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Revisiting large scale factor models for multivariate economic analysis - a FAVAR approach

Danish title: Store faktormodeller til multivariat økonomisk analyse - et FAVAR studie

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Abstract

We consider a clinical econometric implementation of the Factor-Augmented VAR (*FAVAR*) model of Bernanke, Boivin, and Eliasz (2005), examining it in a variety of exercises, including out-of-sample forecasting and structural analysis. We provide evidence of strong parameter instability in the first and second moments of the model-residuals and suggest the implementation of breakpoints and GARCH(1,1)-residuals as to account for instabilities in the first and second moment respectively. Factors are extracted directly from a parametric dynamic factor model, as opposed to the *Principal-Components*-approach considered by Bernanke et al. (2005). We find mixed support for the model. In structural analysis, the explanatory power of the model with dynamic factors is found to be significantly higher than those using conventional principal component factors, both using the standard recursive scheme of Bernanke et al. (2005) and with alternative long-run identifying restrictions. However, we also show that by using formal statistical methods to identify the model structure and incorporating conditional heteroscedasticity, the promising results of Bernanke et al. (2005) are no longer statistically significant. Furthermore, in *out-of-sample* prediction exercises, the model is found to be on par with more conventional, simpler models.

Keywords: Dynamic Factor Models, Factor-Augmented-VAR, high-dimensional analysis, conditional heteroscedasticity, forecasting

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Introduction 1

Few methodologies have had as much influence on empirical macroeconomics as the *vector-autoregressive* (VAR) framework, pioneered by Nobel laureate Sims (1972; 1980). The VAR is a simple yet powerful statistical model that allows the researcher to conduct several useful tasks, uncover statistical and, to some extent, structural relationships, and by extension, reliable forecasts of multivariate systems of variables.

Even though the VAR framework has stood the test of time, it has been subject to critique and has yielded controversial results. In particular, the counter-intuitive finding of Sims (1992), suggesting that hikes in the nominal interest rate increase the general price level - a result commonly dubbed the *»price-puzzle.«* Commonly, researchers believe that the prize-puzzle is a result of neglecting essential variables in the system - *Omitted Variable Bias*.

Solving this puzzle should seem trivial, include the variables causing the bias. However, the VAR-methodology suffers from the so-called *»Curse of dimensionality.«* The number of parameters to be estimated increases as the square of the number of predictors. Therefore, the scarcity of macroeconomic data in the time dimension renders the estimation of a VAR-model with many predictors infeasible very quickly.

However, Bernanke, Boivin, and Eliasz (2005) suggested an alternative approach. Namely, the *factor-augmented vector autoregressive*-framework; abbreviated as the *FAVAR*-model. In the FAVAR-framework, the model's informational structure is significantly increased while keeping the dimensionality of the model to a bare minimum. In the FAVAR dimensionality reduction is performed using static or dynamic factor models. Thus, this approach gracefully combines the simplicity of VAR-analysis with the complex data-rich structure of factor models.

The FAVAR model is a compelling framework for multivariate analysis; however, most fallacies of the regular VAR-model are usually not addressed in the literature. Many (usually) unreasonable assumptions are imposed, such as parameter stability, homoscedasticity, and a firm reliance on a pre-specified model framework. In this present thesis, we take a step back and examine the model in a few clinical econometric exercises. We address the fallacies mentioned above and assess how the model fares in a more clinical setup.

In their seminal paper, Bernanke et al. (2005) propose two approaches to the estimation of FAVARs, the first of which estimates the factors with principal component analysis and spends little time on the actual parameterization of the dynamic factor model. In addition, they propose a one-step estimation technique based on a Gibbs-sampler procedure. Bernanke et al. (2005) explicitly mention the estimation procedure as a possibility for improvement in the model. Several researchers have addressed this question. Bork (2009) uses the *Expectation-Maximization*-algorithm to provide an alternative to the one-step procedure considered by Bernanke et al. (2005). More recently, Bai, Li, and Lu (2016) suggest a two-step maximum-likelihood alternative to PCA, incorporating observable factors into the factor model during estimation.

In this thesis, we will try to extend upon the Bernanke et al. (2005) model, by remapping the factor estimation. In doing so, we first briefly consider the evolution of factor modeling. Stock and Watson (2012) considers three generations of dynamic factor models. The first relies on a fully parametric maximum likelihood estimation, resulting in infeasibility when the dimensions of the cross-section becomes large. Examples of this approach are seen in Quah and Sargent (1993); Watson and Engle (1983). This approach is not appropriate in the present analysis due to the size of our cross-section.

The second-generation of estimators address this issue instead by relying on non-parametric approaches to obtain weighted cross-sectional averages of the time-series of interest. These estimators are usually implemented with principal component analysis, although other methods are available. The validity and asymptotic properties are the subjects of a vast literature, see *e.g.,* Bai (2003); Bai and Ng (2009), showing in particular that as the number of time-series under consideration approaches ∞ the principal component factors span the true factor space. This approach is the one adopted in Bernanke et al. (2005). While promising in theory, simulation studies conducted by Boivin and Ng (2006) suggest that consistency is lost if the idiosyncratic errors are cross-sectionally heteroscedastic, which, as we will see, is indeed the case for our panel.

The third generation relies on ideas from both generations. A particularly interesting idea is presented by Doz, Giannone, and Reichlin (2011), who suggest a two-step estimator relying on the non-parametric approach to obtain initial estimates of parameters. Conditioning on the initial estimates, a fully parametric state-space model can be estimated and filtered using the Kalman-smoother, yielding factors constructed as a weighted average over the cross-section, as in the second generation, and over the time-domain, as in the first generation of models. In this thesis, we adopt the third generation and implement it in the FAVAR-framework. To this end, we will use the Doz et al. (2011) Kalman-smoother approach to extract the factors. We find that especially for structural analysis, this approach results in more reliable results, as the explainability of the panel increases quite significantly.

We present evidence of strong parameter instability of the model, suggesting that the original sample used by Bernanke et al. (2005) is inappropriate due to structural breaks occurring in various time-series. This is a critique often raised in the VAR-literature; due to the scarcity of data in the time-dimension, researchers have historically used all available data, however as time progresses and more data becomes available, we can afford to be more selective when choosing appropriate samples. We find evidence that imposing a structural break in January 1984 stabilizes the mean sufficiently; however, it is insufficient to address strong heteroscedasticity; therefore, we introduce multivariate GARCH(1,1)-residuals during estimation.

Our analysis extends the period of interest further than the considered references.

After the great financial crisis of 2008, the FED conducted expansionary monetary policy to the extent that conventional tools were no longer effective due to the Federal Funds Rate (*FFR*) being bound from below by the effective lower bound (ELB). This makes the FFR a non-continuous series, effectively removing the informational structure of the FFR from 2009 through 2015. On that account, most research in this area, even in recent times, has focused on the period before 2009. One can discuss the current relevance of a time-series model if it is only able to accommodate movements of the considered variables up until a specific point in time. Therefore the present thesis seeks to shed light on the relevance of FAVAR models in the present time. To meet this end, we rely on the results of Wu and Xia (2016), the so-called shadow-rate, to recover the lost informational structure in the *post*-2008 crisis period. We will discuss the usage of the shadow-rate further in the data-section. The central point is that the shadow-rate post-2009 and the federal funds rate before 2008, exhibits similar dynamic correlational and cross-correlational structure with other central variables.

Note that the idea of using the shadow-rate as the monetary policy instrument is not unique. Wu and Xia (2016) verify their proposed shadow-rate by examining it in a FAVAR-model. Recent results closely linked to the validity of the shadow-rate approach are provided by Debortoli, Galí, and Gambetti (2020), who find evidence that the central macro-variables considered in this thesis (among several others) are hardly affected by a binding lower bound. They provide evidence on volatility measures as well as estimated responses in VAR-models.

We center the thesis around the FAVAR, and subset the analysis into three components. In the first part, we assess the model's forecasting performance for three key macroeconomic variables, the nominal interest rate, growth rate of industrial production, and inflation. In the second part, we re-examine the analysis of the seminal paper of Bernanke et al. (2005) using our alternative DFM-implementation. This leads directly up to the third part: implementing a Blanchard and Quah (1989)-type restriction scheme (long-run restrictions) on the model and performing structural analysis. The long-run identification scheme is not a common choice for the trivariate monetary VAR-model; however, in a recent paper, Lanne, Meitz, and Saikkonen (2017) find little support for the more commonly used recursive scheme in a related VAR. This result, as well, as the arbitrary nature of recursive VARs, leads us to believe that there is a benefit to gain by exploring alternatives. A discussion on fallacies of the recursive VAR is presented by Stock and Watson (2001).

2 Methodology

In this chapter, we describe the central methodology used throughout the analysis. This includes (structural)-VAR theory, a description of the factor construction scheme, the linkage between structural VARs and structural FAVARs, and incorporation of conditional heteroscedasticity.

2.1 Vector Autoregression

The main building block of this thesis is the VAR-framework. To form the model let $Y_t = \{y_{1,t}, y_{2,t}, \ldots, y_{n,t}\}$ denote the vector of time-series to be examined. A VAR of order p denoted VAR(p) is then given:

$$Y_t = \alpha + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + \varepsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma) \tag{2.1}$$

Where α is a $n \times 1$-vector of intercepts and $\Phi_i, \forall i = 1, \ldots, p$ is the $n \times n$ matrices of slope coefficients. The covariance matrix of the innovations Σ is a diagonal matrix such that covariances (non-diagonal entries) are all zero, this is a stringent assumption which we will relax later. The VAR-process can be written in lag-polynomial notation,

$$\Phi(L) Y_t = \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \Sigma) \tag{2.2}$$

where:

$$\Phi(L) = \left(\boldsymbol{I}_n - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p\right) \tag{2.3}$$

Where L is the ordinary lag-operator obliging to regular rules. For forecasting purposes an alternative representation of the VAR-model turns out to be very useful. The companion notation lets us rewrite the VAR(p)-system into a VAR(1) representation. To see this consider redefining a new dependent variable constructed such that $\boldsymbol{Y}_t' = \left[Y_t', Y_{t-1}', \ldots, Y_{t-p}'\right]'$. A VAR(1) in $\boldsymbol{Y}_t$ is equivalent to (2.1) with a coefficient matrix on the form:

$$\boldsymbol{\Phi} = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{p-1} & \Phi_p \\ \boldsymbol{I}_n & 0 & \cdots & 0 & 0 \\ 0 & \boldsymbol{I}_n & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & \boldsymbol{I}_n & 0 \end{bmatrix}$$

This matrix is known as the companion matrix and solves:

$$\boldsymbol{Y}_t = \boldsymbol{\alpha} + \boldsymbol{\Phi}\boldsymbol{Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{2.4}$$

The companion form is flexible and is often used to test for VAR stability. The stability condition is fulfilled if all eigenvalues of the companion matrix are strictly smaller than 1 in modulus. Ensuring the series can be represented at each t as the sum of a deterministic part and a discounted sum of stochastic errors, often referred to as the Wold-Decomposition. By backward recursion $\boldsymbol{Y}_t$ can be represented:

$$\begin{aligned} (\boldsymbol{I} - \boldsymbol{\Phi}L)\boldsymbol{Y}_t &= \boldsymbol{\alpha} + \boldsymbol{\varepsilon}_t \\ \boldsymbol{Y}_t &= (\boldsymbol{I} - \boldsymbol{\Phi})^{-1}\boldsymbol{\alpha} + (\boldsymbol{I} - \boldsymbol{\Phi}L)^{-1}\boldsymbol{\varepsilon}_t \\ &= \boldsymbol{\mu} + \sum_{i=0}^{\infty} \boldsymbol{\Psi}^i \boldsymbol{\varepsilon}_{t-i} \end{aligned} \tag{2.5}$$

Where the stability condition ensures that the infinite sum exists, implying decaying memory in shocks, equivalently ensures that the VAR is stationary and invertible and the VMA(∞) exists and is non-explosive.

As it turns out, the companion form is also very convenient for forecasting and structural analysis. Lütkepohl (2005) shows that the forecast minimizing the mean squared forecast error and several other loss-functions for the VAR(1), and by extension the VAR(1)-representation of a VAR(p) is given:

$$\boldsymbol{Y}_{t+h|t} = \left(\boldsymbol{I}_n + \boldsymbol{\Phi}_1 + \ldots + \boldsymbol{\Phi}_1^{h-1}\right)\boldsymbol{\alpha} + \boldsymbol{\Phi}^h \boldsymbol{Y}_t \tag{2.6}$$

2.1.1 Structural relationships

Relaxing the assumption of Σ being diagonal such that the innovations of the model are now covarying. We consider a VAR with a non-identity impact matrix, throughout this part we will, without loss of generality, assume that the constant-vector α is the zero-vector, such that the system given:

$$\Phi_0 Y_t = \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + u_t, \qquad u_t \sim \mathcal{N}(0, \Sigma) \tag{2.7}$$

Where the impact matrix, $\Phi_0 \neq \boldsymbol{I}_n$, implies that contemporaneous effects are present in the system. The covariance matrix of the structural shocks Σ is diagonal and normalized to identity. With lag-polynomial notation:

$$\Phi(L) Y_t = u_t \tag{2.8}$$

where

$$\Phi(L) = \left(\Phi_0 - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p\right) \tag{2.9}$$

Assuming that Φ_0 is non-singular; then $\exists \Phi_0^{-1}$ and the system has a representation:

$$\begin{aligned} Y_t &= \Phi_0^{-1}\Phi_1 Y_{t-1} + \Phi_0^{-1}\Phi_2 Y_{t-2} + \ldots + \Phi_0^{-1}\Phi_p Y_{t-p} + \Phi_0^{-1} u_t \\ Y_t &= B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + B_p Y_{t-p} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma_\epsilon) \end{aligned} \tag{2.10}$$

Where $\Sigma_\epsilon = \Phi_0^{-1}\Sigma\Phi_0^{-1\prime}$ and $B_i = \Phi_0^{-1}\Phi_i \quad \forall i \in \{1, 2 \ldots, p\}$. It is clear that the covariance matrix of the residuals is no longer diagonal, and *ceteris-paribus* inference is not plausible. The model-representation of (2.10) is typically referred to as the *reduced-form VAR* and can be readily estimated with conventional methods. In the reduced-form VAR, the residuals are linear transformations of the structural errors $\epsilon_t = \Phi_0^{-1} t$. Invertibility further ensures the existence of a VMA(∞) representation of both the structural- and the reduced-form VAR

$$Y_t = \Phi(L)^{-1} u_t \tag{2.11}$$

$$Y_t = B(L)^{-1} \epsilon_t \tag{2.12}$$

The challenge facing the econometrician is to determine the impact matrix Φ_0 to recover the structural innovations to the system. We consider several approaches to identify Φ_0^{-1}. In the following sections, we will consider the pros and cons, as well as the theory and main ideas behind these structural restrictions.

2.1.2 Recursive identification

The simplest and most often considered identification of SVAR-systems is the recursive identification scheme. Considering the relation:

$$\Sigma_\epsilon = \Phi_0^{-1} \Sigma \Phi_0^{-1\prime} \tag{2.13}$$

By normalization of the structural errors; $\Sigma = \boldsymbol{I}_n$.

$$\Sigma_\epsilon = \Phi_0^{-1} \Phi_0^{-1\prime} \tag{2.14}$$

Due to the symmetric nature of the covariance matrix, Σ_ϵ contains unique entries given $n\left(n+1\right)/2$, where n indicates the number of endogenous variables. Φ_0^{-1}, on the other hand, is not assumed to be symmetric, and thus contains n^2 unique entries. Therefore the system (2.13) is not uniquely identified without imposing additional restrictions. This is the core of the SVAR-estimation problem.

Unique identification can be recovered by imposing $n^2 - n\left(n+1\right)/2$ restrictions. The most popular method due to computational simplicity is by Cholesky factorization. The Cholesky factorization stems from the result that any positive-semidefinite matrix, such as the considered covariance matrix has a representation given:

$$\Sigma_\epsilon = HH'$$

Where H is a lower triangular matrix. Using the Cholesky factorization, we set $H = \Phi_0^{-1}$. Φ_0^{-1} is computed by Cholesky factorization of Σ_ϵ

$$\Phi_0^{-1} = \mathrm{chol}\left(\Sigma_\epsilon\right)$$

This formulation indicates orthogonal errors, unique to the causal ordering of the system. The main drawback of this approach is the strict restrictions on the contemporaneous impact matrix Φ_0. The first variable in the system is only contemporaneously affected by the first shock; the second variable is contemporaneously related to the first and the second shock, and so on.

The identification scheme holds other disadvantages, namely that some variables are assumed to be "less" endogenous than others; therefore, the econometrician has to order the variables in a meaningful way - based on economic theory. However, this is somewhat challenging with our factor structure as a formal economic interpretation of the factors is difficult to retrieve. Moreover, the contemporaneous ordering remains somewhat arbitrary, as the strictness of the restrictions moves with the sampling intervals. With monthly sampling, the recursive restrictions suggest that the last ordered variable may not affect the other variables within the month, decreasing the sampling frequency to quarterly sampling, the last variable may not affect the others within the quarter. Recursive causal ordering is by far the most common identification scheme, and theoretical justification is often independent of sampling frequency. The strict theoretical justification is thus often vague under the contemporaneous recursive scheme.

The recursive identification scheme, is the one used by Bernanke et al. (2005). The identified model is referred to by Stock and Watson (2001) as a recursive VAR, rather than a structural VAR. If the contemporaneous identification scheme considered contains restrictions incompatible with the recursive structure or non-zero restrictions, an alternative approach is full information maximum likelihood estimation, known as non-recursive timing restrictions, see *e.g.,* Bernanke and Mihov (1998); Gordon and Leeper (1994).

2.1.3 Long-run restrictions

An alternative identification scheme is that of Blanchard and Quah (1989). Where, instead of imposing timing restrictions, we restrict the long-run relations between variables and independent shocks. Consider the VMA-representations in (2.11) and

(2.12), and recall the relationships between the reduced-form- and the structural VAR.

$$\epsilon_t = \Phi_0^{-1} u_t \tag{2.15}$$

$$\Sigma_\epsilon = \Phi_0^{-1} \Phi_0^{-1\prime} \tag{2.16}$$

$$B\left(L\right) = \Phi_0^{-1} \Phi\left(L\right) \tag{2.17}$$

Post-multiply (2.17) with the inverse structural lag polynomial $\Phi\left(L\right)^{-1}$:

$$\Phi_0^{-1} = B\left(L\right) \Phi\left(L\right)^{-1} \tag{2.18}$$

The associated long-run polynomials, *i.e.* setting $L = 1$:

$$\Phi_0^{-1} = B\left(1\right) \Phi\left(1\right)^{-1} \tag{2.19}$$

Where, $B\left(1\right) = \left(\boldsymbol{I}_n - B_1 - B_2 - \ldots - B_p\right)$ and $\Phi\left(1\right) = \left(\Phi_0 - \Phi_1 - \Phi_2 - \ldots - \Phi_p\right)$. Combining (2.19) with (2.16) yields

$$\Sigma_\epsilon = B\left(1\right) \Phi\left(1\right)^{-1} \left(B\left(1\right) \Phi\left(1\right)^{-1}\right)' \tag{2.20}$$

Pre- and post-multiplying both sides with $B(1)'$ and $B(1)^{-1\prime}$ respectively yields the long-run covariance matrix:

$$\begin{aligned} B(1)'\Sigma_\epsilon B(1)^{-1\prime} &= B(1)^{-1} B\left(1\right) \Phi\left(1\right)^{-1} \left(B\left(1\right) \Phi\left(1\right)^{-1}\right)' B(1)^{-1\prime} \\ &= \Phi\left(1\right)^{-1} \Phi\left(1\right)^{-1\prime} \end{aligned} \tag{2.21}$$

$B(1)'\Sigma_\epsilon B(1)^{-1\prime}$ is estimated in the preliminary step as the parameters of the reduced-form VAR. By restrictions on $\Phi\left(1\right)^{-1}$, we pin down enough elements with zero restrictions to reach exact identification. The reduced-form VAR gives us $K(K+1)/2$ elements; to reach exact identification of the K^2 equations, we need to impose $K(K-1)/2$ additional constraints. These can be pinpointed with numerical methods. Alternatively, we can rely on ideas from the recursive identification scheme. Realizing that $B(1)'\Sigma_\epsilon B(1)^{-1\prime}$ is a covariance-matrix, it is positive semidefinite, and therefore obeys the Cholesky factorization. By Cholesky factorization of the covariance matrix, we retrieve exactly $K(K-1)/2$ additional restrictions, and the system is exactly identified.

$$\Phi\left(1\right)^{-1} = \text{chol}\left(B(1)'\Sigma_{\epsilon}B(1)^{-1\prime}\right) \tag{2.22}$$

Note here that $B(1)^{-1}$ only has a closed-form solution for a stable VAR. Hence stability is strictly required for identification by long-run restrictions.

With enough elements restricted in $\Phi\left(1\right)^{-1}$, Φ_0^{-1} is recovered by the relation:

$$\Phi_0^{-1} = B\left(1\right)\Phi\left(1\right)^{-1} \tag{2.23}$$

In the context of long-run restrictions in the SVAR-framework, the source of identification is almost exclusively based on neutrality theories. One can identify shocks that are output-neutral, demand shocks in Blanchard and Quah (1989)-terminology, while supply-side shocks, such as technology shocks, are considered to affect the level of output permanently. When we use the recursive scheme in this context, the idea is that we order the equations in the system that contributes to supply-shocks over output, while the demand shocks are ordered below.

While often considered a more refined identification scheme than the recursive VAR, long-run restrictions are not without limitations. Significant points of critique stem from the discussion of Faust and Leeper (1997); in particular, their discussion on structural shocks has been influential. They argue that the low dimensionality of VARs generally leads to the structural shocks being aggregates several underlying shocks, which might be both supply and demand shocks.

The FAVAR might provide a framework robust to this critique under correct specification. The underlying structural innovations to the factors provide a relatively simple way to disentangle the structural shocks to separate supply/demand concepts. However, the purely statistical nature of the factors implies that one must be careful when assigning formal interpretation to factor shocks in the structural FAVAR.

Another point raised by Faust and Leeper (1997) is the problem arising when conducting inference on the infinite future using a finite sample. The FAVAR in this context is no better suited than a regular VAR model. We acknowledge this, but note that this feature is inevitable and will, almost surely, continue to be so forever.

2.2 Dynamic Factor Models

In Bernanke et al. (2005), two different approaches to the estimation of a FAVAR-model are presented. A single-step Bayesian approach, in which the model likelihood of the dynamic factor model and the VAR are estimated jointly. And in addition, a two-step approach where the factor model is estimated in a first step, whereafter the FAVAR is constructed as a regular VAR-model in latent factors and observables. By the findings of Bernanke et al. (2005), it is not clear which one, if any, is superior to the other. However, the two-step approach seems to provide results more in line with theoretical findings and intuition.

In the present analysis, we will rely solely on a modified two-step approach. The two-step approach is much more computationally efficient; our implementation of the two-step approach is slightly less computationally efficient than the one presented in Bernanke et al. (2005). Essentially we allow parameterized time-domain dynamics by filtering and smoothing the factors during the estimation of the dynamic factor model.

The first part of the computational analysis, consists, therefore, solely on constructing the dynamic factor model (*DFM*) and extraction of smoothed factors. The main idea behind the DFM is that we can use a few latent factors, F, to explain much of the variance and co-movements of a large data-set. We consider the DFM as presented by Stock and Watson (2012). The formal DFM is written:

$$X_t = \lambda\left(L\right) F_t + \eta_t \tag{2.24}$$

Where X_t and η_t are the vectors of time-series and idiosyncratic errors respectively and both of dimension $N \times 1$, F_t is the vector of latent factors with dimension $K \times 1$ where $N \gg K$ and lastly $\lambda\left(L\right)$ is a conformable lag-polynomial of dimension $N \times K$.

In the special case where the lag-polynomial is of order 1, and F_t does not enter (2.24) with any lags, said model is referred to as a static factor model. The estimated coefficients in the lag-polynomial are referred to as the dynamic factor loadings. The product of the lag polynomial and the latent factor for each series in X_t is referred to as the common component of the respective series. By stacking the vectors similar to the companion-representation of the VAR-model in (2.4); (2.24) can be represented as a static factor model:

$$X_t = \mathbf{\Lambda} \boldsymbol{F}_t + \eta_t \tag{2.25}$$

Where $\boldsymbol{F_t} = \left[F'_t, F'_{t-1}, \ldots, F'_{t-p-1}\right]'$ and of dimensions $r \times 1$ with $r = K\,(p-1)$, where r is referred to as the number of static factors. This representation of the model is referred to as the static representation of the dynamic factor model as lags of $\boldsymbol{F}_t$ does not enter (2.25). $\mathbf{\Lambda}$ denotes the stacked loadings matrices.

It is common, and often convenient, to assume that the dynamics of F_t is given by a VAR(p)-process, such that:

$$F_t = \Xi\,(L)\,F_{t-1} + e_t \tag{2.26}$$

Where $\Xi\,(L)$ is a $K \times K$-lag polynomial of finite lag order and e_t is a $q \times 1$-vector of factor innovations where $q = K$, note that this specification implies that the number of dynamic shocks e_t is equal to the number of factors we use in the FAVAR, the usual notation follows $q \leq K$; often, however, empirical work sets $q = K$. Formal tests by Stock and Watson (2005) find that $q = K$ is not an unreasonable assumption especially for smaller K. The common shocks, e_t, in this specification are crucial as they are the main source of large co-movements in X_t. In this set-up, e_t can be interpreted as the aggregate shocks to the economy, affecting most variables all at once. The idiosyncratic errors η_t can be thought of instead as series-specific, local shocks driving variance of only one (exact dynamic factor model) or a small subset of variables (approximate dynamic factor model).

