zero Filling Reconstruction Algorithm
  - Zero get filled in the missing entries of k-space data, directly making missing entries to zero without rebuliding.
  - It works well when almost full k-space data is available.
  - *Pros*: Simple.
  - *Cons*: Not an acceptable technique when there is no full k-space data.

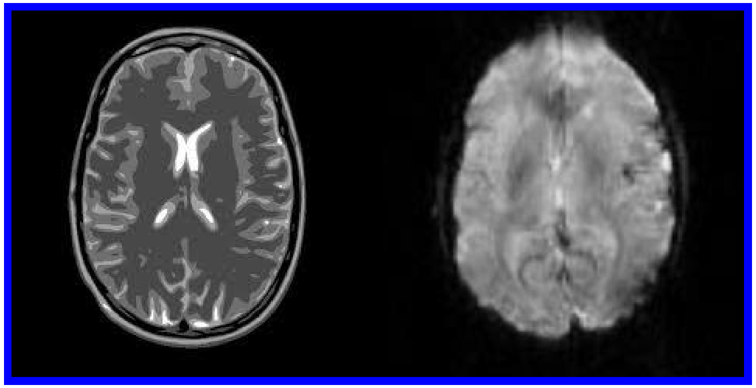

Figure: Original Phantom Image (U) and Reconstructed Phantom Image (D) after applying Zero Filling Algorithm

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Classical Algorithms for MRI Image Reconstruction

- Phase Correction using Conjugate Symmetry
  - This method fills these entries by considering the property of conjugate symmetry of k-space.
  - The algorithm takes input as partial k-space data and produces reconstructed image.
  - *Pros*: Better than zero filling.
  - *Cons*: It does not work when most of the k-space data entries are real valued. It is not possible to get the conjugate symmetry of real numbers.

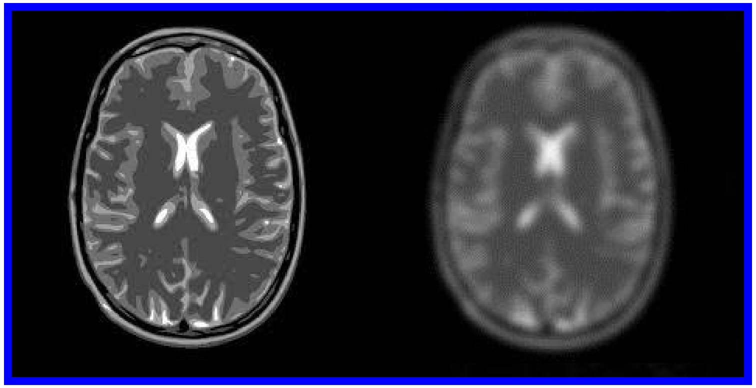

Figure: Original Phantom Image(U) and Reconstructed Phantom Image(D) after applying Phase Correction using Conjugate Symmetry

