Risk Quantification in Deep MRI Reconstruction

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Introduction

- Yet despite the importance of assessing risk in medical image reconstruction, little work has explored the robustness of deep learning (DL) architectures in inverse problems, and there is a lack of established methods for quantifying model uncertainty.
- We develop a variational autoencoder (VAE) model for MR image recovery, which is notable for its low error and probabilistic nature.
- We then introduce Stein's Unbiased Risk Estimator (SURE) as a means of assessing uncertainty of the DL model, which we find effectively approximates MSE and serves as a valuable tool for assessing risk when the ground truth reconstruction (i.e. fully sampled image) is unavailable.

Preliminaries and Problem Statement

- The goal is to recover the true image $x_0 \in \mathbb{C}^n$ from undersampled k-space measurements $y \in \mathbb{C}^m$ with $m \leq n$ that admit:

$$y = \Phi x_0 + v \tag{1}$$

- where Φ in general includes the acquisition model with the sampling mask Ω, the Fourier operator F, as well as coil sensitivities. The noise term v also accounts for measurement noise and unmodeled dynamics.
- It is necessary to incorporate prior information to obtain high-quality reconstructions.
- One risk is the introduction of realistic artifacts, or so-termed ”hallucinations”.

VAE for Medical Image Recovery

- latent code vectors are sampled from a normal distribution $z \sim \mathcal{N}(\mu_z; \sigma_z)$ to generate new reconstructions.
- An affine projection based on the undersampling mask was applied, which we found essential for high SNR.
- The loss function in training was based on the mixutre of pixel-wise l_2 with a KL-divergence term (weighted by constant η that constrains the latent code.

$$\min \mathbb{E}_{x,y}[\|\hat{x} - x_0\|_2^2] + \eta \mathrm{KL}(\mathcal{N}(\mu_z, \sigma_z) \| \mathcal{N}(0, 1)) \tag{2}$$

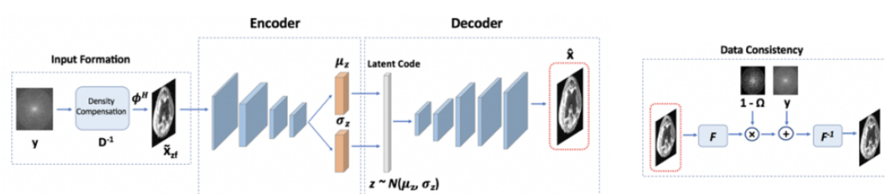

Figure: The model architecture, with aliased inputs feeding into the VAE encoder, the latent code feeding into the VAE decoder, and data consistency applied to obtain the output reconstruction.

SURE for Uncertainty Analysis: Denoising SURE

- It is not possible to know the fully-sampled ground truth corresponding to a given reconstruction, which motivates the use of **SURE**.
- With zero-filled (aliased) input x_{zf} and reconstruction $\hat{x}$ with dimension n, SURE can be expressed as follows (where $r \sim \mathcal{N}(0, \sigma^2 I)$ is the noise process that describes the difference between the input and the ground truth):

$$\mathrm{SURE} = -n^2 + \underbrace{\|\hat{x} - x_{zf}\|^2}_{RSS} + \sigma^2 \underbrace{\mathrm{tr}\left(\frac{\partial \hat{x}}{\partial x_{zf}}\right)}_{DOF} \tag{3}$$

- It approximates the DOF with the trace of the end-to-end network Jacobian $J = \hat{x}/\partial x_{zf}$, since the Jacobian represents the network sensitivity to small input perturbations and has been previously utilized to analyze robustness in computer vision tasks.
- σ^2 can be estimated by setting the sum of the first two terms in the SURE expression to zero, yielding the following expression:

$$\mathrm{SURE} = \sigma^2 \mathrm{tr}(J) \tag{4}$$

SURE for Uncertainty Analysis: Gaussian residuals with density compensation

- The key assumption behind SURE is that the residual model is Gaussian with zero mean. However, it is not safe to assume that the undersampling noise in MRI reconstruction inherits this property.
- We introduce density compensation on the input image as a way of enforcing zero-mean residuals. This approach has the added benefit of making artifacts independent of the underlying image and we find that it significantly increases residual normality
- Given a 2D sampling mask Ω, we can treat each element of the mask as a Bernouli random variable with a certain probability $D_{i,j}$, where $\mathbb{E}[\Omega] = D$. We can define a density compensated zero-filled image as follows:

$$\hat{x}_{zf} = x_0 + \underbrace{(F^{-1}D^{-1}\Omega F - I)x_0}_{:=r} \tag{5}$$

- In addition,

$$^2 = \mathrm{tr}(x_0^H(F^{-1}D^{-1}\Omega F - I)x_0) \tag{6}$$

- where x_0 is the groundtruth. In practice we do not have access to the ground truth image x0, and instead rely on the approximation for σ^2 for the residual variance.

Empirical Evaluations

- Dataset
  - The Knee dataset: 19 patients with a 3T GE MR750 scanner.
  - Each volume consisted of 320 2D slices of dimension 320×256
- Network
  - The VAE encoder was composed of 4 layers formed through a sequence of strided convolution operations followed by ReLU activations and batch normalization.
- Residual Distrubution with Density Compensation
  - We produce histograms and Q-Q plots of the residuals at various undersampling rates, by considering the differences for individual pixels across test images.
  - To overcome the lack of normality in the residuals, we apply our density compensation method.
- SURE results
  - we produce correlations of SURE versus MSE.
  - the results show that even with relatively high undersampling, SURE can be used to effectively estimate risk in medical image reconstruction.