Several approaches can be used to retrieve latent factors. We extend the two-step approach proposed by Bernanke et al. (2005). The original two-step approach uses the Stock and Watson (2002b) principal-component estimator of the factor-model to extract factors in the first step. The second step consists of estimating a regular VAR in $[F_t', Y_t']'$. The PCA-estimator of the factors has been shown to consistently recover the true factor space, as N and T, tends to ∞. However, as shown by Boivin and Ng (2006), the estimator loses efficiency when the idiosyncratic component η_t is large relative to the common component, and when the volatility of η is varying over the cross-section (cross-sectional heteroscedasticity).

Our approach introduces an intermediary step, which is the Kalman-smoother based estimator of Doz et al. (2011). The consistency issue of PCA-factors with cross-sectional heteroscedasticity is addressed in the paper of Doz et al. (2011) proving that the bias arising from mild misspecification of the variance/covariance matrix of the idiosyncratic component, is negligible for large N (cross-section). Nevertheless, the principal component factors still bear a significant load in our analysis, as they are used to parameterize the estimator of Doz et al. (2011). Therefore we briefly present it below.

The regular PCA-estimation method follows from the nonlinear least-squares estimator

$$V\left(\tilde{\boldsymbol{F}}_t, \tilde{\boldsymbol{\Lambda}}\right) = (NT)^{-1} \sum_i \sum_t \left(x_{it} - \tilde{\boldsymbol{\Lambda}}_i \tilde{\boldsymbol{F}}_t\right) \tag{2.27}$$

A tilde denotes hypothetical, non-identified, values. An analytical solution is found by concentrating out one of the unidentified parameters, either the loadings $\boldsymbol{\Lambda}$ or the factors $\boldsymbol{F}_t$. This is done by considering the theoretical values of either the factors or loadings yielding the maximum value of the objective function $V(\cdot)$, we denote this matrix with a hat. Conditional on the optimum factors $\hat{\boldsymbol{F}}_t$, Stock and Watson (2002b) show that maximizing (2.27) is equivalent to minimizing trace $\left(\tilde{\boldsymbol{\Lambda}}' X' X \tilde{\boldsymbol{\Lambda}}\right)$ s.t. $\tilde{\boldsymbol{\Lambda}}'\tilde{\boldsymbol{\Lambda}}/N = \boldsymbol{I}_r$, where r denotes the number of static factors. The normalization constraint is imposed to ensure a unique identification of the factors, as they are identified up to a normalization, and in general holds no natural unit of measurement nor known scale. This normalization is standard to related literature.

The analytical solution as given by Stock and Watson (2002b) is found by setting $\tilde{\boldsymbol{\Lambda}}$ equal to the normalized eigenvectors associated with the r largest eigenvalues of XX', and factors are backed out as:

$$\hat{\boldsymbol{F}} = X'\hat{\boldsymbol{\Lambda}}/N \tag{2.28}$$

In the factor model literature, our specification is known as a large ($N \gg K$) approximate (no orthogonality restriction on idiosyncratic errors) dynamic (common shocks affect X_t both contemporaneously and with lags) factor model. We treat the idiosyncratic components as a homoscedastic stochastic process during estimation, but we note that, as shown by Doz et al. (2011), even under mild misspecification of η their estimator remains consistent, efficient and unbiased.

A problem arising in factor modeling is to determine the optimal number of factors. In a static factor environment, Cattell (1966) suggested that one can visually inspect so-called scree-plots, plotting the additional proportion of variance explained by each additional factor, to determine the optimal number of factors. While scree-plots might provide intuition and a rough idea about the appropriate number of factors, this approach is invalid in the present analysis as it is designed for static factor analysis. We are interested in the number of dynamic shocks or primitive factors. Regular information criteria, such as those of Akaike (1973); Schwarz (1978) are not applicable for panel-data, and those proposed by Bai and Ng (2002), are not applicable in our case, as they similar to the scree approach are developed to accommodate static factor models and only under strict assumptions can accommodate DFMs (Hallin & Liška, 2007). To see the problem, we follow Breitung and Choi (2013) and consider a simple one-factor dynamic factor model in one lag:

$$\begin{aligned} x_t &= \lambda_1 F_t + \lambda_2 F_{t-1} + \eta_t \\ F_t &= \Xi F_{t-1} + e_t \end{aligned}$$

The true number of primitive factors and dynamic shocks is one, F_t and e_t, while criterion developed for static factor models, would consider the lagged factor a separate factor and suggest the presence of two factors. This suggests that in the presence of intertemporal dynamics, the Bai and Ng (2002) criterion are likely to overestimate K, the number of dynamic factors, a result confirmed by a Monte-Carlo study by Hallin and Liška (2007). We are interested in the number of dynamic shocks. Therefore we rely on the framework for dynamic factor models suggested by Hallin and Liška (2007). To consistently recover the true number of dynamic shocks Hallin and Liška (2007) considers the generalized PC estimator of Mario, Reichlin, Hallin, and Lippi (2000), their framework relies on the fact that the number of primitive shocks equals the number of, at least linearly, diverging eigenvalues of the sample spectral density matrix when the cross-section of the panel, N, tends to ∞.

Having determined an estimate of the optimal number of factors, we are interested in the lag-structure of the dynamic factor model. To obtain a reliable estimate, we treat the latent principal component factors, as the true ones; the validity of this approach is proven by Stock and Watson (2002a) showing that under appropriate assumptions, the PCA-factors are a consistent estimate of the true DFM factors. We then consider the auto-correlational structure of F_t; we use correlelograms and regular information criterion for a formal way to determine optimum lag-structure of (2.26). With the structure of the dynamic factor model determined, we recast it into a parameterized state-space representation and smooth the estimates with a Kalman procedure. The explicit parameterization and filter are laid out in the following section.

2.2.1 Kalman Smoother in the factor model

This section will map out the procedure adapted and modified from Doz et al. (2011). Overall the methodology proposed by Doz et al. (2011) consists of two steps, in the first step, we use PCA to extract preliminary factor estimates, then the parameters of the DFM are estimated by regressing factors on the X_t panel and the factors on factor lags. The second step consists of casting the parameterized model into a state-space form and extracting the factors by a single run of the Kalman filter/smoother.

We consider a state space system representation of the DFM as presented above such that:

$$X_t = \lambda(L) F_t + \eta_t \qquad \eta_t \sim \mathcal{N}(0, R) \tag{2.29}$$

$$F_t = \Xi(L) F_{t-1} + e_t \qquad e_t \sim \mathcal{N}(0, Q) \tag{2.30}$$

Where (2.29) is the signal-equation and (2.30) is the state-equation. Both the signal noise and state innovations are assumed to be Gaussian, this implies that the system is a linear Gaussian state-space model. The Gaussianity assumption is crucial as it ensures that the Kalman smoother is the optimal MSE linear filter.

Rewriting (2.29) and (2.30) to companion representations:

$$X_t = \boldsymbol{\Lambda} \boldsymbol{F}_t + \boldsymbol{\eta}_t \tag{2.31}$$

$$\boldsymbol{F}_t = \boldsymbol{\Xi} \boldsymbol{F}_{t-1} + \boldsymbol{e}_t \tag{2.32}$$

The Kalman Filter utilizes the joint Gaussian over the T-observations. Applying the Kalman Smoother to the filtered estimates of F_t, we obtain a series of the optimal estimated factors utilizing all available information. The recursions are subset into three parts. Two forward recursions; the prediction step and the filtering step, and one backward recursion; the smoothing step.

The prediction step uses all available information up to $t-1$ to produce an *a priori* estimate of the conditional mean and covariance matrix of the state at time t. To start the recursion, we have to provide the filter with an initial estimate of the mean and the covariance matrix of the state. The filter is susceptible to the initial estimates, but under the right conditions, the estimate is consistent and efficient; therefore, it is of importance that the initial values and the parameterization we provide to the recursion are optimal. We will discuss this in a moment; but first, we present the prediction recursions, these are readily available from equation (2.29):

$$\boldsymbol{F}_{t|t-1} = \boldsymbol{\Xi} \boldsymbol{F}_{t-1|t-1}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{\Xi} \boldsymbol{P}_{t-1|t-1} \boldsymbol{\Xi}' + \boldsymbol{Q}'$$

Where $\boldsymbol{P}_{t|t-1}$ is the predicted covariance of the estimate and $\boldsymbol{F}_{t|t-1}$ is the conditional mean or predicted value of F_t.

From the prediction step we have obtained the *a priori* state estimates, $\boldsymbol{P}_{t|t-1}$ and $\boldsymbol{F}_{t|t-1}$. Entering period t we obtain additional information, namely, that which stems from new observations of X. The Kalman filter uses this information to correct the *a priori*-estimates. This is done by weighing the estimation error of X_t in equation (2.29), obtained using the *a priori* estimate. The recursion is given:

$$\begin{aligned}\boldsymbol{F}_{t|t} &= \boldsymbol{F}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{\Lambda}'\boldsymbol{L}_t\left(X_t - \boldsymbol{\Lambda}\boldsymbol{F}_{t|t-1}\right)\\ \boldsymbol{P}_{t|t} &= \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1}\boldsymbol{\Lambda}'\boldsymbol{L}_t\boldsymbol{\Lambda}\boldsymbol{P}_{t|t-1}\end{aligned} \tag{2.33}$$

Where $\boldsymbol{L}_t$ is the error correction term given by,

$$\boldsymbol{L}_t = \left(\boldsymbol{\Lambda}\boldsymbol{P}_{t|t-1}\boldsymbol{\Lambda}' + \boldsymbol{R}\right)^{-1} \tag{2.34}$$

And can be seen as an importance weight of the t'th residual vector.

A single run of the smoother is then utilized to update the filtered state estimates using all available information, the smoothing procedure ensures minimum MSE estimates and as shown by Doz et al. (2011) leads to consistent estimates of the factors even with mild misspecification of the idiosyncratic component, such as our Gaussian white noise assumption.

The smoother recursions is given by:

$$\begin{aligned}\boldsymbol{F}_{t|T} &= \boldsymbol{F}_{t|t} + \boldsymbol{J}_t\left(\boldsymbol{F}_{t+1|T} - \boldsymbol{F}_{t+1|t}\right)\\ \boldsymbol{P}_{t|T} &= \boldsymbol{P}_{t|t} + \boldsymbol{J}_t\left(\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t}\right)\boldsymbol{J}_t'\end{aligned} \tag{2.35}$$

Where the smoother gain is given by:

$$\boldsymbol{J}_t = \boldsymbol{P}_{t|t}\boldsymbol{\Xi}'\boldsymbol{P}_{t+1|t}^{-1} \tag{2.36}$$

Having laid out the filtering and smoothing process, we are to determine initial state estimates. Doz et al. (2011) shows that preliminary state estimates can be consistently recovered using PCA-analysis, using the following procedure:

1. Extract Principal component factors, $\hat{F}_t$, from X_t, assume for now they are true factors
2. Using $\hat{F}_t$, specify the DFM and determine lag orders in (2.31) and (2.32), using regular methods; information criterion and correlograms.
3. With ordinary least-squares regressions obtain estimates of $\boldsymbol{\Lambda}$, $\boldsymbol{\Xi}$, $\boldsymbol{R}$ and $\boldsymbol{Q}$

For this analysis, the factors extracted from the Kalman Smoother will be treated as the true factors, implying they can be treated as data in the VAR-specification. Ensuring that regular SVAR-identification schemes are applicable. Stock and Watson (2005) provides an overview of the implications such a choice may have on the analysis. The main purpose of this analysis is to assess the observables in the VAR-part; the factor's main purpose in the model is the contribution of additional information to forecasts and impulse response functions. Therefore, we impose a bare minimum of restrictions; when we reconsider the Bernanke et al. (2005), we follow their identification, a fast/slow-scheme, slow variables in X_t reacts to monetary policy shocks only with a lag, while fast variables may react contemporaneously. This identification follows the following scheme:

1. Estimate factors $\hat{F}_t^{all}$ from the full X_t-set
2. Estimate factors $\hat{F}_t^{slow}$ from the subpanel by indicator $slow = 1$.
3. Run regression $\hat{F}_t^{all} = \alpha + \beta_s \hat{F}_t^{slow} + \beta_{FFR} FFR_t + \epsilon_t$
4. The recursive factors are estimated $\hat{F}_t = \hat{F}_t^{all} - \beta_{FFR} FFR_t$
5. VAR in $\left[\hat{F}_t', Y_t'\right]'$ is recursively identified by Cholesky factorization

Bernanke et al. (2005) notes that extracting factors separately from the slow and fast subpanels individually might be superior if components in the factors does respond contemporaneously to monetary policy shocks. Such that the VAR is recursively identified in $\left[\hat{F}_t^{slow,\prime}, Y_t', \hat{F}_t^{fast,\prime}\right]'$. However they note that because factors extracted solely from the fast variables are strongly correlated with the federal funds rate this approach introduces strong multicollinearity in the VAR.

For the long-run identification, we consider a different approach. We try to extract some economic sense from the factors by considering regressions of $\hat{F}_t$ on X_t. We then extract the top ten loading series as measured by R^2. This gives a quantitative measure of the underlying driving forces behind each factor, *i.e.*, if many productivity-series are loading heavily on a specific factor, we expect that the associated factor shock can be thought of as a component of the aggregate supply shock.

1. Estimate factors $\hat{F}_t^{all}$ from the full X_t-set

2. Run regressions $X_{j,t} = \beta_{\hat{F}_i^{all}}(L)\,\hat{F}_{i,t}^{all} + \epsilon_t$ and extract $R^2_{j,i}$

3. For each i extract the top ten series as measured by R^2_j, and qualitative assign most probable shock interpretation by grouping

We then order the FAVAR accordingly such that the latent factors, in general, are ordered above the associated shock, *e.g.,* shock to a productivity factor is ordered just before a shock to productivity as measured by the industrial production index. This accounts for factor shocks being a more general component of the aggregate shock and implies that the forces represented by the factor shock may affect the associated observable in the long-run. As the factors enter the VAR as I(0)-variables, they are not affected in the long-run by any shocks.

Note that forecasts unconditional on future shocks are independent of structural restrictions, this follows from the reduced-form specification in (2.10), as the reduced-form (FA)VAR residuals, are in general cross-correlated *ceteris paribus* assesment is infeasible. However, the reduced-form (FA)VAR is sufficient for forecasts unconditional on structural shocks as the reduced-form residuals, though cross-correlated, are zero-mean and serially uncorrelated processes.

2.3 FAVAR

The reduced-form FAVAR-model of Bernanke et al. (2005) is specified in state-space form as:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \Lambda(L) \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \eta_t \qquad \eta_t \sim \mathcal{N}(0, \Sigma_F) \tag{2.37}$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = B(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}\left(0, \Sigma_{\epsilon,t|t-1}\right) \tag{2.38}$$

From this specification, the link between the FAVAR and the DFM should be clear, in essence, (2.37) can be seen as a DFM augmented with observable factors, this implies in turn that common component of Y_t in (2.37) accounts for 100% of the variance, and that Y_t loads exclusively on Y_t, *i.e.,* the lower-left block of Λ is identity. All uncertainty of Y_t stems from the law of motion for the factors in (2.38). While the uncertainty around X_t stems from both common shocks ϵ_t and individual idiosyncratic components η_t. We allow some dynamics in the volatility of the FAVAR-residuals; hence the time dynamics are introduced, while the idiosyncratic components η_t are assumed to be homoscedastic and unconditionally Gaussian.

Both data-sets Y_t and X_t are demeaned and standardized prior to estimation, hence the lack of a intercept in (2.37) and (2.38). (2.37) represents a dynamic factor model while (2.38) represents a VAR(p)-model in observable factors Y_t and latent factors F_t, and thus admits to regular VAR-identification schemes.

2.3.1 Structural instabilities

A significant critique of the traditional VAR-approach to empirical macroeconomics is that estimation is often conducted on long samples, and model stability is assumed. This assumption relies on constant parameters in the implied policy-functions of the model-economy, which, as shown by Lucas (1976), is an assumption difficult to justify both empirically and theoretically; this result is widely known as the Lucas Critique.

We assess the stability of the VAR-model, with Cusum- and Chow tests equation-by-equation on the full-sample using an FAVAR. In presence of parameter instability, we subsample accordingly. As the main focus of the analysis is the current relevance of the FAVAR, we use the latter sample as the reference point. For assessment of robustness to instabilities we use a breakpoint specification:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{cases} \Lambda_1 (L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \eta_{1,t} & \text{if } t < BP \\ \Lambda_2 (L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \eta_{2,t} & \text{if } t \geq BP \end{cases} \tag{2.39}$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \begin{cases} B_1 (L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \epsilon_{1,t} & \text{if } t < BP \\ B_2 (L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \epsilon_{2,t} & \text{if } t \geq BP \end{cases} \tag{2.40}$$

Where BP is a pre-specified breakpoint indicating a regime-switch in the mean- and/or the volatility-model. It is clear that in the special case where $B_1(L) = B_2(L)$ and $\Lambda_1(L) = \Lambda_2(L)$, this general form reduces to the FAVAR presented in (2.37) and (2.38). Note here that it is possible to add additional breakpoints during estimation, taking into account the degrees of freedom lost for each additional breakpoint.

As to further stabilize the model parameters, and purge implied instability in the second moment, we allow the univariate FAVAR-residuals of the model to follow GARCH(1,1)-processes and jointly obey a *Dynamic Conditional Correlation*- DCC-structure, a multivariate-GARCH model proposed by Engle (2002). As is usually the case with multivariate GARCH-models, especially with a rather sophisticated mean model, full information maximum likelihood is computationally infeasible, therefore we follow Engle (2002); Lütkepohl and Schlaak (2019) and estimate the

DCC-model on the estimated FAVAR GARCH(1,1)-residuals in a quasi-maximum likelihood approach, note that the MGARCH considered by Lütkepohl and Schlaak (2019) is a Generalized Orthogonal- (GO)-GARCH rather than a DCC-GARCH. We choose the DCC-GARCH model as it is a relatively computational efficient method for incorporating multivariate heteroscedasticity. The incorporation of the DCC-GARCH-model for the conditional variance acts primarily as a stability filter for the model and does not affect the mean-model $FAVAR$ directly, in any other way than adaption of the bootstrapping procedure and numerical rather than analytical maximum likelihood estimation.

We infer structural instability according to formal tests and, in addition, macroeconomic events occurring mid-sample. As the allowed form of heteroscedasticity is quite general, breakpoints act primarily to stabilize the mean-model.

2.3.2 Conditional Heteroscedasticity

This section briefly lays out the Multivariate GARCH model we consider driving the FAVAR-innovations. Engle (2002) proposed the DCC-model as a feasible alternative to fully parameterized Multivariate GARCH-, $MGARCH$-models. The idea is that individual residuals $\epsilon_{i,t}$ are jointly conditionally Gaussian:

$$\epsilon_{t|t-1} \sim \mathcal{N}\left(0, \Sigma_{\epsilon,t|t-1}\right) \tag{2.41}$$

Now assume that the independent residuals follow univariate GARCH(1,1)-marginals. That is:

$$\epsilon_{i,t} = \sigma_{i,t} z_{i,t}, \qquad z_{i,t} \sim \mathcal{N}\left(0, 1\right) \tag{2.42}$$

$$\sigma^2_{i,t} = \omega_i + \alpha_i \epsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \tag{2.43}$$

Obeying to standard GARCH stationarity conditions $0 \leq \{\alpha, \beta\} \leq 1$ and $\alpha + \beta \leq 1$. We then define the unconditional correlations matrix (*from the standardized residuals*):

$$\mathbb{E}\left[z_t z_t'\right] = \bar{R} \tag{2.44}$$

From (2.41) retrieve the individual standard deviations, *i.e.,* $D_t = \mathrm{diag}\left(\sqrt{\Sigma_{\varepsilon,t}}\right)$ and $\mathrm{diag}\left(\Sigma_{\varepsilon,t}\right) \sim \mathrm{GARCH}\left(1,1\right)$. The conditional covariance matrix is then decomposed:

$$\Sigma_{\epsilon,t} = D_t R_t D_t \tag{2.45}$$

Where D_t is simply estimated by Gaussian maximum likelihood estimation of the univariate GARCH-processes under the assumption of $R_t = \mathbf{I}_n$ (Hence Quasi-ML instead of ML). To recover the conditional correlation matrix, we state the relation between covariance and correlation:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\left(q_{ii,t}q_{jj,t}\right)^{1/2}} \tag{2.46}$$

We assume that the conditional covariance matrix of the standardized residuals: $Q_t = \{q_{ij,t}\}$, is well modelled by a GARCH$(1,1)$-process, hence a law of motion is constructed:

$$Q_t = \bar{R}\left(1 - a - b\right) + a\left(z_{t-1}z'_{t-1}\right) + bQ_{t-1} \tag{2.47}$$

We assume here, as we did in the univariate marginals that regular GARCH stationarity constraints hold. To transform the estimated covariance, Q_t, back to its correlation-coefficient equivalent, R_t, we use the matrix representation of (2.46).

$$R_t = \mathrm{diag}\left(Q_t\right)^{-1/2} Q_t \,\mathrm{diag}\left(Q_t\right)^{-1/2} \tag{2.48}$$

Note a typo in the paper by Engle (2002) in their equation (25) raises Q_t to the power of -1 instead of $-1/2$. Our computational code follows the log-likelihood as derived by Engle (2002) but augmented with a FAVAR conditional mean-model. To estimate the conditional correlation we utilize the quasi-maximum likelihood estimator of Engle (2002) an estimation method suitable for relatively large systems. However, being a quasi-maximum likelihood approach the estimator looses efficiency but remains consistent and unbiased. The overall model estimation can thus be summarized in parts:

1. Estimate the factors F_t using the Kalman-smoother using the Doz et al. (2011)-estimator.

2. Using the extracted factors jointly estimate the mean- (FAVAR) and the volatility- (GARCH(1,1))-models equation-by-equation, using numerical maximum likelihood under the assumption of identity correlation $R_t = \boldsymbol{I}_n$.

3. Using the GARCH(1,1)-marginals estimate the dynamic conditional correlation model by numerical quasi-maximum likelihood, retrieve conditional correlation, $R_{t|t-1}$, and conditional covariance matrix $\Sigma_{\epsilon,t|t-1},\ \forall t = 1, \dots T$.

For the second and third step we use the built-in non-linear unconstrained[1] optimization procedure in MatLab, `fminunc`; we use the closed-form Gaussian analytical maximum-likelihood coefficients, $B = (X'X)^{-1} X'Y$, as the starting values for the mean model, while the starting values of the GARCH-coefficients are chosen following typical convention in statistical packages; $\alpha = 0.1$ $\beta = 0.8$ and $\omega = \operatorname{var}(\varepsilon_i)(1 - \alpha - \beta)$, where $\operatorname{var}(\varepsilon_i)$ is the empirical variance of the residuals. For the initial values of the DCC-parameters we initialize in a similar way following statistical packages $a = 0.05$ and $b = 0.90$.

To achieve structural identification in the conditional heteroscedastic FAVAR, we follow the likes of Lütkepohl and Schlaak (2019; 2018), and achieve structural identification using the estimator of the unconditional covariance of the GARCH-process, such that $\hat{\Sigma}_\epsilon = T^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t \hat{\epsilon}_t'$. They reach identification using heteroscedasticity, we do so with recursive-contemporaneous and long-run restrictions.

2.4 Structural Analysis of the FAVAR

As is the case with usual SVAR-systems, we can perform structural analysis on the estimated structural FAVAR.

Having imposed structural restrictions and recovered the contemporaneous impact matrix ϕ_0, we have the usual SVAR-tools at our disposal. We will focus on two,

[1]DCC- and GARCH-stationarity constraints are softly imposed directly in the objective function.

namely impulse-response analysis and forecast error variance decomposition. The next subsections will briefly review the modifications necessary to transform SVAR results into structural FAVAR results.

2.4.1 Impulse Response Functions

Impulse-response functions (IRF) is a common point of interest of identified SVAR-models. The IRFs trace out the movements in the endogenous variables following a shock to the stochastic u_t in (2.11).

The typically SVAR-model impulse response analysis is straightforward and given by the estimated coefficients of the VMA-representation of the VAR model. Impulse response functions for the FAVAR is a straightforward extension of the VAR-IRF, namely still given by the VMA(∞) representation of the FAVAR, such that:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Theta\left(L\right) u_t \tag{2.49}$$

The responses of variables in X to shocks in u_t are readily obtained by projecting the VAR-responses into equation (2.37). Note that this accounts only for the common component for each series in X even though the idiosyncratic component may be substantial. As the present analysis is mostly interested in variables in Y, we will focus on the uncertainty on the responses of Y by bootstrapping with the estimated FAVAR-residuals.

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \mathbf{\Lambda}\Theta\left(L\right) u_t \tag{2.50}$$

(2.50) implies a very convenient and simple mapping between the VAR-results and the factor model responses. The confidence of the IRFs on X however, is, by construction, weaker as we add an extra layer of uncertainty through the idiosyncratic errors. Again determined by the variance of each series in X explained by the common component. Series in Y is assumed observable, and their common component in the state-space representation explains by construction 100% of the variability. Thus uncertainty is recovered solely through the VAR residuals. In other words, the IRFs of Y is equivalent to those recovered from a no-factor VAR, except for the extra information contained in F.

As we do allow conditional heteroscedasticity in the form of GARCH(1,1)-residuals standard non-parametric bootstraps are not asymptotically valid. Using regular non-parametric bootstraps, *i.e.,* random sampling with replacement; may lead to the estimated chains of residuals no longer obeying the GARCH-structure. Instead we use a residual-based GARCH bootstrap procedure similar to that considered by Jeong (2017); Lütkepohl and Schlaak (2019).