Classical Algorithms for MRI Image Reconstruction

- Phase Correction using Conjugate Symmetry Algorithm

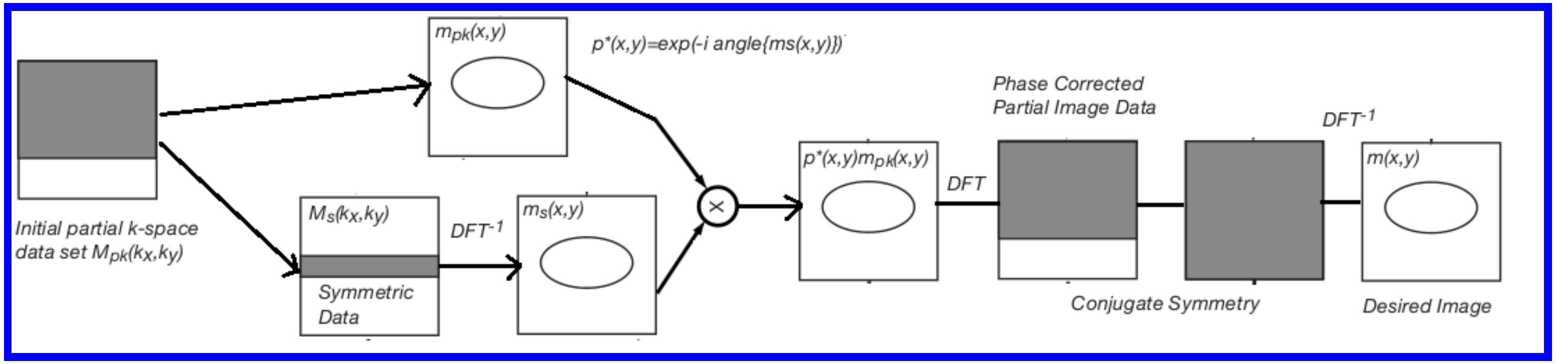

Figure: Pictorial Representation of Phase Correction using Conjugate Symmetry Algorithm

Classical Algorithms for MRI Image Reconstruction

- Homodyne Reconstruction Algorithm
  - The previous approach have to compute FT and IFT multiple times. Homodyne overcomes this and requires the FT and IFT to be done only once.
  - It used a weighing function that splits the real and imaginary part of the partial k-space data into anti-symmetric and symmetric components. The weighing function is to get the uniform distribution of spatial frequency in k-space data that are usually not uniform.
  - The nature of weighing function can be seen as follow:

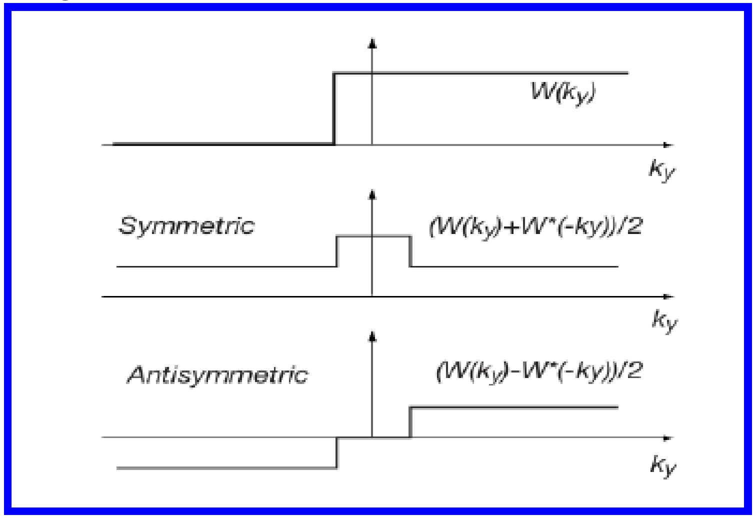

Figure: Weighing function is performing uniform distribution

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Classical Algorithms for MRI Image Reconstruction

- Homodyne Reconstruction Algorithm
  - Both algorithm perform well when the variation of phase is minimal in both.
  - The Homodyne algorithm performs phase correction after conjugate synthesis while phase correction using conjugate symmetry perform the phase correction before conjugate synthesis.