The algorithm we follow are summarized:

1. Estimate Reduced-form FAVAR-parameters $\hat{B}\left(L\right)$, GARCH-model parameters $\hat{\omega}$, $\hat{\alpha}$, $\hat{\beta}$ and obtain estimates of $\hat{\phi}_0^{-1}$.

2. Using obtained parameters obtain FAVAR reduced-form residuals $\hat{\epsilon}_t = Y_t - \hat{B}\left(L\right)Y_{t-1}$ and filtered volatility $\hat{\sigma}_t^2 = \hat{\omega} + \hat{\alpha}\hat{\epsilon_{t-1}}^2 + \hat{\beta}\hat{\sigma}_{t-1}^2$

3. Using filtered volatility obtain standardized residuals $\hat{z}_t = \text{diag}\left(\hat{\Sigma}_{\epsilon,t|t-1}\right)^{-0.5}\hat{\epsilon}_t$

4. Draw bootstrap sample $\hat{z_t^*}$, transform them to reduced form errors $\hat{\epsilon}_t^* = \text{diag}\left(\hat{\Sigma}_{\epsilon,t|t-1}\right)^{0.5}\hat{z}_t^*$

5. Generate bootstrap simulations $\hat{y^*}_t = \hat{B}_1 y_{t-1}^* + \hat{B}_2 y_{t-1}^* + \ldots + \hat{B}_p y_{t-1}^* + \hat{\epsilon}_t^*$

Repeat steps 3 through 5, k times. In this analysis we use $k = 1.000$ bootstrap replications.

2.4.2 Variance Decomposition

As is the case with the IRFs the linkage between the DFM and the FAVAR allows for a rather simple adjustment to the regular forecast error variance decomposition (FEVD) to accommodate series in X, as shown by Bernanke et al. (2005). For a standard no-factor SVAR model the FEVD is given:

$$VD_i = \frac{\operatorname{var}\left(Y_{t+h|t+h} - Y_{t+h|t} \mid u_{i,t}\right)}{\operatorname{var}\left(Y_{t+h|t+h} - Y_{t+h|t}\right)} \tag{2.51}$$

That is the ratio between the error variance conditional on a given shock in the VAR and the unconditional forecast error variance in the VAR. The structural FAVAR adjustment:

$$VD_{i,\text{FAVAR}} = \frac{\mathbf{\Lambda}_i \operatorname{var}\left(Z_{t+h|t+h} - Z_{t+h|t} \mid u_{i,t}\right) \mathbf{\Lambda}_i'}{\mathbf{\Lambda}_i \operatorname{var}\left(Z_{t+h|t+h} - Z_{t+h|t}\right) \mathbf{\Lambda}_i'} \tag{2.52}$$

Where $Z_t = [F_t', Y_t']'$ and $\mathbf{\Lambda}_i$ denotes the block of parameters in $\mathbf{\Lambda}$ linking the i'th series in X_t to Z_t, in the companion representation.

This formulation yields the percentage of the common component in the factor model explained by each structural innovation in the FAVAR system. Thus, the larger the fraction of variance of $X_{i,t}$ explained by the common component, the larger the confidence in the FEVDs ability to explain the true variance decomposition of $X_{i,t}$. Thus for Y_t, the FEVD is similar to that retrieved from a regular SVAR, as it contains no idiosyncratic error by construction.

2.5 Computational execution

All functions and the main scripts are written in MatLab. A subpart of the codes builds upon the SVAR-toolbox made available by Cesa-Bianchi (2015). This toolbox provides a convenient VAR-structure; the toolbox is heavily modified to accommodate the FAVAR and extended with numerical maximum likelihood estimation procedures, conditional heteroscedasticity, and the GARCH residual-based bootstrap. Most functions rely on the modified FAVAR-structure, and are written specifically for this thesis, but are general in the sense that they can be used to estimate other FAVAR-systems. All computer procedures used along with a replication guide are readily available at `https://github.com/RasmusJensen96/FAVAR` and a summarizing procedure table is available in the appendix, table B.1.

Data 3

Various literature on monetary VAR-systems are considering different sample-spans, we sample an, in contrast to many other studies, extended sample, such that we can subsample according to implied instability. Our total sample thus spans monthly observations of 128 time-series from 01:1959 to 10:2019.

Other authors argue that a structural break in the monetary policy response function occurs in the sample in the midterm of the monetary-regime of Paul Volcker in the first quarter of 1984, see for example Boivin and Giannoni (2006); Clarida, Galí, and Gertler (2000), A figure with the Volcker-recessions shaded is presented in figure A.1 in the appendix. The sample utilized by Bernanke et al. (2005) spans 01:1959 to 08:2001 and assumes model stability over the period. If indeed, population parameters are unstable, we obtain more reliable results if we accommodate different regimes. To this end, if we find evidence of a structural break, we subsample accordingly.

3.1 Factor Data

We retrieve the data-panel for extraction of the latent factors from the *FRED Monthly*-database of McCracken and Ng (2016). The data panel consists of 128 time-series, which can be subset into 8 groups, these are presented in table 3.1.

Table 3.1
FRED-MD summary

Group	Output and Income	Labour Market	Consumption and Orders	Orders and Inventories
No. Series	17	32	10	14
Slow	Yes	Yes	Yes	Yes
Group	Money and Credit	Interest rate and Exchange Rates	Prices	Stock Market
No. Series	14	22	21	4
Slow	No	No	Yes	No

1 "Slow" is an indicator variable utilized under the Bernanke et al. (2005)-identification scheme.

As the panel is unbalanced and has missing observations/mixed frequency-sampling, we impute missing observations implicitly in an initial PCA-procedure with the EM-algorithm as presented in Stock and Watson (2002b). The procedure for the imputation is supplied along with the data in the companion-repository of McCracken and Ng (2016). A variable in the factor-set is defined as slow-moving if it is assumed to not react contemporaneously to monetary policy shocks. This indicator-variable is only used in the present analysis when we revisit the results of Bernanke et al. (2005). To enforce stationarity, we follow the transformations of McCracken and Ng (2016). A table listing variables in the factor data set along with transformations and brief descriptions are presented in appendix C. After transformation to stationarity, the data are demeaned and standardized to unit-variance.

3.2 VAR Data

The observable factors are retrieved directly from the FRED-database. Due to the factor-structure of the model, we can use a scarce Y-matrix without loss of informational content. The Y-matrix consists, therefore, of 3 (semi)-observable time-series, the industrial production index, consumer price index, and the (shadow-rate augmented) federal funds rate. In the BBE benchmark specification, only the federal funds rate is assumed to be observable; we, however, extend the model to accommodate the benchmark trivariate VAR-specification of Stock and Watson (2001), this is necessary for the long-run structural identification scheme we consider later in the analysis.

The trivariate VAR-system consisting of a real activity measure, an inflation measure, and a measure of monetary policy is theoretically justified by the well-famous Taylor rule presented by Taylor (1993), linking the nominal interest rate to output and inflation. We use a specification similar to that of Stock and Watson (2001):

$$i_t = f_i\left(y_t, \pi_t, X_t\right) + \epsilon_t \tag{3.1}$$

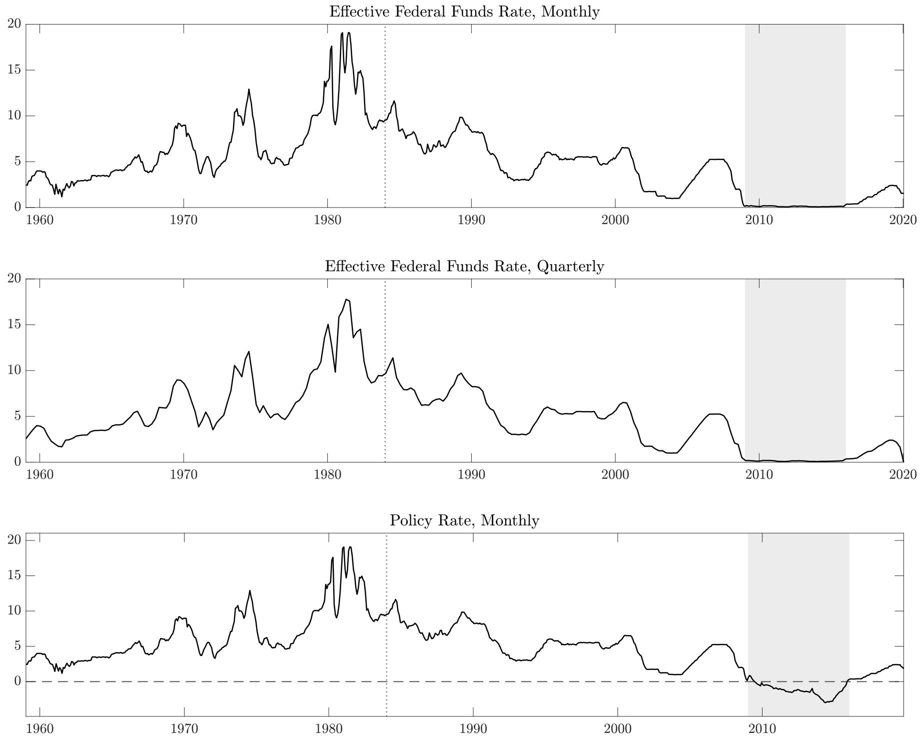
Where f_i is an unknown but linear function, X_t denotes additional explanatory variables, including lags of other regressors and latent factors. Deviations from this rule in the model framework are captured as the prediction error terms of the nominal interest rate equation ϵ_t. Under the BBE specification, shocks to the interest rate equation are interpreted as unexpected monetary policy shocks. This specification of the Taylor-rule is known as a backward-looking policy-rule because predictions of i_t rely only on current and lagged values of the predictors. Even though the use of a backward-looking Taylor rule is the most common to literature, Stock and Watson (2001) considered a forward-looking alternative. In the forward-looking rule, Y_t and π_t are replaced by conditional forecasts from the reduced-form VAR. We will consider only a backward-looking rule, as it is the most common implementation in literature, ensuring comparative benchmarks in the dominant literature on similar VARs.

3.2.1 Federal Funds Rate

One of the primary data concerns is that of the federal funds rate (FFR). There is little to no doubt that the federal funds rate contributes valuable information to any macroeconomic system, specifically concerning conducted monetary policy.

However, the time-series of FFR exhibits irregular behavior, and as it closely tracks the target rate as determined solely by the FED, it remains near-constant over several time-domains. To overcome the obstacle of local deterministic behavior, we consider a monthly sampling frequency; in quarterly estimates of the FFR, noise is filtered away, as illustrated in figure 3.1. From the figure, it is clear that the monthly

FIGURE 3.1
Effective Federal Funds Rate - Levels



1 The light grey shaded area denotes the period in which the effective federal funds rate is bound from below by the effective lower bound. The third panel is constructed as the shadow-rate augmented effective federal funds rate. The dotted line marks the end of the Volcker recessions

time-series is noisier than the quarterly, and depending on the type of noise, the quarterly series has less informational content. From figure 3.1, another source of local deterministic behavior is revealed; namely, that following the *»great financial crisis«*, the federal funds rate is bound from below by the effective lower bound. This is problematic from a modeling perspective, as all of the monetary regimes' expansionary efforts are not reflected in the series. Effectively the time-series is purged of all informational content during this period. To recover informational content about monetary policy conducted after 2009 from the federal funds rate, we splice FFR-observations with the implied shadow-rate as constructed by Wu and Xia (2016). When the FFR is bound from below, the central bank cannot conduct conventional monetary policy; therefore, they have to rely on unconventional policy such as the *quantitative easing*-programmes seen in the US after the great financial crisis. This is captured by the shadow-rate. More importantly, we expect that the economy reacts similarly to the conducted unconventional policy as it does to conventional policy. As shown by Debortoli et al. (2020); Wu and Xia (2016), the shadow-rate and macro variables of interest exhibit cross-moments similar to those between FFR and the same macro variables outside the constrained time span.

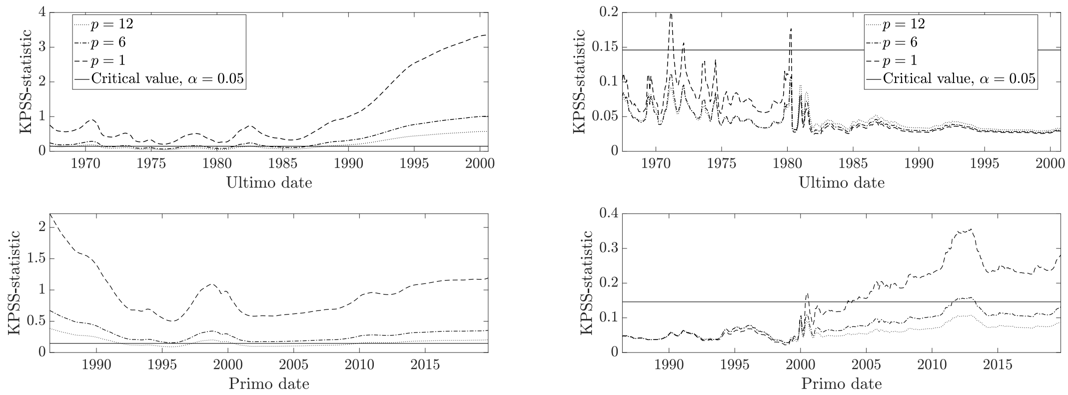
The stationarity of the federal funds rate is another concern. It is clear from figure 3.1 that FFR exhibits properties uncommon to most, if not all, covariance stationary time-series. The stationarity of the federal funds rate is widely discussed in academia and is the subject of many studies. Bec and Bassil (2009) finds that the FFR in levels can be assumed to be stationary if one takes structural breaks into account. Other studies suggest differencing; however, as the series is almost constant over several time-domains, this approach leads to strong amplification of heteroscedasticity; this is the suggested approach of McCracken and Ng (2016). The FAVAR studies of Bernanke et al. (2005); Stock and Watson (2005) considers FFR in standardized level-units. As we incorporate conditional heteroscedasticity on a quite general form into the model, we use the first-difference approach.

To formally infer stationarity, we consider the Kwiatkowski, Phillips, Schmidt, and Shin (1992)-test (KPSS-test) and the Augmented Dickey Fuller-test (ADF-test). Using an iterative KPSS procedure, considering different breakpoints reinforces the hypothesis that the leveled federal funds rate contains a stochastic trend even when subsampled, as revealed in figure 3.2. With one lag in the test equation, we reject the null of trend-stationarity and can thus not reject the presence of a unit-root, for all subsamples. Increasing the number of lags in the test lets changes the results marginally with the most significant effect in the bottom-tail sample. Overall it is hard to argue that the federal funds rate in levels is stationary or trend-stationary in the full sample, see table A.3 in appendix.

As can be seen in the last panel in figure 3.2b, in the isolated post-crisis sample the KPSS-test with 12-test lags can not reject the null of an underlying stochastic trend. Considering leveled ADF-tests in figure 3.3a suggest non-stationarity at almost

Figure 3.2
Iterative KPSS-test

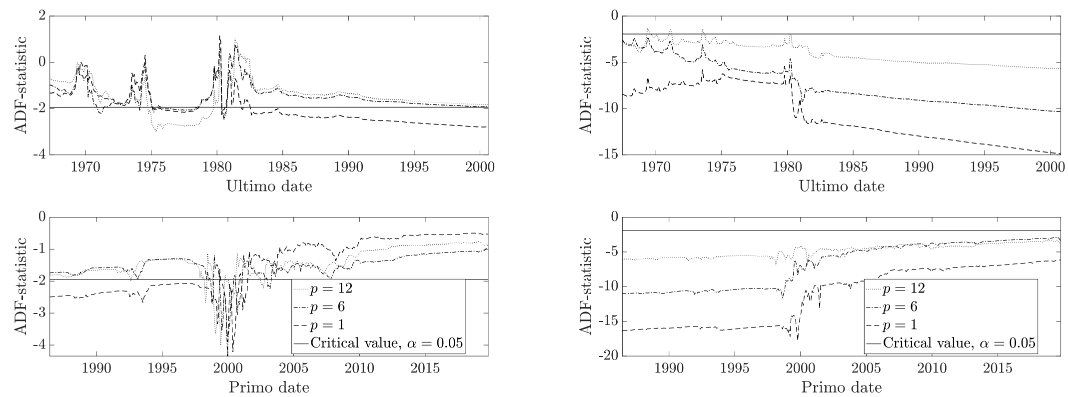
(a) Policy Rate, Level (b) Δ Policy Rate



1 The upper panel shows the iterative calculated KPSS-statistic for deterministic trend with three different lag lengths, $p \in \{1, 6, 12\}$, where the subsample spans 01:1959 - Ultimo. The bottom panel contains the same statistic over the subsample Primo - 10:2019. *Statistic* $>$ *crit*, provides evidence against the null, and is thus evidence supporting a stochastic trend in the subsample under consideration.

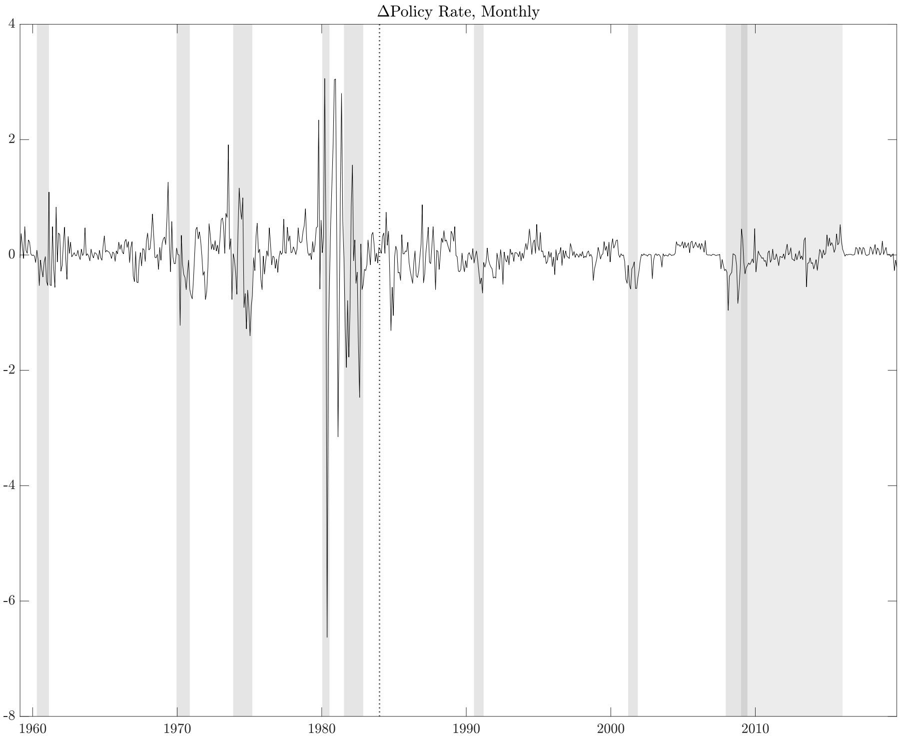
Figure 3.3
Iterative ADF-test

(a) Policy Rate, Level (b) Δ Policy Rate



1 The upper panel shows the iterative calculated ADF-statistic for stochastic trend with three different lag lengths, $p \in \{1, 6, 12\}$, where the subsample spans 01:1959 - Ultimo. The bottom panel contains the same statistic over the subsample Primo - 10:2019. *Statistic* $<$ *crit*, provides evidence against the null, and is thus evidence against a stochastic trend in the subsample under consideration.

Figure 3.4
ΔPolicy rate



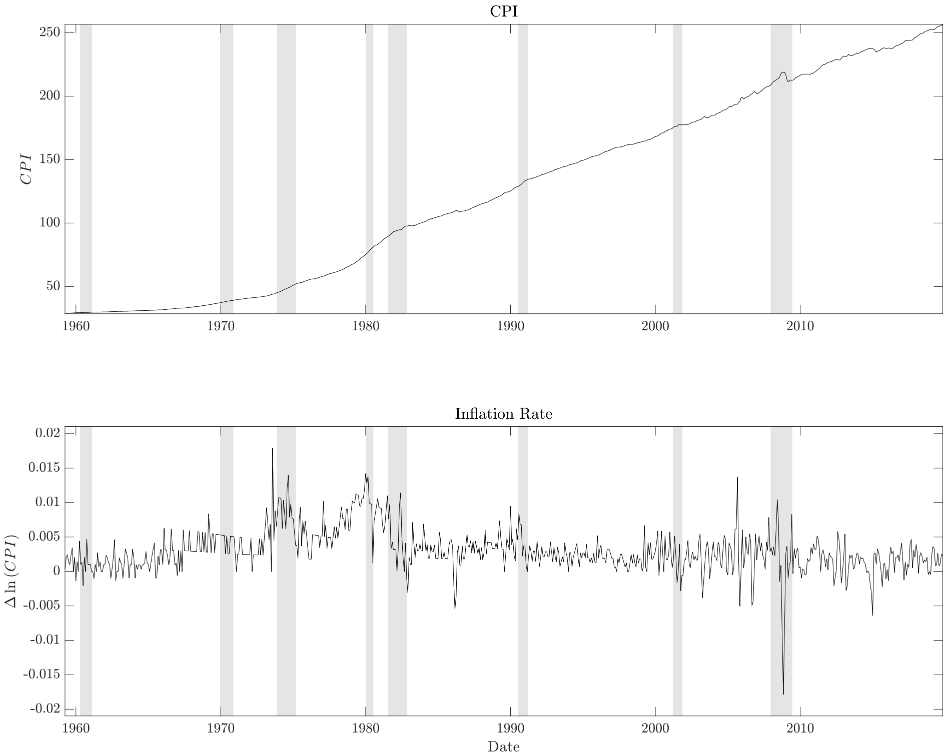
1 The darker grey areas, denotes NBER-recessions. The light grey shaded area denotes the period in which the effective federal funds rate is bound from below by the effective lower bound, and the series is spliced with the shadow-rate. The dotted line at 1984 is a hypothesized breakpoint.

all sample splits with few exceptions. The ADF-test, when considering the first difference of FFR in figure 3.3b, suggests that all splits are stationary. Therefore we consider the first differenced policy-rate, where both the KPSS- and ADF-test implies stationarity through different sample-splits. In addition full-sample test provides the same result. Following this evidence we choose a first difference transformation, as evidence at large suggests an $I(1)$ series. The transformed series for the full sample is presented in figure 3.4.

3.2.2 Inflation Rate

We calculate the headline inflation rate as the change in the general price level, as measured by the consumer price index. This serves two purposes first and foremost,

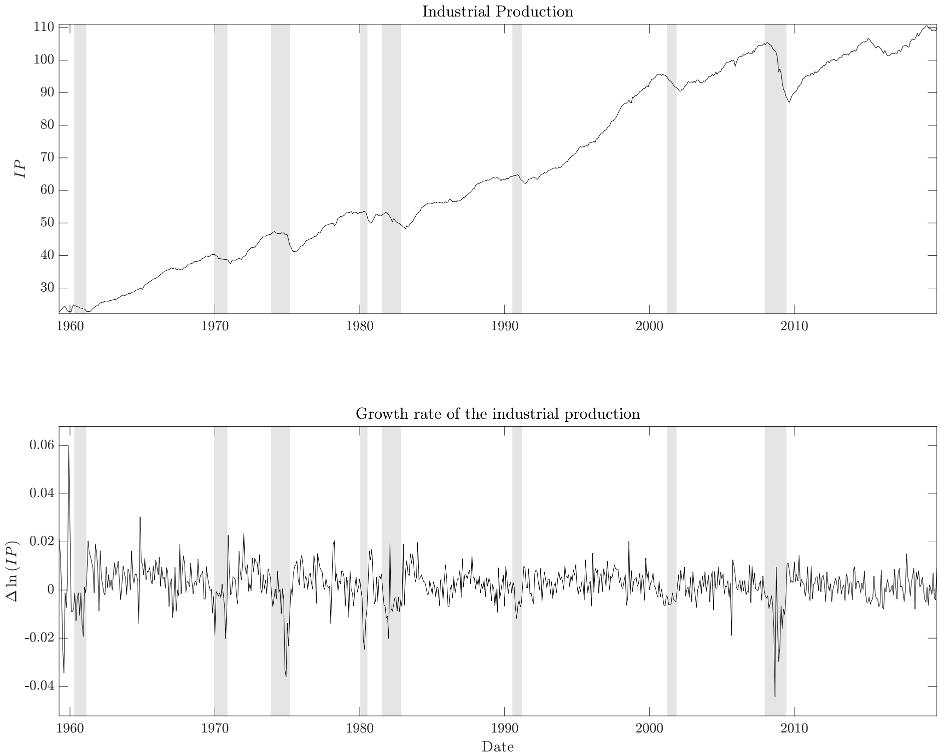
Figure 3.5
Consumer Price Index series



1 The light grey shaded areas denotes NBER-recessions

the consumer price index exhibits strong trending behavior, and can thus by no means be considered stationary. Secondly, it admits a simple transformation between price level in the economy and inflation rate; by conducting a log first-difference transformation of the CPI, we obtain a reasonable approximation of the inflation rate. Which conveniently exhibits stationarity. This transformation is consistent with that of Bernanke et al. (2005). The transformed and untransformed series are reported in figure 3.5. Alternatively, we could have used the GDP-deflator given by the ratio of real and nominal GDP as an inflation measure, however being available only on a quarterly sampling frequency, we choose the CPI measure. The series for the full sample is presented in figure 3.5

Figure 3.6
Industrial Production Index series



1 The light grey shaded areas denotes NBER-recessions

3.2.3 Economic Activity Measure

As our last observable in Y_t, we include a measure of real economic activity. Here a variety of variables can be considered. We choose the industrial production index and again we we use a log first-difference transformation to retrieve stationarity. The transformed series can again be interpreted as the percentage growth rate of industrial production for small changes in the index. The series for the full sample is presented in figure 3.6

3.2.4 VAR data summary

The observables used are similar to those of Bernanke et al. (2005) apart from the transformation of the Federal Funds Rate to stationarity. All series apart from the

Table 3.2
Augmented Dickey Fuller Tests

Variable	*Order of integration*	*Test-Statistic*	*p-value*
CPI	$I(0)$	8.182	0.999
	$\log I(1)$	−4.199	0.001
IP	$I(0)$	2.697	0.998
	$\log I(1)$	−9.526	0.001
Policy Rate	$I(0)$	−1.418	0.146
	$I(1)$	−14.362	0.001
Critical Value		−1.941	

1 Test regression specified with a lag-order of 4 and no deterministics. The results are robust to changes in lag order, tested on 0:12 lags. 01:1959-10.2019

FFR are seasonally adjusted from origin using the Census X-13 ARIMA adjustment routine.

Stationarity is checked using an Augmented Dickey-Fuller test, verifying that the transformations described above results in stationarity. The results of the ADF-tests are reported in table 3.2. Full sample tests with various lag specifications are reported in appendix, tables A.3 and A.4. All of which indicating relevant transformations result in stationarity.

In the data-set of McCracken and Ng (2016) series equivalent to our Y_t are provided, these are removed prior to factor extraction to avoid multi-collinearity in the VAR-estimation, even though unreported results show that this does not alter our results quantitatively.

As can be seen from the data-summary table A.1 in the appendix, the magnitude difference in variance between the variables is huge, to accommodate all data are demeaned and standardized; note that this is the conventional approach in much literature, for example, Bernanke et al. (2005); Stock and Watson (2001). However, we do not accommodate the skewness and excess kurtosis, noting that the data is not Gaussian is sufficient for our analysis. All the observables, transformed to stationarity and standardized are plotted in appendix, figure A.1.

Analysis 4

4.1 Model identification

In this section, we identify the subcomponents of the model; the dynamic factor model, and the FAVAR. The identification follows the methodology laid out in the prior section. Note that throughout the analysis, we assume that the reduced-form residuals are well-characterized by a DCC-GARCH(1,1), and no further identification is sought here. We believe that this assumption is more sensible than the typically assumed strictly Gaussian innovations. This follows from GARCH(1,1)-processes being capable of entailing a wide variety of conditional heteroscedasticity.

4.1.1 Factor model specification

To determine the optimal number of factors, we will be relying on the methodology of Hallin and Liška (2007). This holds several advantages; first and foremost, we keep the dimensionality of the model somewhat low, leaving us with more degrees of freedom and higher model tractability. A second advantage is that we can compare our findings to those of Bernanke et al. (2005) more easily. Additionally, the framework upon which the criterion is built is a dynamic factor model as opposed to the static factor model used by Bai and Ng (2002), despite this, most studies concerned with the FAVAR utilize Bai and Ng (2002) the justification being that because they cast the DFM into a static representation, the Bai and Ng (2002)-criteria should then be able to recover the true number of factors consistently. We argue that this justification is only valid if the number of dynamic factors is equal to the number of static factors, which in general should not be the case, in that particular case, the dynamic factor model reduces to a static factor model. Bai and Ng (2007); Stock and Watson (2005) acknowledged this fallacy, and extended the Bai and Ng (2002)-criteria to accommodate dynamic factors in a general setting; however, Hallin and Liška (2007) shows that this approach is only valid under strict assumptions.

A plot of the Hallin and Liška (2007)-criterion for our model is presented in the appendix figure A.3. As the authors show in their paper, for low values of the penalty weight, c their criterion severely overestimates the number of factors, while a high value of the penalty weight leads to underestimation of the number of factors. We consider a grid on $c \in \{.001, .002, \ldots 5.00\}$ and find satisfying convergence around a penalty weight of 2, suggesting three dynamic factors. This is consistent with the specification of Bernanke et al. (2005) who considers 1, 2, 3, and 5 latent factors in their FAVAR-specification, although they do not include any lags in the measurement equation.

It is important to note that different studies have different approaches to the number of factors in the FAVAR and that the numbers used are extremely sensitive to the methodology used. And indeed, we find that using the Bai and Ng (2002) information criterion criteria suggest between 6 and 8 latent factors, which is consistent with the FAVAR-implementation of Stock and Watson (2005). In the FAVAR literature, a clear method for determining the optimal number of factors has yet to be determined and is often chosen somewhat ad hoc between three and nine, see *e.g.,* Bernanke et al. (2005); Mumtaz and Surico (2009); Stock and Watson (2005); Wu and Xia (2016).

One thing to keep in mind is that the number of primitive factors determined by Hallin and Liška (2007) is based on a dynamic factor model with all factors being unobservable. The additional structure imposed by the FAVAR, observable factors in the DFM, is not accounted for in this criterion. This is not limited to our approach but is also the case in alternative criteria such as those by Bai and Ng (2002). An argument related to this fallacy is explored briefly by Bernanke et al.

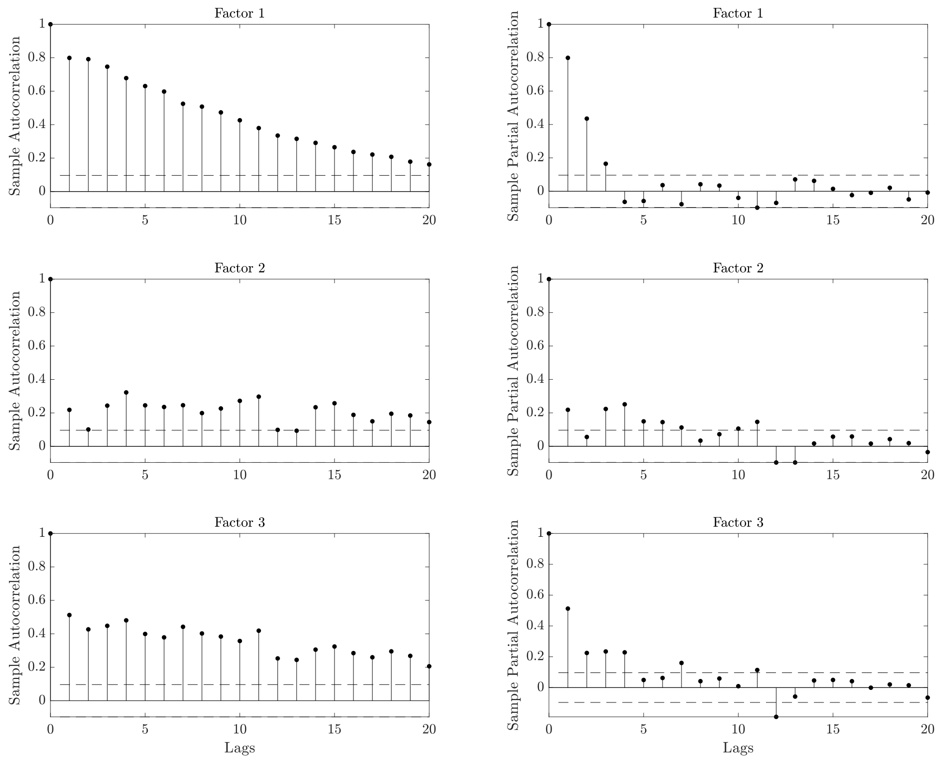
(2005), who notes that while criterion developed for factor models gives an indication of the optimal number of factors for the factor model, they might not necessarily correspond to the optimal number of factors in the VAR-specification. Possible solutions are scarce, and in general, no solution has emerged in the literature. We follow the literature and use a regular information criterion developed for dynamic factor models.

As with other regression-based techniques neglecting a significant factor, whether it is observable or not, results in biased estimates. While the opposite is not true, including an insignificant factor will not result in biased and inconsistent estimates; it will, however, lose efficiency. To that end, some researchers include many factors, where it may be speculated that the true number of dynamic factors is fewer.

From the initial factor estimates extracted with PCA, we compute the (partial) autocorrelation function for each of the factors, to determine the length of the VAR-filter, we are to impose when filtering/smoothing and thereafter estimating the FAVAR. The resulting (P)ACF's are presented in 4.1. It is apparent that the conditional mean of the first and third factor, are well-described by autoregressive-processes with $p = 3$ and $p = 4$. The second factor seems not to be presentable as a simple univariate ARMA-process. To filter the DFM-factors we choose an order of three to accommodate the fact the factor 1 accounts for the majority of explained variability of X_t. To verify that the univariate (P)ACF-measure yields correct specification we consider instead regular information criterion for the VAR in F_t, reported in table 4.1. Indeed according to the Schwarz- and the Hannan-Quinn information criterion a lag order of three seems optimal in the state-equation (2.26).

Thus we have three dynamic factors, affecting X_t contemporaneously and with two additional lags. Equivalent to nine factors in a static representation, this is remarkable close to the criterion of Bai and Ng (2002) that suggest eight static factors on the same sample. Satisfied with the findings we proceed to identification

Figure 4.1
(P)ACF of the Factors, 01:1959-10:2019



1 Confidence intervals $\pm 2\sigma$, the left column contains the ACF while the right column plots the PACF

of the VAR in factors and observables.

4.1.2 FAVAR specification

To identify the optimal VAR-lag specification we consider the three usual information criteria, that is the Akaike (1973), the Schwarz (1978) and the Hannan and Quinn (1979) Information-criterion. The calculated values are presented in table 4.2.

While all of the criteria see use, we shall rely on only one. Due to our large sample size, the efficiency of the estimator weighs less for our study than consistency. Therefore we shall focus on either the HQC or SIC, which are both consistent estimators of the true lag-length. Several studies provide evidence of the finite sample superiority

Table 4.1
Information Criterion, DFM-state equation

Lags	AIC	SIC	HQC	Log Likelihood
1	-1.5973	-1.5104	-1.563	342.8394
2	-1.9633	-1.7895	-1.8946	428.3205
3	-2.0892	**-1.8286**	**-1.9862**	463.6521
4	-2.0865	-1.739	-1.9491	472.0858
5	-2.103	-1.6686	-1.9313	484.537
6	-2.1077	-1.5863	-1.9016	494.503
7	-2.0872	-1.479	-1.8467	499.2189
8	-2.0712	-1.3761	-1.7964	504.8785
9	-2.0614	-1.2794	-1.7523	511.831
10	-2.0717	-1.2028	-1.7282	522.9791
11	-2.117	-1.1612	-1.7391	541.4429
12	**-2.1176**	-1.075	-1.7055	550.5877

1 Minimized values denoted in bold where meaningful, *(LLH is non-decreasing in p)*

of the SIC for models of the VAR-class, see Lütkepohl (1985); Raffalovich, Deane, Armstrong, and Tsao (2008).

This approach to the lag-specification is an attempt to let the data decide as much as possible before imposing theoretical restrictions. A researcher having much faith in a DSGE-representation of the model economy will argue that a severe truncation bias may arise as the linear representation of DSGE-models is known to be VARMA, and thus the VAR-representation, if it exists, is of order ∞, see *e.g.,* Christiano (2012). Our approach imposes no prior knowledge of the DGP, other than it is well-approximated by a (log)-linear FAVAR-model of finite-order in latent and observable factors.

With that in mind, we identify the lag-structure of our VAR using the Schwarz (1978) criterion. The specified FAVAR structure is thus determined as a three latent factor model with three lags in the VAR polynomial, and contemporaneous factor plus two additional factor lags in the measurement equation (2.37). Also, we will

Table 4.2
Information Criterion, 3 factor DFM-FAVAR, 01:1984-10:2019

Lags	AIC	SIC	HQC	Log Likelihood
1	5.0296	5.3771	5.167	-1015.1775
2	3.0709	3.766	3.3457	-569.8103
3	1.2139	**2.2566**	1.6261	-145.7053
4	0.96133	2.3515	**1.5109**	-56.9169
5	**0.85226**	2.59	1.5392	1.8768
6	0.88076	2.9661	1.7051	31.9215
7	0.93011	3.363	1.8919	57.6069
8	0.99759	3.778	2.0968	79.5034
9	0.97602	4.104	2.2126	120.0112
10	0.97525	4.4508	2.3492	156.1728
11	1.0298	4.8529	2.5412	180.7683
12	1.0857	5.2563	2.7344	205.0981

1 Minimized values denoted in bold where meaningful, *(LLH is non-decreasing in p)*

2 Full-sample in appendix table A.5

refer to the FAVAR with Kalman-smoother factors *DFM-FAVAR*, while the model with principal component factors is referred to as the *PCA-FAVAR*.

4.1.3 Model assesment

With model identification fixed as described above, we conduct a series of stability tests to determine optimum breakpoints. We use the test of Chow (1960) to make equation-by-equation assessments in an iterative style. In figures A.9 and A.10, test-statistic along with critical value are reported. We reject the null hypothesis of equal coefficients in the full sample for IP, CPI, and FFR. Many studies concerning structural instabilities impose a structural break after the Volcker-recessions at the beginning of 1984. Imposing a breakpoint in the first month of 1984 and re-conducting the Chow-tests leads to approximate stability of the policy-rate while evidence arises for instability during the financial crisis for the industrial production growth rate series, the test provides evidence against the stability of the inflation rate equation even in the 01:1984-10:2019 subsample. Unfortunately, incorporation of conditional heteroscedasticity amplifies evidence of mean-instability in the Chow-tests.

To further explore structural instability and optimal breakpoint, we again run equation-by-equation tests using the Cusum- and Cusum square-test for a break in mean and break in volatility, respectively; The results are reported in the appendix in figures A.11 through A.14. The Cusum-test suggests one structural break in the mean on the full-sample and a single breakpoint in the second moment on the full sample in the observables. The factors also exhibit a single break in the first moment; further, the very narrow area where the test-statistic seems to exceed the critical value in modulus is almost exclusively during the Volcker recessions. We, therefore, subsample accordingly and denote the 01:1984-10:2019-sample as our primary sample. In the primary sample, both the first moment and the GARCH-filtered second moments are stable according to the Cusum-tests. Stability in the second moment is rejected under unconditional Gaussianity. Following the evidence, we implement a single breakpoint in January 1984.

Conducting the trace-test of Johansen (1991) on Y_t yields mixed results, in particular for the full-sample 01:1959-10:2019, the Johansen-test (4 lags, no deterministics) suggests one cointegrating-relationship. When the test is conducted with a single breakpoint in 1984, the *pre*-split sample shows evidence of two cointegrating relationships, while the latter period, spanning 01:1984-10:2019, shows evidence of none. The test results are provided in table 4.3. Assuming these are neither false positives nor false negatives, this evidence provides further evidence of structural breaks in the series. With 01:1984-10:2019 being the primary sample of interest, we follow the test and do not impose any cointegrating relationships in the model.

We use the trace-test rather than the maximum eigenvalue test as Monte-Carlo simulations presented by Lütkepohl, Saikkonen, and Trenkler (2001), suggests that the tests exhibits equal power in larger samples with situational superiority of the Trace-test.

TABLE 4.3
Johansen (1991) Trace-test for cointegration

		Pre, 01:1959-12:1983		*Post, 01:1984-08:2019*		*Full, 01:1959-08:2019*	
r	Critical Value	Trace Stat	p-Value	Trace Stat	p-Value	Trace Stat	p-Value
0	29.798	59.584	0.001	22.427	0.302	72.645	0.001
1	15.495	28.550	0.001	4.900	0.819	6.881	0.623
2	3.842	2.526	0.113	0.004	0.949	0.755	0.537

1 Null-hypothesis, *number of cointegrational relationships k higher than r* vs. the alternative $k = r$. Hence the first rejection is used as the number of cointegrating relationships.

2 The test are performed on the untransformed data-set. Critical value calculated with significance level of 5% on three lags.

4.1.4 In-sample model assesment

In literature, the estimated VAR-polynomials are rarely reported. The dynamic complexity of the model, leads the estimated coefficients to be of little interest. Our three-factor reduced-form DFM-FAVAR contains a total of 2,432[1] parameters excluding, variance/covariance matrices. Instead, to determine how the flow of informational dependence goes through the system, we conduct a series of Granger (1969) tests, the results are reported in table 4.4. These show that the factor structure is contributing to predictions in the system. The first factor is the main contributor to the observables, Granger-causing both industrial production and the nominal interest rate at all relevant significance levels p-values of less than 0.001).

It follows that the two other factors mainly contribute by contributing predictability to the first factor. The exception being the third factor, as it is Granger-causing the consumer price index (p-value of 0.03). Considering the observables, we see how the federal funds rate helps to predict the industrial production and the consumer price index (p-value of 0.032), as well as the industrial production index (p-value of 0.011). While the industrial production index Granger-causes the nominal interest rate, but not the inflation-rate.

In some FAVAR-implementations, authors impose restrictions on the Granger-

[1] 128×18 in the measurement equation. 18×6 in the state-equation, 2 DCC-parameters and 3×6-GARCH-parameters

causality links. In Stock and Watson (2005), as an example, it is imposed that the observables Y_t does not Granger-cause F_t given lagged F_t. That is zero restrictions on the parameters in $\Phi(L)$ in (2.38), linking F_t to Y_t. In our case, we see how removing the observables from the informational set of latent factors would worsen predictions. It is relevant to mention here that Stock and Watson (2005) tests these over-identifying restrictions on all (1,188 in their case) exclusion constraints, and finds rejection of the null-hypothesis more frequent (25% of p-values less 0.057) than one would expect under correctly imposed restrictions (5% being less than 0.05). As such, we will not impose excluding restrictions of this kind.

Table 4.4
Granger Causality Tests

	Cause					
Effect	Factor 1	Factor 2	Factor 3	CPI	IP	FFR
Factor 1	0	<0.001	0.176	0.344	0.011	<0.001
Factor 2	0.028	0	<0.001	<0.001	0.472	0.541
Factor 3	<0.001	<0.001	0	<0.001	0.143	<0.001
CPI	0.234	0.951	0.030	0	0.484	0.032
IP	<0.001	0.451	0.098	<0.001	0	0.011
FFR	<0.001	0.0678	0.189	0.253	<0.001	0

1 Granger Causality tests, p-values associated with the F-test. Each column represents the regressor under consideration for the dependent variable (rows).

2 Model specification: $Y_t = \{\Delta \log IP,\ \Delta \log CPI,\ \Delta FFR\}$, $p = 3$, $K = 3$. On the 1984:2019 sample.

Additional in-sample measures are presented in appendix, including a plot of the filtered mean in A.4, the filtered volatility A.5, in-sample residuals A.6 and the conditional correlation A.7.

4.1.5 Out-of-Sample model assessment

Rewriting the FAVAR to companion form allows for easy assessment of the out-of-sample properties of the model unconditional of structural shocks. This is not the sole purpose of the analysis, but good out-of-sample properties of a model is a preferable trait.

As true out-of-sample evaluation is infeasible, we conduct *pseudo*-out-of-sample forecasting. For each window with ultimo period t, factors and parameters are re-extracted/estimated to minimize forward-looking bias. We can not eliminate the forward-looking bias, the reason being that real-time estimates of the series in $[X', Y']'$ might differ from the revised data available to us. We acknowledge that there is little we can do to retrieve true real-time data, and therefore proceed with the analysis.

Due to the implied structural instability of the model parameters, we will be conducting predictions in a rolling window style. As we use a rolling window, we do not accommodate explicitly a structural break, as this is done implicitly in the procedure. For multi-periods ahead, we will be using recursive forecasting.

$$\begin{bmatrix} \boldsymbol{F}_{t+h|t} \\ \boldsymbol{Y}_{t+h|t} \end{bmatrix} = \boldsymbol{B}^h \begin{bmatrix} \boldsymbol{F}_t \\ \boldsymbol{Y}_t \end{bmatrix} + \boldsymbol{\varepsilon}_{t+h} \tag{4.1}$$

Where $\boldsymbol{B}$ is the companion matrix of parameters of the VAR(1) representation of the reduced-form FAVAR, note that unreported goes the alternative specification of the FAVAR, in which we include a constant-term, but the quantitative results are similar, due to the demeaned nature of the series. All conclusions thus remains equivalent when we include a constant.

Using the FAVAR we can obtain forecasts of the latent factors as well. However, with factors being unobservable at time $t + h$, we are not able to evaluate performance. Therefore factor forecasts will not be of interest here. We restrict our focus to the observable factors in Y_t. Strictly speaking, because we utilize a rolling window training set, the estimated parameters are affected by ten years worth of fluctuations, and thus the initial sample is completely replenished after 120 iterations.

From table 4.5, we see the performance of various models relative to that of the benchmark DFM-FAVAR in factors and three lags. Firstly, we note that the DFM-FAVAR performs relatively well out-of-sample; it outperforms the random walk by

TABLE 4.5
Relative root mean squared prediction error

	Inflation Rate				*IP-Growth*				*Policy Rate*			
h	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*
DFM $K = 3$, $p = 3$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DFM $K = 3$, $p = 1$	0.99	0.99	1.02	1.01	0.92	0.96	0.97	0.93	0.97	0.94	1.01	1.01
DFM $K = 1$, $p = 3$	0.98	0.98	1.00	1.07	0.96	1.00	1.06	1.12	0.97	0.94	1.01	1.01
PCA $K = 8$, $p = 3$	1.08	1.04	1.08	1.07	1.08	1.06	1.07	1.17	0.97	1.07	1.05	1.02
PCA $K = 3$, $p = 3$	1.01	0.99	0.99	0.98	0.99	0.99	0.99	0.95	1.00	0.99	0.99	1.00
VAR $p = 3$	0.96	0.95	0.96	0.97	0.99	1.00	1.00	0.90	0.97	0.97	1.01	0.99
AR $p = 3$	1.06	0.97	1.01	1.05	0.98	0.99	1.04	1.00	1.13	0.96	1.09	1.07
Random Walk	1.05	1.14	1.17	1.20	1.18	1.25	1.32	1.32	1.08	1.41	1.46	1.52

1 Root mean squared forecast errors of various models relative to the three-factor, three-lag DFM-FAVAR. *DFM* and *PCA* both refer to a FAVAR, where factor estimation is carried out with the two-step Kalman procedure and Principal components respectively.

2 Rolling window *pseudo OoS* forecasts over 1959-2019

a significant margin. However, the correlational-based models all seem to perform relatively well, with no clear superiority of one over the other; all robust to window size changes. In addition, we see that the parsimonious model tends to perform slightly better than the unrestricted models suggested by the information criterion.

The exceptions are the inflation-rate, where the VAR-forecast is strictly better, and the industrial production growth where the three-factor PCA-FAVAR is strictly superior. However, for the case of the policy-rate, we also see poor performance of the no-factor VAR, suggesting that the policy-rate seems to be better described by a univariate process than the multivariate specification. It does seem as though the regular no-factor VAR, might provide slightly superior out-of-sample predictions.

It is worth noting that the results are sensitive to different specifications. Therefore we consider a broad spectrum of specifications nested in the FAVAR-framework. The DFM $k = 3$ is the specification we use throughout, it is identified using the econometric procedure described above, DFM $k = 1$ is identified with the same procedure but restricting the number of factors to one. PCA $k = 3$ follows DFM $k = 3$ but extracts three static factors with PCA, while PCA $k = 8$ is the specification suggested by the Bai and Ng (2002) information criterion often used in the PCA-FAVAR framework, however, note here, that this specification along with the random walk is *not* nested in the three factor DFM-FAVAR. The three-factor PCA-FAVAR is nested in the DFM-FAVAR by zero restrictions in the lag-polynomial of (2.37), and zero-restrictions in the state noise covariance-matrix of the Kalman-filter. In the dynamic factor extraction, the lag-order of the factors in the measurement and state-equation are fixed at three.

To assess the predictive power of the DFM-FAVAR relative to the various models, we conduct a series of Clark and West (2007), CW-tests. The CW-test tests the null of equal out-of-sample predictive power of the models under consideration in population. Rejection of the null, suggests that the unrestricted model, in this case, the DFM-FAVAR, produces lower mean squared prediction errors than the parsimonious model under consideration. Because the models are nested, it follows that under the null of equal predictive power in population, that the restricted model is the true model. We conduct the test on the same variety of models, excluding those not nested in the DFM-FAVAR.

p-values are reported in table 4.6. Rejection of the null on a 5%-significance level at all horizons and all variables when comparing the DFM-FAVAR to a random walk indicates that the various FAVAR specifications outperform the random walk. Similar conclusions arise for the univariate autoregression and the no-factor VAR, with the exceptions of inflation forecasts on a 12-month horizon, industrial production forecasts at 3- and 6-month horizons, and the policy rate at 6- and 12-month horizons.

Overall the factor models seem to be well-performing when considering out-of-sample forecasts. However, it is not apparent whether the DFM approach is strictly better when considering the two different variants, PCA and DFM. Considering first the policy rate, we cannot reject the null of equal predictive power at any horizon at any reasonable significance level. For the growth rate of the industrial production index, we can reject the null only on a 6-months horizon and for the inflation rate only on 1- and 12-month horizons.

Table 4.6
Clark and West (2007)-tests for predictive accuracy

	Inflation Rate				*IP-Growth*				*Policy Rate*			
h	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*
PCA $K = 3$, $p = 3$	0.005	0.628	0.289	0.626	0.636	0.645	0.728	0.681	0.696	0.833	0.842	0.786
DFM $K = 1$, $p = 3$	0.011	0.135	0.089	0.31	0.002	0.874	0.15	0	0.021	0.121	0.02	0.017
DFM $K = 3$, $p = 1$	0	0.042	0.693	0	0.019	0.654	0.001	0.222	0.413	0	0.653	0.475
AR $p = 3$	0	0	0	0.003	0	0.035	0.001	0	0.003	0.002	0.007	0.018
VAR $p = 3$	0.119	0	0.065	0.763	0	0.138	0.555	0.001	0.023	0	0.238	0.124
Random Walk	0	0	0.004	0	0	0.003	0	0	0.002	0	0	0.002

1 p-values, failure to reject the null-hypothesis suggests equal predictive accuracy of the two models in population. The benchmark model is the DFM-FAVAR with three factors. Rejection of the null implies that the DFM-FAVAR produces lower MSPE.