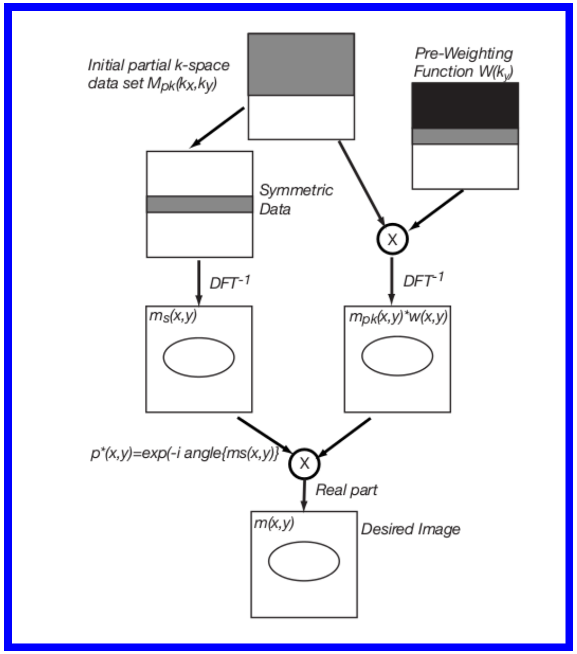

Figure: Pictorial Representation of Homodyne Reconstruction Algorithm

Modern Algorithms for Image Reconstruction

- SVD and Dictionary Learning are modern approaches that rebuild the given k-space data efficiently.
- Projection onto Convex Set Algorithm
  - It uses a mathematical model which is used to determine points of convergence between two convex sets, one set contains determined phase constraints and the other contains image constraints.
  - This approach guesses the k-space data by applying phase correction using conjugate symmetry algorithm iteratively.
  - The information about an image is near to central region. The central region is bounded for iteration with fix point.
  - Pros: It produces better quality images. It changes the conjugate symmetry and phase correction iteratively.
  - Cons: It fails in filling entries in k-space if the k-space is quite more asymmetrical.

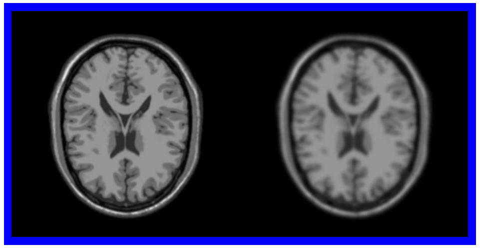

Figure: Original Brain Phantom Image (U) and Reconstructed Phantom Image (D) after applying POCS Reconstruction Algorithm

Modern Algorithms for Image Reconstruction

- Projection onto Convex Set Algorithm Assumptions:
  - $m(x, y)$ represents true image; $M(k_x, k_y)$ represents full k-space data
  - Since, $m(x, y) = IDFT\{M(k_x, k_y)\}$, $M_p(k_x, k_y)$ represents low frequency region (central region)
  - A new sub k-space is defined from $k_{x1} \leq k_x \leq k_{x2}$ can be given as:
  - $M_p(k_x, k_y) = \begin{cases} M(k_x,\ k_y) & \mathit{if}\ (k_{x1} \leq k_x \leq k_{x2}) \\ 0 & \mathit{otherwise} \end{cases}$

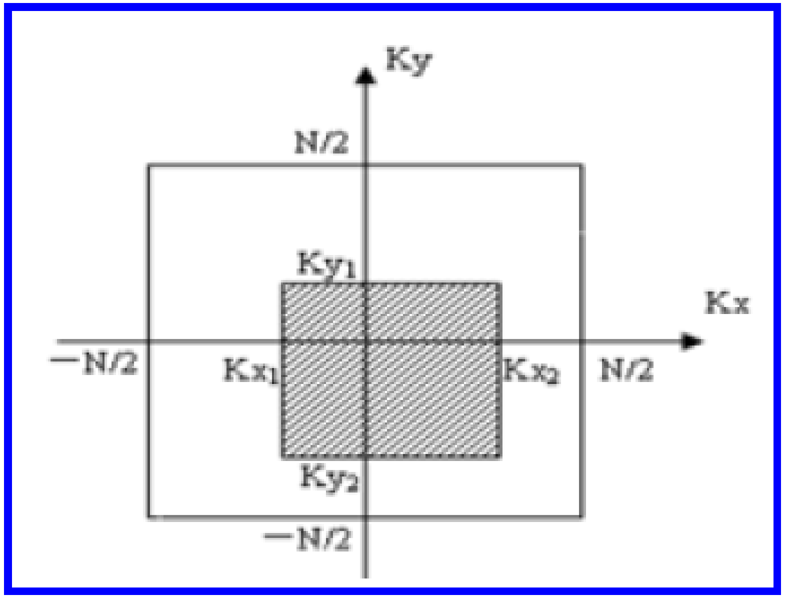

Figure: k-Space data emphasizing on Central Region

Modern Algorithms for Image Reconstruction

- Projection onto Convex Set Algorithm Details:

Input: k: k-space raw-data.
Output: m_r: The reconstructed image.
if *k is not Conjugate Symmetric* **then**
  while *phase started varying slowly (Almost conjugate assymmetric k-space is obtained)* **do**
    $m_p(x, y) = IDFT(M_p(k_x, k_y))$; `/* Apply` $IDFT$ `on` M_p`, we will get partial reconstructed image` m_p `*/`
    $m_r(x, y) = |m_p(x, y)| \exp(i, \phi(x, y))$; `/* Find phase constraint for each entry, after phase shift */`
    $M_r(k_x, k_y) = DFT\{m_r(x, y)\}$;
  end
else
  $m_r = IDFT M_r(k_x, k_y)$;
end