A visual subsection of the point out-of-sample 1- through 12-period-ahead forecasts of the FAVAR plotted against observed values are presented in figure A.8 in the appendix. From the figure, it is apparent that the forecasts generated during the 2008 financial crisis, especially at longer horizons, are performing relatively well when taking the severity of outliers into account.

4.2 Revisiting the results of Bernanke et al. (2005)

To re-examine the work of Bernanke et al. (2005) (BBE), we sample our data to recover a subsample spanning the period under consideration by BBE, that is 01:1959-08:2001. This section aims to determine whether or not the model is robust to identification using statistical measures as opposed to the theoretical justification of BBE. While also examining whether we can verify the results of BBE using the proposed two-step Kalman-approach for factor extraction as opposed to the PCA-approach.

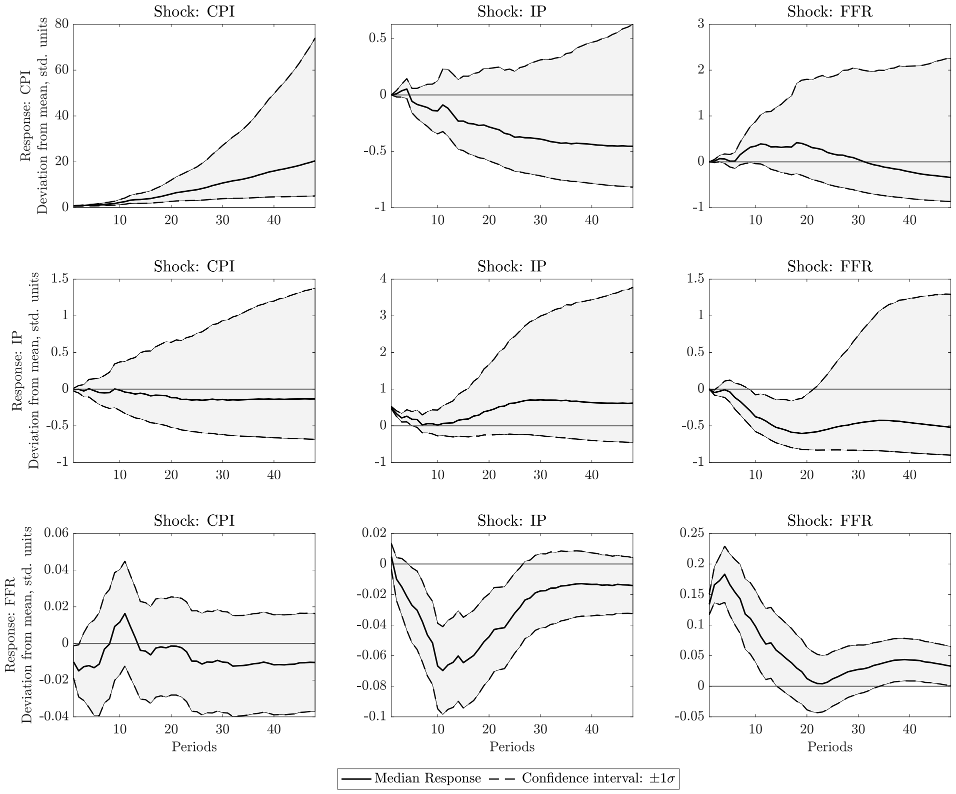
Our primary tool for this part of the analysis is the impulse response functions. Impulse response functions can be regarded as forecasts conditional on specific shocks in the economy. In this approach we set up a theoretical scenario in which the model is in a steady-state and in the complete absence of prior shocks in the infinite past, this is crucial as the IRF are in essence shocks to a VMA(∞) representation of the FAVAR; therefore shocks can carry information, albeit small, from impact to the infinite future. The dynamics revealed by the IRFs can be interpreted as theoretical forecast conditional on a particular chain, most often a single impulse, of innovations/events in the model economy.

The model identification and sample-span used by BBE (extended with GARCH-residuals and Kalman factors), produces IRFs as can be seen in figure 4.2. BBE identifies only shocks to the FFR and thus only reports the final column of 4.2[2]. The main takeaway from these impulse-responses by BBE is that the FAVAR can significantly diminish the prize-puzzle. Where the traditional three variable recursive VAR implies solely non-negative responses of inflation to a monetary policy shock, the FAVAR-median response shows a temporary increase followed by the expected decrease in inflation, however, at the one standard deviation confidence mark, these results are statistically insignificant in the present analysis. The structure of the BBE FAVAR is quite arbitrary; they utilize a lag structure of 13 lags. This is most likely to diminish the truncation bias arising from log-linearizing a theoretical model to a VAR(∞) representation, although this is not discussed in their paper. The formal information criterion suggests between two to five lags of the FAVAR for the BBE-sample. Unreported results shows that the reduced form residuals do imply that 13-lags lead to a slightly less light-tailed distribution of especially the latent factors and FFR residuals, closer to the desired normal distribution of the residuals, however filtering GARCH-effects from residuals during estimation corrects the distribution even at lower lag-lengths.

Another thing to note is that re-estimating the model, using the IC suggested lag-order and a first-difference transformation of the FFR, leads to a disentanglement of the otherwise promising results concerning the prize-puzzle, see figure 4.3. We see a dampening of the economic activity following a contractionary innovation (increase)

[2]However by the Cholesky-factorization the remaining shocks are statistically identified (but bears no explicit economic interpretation)

Figure 4.2
Impulse response functions recursive FAVAR Bernanke et al. (2005)-specification



1 Three factor, 13-lag FAVAR, Full Bernanke et al. (2005)-sample. 1,000 draws 1 standard deviation GARCH-residual-based bootstrap confidence intervals.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta \log CPI,\ FFR\}$

3 Bands symmetric on a log-scale (except FFR as it is symmetric in levels)

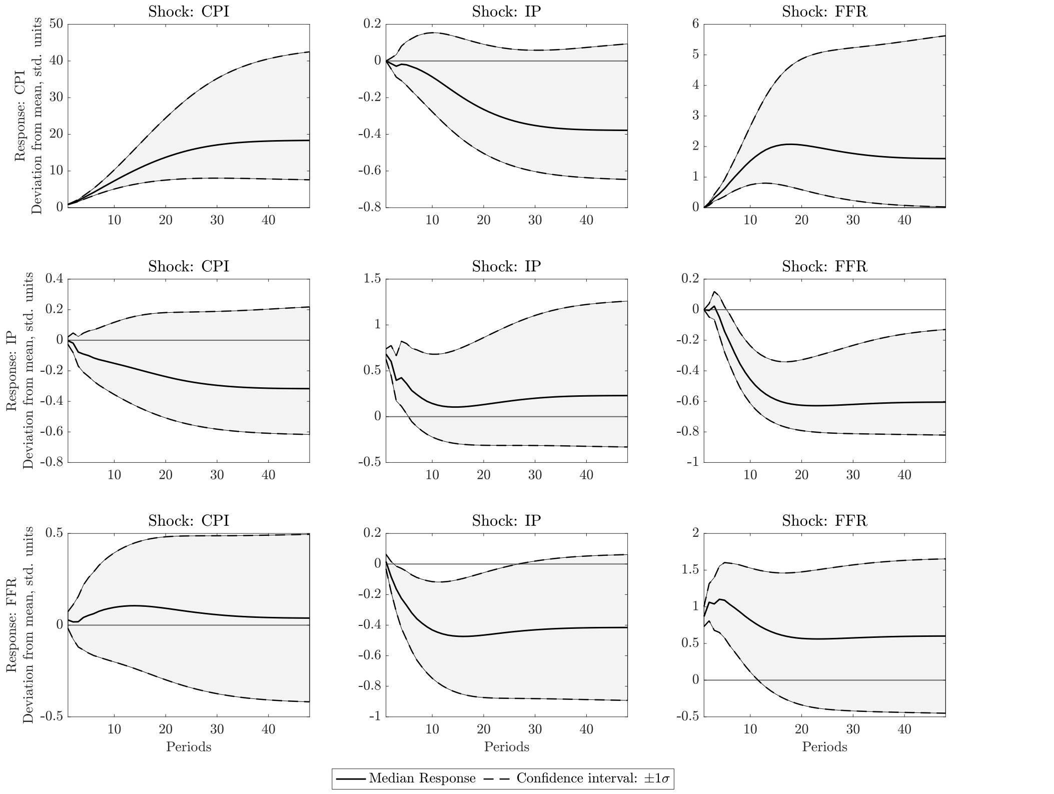
in the FFR, but in contrast to our expectations, we see an increase in the general price level. Notice that as we do a detransformation of the IRFs before plotting, the responses track movements in the industrial production index, consumer price index, and federal funds rate in standardized levels.

The 13-lag structure leads the model to find responses mostly in line with theory; unfortunately, the data does not justify that lag structure following standard procedures. Even more unfortunate, the findings are not robust to changes in lag-order. Furthermore, correcting for heteroscedasticity renders the results, even with a lag structure of 13, statistically insignificant.

We illustrate the differences between the FAVAR-model estimated with the Kalman

Figure 4.3

Impulse response functions recursive FAVAR, alternative specification



1 Three factor, three lag DFM-FAVAR, Full Bernanke et al. (2005)-sample. 1,000 draw one standard deviation GARCH-residual-based bootstrap confidence intervals.

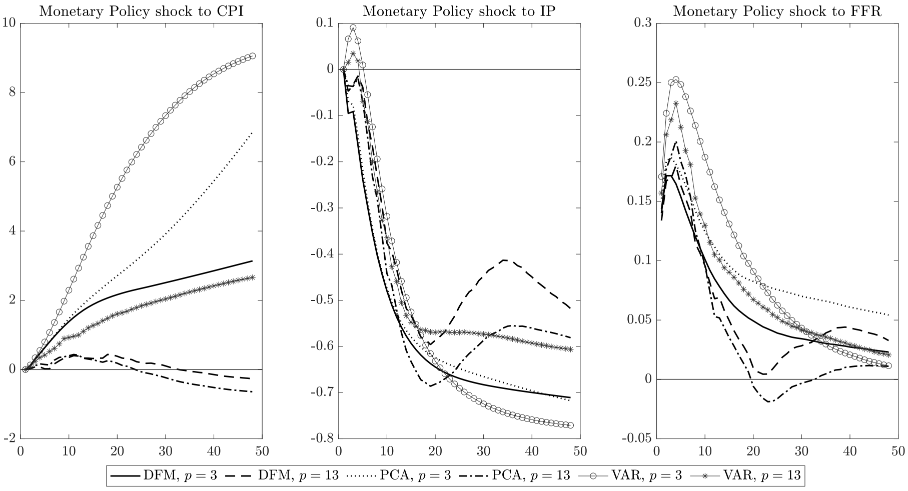
2 $Y = \{\Delta \log CPI,\ \Delta \log IP, \Delta FFR\}$

smoother and PCA as well as no-factor VAR-models by considering the median impulse response function estimated for each model. The models are estimated with 13 lags, as suggested by BBE and three lags, as suggested by the Bayesian information criterion. As in BBE, we now consider only a monetary policy shock. The responses are reported in figure 4.4. The first column showing how the CPI is affected by the monetary policy shock is the key-finding of BBE; they seemingly solved the price-puzzle, which is highly present in the regular VAR-specifications. However, as we see in the figure, this result deteriorates using the specification suggested by the information criterion, the three-lag FAVARs both under the PCA and DFM-extraction scheme.

Another thing to note in the broader spectrum of the figure, is the differences between

Figure 4.4

Responses to a monetary policy shock, various recursive models



1 *DFM* and *PCA* both refers to a three factor FAVAR where the factors are retrieved with the Kalman smoother and PCA respectively.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ FFR\}$

the PCA-FAVAR and the DFM-FAVAR. They seem to qualitatively provide the same results with small quantitative differences. This suggests that the conclusions one might draw from the different models are similar. And that it stands as no surprise that the DFM-FAVAR is able to recreate the results of BBE.

After noting that the DFM-approach did not affect the shape of the responses by a large margin over the PCA approach, as we saw in figure 4.4. The improvement gained by this more cumbersome estimation method is, therefore, primarily stemming from a higher explanatory power of the factor-panel variables in X_t, by decreasing the magnitude of the idiosyncratic component, and thus lower uncertainty surrounding the IRFs. The Kalman Smoother approach, in contrast to the PCA-approach used by BBE, shifts the proportion of X_t explained by the common component. The explicit DFM-extraction implemented in this analysis increases the explanatory power of the FAVAR, using a selection of 9 variables in X_t table 4.7 shows the R^2 for both the PCA-FAVAR and the DFM-FAVAR. Over the nine variables selected, the mean increase in R^2 is .051, indicating a 5.1% increase in explainability by the common component. Over the entire 125-variable factor panel, this number increases to .077 or 7.7%. The model used here is the three factor DFM-FAVAR with three lags and

FFR in first difference; the specification justified using statistical methods.

Table 4.7
Common Component, select series, R^2, 01:1959-08:2001

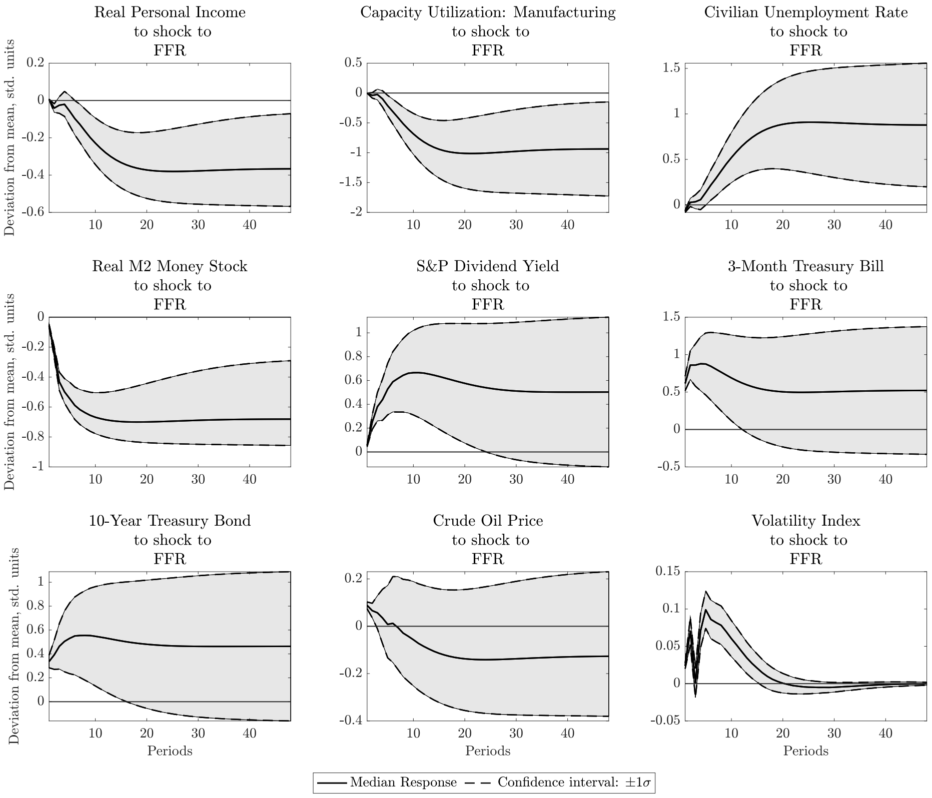
Series:	*PCA*	*Kalman Smoothed*
Real Personal Income	0.166	0.181
Capacity Utilization: Manufacturing	0.914	0.925
Civilian Unemployment Rate	0.392	0.395
Real M2 Money Stock	0.476	0.548
S&P Dividend Yield	0.428	0.521
3-Month Treasury Bill	0.728	0.813
10-Year Treasury Bond Rate	0.636	0.782
Crude Oil Price	0.023	0.100
Volatility Index	0.168	0.172
Consumer Price Index[1]	1.00	1.00
Industrial Production Index[1]	1.00	1.00
Federal Funds Rate[1]	1.00	1.00

1 R^2 denotes the fraction of variance in each series in X_t explained by the common component, the remaining variance stems from the idiosyncratic component. By construction the observable factors in the DFM are explained solely by their respective common component with identity loading.

We plot the responses of the same nine variables to a monetary policy shock in figure 4.5. Remember here that the responses are for the common component of the series, and results are dependent on the coefficient of determination from table 4.7. The IRFs again are at large in line with our theoretical expectations. In particular, the responses of treasury bill rates, closely resembling the responses of the FFR itself. In addition to being in line with theoretical and empirical knowledge, the results are quite solid, seeing how the common component of the treasure bill rates explains between 70% and 80% of the total variation. We see the short maturity bonds reacting very strongly to monetary tightening, while the longer maturity bonds contemporaneous response is around half in magnitude, with a flatter response curve, suggesting higher persistence in the monetary policy shock. The response of dividend-yields is a result of decreasing stock prices; this is verified by computing the IRFs associated with the S&P 500 Common stock price index; these are presented in figure 4.6.

Figure 4.5

Impulse response functions X_t, FAVAR Bernanke et al. (2005)-identification

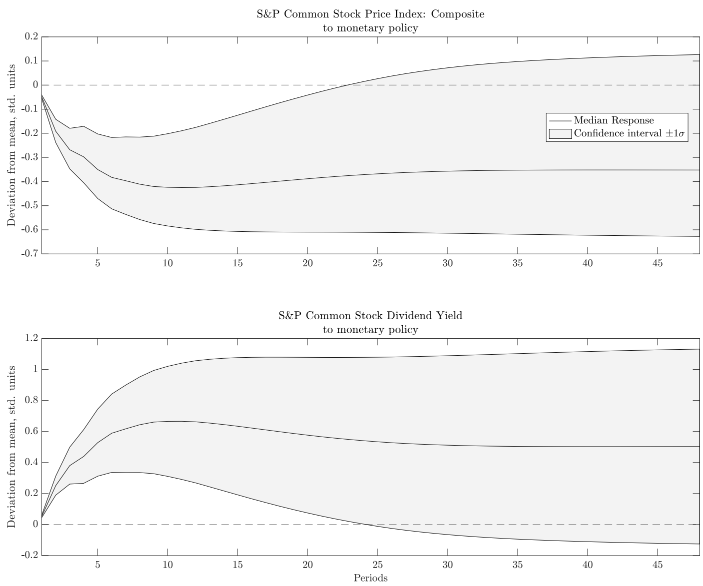


1 Three factor, three lag DFM-FAVAR. Full Bernanke et al. (2005)-sample. 1,000 draws 1 standard deviation GARCH-residual-based bootstrap confidence intervals.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$.

Figure 4.6

S&P-500 responses to monetary policy shock



1 Recursive three factor, three lag DFM-FAVAR, Full Bernanke et al. (2005)-sample. 1,000 draws 1 standard deviation GARCH-residual-based bootstrap confidence intervals.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$.

The real economy is affected as one would expect; following a contractionary policy shock, we see dampening of economic activity, rising unemployment, lowering of real income, firms decreasing production, and a sharp decrease in the money supply. Considering the spike in volatility following a tightening of monetary policy is interesting. The suggestion of increasing volatility following contractionary monetary policy is consistent with empirical results of Bekaert, Hoerova, and Lo Duca (2013); Mallick, Mohanty, and Zampolli (2017), suggesting that contractionary monetary policy is associated with an increase in risk-aversion. However, while the intuition holds, the persistence of the volatility spike is controversial; although it seems to emulate a GARCH-process, it does hold a 20-month recovery time following a one-standard-deviation shock to the federal funds rate, which seems excessive.

4.3 Long-run identified structural FAVAR

The recursive scheme is often used in trivariate (FA)VAR-systems such as ours. However, we believe that the superiority of the recursive scheme in the context of policy shocks is somewhat prone to critique. The primary justification of the recursive scheme is that the policy innovations should only affect price level and economic activity with a lag due to sticky prices and wages. This is a sensible assumption in the very short run; however, as the sampling frequency decreases, it becomes less and less plausible that prices should be rigid contemporaneously. Usually, sampling occurs once a month or once a quarter. This implies that the model economy is indifferent for at least a month following a shock to monetary policy, while this can be justified in theory, we would prefer not to impose such restriction. Therefore we will consider the system with a different identification scheme, namely that of Blanchard and Quah (1989). In this part of the analysis, we fix the lag order of the model to those suggested by our statistical approach, three lags in the factor model, and three lags in the FAVAR. That is, we do not consider the 13-lag FAVAR of BBE.

The restrictions we impose on $[F_t', Y_t']'$ is centered around the hypothesis of long-run money neutrality, or equivalently a vertical long-run Phillips curve. Bullard (1999) presents an overview of the empirical findings regarding the long-run neutrality of money; the conclusion is that across industrialized countries, empirical evidence is largely in favor of the hypothesis.

Thus at the core of our identification, monetary policy shocks should not have lasting effects on real variables such as the industrial production index. Consumer price index enters the FAVAR in $I(1)$, however using that the headline inflation rate is well approximated by $\Delta \log CPI$, we can by a switch of notation assume that the inflation rate enters the FAVAR as an $I(0)$ variable. The ordering suggests that spending and supply-shocks can have a permanent influence on price-level, but not inflation. This allows the short-run Phillips curve to shift due to money-illusion following monetary stimulus while ensuring convergence of productivity back to the full-employment level in the long-run. The implication being, following monetary stimulus agents in the model economy, realizes rising nominal wages but not inflationary pressure, hence increasing labor-supply until they realize that their real wage is unchanged, the labor supply decreases to the long-run equilibrium, full-employment. This scheme is equivalent to imposing verticality of the long-run Phillips curve.

To follow the identification scheme, consider the simple stochastic AS-AD-model augmented with the Taylor Rule. For now, disregard the latent factors. We follow the theoretical framework of Mihira and Sughihara (2000). The model equations are given:

$$y = f_{y^d}(i) + u_{IS} \tag{IS}$$

$$i = f_i(y, \pi) - u_{MP} \tag{Taylor Rule}$$

$$y = f_{y^s}(\pi) + u_{AS} \tag{SRAS}$$

$$y = y^{LR} + u_{AS} \tag{LRAS}$$

$f_{(\cdot)}$ denotes unknown but (log)-linear functionals. In general, $y \neq y^{LR}$ due to rigidities in the pricing and wage kernel. However with prices being perfectly flexible in the long-run and with output being determined only by supply-side components (by Say's Law), the functional in the short-run aggregate supply-, (SRAS)-equation reduces to a constant y^{LR} in the long-run. By recursive comparative statics, we work out the theoretical long-run effect of each u on the system. Starting with the differential of the long-run aggregate supply curve:

$$dy = du_{AS} \tag{4.2}$$

From the IS equation, we solve for i by again taking the differential (note we specify the partial derivative of $f_{(\cdot)}$, with respect to j as $f_{(\cdot),j}$)

$$\begin{aligned} di &= (-dy + du_{IS}) / f_{y^d,i} \\ &= (-du_{AS} + du_{IS}) / f_{y^d,i} \end{aligned} \tag{4.3}$$

From the Taylor rule, we follow the same procedure:

$$\begin{aligned} d\pi &= (-f_{i,y} dy + di + du_{MP}) / f_{i,\pi} \\ &= (-f_{i,y} du_{AS} + (-du_{AS} + du_{IS}) / f_{y^s,i} + du_{MP}) / f_{i,\pi} \end{aligned} \tag{4.4}$$

The implication of this theoretical model in the VAR is that monetary policy shocks no longer corresponds to unforeseen changes in the interest rate i but instead through impulses to the π equation in the model. Whereas the structural shock associated with the nominal interest rate are shocks to the investment–savings curve in the FAVAR-model, this is a well-established result in long-run-VAR literature, see, *e.g.,* Keating (1992).

From the signs of theoretical effects, we can recover the expected sign of the long-run effect to positive innovations:

$$\begin{bmatrix} dy \\ di \\ d\pi \end{bmatrix} = \begin{bmatrix} + & 0 & 0 \\ - & + & 0 \\ - & + & + \end{bmatrix} \begin{bmatrix} du_{AS} \\ du_{IS} \\ du_{MP} \end{bmatrix} \tag{4.5}$$

Even when the factor structure of the model is recovered and justified, we cannot retrieve expected signs of the factors, as they are identified only up to a sign. From this setup, support for the model is acquired statistically if:

1. The aggregate supply shock (positive shock to IP) results in a permanent increase in IP and a permanent decrease in the interest rate and prices.

2. The spendings shock (positive shock to the interest rate) has no permanent effect on real variables (IP) and a positive effect on prices and the interest rate.

3. The expansionary monetary policy shock (positive shock to inflation) has no permanent effect on interest rate nor real economic activity (IP).

To infer factor ordering is more tricky as textbook macroeconomic-models do not incorporate concepts of big-data. This is addressed by intuition in the paper of Bernanke et al. (2005), with their quick/slow-identifying approach, *i.e.,* financial series should move contemporaneously with monetary policy shocks, while real variables are denoted slow, and thus only responds with a lag. Some authors suggest that because no clear economic interpretation can be retrieved, that one might just as well do a block-decomposition $[F', Y']'$ and order the factors arbitrarily. However, this approach is invalid, as the shocks recovered from factors and observables while orthogonal have no economic interpretation, corresponding to a strictly statistical identification.

Instead, we try to recover some economic sense of the factors. VAR-shocks with long-run restrictions are thought of as representations of structural forces able to shift equilibrium values of the observables, (driving the stochastic trend in the $I(1)$-variables), the problem is then determining to which stochastic trends each factor-shock may contribute. To infer some structure to the factors, we do single-factor regression of the factors on all variables equation-by-equation to determine where the factors are loading heaviest as measured by R^2, *e.g.,* a factor explaining the majority of variance in production-variables can be thought of as a productivity-factor. Table 4.8 reports the ten variables upon which the factors respectively load the heaviest, *i.e.,* the variables best explained by the dynamic factors.

TABLE 4.8
Part of total variance explained by a single factor R^2, Dynamic factor model

Factor 1		*Factor 2*		*Factor 3*	
PAYEMS [2]	0.758	*CUSR0000SA0L5* [7]	0.750	*CUSR0000SAC* [7]	0.711
IPMANSICS [1]	0.718	*CUSR0000SA0L2* [7]	0.750	*CUSR0000SA0L2* [7]	0.710
USGOOD [2]	0.711	*CUSR0000SAC* [7]	0.749	*CPITRNSL* [7]	0.702
CUMFNS [1]	0.647	*CPIULFSL* [7]	0.736	*DNDGRG3M086SBEA* [7]	0.698
SRVPRD [2]	0.614	*DNDGRG3M086SBEA* [7]	0.728	*CPIULFSL* [7]	0.698
MANEMP [2]	0.594	*CPITRNSL* [7]	0.724	*CUSR0000SA0L5* [7]	0.698
IPFPNSS [1]	0.594	*PCEPI* [7]	0.553	*PERMITW* [3]	0.547
DMANEMP [2]	0.591	*WPSFD49502* [7]	0.542	*HOUST* [3]	0.537
USWTRADE [2]	0.576	*WPSFD49207* [7]	0.532	*HOUSTW* [3]	0.536
USTPU [2]	0.545	*PERMITW* [3]	0.530	*PERMIT* [3]	0.520

1. Superscript denotes group; 1: Output and income, 2: Labour market, 3: Consumption and Orders, 7: Prices
2. R^2 from the regressions $x_t = \boldsymbol{\lambda}_i \boldsymbol{F}_i$, (contemporaneous and 2 additional lags of the i'th latent factor on x_t) .
3. The subset of the 10 variables loading heaviest on each factor.
4. The high degree of interdependence between factor 2 and 3, seems to stem from serial cross-variable correlation. Using only contemporaneous values of F_t, (assumed static relationship), yields different results, see table A.8 in appendix. $F_{2,t} = \alpha F_{3,t-1} + e_t$ yields $R^2 = 0.29$.

The results suggest a non-exhaustive specification of the factors, which we do consider it sufficient for this analysis. The specification does suggest a natural ordering of the factors. Namely, the first factor and, by extension, the associated structural shock, being closely related to productivity measures, can be interpreted as a component of the aggregate supply shock. While the two additional factors are loading the heaviest on price-series and can thus be associated with the monetary policy shock (CPI).

With this ordering our system represented as a VMA(∞) can be expressed:

$$\begin{bmatrix} F_1 \\ \Delta \log IP_t \\ \Delta R_t \\ F_2 \\ F_3 \\ \pi_t \end{bmatrix} = \sum_{j=0}^{\infty} \begin{bmatrix} \Theta_{11,j} & \Theta_{12,j} & \Theta_{13,j} & \Theta_{14,j} & \Theta_{15,j} & \Theta_{16,j} \\ \Theta_{21,j} & \Theta_{22,j} & \Theta_{23,j} & \Theta_{24,j} & \Theta_{25,j} & \Theta_{26,j} \\ \Theta_{31,j} & \Theta_{32,j} & \Theta_{33,j} & \Theta_{34,j} & \Theta_{35,j} & \Theta_{36,j} \\ \Theta_{41,j} & \Theta_{42,j} & \Theta_{43,j} & \Theta_{44,j} & \Theta_{45,j} & \Theta_{46,j} \\ \Theta_{51,j} & \Theta_{52,j} & \Theta_{53,j} & \Theta_{54,j} & \Theta_{55,j} & \Theta_{56,j} \\ \Theta_{61,j} & \Theta_{62,j} & \Theta_{63,j} & \Theta_{64,j} & \Theta_{65,j} & \Theta_{66,j} \end{bmatrix} \begin{bmatrix} u_{F1,t-j} \\ u_{AS,t-j} \\ u_{IS,t-j} \\ u_{F2,t-j} \\ u_{F3,t-j} \\ u_{MP,t-j} \end{bmatrix} \tag{4.6}$$

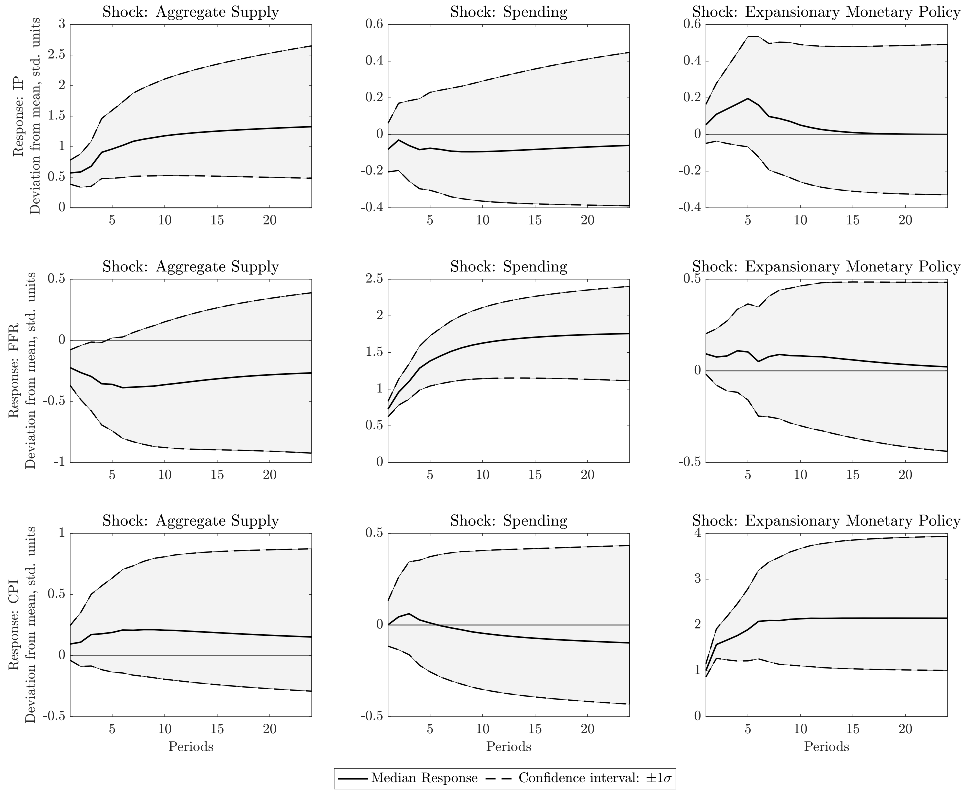
Where π_t is defined as $\Delta \log CPI_t$. We restrict the infinite sum of the coefficients, $\boldsymbol{\Theta}(1)$ to be lower triangular. The restrictions we impose are thus: $\sum_{j=0}^{\infty} \Theta_{ki,j} = 0$ for

$i > k$. The restrictions on factor innovations can be regarded as controversial, as the imposed restrictions assume that factor shocks should be regarded solely as supply shocks (first factor) or demand-driven shocks (second and third factor) while they in-fact might be an aggregation of both demand and supply shocks. Unfortunately, no obvious way to circumvent this issue is available at the present time. We note here that this scheme is robust to reordering the latent factors, such as the clear block ordering, all latent factors ordered before observables.

Figure 4.12a reports level responses of the observables to one standard deviation shocks to the system. The smooth responses are a consequence of the low lag order. The first thing to note is the large confidence bands, indicating a considerable uncertainty surrounding the median responses; secondly, the confidence bands span in large the first axis and are thus not statistically significantly different from zero.

We now consider the result against our proposed theoretical hypothesis. As expected, the aggregate supply shock leads to a permanent increase in the industrial production index, the responses of the interest rate holds the expected sign at the median response, but quickly reaches statistical insignificance. The price index is insignificant at the one standard deviation mark throughout all periods, and the median holds the wrong sign. The spending shock leads to a zero long-run estimate of productivity but is highly insignificant through all periods. The short-run effect of the spending shock to industrial productivity, suggests a slight dampening in real economic activity. The theoretical model suggests that such shock should lead to productivity increases in the short run; however, the confidence interval is almost centered around the first axis, making any inference infeasible. Likewise, the price response to a spending shock is highly insignificant, and inference is infeasible. The leveled policy rate responds by a persistent accelerating increase over eight to ten periods post-shock. Note here that this behavior is consistent with the series, as can be seen in figure 3.1, shocks seem very persistent, and in 1990-2010 are the series almost deterministic-looking around a few innovations. A positive shock to monetary policy likewise results in insignificant responses. However, by considering the median responses, we see an

FIGURE 4.7
Impulse Response Functions, Long-run identified FAVAR



1 Three factor, three lag DFM-FAVAR, 01:1984-10:2019-sample. 1,000 draw residual-based GARCH bootstrap confidence intervals, one standard deviation. All responses transformed to standardized levels.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

increase in economic activity in the short-run < 20-periods; after this increase as agents in the economy realize that their increased prosperity is only nominal, they lower their labor-supply to steady-state supply. As theoretically, the structural shock recovered is equivalent to a negative Taylor-rule innovation; we expect a decrease in the interest rate; however, the median response is positive but strongly insignificant. We find that expansionary monetary policy leads to a permanent increase in the economy's price level following a single impulse. The permanent increase in the consumer price index is equivalent to a temporary increase in headline inflation, with convergence towards steady-state value, a non-permanent effect on inflation-rate.

Table 4.9
Common Component, select series, R^2, 01:1984-10:2019

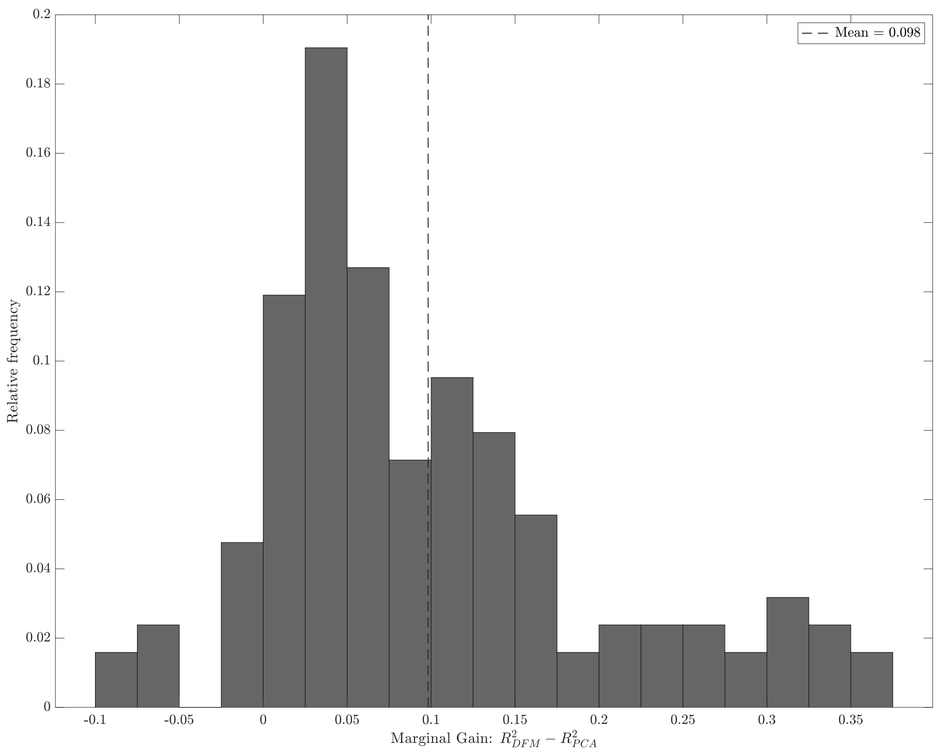
Series:	*PCA*	*Kalman Smoothed*
Real Personal Income	0.130	0.190
Capacity Utilization: Manufacturing	0.597	0.831
Civilian Unemployment Rate	0.297	0.316
Real M2 Money Stock	0.216	0.527
S&P Dividend Yield	0.160	0.177
3-Month Treasury Bill	0.294	0.625
10-Year Treasury Bond Rate	0.168	0.425
Crude Oil Price	0.188	0.305
Volatility Index	0.288	0.374
Consumer Price Index[1]	1.00	1.00
Industrial Production Index[1]	1.00	1.00
Federal Funds Rate[1]	1.00	1.00

1 R^2 denotes the fraction of variance in each series in X_t explained by the common component, the remaining variance stems from the idiosyncratic component. By construction the observable factors in the DFM are explained solely by their respective common component with identity loading.

Considering the factor model's responses, following contractionary monetary impulses to the FAVAR, we examine the same nine series plotted in figure 4.12a. Again to determine the reliability of the IRFs, we compute marginal R^2-gained from the Kalman-Smoother approach. We report these for the nine series of interest in table 4.9; furthermore, we report histogram for all 125-variables in the panel in figure 4.8. Once again, we have (mostly) gains in terms of explanatory power. For our nine chosen variables in X, the mean R^2-gain is 11.8%, while for all 125 variables the mean R^2-gain is 9.8%. Therefore, the point precision of the DFM-IRFs is (mostly) higher than those from the PCA-FAVAR.

Contrary to IRFs under the recursive scheme, we see in general that confidence intervals reach insignificance quicker. We see very fast convergence of market volatility after a monetary policy shock, longer readjustment times for real economic disequilibrium as measured by real personal income, and the unemployment rate. Unexpected, however, is the highly significant decrease of the real money stock, following an unanticipated monetary stimulus. One would expect short-run increases

Figure 4.8
Histogram, R^2-improvement of DFM over PCA, 01:1984-10:2019

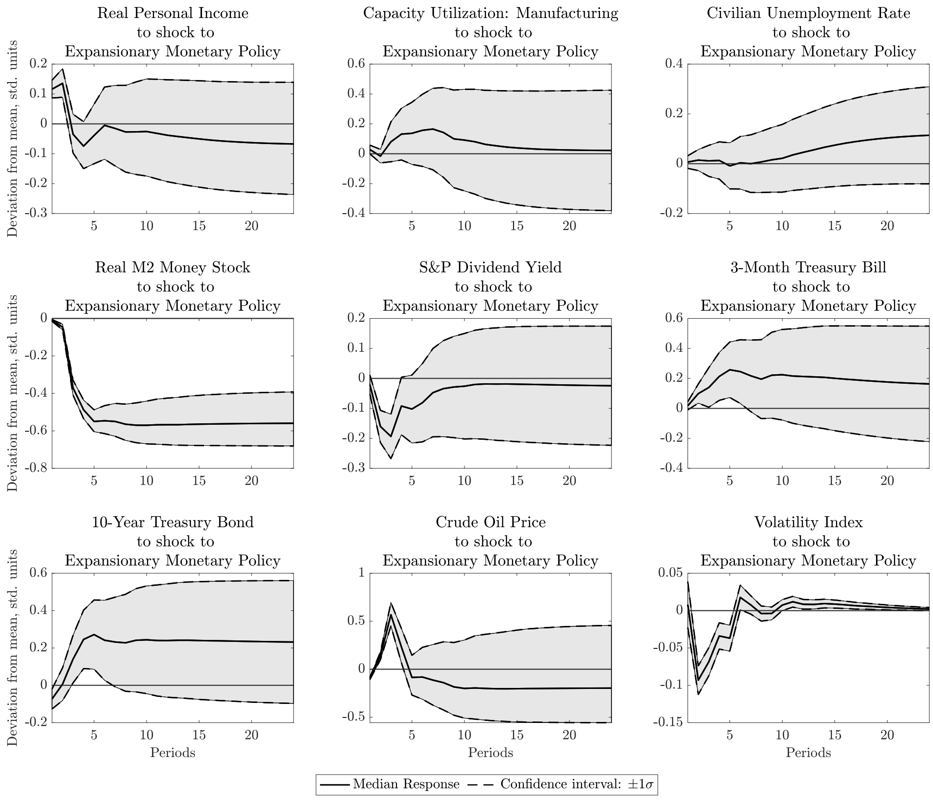


1 Histogram showing the difference in R^2 for each variable in X_t when using the two-step Kalman approach over the PCA-approach.

and no long-run effect on the real money supply (the increased nominal money supply, would be neutralized by corresponding increases in inflation). Considering financial assets, we see a significant temporary increase in the price of oil; however, the effects quickly die out.

Similarly, the negative response in dividend-yields is significant only so for three months post-shock. This implies that the model supposedly captures the efficiency of financial markets; a sudden unexpected monetary expansion generates economic stimulus and expectations hereof; as a result, stock-prices increases, dividend-yield decreases, and volatility decreases. Empirically these volatility clusters rarely last longer than a few months, and this is also the case here. Another unexpected result is the response of the treasury bill, increasing monetary policy shocks. It is in line with the similarly unexpected result that an expansionary monetary policy shock increases (insignificantly) the federal funds rate. When these expectations have been priced into the market, the prices should stabilize, and volatility increases back to a steady-state level.

FIGURE 4.9
Impulse response functions X_t expansionary monetary policy

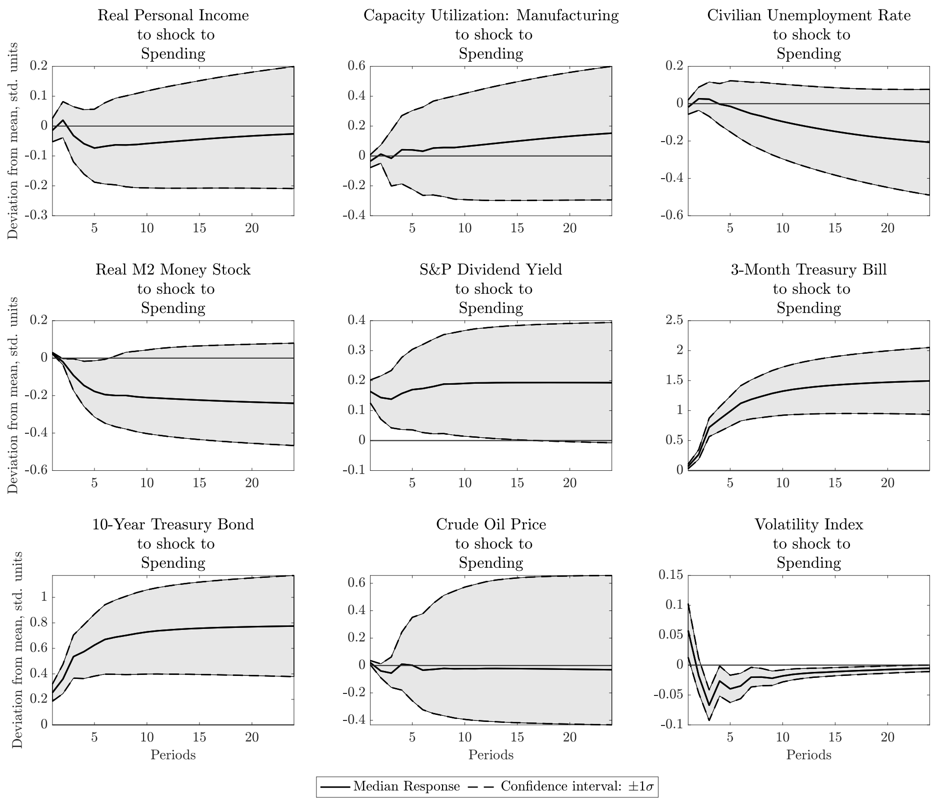


1 Three factor, three lag DFM-FAVAR, 01:1984-08:2019-sample. 1,000 draw residual-based GARCH bootstrap confidence intervals, one standard deviation. All responses transformed to standardized levels.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

Recalling how monetary policy shocks under this identification only have transitionary effects on the nominal interest rate, while the stochastic trend is driven by supply- and spending shocks. We, therefore, consider the spending shock in figure 4.10. We find that the permanent response of the nominal interest rates to a spending shock correctly translates to increases in the treasury bill rates, with the larger response in the 3-month rate and the longer 10-year bond rate tracing the same response at a smaller magnitude, these results, fortunately, are significant on the one standard deviation-confidence mark. Moreover, the T-bill rates are empirically well explained by the FFR, see table A.7 in the appendix, showing an R^2 of above 0.5 solely by the differenced FFR for the short-rates.

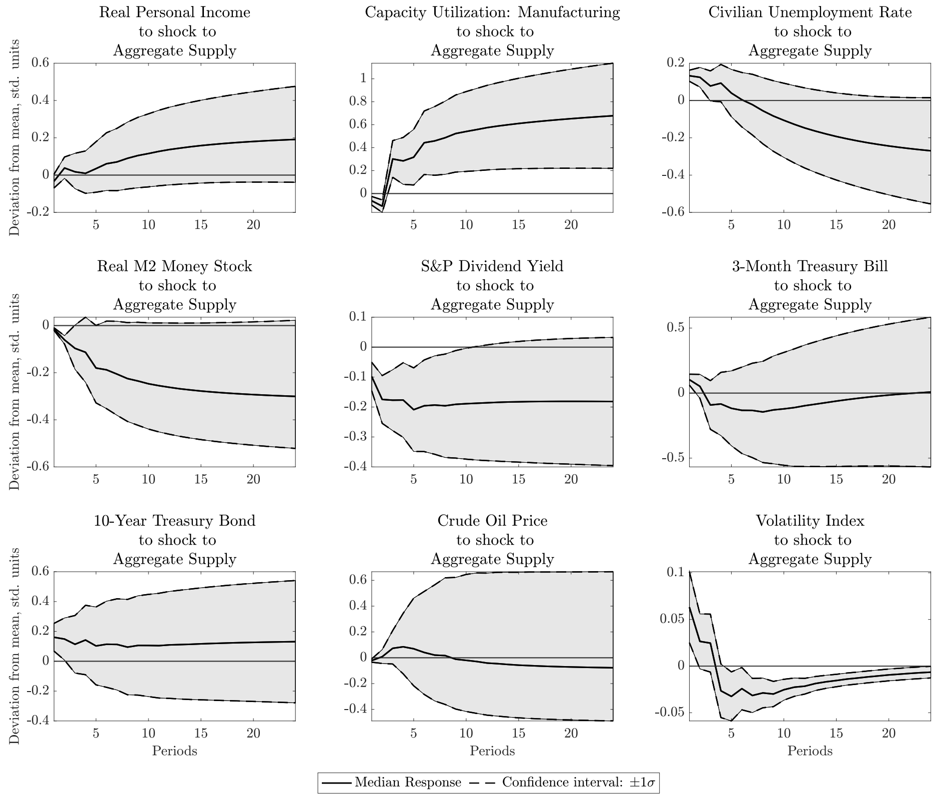
Figure 4.10
Impulse response functions X_t spending shock



1 Three factor, three lag DFM-FAVAR, 01:1984-08:2019-sample. 1,000 draw residual-based GARCH bootstrap confidence intervals, one standard deviation. All responses transformed to standardized levels.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

Figure 4.11
Impulse response functions X_t aggregate supply shock



1 Three factor, three lag DFM-FAVAR, 01:1984-10:2019-sample. 1,000 draw residual-based GARCH bootstrap confidence intervals, one standard deviation. All responses transformed to standardized levels.

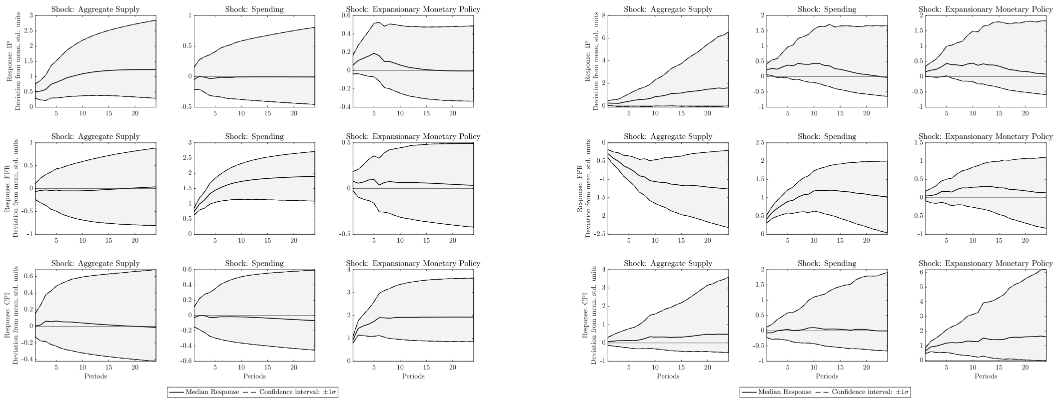
2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

Figure 4.11 shows the responses of our selected 9 variables to a positive supply-shock. The results are inline with what one would suspect. The capacity-utilization are significantly increasing, the dividend yields are slightly significantly decreasing. The other results are statistically insignificant at the one standard deviation mark, but with median-responses inline with theoretical expectations. We see a short-run increase in unemployment, but very quickly the effects decline and the unemployment decreases below steady-state level. The median response, suggests a steepening of the yield-curve, but highly insignificant.

To test the robustness to misspecification, we consider the impulse-responses of Y under two alternative specifications, these are reported in 4.12. Quite contrary to the

Figure 4.12
FAVAR impulse responses alternative specifications

(A) 5 Factors, 3 lags (B) 3 Factors, 13 lags



1 Long-run restrictions in the DFM-FAVAR, considering different model specifications

2 Three factor, three lag DFM-FAVAR, 01:1984-10:2019-sample. 1,000 draw residual-based GARCH bootstrap confidence intervals, one standard deviation. All responses transformed to standardized levels.