Modern Algorithms for Image Reconstruction

- Reconstruction using Singular Value Decomposition
  - Fundamentals of Singular Value Decomposition:
    - $M = U\Sigma V^*$, M is given matrix
    - U is an $m \times m$ size unitary matrix,
    - Σ is an $m \times n$ size diagonal matrix
    - V^* is conjugate transpose of V unitary matrix of size $n \times n$
    - Its application is of finding pseudo inverse of a matrix. $M^+ = V\Sigma^+U^*$
    - Σ^+ is obtained by replacing every non-zero diagonal entry by its reciprocal and transposing the resulting matrix.
  - Pros: It gives quite better result and if there is priory knowledge about the Fourier coefficient matrix.
  - Cons: High time and space complexity.

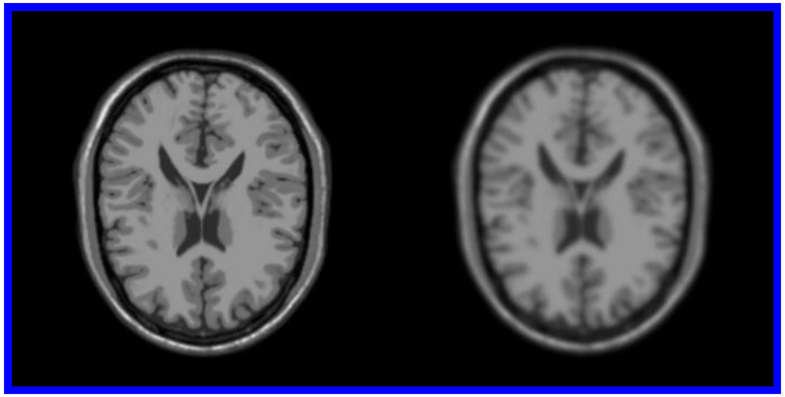

Figure: Original Brain Phantom Image (U) and Reconstructed Phantom Image (D) after applying SVD Reconstruction Algorithm

Modern Algorithms for Image Reconstruction

- Reconstruction using Singular Value Decomposition Algorithm:
  - The k-space in the frequency domain or spatial can be written as:

$$S(k_x, h_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} X(x, y) e^{-i2\pi(\frac{k_x x}{M} + \frac{k_y y}{N})} e^{i\phi(x,y)} \tag{9}$$

  - In terms of SVD k-space can be written as:

$$S_{MN\times 1} = A_{MN\times M^2} \times X_{M^2\times 1} \tag{10}$$

  - A is the matrix of Fourier coefficient whose values are phase corrected, S is the k-space data, and X is the value of intensity that need to be reconstructed.
  - The idea of SVD in the reconstruction is to make the matrix regular. Regular signifies that there should be only one value corresponding to each row and column.

Modern Algorithms for Image Reconstruction

- Reconstruction using Dictionary Learning
  - It is used for sparse representations of images/signals.
  - It assembles the collection of atom. These atoms are collection of column vectors of some fixed length.
  - Let the length of atom be N here and the total number of atoms are K. A dictionary D can be created of size $N \times K$
  - X is a vector and represents a signal/image in terms of linear combination of some of the Dictionary atoms and Sparse Random Vector can be given as:

$$X = Dw \tag{11}$$

  - w is a Sparse Random Vector.
- The quality of reconstructed images can be improved in two ways
  - increase the number of atoms in dictionary learning upto blocks.
  - Divide the dictionary learning scheme in two levels, a low resolution learning and high resolution learning.
- The result is quite similar to the method using SVD.

Conclusions

- Zero filled method is a trivial method, works fine when full k-space is available.
- The phase coorection using conjugate symmetry, homodyne algorithm and projection onto convex set (POCS) work well only if variation in phase in the image is low.
- When the degree of phase variation is high, these algorithms give ghosting effect in an image.
- SVD works better even if phase variation is there is k-space data.
- Dictionary Learning is the modern approach. We can use pre-build dictionary that can be used for reconstructing images. Gives better result when the input data is sparsed.