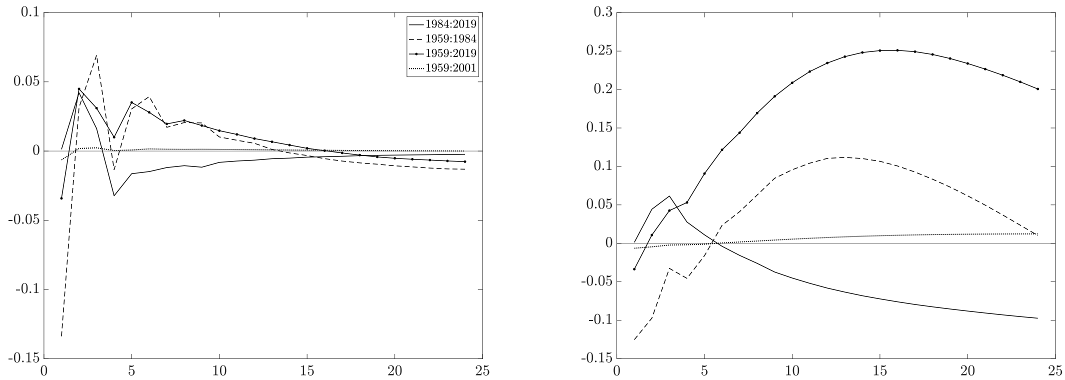
3 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

recursive identification scheme, the long-run identification is robust to changes in lag-order; that is, we obtain very similar results using a lag order of 13. Similarly to the finding of BBE, we see that the model is robust to increasing the number of dynamic factors. We see similar results using five dynamic factors instead of three. This suggests that three dynamic factors capture movements in X_t sufficiently, and adding further factors only contribute by decreasing efficiency in estimation. Though robust to changes in the specification, the scheme at large is not robust to large changes in the sample under consideration. This puts further emphasis on the presence of structural breaks. To illustrate the sensitivity of the structural analysis to the sample used in estimation, we consider the widely controversial prize-puzzle. In figure 4.13, the median response of the inflation rate and, by extension, the price-level of the model economy to a positive spending shock is plotted. From the figure, the issue arising when not addressing parameter instability is apparent. In our benchmark sample 01:1984-10:2019, the model yield results in contrast with the underlying theoretical model; we see decreasing prices following a positive spending shock. While all the other samples are providing results in line with the theoretical framework, upon which the FAVAR is built. This point shows that failure to accommodate

Figure 4.13

Median response to spending shock, different samples

(a) Inflation Rate (b) Consumer Price Index



1 Long-run restrictions in the DFM-FAVAR with 3 lags in the VAR and 3 lags in the DFM; considering different estimation samples.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

3 $\pi = \Delta \log CPI$, where π is defined as the inflation rate.

structural instabilities possibly leads to misleading conclusions in a framework as sensitive as the SVAR and, by extension, the structural FAVAR. For our model, the accommodation of structural instabilities leads to rejection of our proposed model.

4.3.1 Variance Decomposition

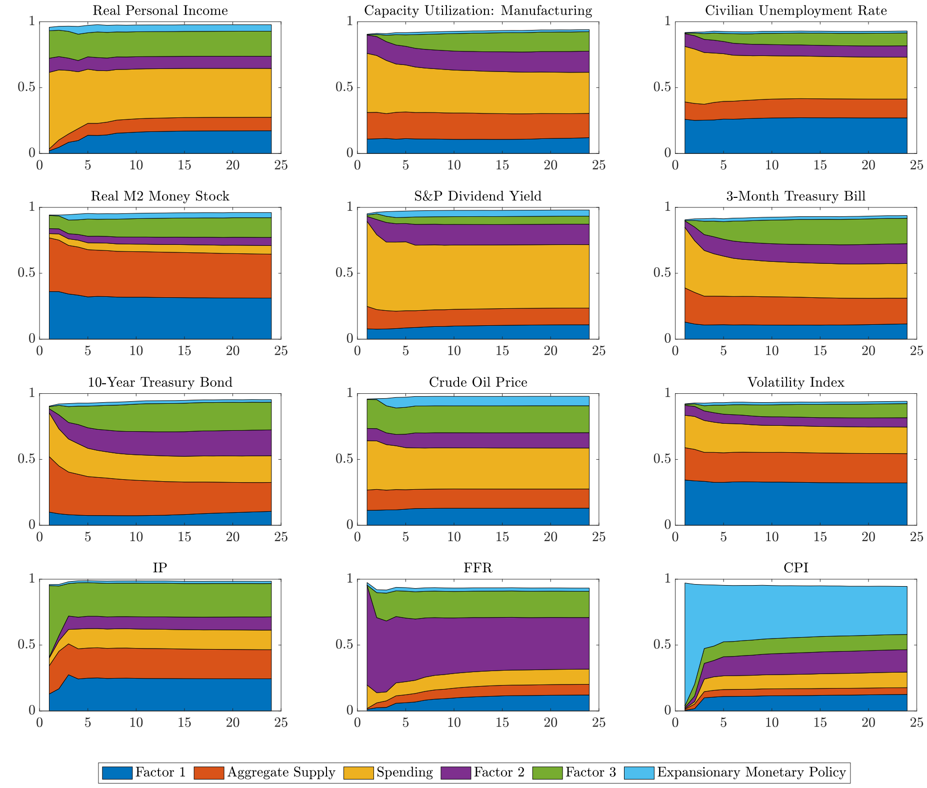
In the FAVAR framework, we can decompose the error-term for each equation into structural shock components in a fashion similar to that of SVAR. However, it does allow for a very convenient modification, by projecting into the DFM, we can decompose the common component of each series in X_t into shock innovations as described in section 2.4.2. A subset of 9 series, in addition to the three observables, are plotted in figure 4.14.

First and foremost, these suggest that the factors do contribute significantly. Considering first industrial production, where the first factor is explaining 12.9% of the error variance in the first period, the identified aggregate supply shock contributes 21.4% in comparison. This indicates that the shock associated with the first factor under this identification accounts for more variance than does the shock associated with the industrial production index. The shock to the third factor is the most influential disturbance on impact peaking at 54.4%. Consistent with other literature on long-run effects, the supply-components account for an increasing fraction as we increase the horizon. The restrictions imposed suggests that in the limit, the shock to the first factor and industrial production (Aggregate Supply) should tend to unity; account for all variability in industrial production.

The federal funds rate is accounted for mainly by shifts in the IS-curve and shocks to the second factor; spending-shocks and shocks to the second factor in the short run respectively accounting for 17.8%- and 75.7% on impact, at which point the permanent supply shocks take over with the first factor and aggregate supply (IP) contributing 20.2% 24 months after impact, the spending shock decreasing to 11.5% after 24 months. The spending shock accounts for short-run fluctuations, and the major part of the remaining variance, interestingly and somewhat disappointing, the monetary policy-shock accounts for merely ≈2% of the error variance. However this can be due to the disentanglement of shocks, our ordering sets the second and third factor directly above the consumer price index as we saw high loadings on the price series, it is therefore very likely that the shocks of the second and third factor can be interpreted as components of a monetary policy shock. The FEVD of the FFR concerning the last two factors spans between 39.2% and 19.9% 24-months post-impact, suggesting that the aggregate monetary policy shock may account for as much as 61% of the long-run variance of the interest rate.

It follows then that under this scheme, the price-puzzle is no longer present, that is, an expansionary monetary policy shock results in the expected price increase. Due to the structure, however, this is more-or-less by construction, we see that monetary policy accounts for between 93.3% (short-run) and 36.5% (long-run) of the inflation error variance. The remaining long-run variance is accounted for by the remaining factors and observables; in particular, we note that the two supply components account for an increasing fraction of total error variance increasing from 1.3% (short-run) to 17.7% (24 months post-impact).

FIGURE 4.14
Forecast error variance decomposition, long-run restricted DFM-FAVAR



1 Median variance decomposition of the three factor, three lag DFM-FAVAR identified with long-run restrictions.

2 $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

Over the entire set of series, consistent with other literature, the monetary policy stand-alone accounts for a relatively small fraction of total error variance. However, by revisiting the idea, that some of the aggregate monetary policy shock may be accounted for by the second and third factor we find that a relatively large part of the considered variables are accounted for by those factors, that is the crude oil price, which is known to be sensitive to monetary policy, unemployment rate, RPI, capacity utilization, volatility and treasury bonds. Especially treasury bonds are known to follow the monetary policy closely by tracking the interest rate.

TABLE 4.10
FEVD: Industrial Production

	Supply components		*Demand Components*			
horizons	Factor 1	Aggregate Supply	Spending	Factor 2	Factor 3	Expansionary Monetary Policy
1	12.86 (6.5, 20.3)	21.41 (11.7, 33.7)	5.87 (1.5, 13.3)	0.66 (0.1, 2.6)	54.35 (47, 61.7)	0.8 (0.1, 3.1)
2	16.8 (8, 27.6)	28.44 (15.3, 43.1)	7.95 (2.2, 17.2)	3.91 (1.7, 6.9)	37.74 (32.2, 43.2)	1.19 (0.3, 3.5)
3	27.49 (15.3, 37.5)	23.45 (11.7, 36.8)	10.92 (6.8, 15.5)	10.19 (5.4, 15.8)	24.63 (20.8, 29.3)	1.39 (0.5, 3.4)
6	25.08 (14.5, 34.3)	22.91 (12.1, 34.7)	14.53 (8.8, 20.3)	9.36 (5.4, 14)	25.23 (21.3, 30.2)	1.57 (0.7, 3.3)
12	24.62 (14.1, 33.7)	22.75 (12.2, 34.4)	14.68 (8.8, 20.7)	9.36 (5.4, 13.7)	25.52 (21.5, 30.6)	1.74 (0.9, 3.5)
24	24.39 (14.5, 33.2)	22.09 (12.2, 33.7)	14.89 (8.9, 21.1)	10.03 (6, 14.6)	25.33 (21.4, 30.7)	1.72 (0.8, 3.4)

1 Forecast error variance decomposition of the FAVAR; brackets below the point estimate shows one standard deviation error bands

TABLE 4.11
FEVD: Consumer Price Index

	Supply components		*Demand Components*			
horizons	Factor 1	Aggregate Supply	Spending	Factor 2	Factor 3	Expansionary Monetary Policy
1	0.5 (0.1, 2.1)	0.75 (0.1, 3)	0.54 (0, 2.5)	0.79 (0.1, 3.3)	1.26 (0.1, 4.4)	93.26 (88, 96.7)
2	1.69 (0.4, 5.2)	3.56 (1.2, 8.1)	1.77 (0.4, 5.2)	4.32 (1.5, 8.9)	8.91 (4.8, 14.5)	75.87 (68.4, 81.8)
3	9.96 (5.2, 15.8)	4.7 (2, 9.5)	9.52 (4.2, 17.9)	11.92 (7.6, 17.5)	11.28 (6, 18.2)	48.4 (43.3, 53.4)
6	11.04 (6.1, 17.5)	5.29 (2.6, 10.1)	10.38 (4.7, 19.7)	14.66 (9.7, 20.3)	11.35 (5.9, 19.3)	42.47 (37.9, 47.2)
12	11.61 (6.3, 18.7)	5.29 (2.6, 10.1)	10.81 (4.9, 20.3)	16.24 (11, 22.1)	11.62 (5.9, 19.7)	39.51 (35.2, 43.8)
24	12.53 (6.8, 20.3)	5.12 (2.6, 10)	11.81 (5.2, 21.4)	17.01 (11.9, 22.9)	11.58 (5.9, 20.1)	36.45 (32.3, 41.1)

1 Forecast error variance decomposition of the FAVAR; brackets below the point estimate shows one standard deviation error bands

Table 4.12
FEVD: Federal Funds Rate

	Supply components		Demand Components			
horizons	Factor 1	Aggregate Supply	Spending	Factor 2	Factor 3	Expansionary Monetary Policy
1	1.26	0.59	17.81	75.69	0.19	1.9
	(0.1, 4.4)	(0, 2.7)	(9.7, 27.4)	(65.7, 84)	(0, 0.9)	(0.2, 5.1)
2	2.44	3.69	7.71	57	18.97	2.22
	(0.6, 9.9)	(0.8, 10.6)	(4.2, 15)	(40.9, 70.8)	(10.9, 28.6)	(0.5, 6.5)
3	2.69	4.97	6.8	53.84	21.12	2.36
	(0.7, 10.8)	(1.4, 11.9)	(3.6, 14)	(37.1, 69)	(12.2, 31.4)	(0.6, 6.7)
6	6.81	6.47	10.02	46.46	20.67	2.55
	(3.3, 14.8)	(2.7, 13.2)	(5.5, 17.9)	(31.7, 60.8)	(12.2, 29.6)	(1, 6.1)
12	10.62	7.78	11.29	41.22	19.99	2.53
	(5.1, 21.5)	(3.6, 14.1)	(5.8, 20.5)	(27.2, 54.2)	(11.8, 28.8)	(1, 5.6)
24	12.09	8.11	11.53	39.18	19.91	2.44
	(5.6, 24.2)	(3.8, 14.8)	(5.9, 21.3)	(25.5, 52.4)	(11.7, 28.8)	(1, 5.3)

1 Forecast error variance decomposition of the FAVAR; brackets below the point estimate shows one standard deviation error bands

5 Conclusion

Throughout the analysis, we have uncovered several interesting properties and certain concerns that may arise in a framework like the FAVAR.

Being a natural extension of the dynamic-factor model framework, it should be no surprise that the FAVAR is producing satisfactory forecasts out-of-sample, as it is well-established that factor models, in general, are efficient at forecasting macroeconomic time-series. These results are in line with several studies examining the forecasting performance of various competing factor models. In predicting Y_t out-of-sample, the DFM-FAVARs improvement over the PCA-FAVAR is questionable. The PCA-approach still seems to dominate academic literature on FAVAR-models, likely due to the computational simplicity associated with estimation.

Whether or not the extra layer of complexity is well worth the trouble for future FAVAR-estimation depends on the goal of the analysis. For credible high dimensional structural inference, the DFM-FAVAR appears superior as predictability of the panel is increased by a significant margin. The benefits of the FAVAR over traditional VARs are two-fold. On the one hand, the informational content for the observables is increased, adding predictability and diminishing omitted variable bias, on the other hand, we can conduct structural analysis on a vast number of variables in the factor data-set. For structural analysis on X_t, it is crucial that the idiosyncratic component does not dominate the common component, thereby drowning estimated effects in uncertainty. In the sample spanning 01:1984-10:2019, we have shown a mean improvement of 10%, over the PCA-approach, in explaining each variable in X_t by merely extending the model with time-dimensional smoothing. The ability to track movements in variables in the factor panel with higher precision is especially useful for structural analysis in high dimensions. We consider this to be a positive result from this study. Due to the scarcity of replication studies in this area, we find relevance in the inclusion of the BBE-replication-part. We find evidence that the model by adopting the specification of BBE, unsurprisingly, is still able to reproduce their findings, even though the results themselves are insignificant when we accommodate conditional heteroscedasticity. It does, however, seem as though the extended panel we use, might be partly to blame for the statistical insignificance, as we also find quicker convergence to insignificance than BBE, even when using the exact specification of BBE (PCA-extraction, recursive scheme, 13 lags, level FFR and assumed homoscedastic residuals).

The BBE specification furthermore is empirically questionable, the data exhibits evidence of unit-roots, and their chosen lag-length can not be statistically justified using conventional methods. Unfortunately, by correcting the lag-length and transforming variables to implied stationarity, the promising point-estimate results deteriorate. We, therefore, find it to be of particular interest to explore alternative identification schemes.

We address this, in the third section, by proposing an alternative long-run identification scheme that suggests point-estimates of the dynamics that are mostly in line with economic theory. Unfortunately, they are, similarly to the recursive scheme, largely statistically insignificant even at a relatively low confidence level. The recursive identification scheme still stands as the dominant scheme in these types of FAVAR-models in other literature, yet as we argue, it is not without issues. We, therefore, believe that showing that it is indeed possible to represent the structural model by imposing long-run restrictions, we add to the limited literature on non-recursive FAVAR-models. An interesting approach to improve the identification further could be to separate the panel into subpanels containing supply- and demand-driven time-series, respectively. Factors extracted from those panels individually would yield a more natural ordering scheme in the long-run Cholesky factorization, thereby disentangling the structural shocks further and, thus, being robust to the critique of long-run-identification of Faust and Leeper (1997). Another possibility that could be particularly exciting for future research is the possibility of long-run dynamics in the factor panel. In the analysis, we followed the work of McCracken and Ng (2016) and transformed the panel to stationarity. In doing so, we lose out on the possibility of exploiting cointegrating relationships, and thus inherent long-run relationships in the factor panel. Recent work of Banerjee, Marcellino, and Masten (2017) exploits this when constructing their proposed *Factor-Augmented Error Correction*-model.

An alternative to both identifications is that of Uhlig (2005), namely sign-restrictions, while promising in theory, sign-restrictions holds several disadvantages in our framework. As the latent factors are unique only up to a sign, we have no reasonable way to impose sign-restrictions on latent factors, as they are identified up to sign, making structural inference on the FAVAR infeasible. Work to circumvent this issue is presented by Amir Ahmadi and Uhlig (2015); however, sign-restrictions still do not provide us with a point estimate of a structural model but rather a confidence set of models consistent with the imposed restrictions.

Our findings concerning filtered volatility of the FAVAR-residuals, suggest another possibility currently unexplored in FAVAR-literature but is gaining traction and popularity in many recent SVAR-studies. Namely, that the model-residuals suffer greatly from volatility clusters and are seemingly well-behaved around conditional heteroscedasticity on the GARCH-form, this key finding provides promising grounds for the possibility of structural identification by heteroscedasticity in the tradition of Lanne et al. (2017); Lütkepohl and Schlaak (2019).

One may note, that the results might improve if one were to address model instabilities in the mean separately for each model-equation. A way to do so is by assuming a law-of-motion for the model parameters, and thereby obtaining a *time-varying* FAVAR-model. However, inference and estimation of such a model are more tricky as one has to rely on Bayesian inference and, to that end, impose priors. Even though more-or-less standard priors are present in the literature now, Bayesian estimation is still heavily dependent on the actual prior used, posing the risk of a poorly chosen prior that might significantly shift estimates and confidence sets; Bayesian estimation is thus very prone to misspecification.

So, while Bayesian estimation is an option for these type models, it is still convenient and relevant for many applications to offer a frequentist alternative. Another thing to consider in this context is that the seminal paper by BBE estimated a time-invariant Bayesian FAVAR without apparent improvement over the frequentist approach. It might be that the sheer number of parameters to be estimated and, by extension, the generalization of the priors lead to less-than-optimal parameter estimates.

Over a decade has passed since the introduction of the factor-augmented VAR model; yet, the framework is still at a relatively early development stage. The need for accommodation of big data is as big as ever in nearly every field. For this purpose, in empirical macroeconomics, the FAVAR looks promising. To increase the reach of the framework and to stimulate further research, a set of more or less standardized workflows and methods should be constructed; this is especially true for the underlying factor-model as methods for structural inference are more-or-less limited to the approach of Bernanke et al. (2005) and ad hoc solutions such as the one used in the present long-run scheme. To this end, theoretically justified restrictions are often challenging to obtain as macroeconomic models usually do not include concepts even approximately corresponding to factors.

A well built theoretical framework incorporating big-data, is likely the road to success in this framework, along with more general statistical tools to conduct the empirical analysis. With those tools in hand, however, the FAVAR-model is likely a potent model capable of important findings.

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Additional Figures and Tables A

Table A.1
Summary statistics

	Obs	*Mean*	*Median*	*Std*	*Minimum*	*Maximum*	*Skewness*	*Kurtosis*
				Full, 01:1959-10:2019				
Δ log(CPI)	728	0.0030	0.0026	0.0031	-0.0179	0.0179	0.3124	7.5803
Δ log(IP)	728	0.0024	0.0024	0.0080	-0.0443	0.0600	-0.3373	10.350
ΔPolicy Rate	728	-0.0008	0.0100	0.5152	-6.6300	3.0600	-2.3007	49.509
				Pre, 01:1959-12:1983				
Δ log(CPI)	298	0.0042	0.0032	0.0035	-0.0031	0.0179	0.7667	3.3422
Δ log(IP)	298	0.0028	0.0036	0.0101	-0.0361	0.0600	-0.0625	7.5809
ΔPolicy Rate	298	0.0236	0.0300	0.7572	-6.6300	3.0600	-1.8277	26.154
				Post, 01:1984-10:2019				
Δ log(IP)	430	0.0016	0.0021	0.0061	-0.0443	0.0203	-1.5710	12.247
ΔPolicy Rate	430	-0.0186	0.0000	0.2286	-1.3100	0.8700	-0.8954	7.4587
Δ log(CPI)	430	0.0022	0.0022	0.0025	-0.0179	0.0137	-1.4169	14.879
				Bernanke et al. (2005), 01:1959-08:2001				
Δ log(CPI)	510	0.0036	0.0029	0.0030	-0.0055	0.0179	1.0545	4.7262
Δ log(IP)	510	0.0027	0.0032	0.0084	-0.03605	0.0600	0.0217	9.3772
ΔPolicy Rate	510	0.0024	0.0200	0.6031	-6.6300	3.0600	-2.0468	37.682

Table A.2
Summary statistics, untransformed

	Mean	*Median*	*Std.*	*Minimum*	*Maximum*	*Skewness*	*Kurtosis*
				Full, 1959:2019			
CPI	67.1703	63.3748	27.0244	22.7081	110.5516	0.064096	1.6089
Industrial Production	126.9279	124.5	74.4961	28.97	258.444	0.13757	1.645
Policy Rate	4.8195	4.795	3.8817	-2.9856	19.1	0.63987	4.0533
				Post, 1984:2019			
CPI	85.1463	92.0785	18.0563	49.2654	110.5516	-0.41898	1.6677
Industrial Production	178.2793	177.4	46.9065	98.1	258.444	-0.022236	1.7593
Policy Rate	3.6661	3.99	3.4173	-2.9856	11.64	-0.0057939	2.0973
				Pre, 1959:1983			
CPI	39.6156	39.8118	9.6269	22.7081	53.5053	-0.25837	1.8457
Industrial Production	48.2907	40	20.7147	28.97	98.1	1.0873	2.9787
Policy Rate	6.5968	5.31	3.8856	1.17	19.1	1.2518	4.0741

Table A.3
KPSS-test, 01:1984-10:2019

	KPSS-test statistic, critical value = 0.146					
Lags	*1*	*2*	*3*	*4*	*5*	*6*
$\Delta \log IP$	0.1629	0.1370	0.1155	0.0999	0.0894	0.0812
$\Delta \log CPI$	0.0544	0.0494	0.0479	0.0470	0.0468	0.0470
Δ*Policy Rate*	0.1126	0.0905	0.0784	0.0710	0.0651	0.0603
	7	*8*	*9*	*10*	*11*	*12*
$\Delta \log IP$	0.0757	0.0713	0.0677	0.0648	0.0626	0.0609
$\Delta \log CPI$	0.0472	0.0475	0.0478	0.0476	0.0471	0.0472
Δ*Policy Rate*	0.0566	0.0536	0.0510	0.0489	0.0471	0.0457

1 Green coloration indicates failure to reject null-hypothesis on a 5% significance level. Notice that in the KPSS-test inability to reject H_0 is evidence against a unit root. That is the H_0 is presence of deterministic trend. While H_1 is unit root.

Table A.4
ADF-test, 01:1984-10:2019

	ADF-test statistic, critical value = -1.9413					
Lags	*1*	*2*	*3*	*4*	*5*	*6*
$\Delta \log IP$	-10.4303	-7.2244	-5.7929	-5.4434	-5.0049	-5.5233
	(2.1026)	(2.0773)	(2.0030)	(1.9982)	(1.9834)	(2.0001)
$\Delta \log CPI$	-12.9110	-10.2652	-8.8209	-8.4788	-7.5061	-6.8291
	(1.9765)	(2.0020)	(1.9963)	(1.9941)	(2.0015)	(1.9984)
Δ*Policy Rate*	-9.5218	-8.3182	-7.6236	-6.3669	-5.7428	-5.8140
	(2.0121)	(1.9928)	(1.9578)	(1.9983)	(1.9856)	(1.9504)
Lags	*7*	*8*	*9*	*10*	*11*	*12*
$\Delta \log IP$	-5.3522	-4.9964	-4.8417	-4.8881	-5.0354	-5.2783
	(1.9981)	(1.9943)	(1.9974)	(2.0012)	(2.0049)	(1.9995)
$\Delta \log CPI$	-6.5672	-5.9200	-5.0941	-4.8050	-5.5658	-5.2287
	(1.9981)	(2.0089)	(2.0051)	(1.9908)	(1.9775)	(2.0083)
Δ*Policy Rate*	-5.1216	-4.5542	-4.6763	-4.9628	-4.8354	-4.7537
	(2.0129)	(2.0048)	(1.9674)	(1.9329)	(2.0121)	(2.0043)

1 Brackets below indicates Durbin Watson-statistic (Durbin & Watson, 1950) for each equation. Green colouration indicates significance at a five percent critical level $\alpha = 0.05$. Durbin-Watson statistics suggests no serial correlation in the test residuals.

Table A.5
Information Criterion, 3 factor DFM-FAVAR 01:1959-10:2019

Lags	AIC	SIC	HQC	Log Likelihood
1	2.2867	2.5167	2.3755	-782.647
2	1.6296	**2.0895**	1.8072	-511.4031
3	1.5085	2.1984	**1.7749**	-432.0498
4	1.4745	2.3943	1.8297	-383.8612
5	1.4155	2.5653	1.8595	-326.7382
6	1.3856	2.7653	1.9184	-280.0354
7	1.3747	2.9845	1.9963	-240.1581
8	1.3427	3.1824	2.0531	-192.6943
9	1.3287	3.3983	2.1279	-151.6628
10	1.3294	3.629	2.2174	-115.9184
11	**1.3205**	3.8501	2.2973	-76.7442
12	1.3476	4.1072	2.4133	-50.4576

1 Minimized values denoted in bold where meaningful, *(LLH is non-decreasing in p)*

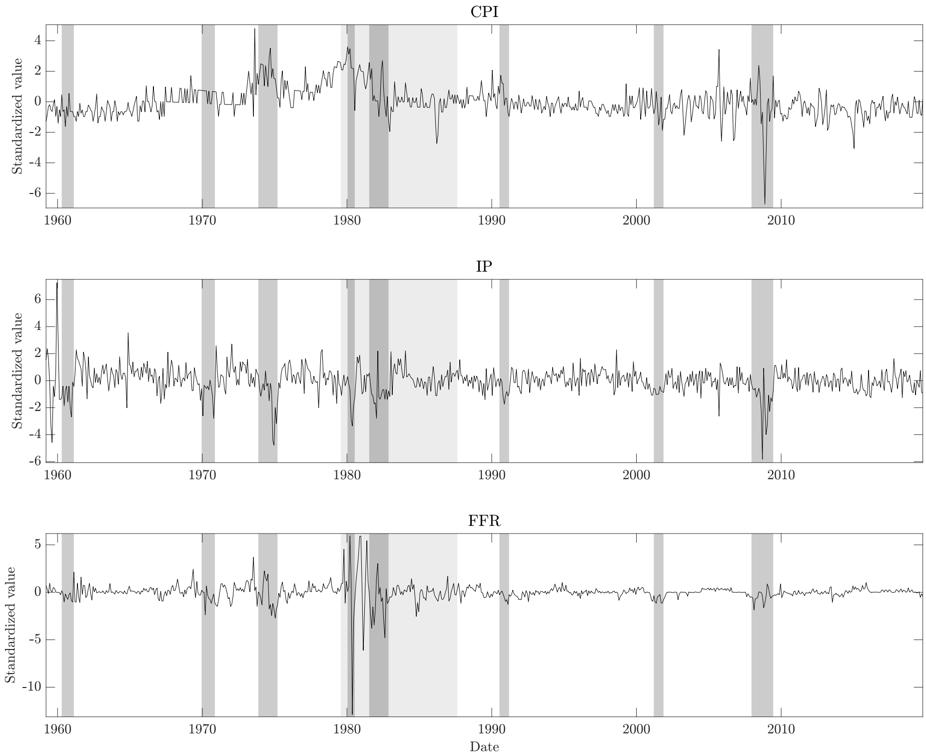
Table A.6
Sign prediction accuracy, %

	Inflation Rate				*IP-Growth*				*Policy Rate*			
h	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*	*1*	*3*	*6*	*12*
DFM $K=3$, $p=3$	67.7	61.4	60.3	61.4	61.8	63.3	61.3	54.9	66.2	60.8	62.6	59.1
DFM $K=3$, $p=1$	66.2	52.0	51.7	51.0	62.0	62.1	59.1	54.2	69.4	65.8	63.3	57.6
DFM $K=1$, $p=3$	68.0	63.0	63.1	59.8	62.3	61.1	58.9	55.1	68.2	64.6	64.1	56.4
PCA $K=8$, $p=3$	68.0	62.6	63.8	63.3	62.0	60.1	58.9	55.2	66.3	62.0	59.3	60.8
PCA $K=3$, $p=3$	67.7	62.5	61.4	60.8	62.3	62.3	61.4	53.7	65.7	62.6	63.3	57.9
VAR, $p=3$	67.8	62.6	62.8	55.2	61.3	60.3	55.7	54.2	64.6	63.0	56.4	54.5
AR, $p=3$	67.8	60.3	59.3	54.0	59.9	58.4	55.7	50.0	62.1	57.9	53.5	56.6
Random Walk	69.2	66.5	66.8	60.9	57.1	58.8	57.6	49.2	69.7	63.6	59.8	58.8

1 Percentage of forecasts over different horizons bearing same sign as realized value. *DFM* and *PCA* both refer to a FAVAR, where factor estimation is carried out with the two-step Kalman procedure and Principal components respectively.

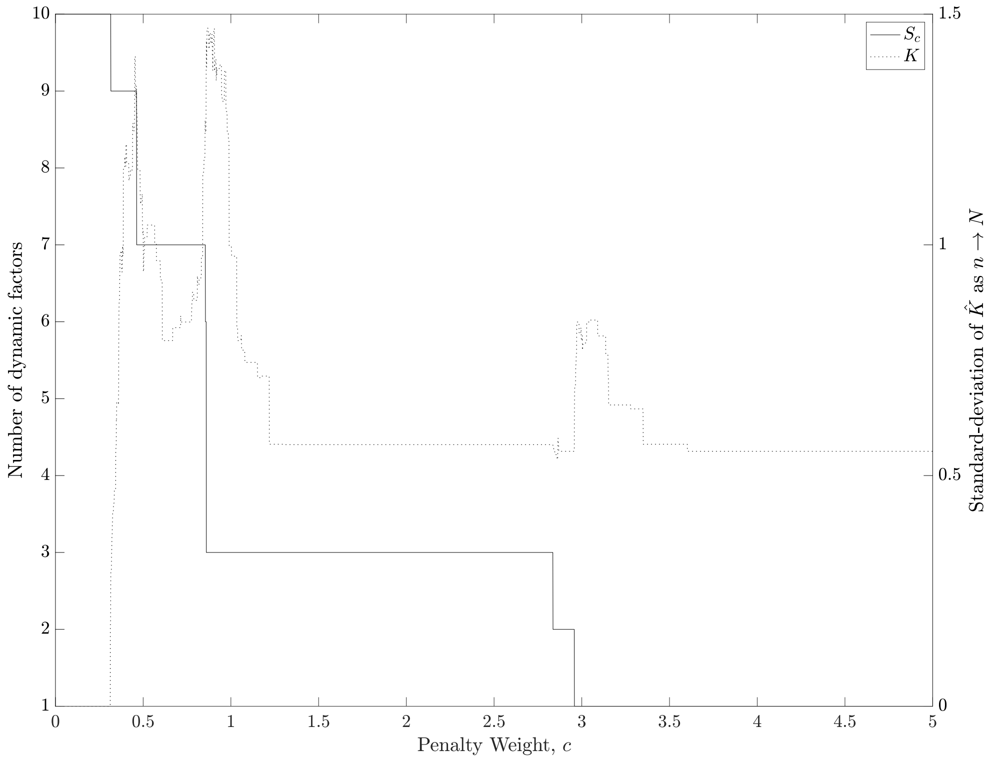
2 Rolling window *pseudo OoS* forecasts over 01:1959-10:2019

Figure A.1
Observables full sample, transformed



1 Full sample movements in the observable factors $\boldsymbol{Y}$ of the model. Notice the two overlaps of the Volcker-regime and recessions. These periods are often referred to as the Volcker recessions, as it is widely believed that they stem from Volckers fight against inflation, conducting contractionary monetary policy in an already relatively high interest rate environment.

Figure A.2
Hallin and Liška (2007) Number of primitive factors in the DFM, 01:1959-10:2019

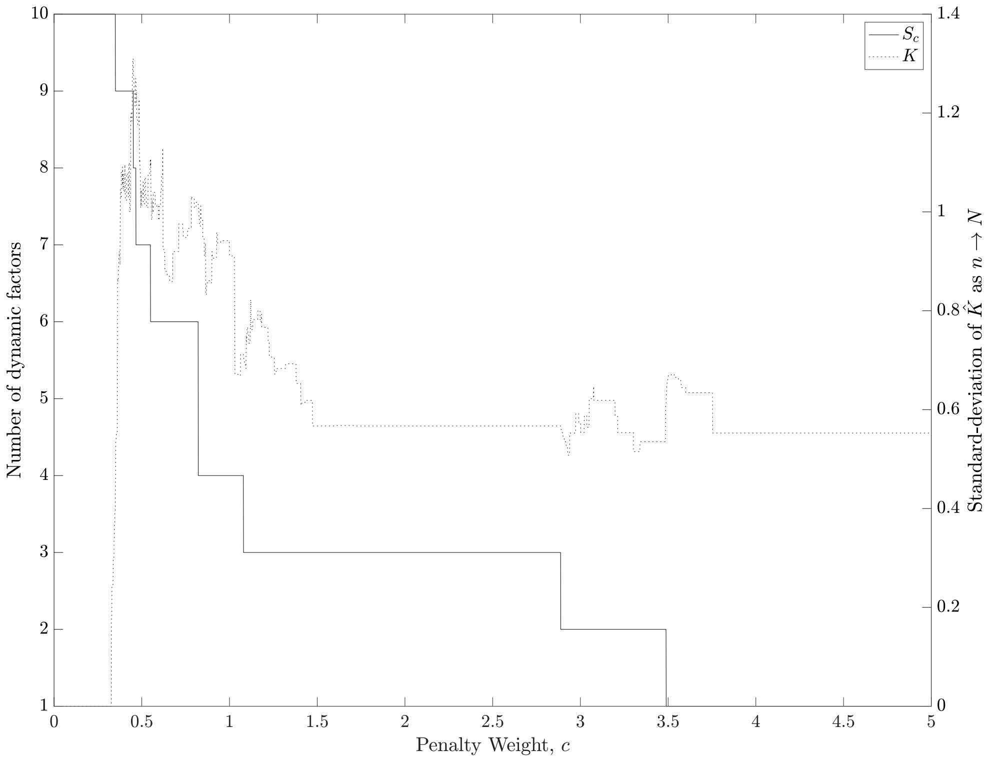


1 Hallin and Liška (2007) Information criterion (log-criterion 2) for number of primitive dynamic factors.

2 S (dotted line) on the right axis is an indication of the standard-deviation associated with the current estimate of number of factors (estimated for a number of time-series, (n), as n increases from a lower threshold (94 series) to the number of series (125)), K (Solid line) shows the estimated number of factors for each value of the penalty weight on an equally distanced linespace.

Figure A.3

Hallin and Liška (2007) Number of primitive factors in the DFM, 01:1984-10:2019

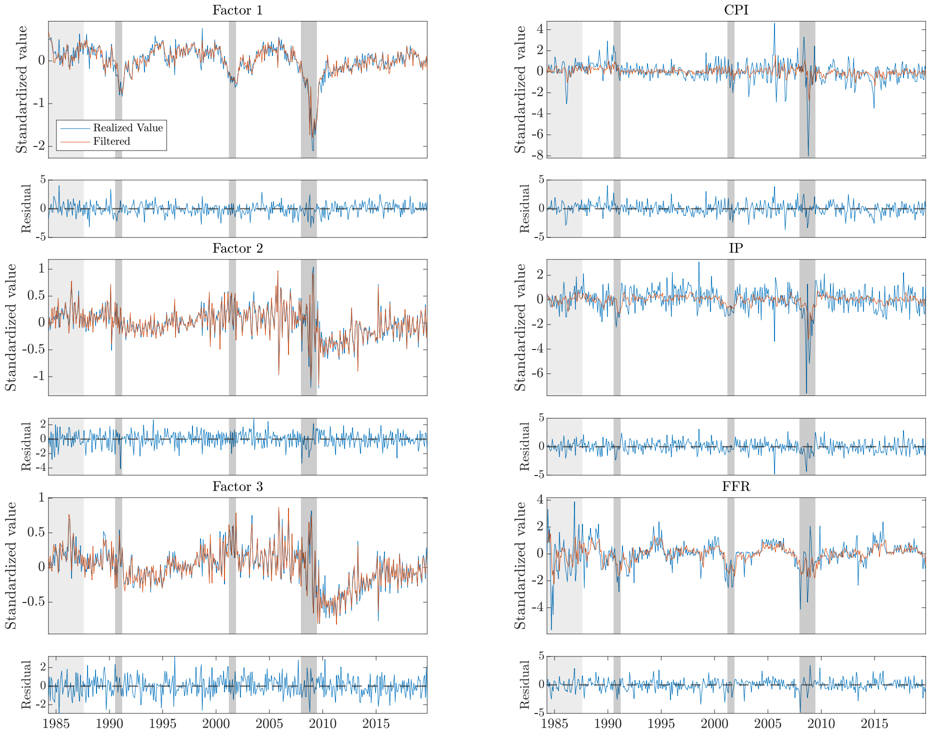


1 Hallin and Liška (2007) Information criterion (log-criterion 2) for number of primitive dynamic factors.

2 S (dotted line) on the right axis is an indication of the standard-deviation associated with the current estimate of number of factors (estimated for a number of time-series, (n), as n increases from a lower threshold (94 series) to the number of series (125)), K (Solid line) shows the estimated number of factors for the full panel 125 series for each value of the penalty weight on an equally distanced linespace.

FIGURE A.4

Filtered values of the DFM-FAVAR, 01:1984-10:2019

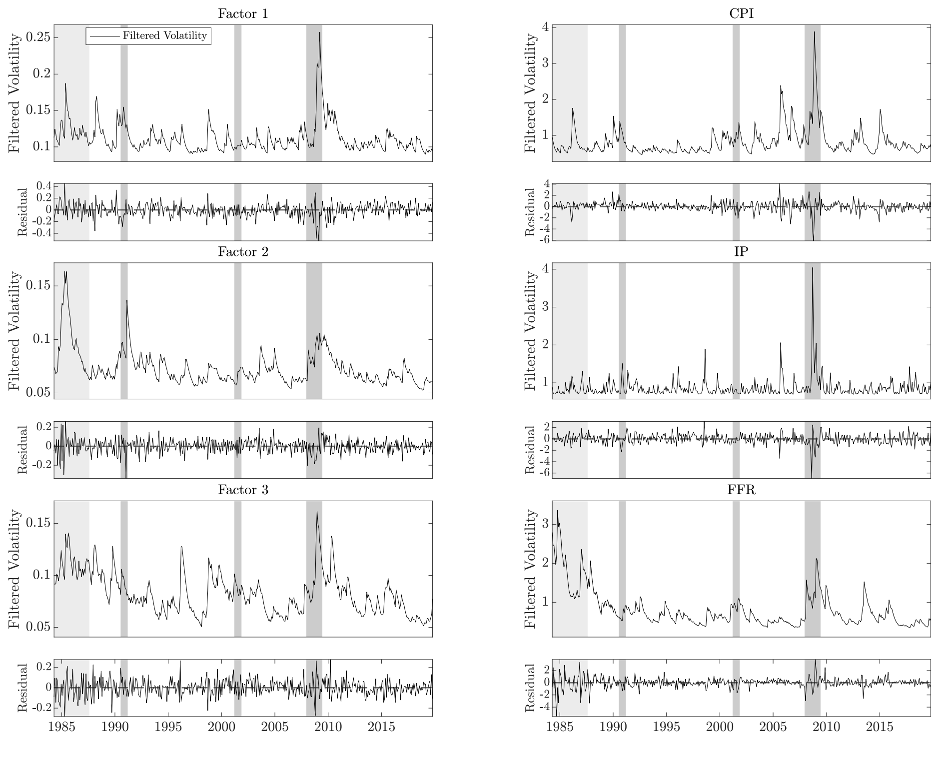


1 In-sample fit of the three factor, three lag DFM-FAVAR model.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$

Figure A.5

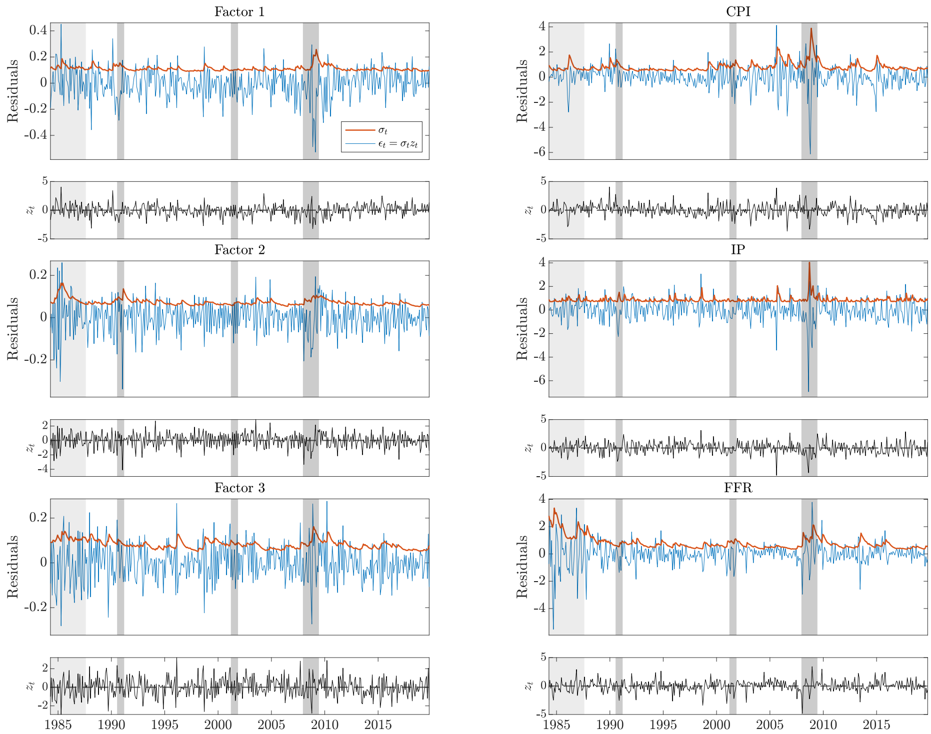
Filtered volatility of the DFM-FAVAR residuals, 01:1984-10:2019



1 Conditional GARCH(1,1) volatility in the three factor, three lag DFM-FAVAR model.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$

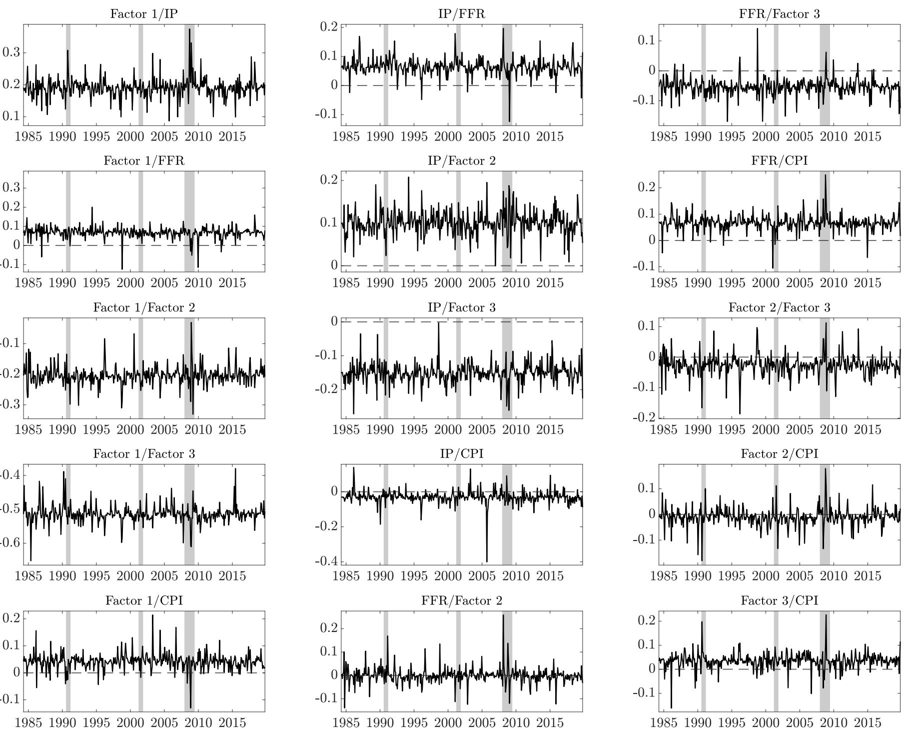
Figure A.6
Residuals of the DFM-FAVAR, 01:1984-10:2019



1 Residuals of the three factor, three lag DFM-FAVAR model. The black line shows the GARCH-filtered residuals, z_t, while the blue line shows the mean-model residuals ϵ_t, and the orange line shows the filtered GARCH-volatility

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$

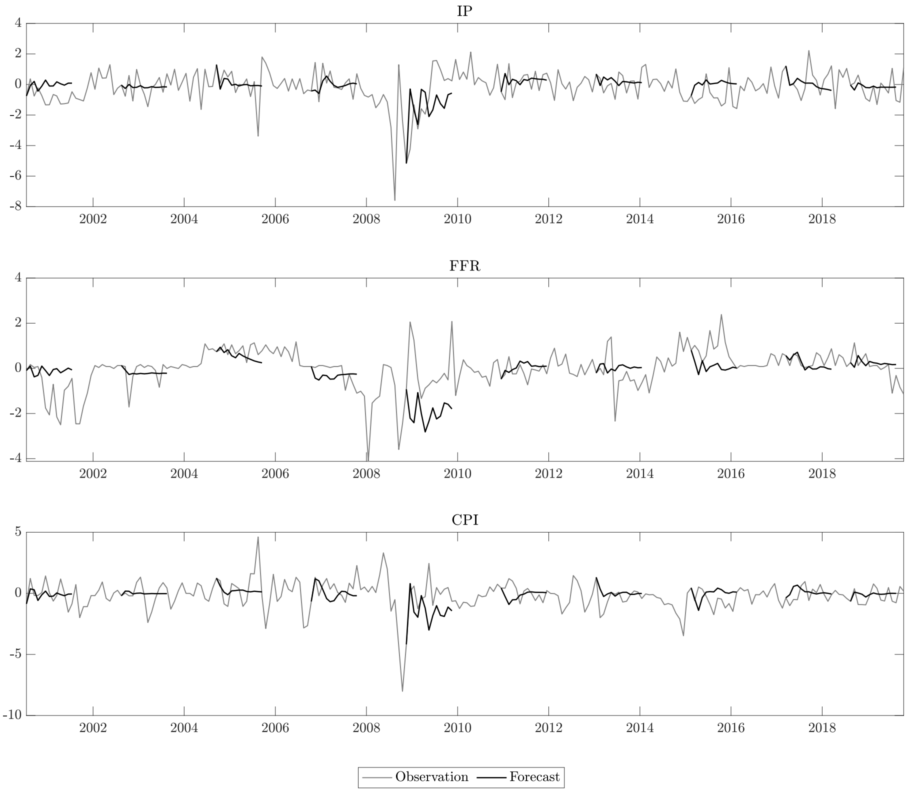
Figure A.7
Conditional Correlation FAVAR residuals, 01:1984-10:2019



1 Conditional Correlation DCC(1,1) in the three factor, three lag DFM-FAVAR model.

2 $Y = \{\Delta \log CPI,\ \Delta \log IP,\ \Delta FFR\}$

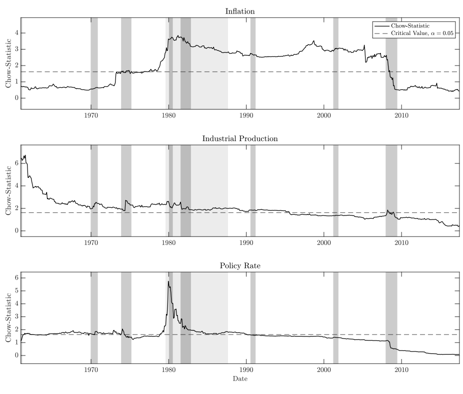
Figure A.8
Monthly growth-rate forecasts of the DFM-FAVAR



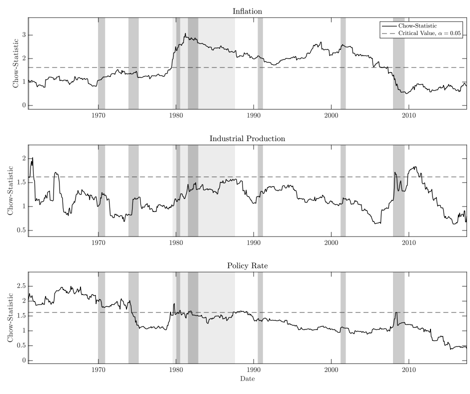
1 Recursive *pseudo* out-of-sample forecasts of the three factor, three lag DFM-FAVAR for the last 17 years of data, computed on a 120 period rolling window for $h \in \{1, 2, ..., 12\}$

Figure A.9
Chow-tests 01:1959-10:2019

(a) Gaussian residuals



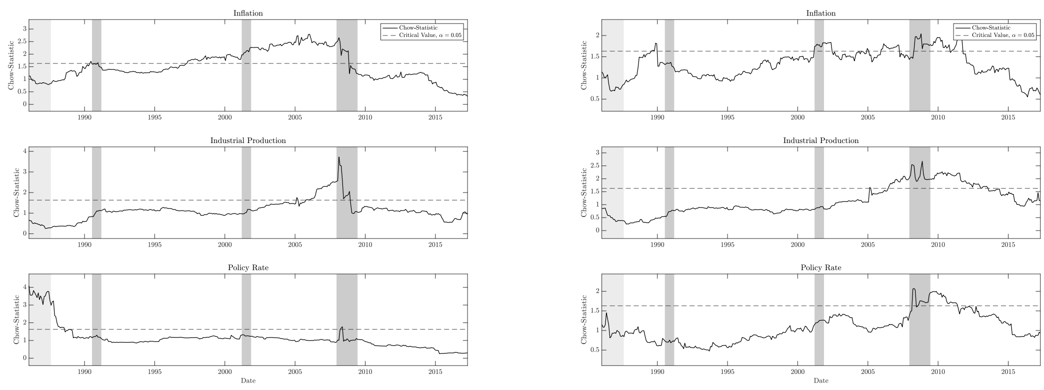
(b) GARCH(1,1)-residuals



1 The Chow-test splits the sample *01:1984-10:2019* into two subsamples and conducts a regression (with identical specification) on both subsamples, here we use the FA(3)-VAR(3), to check whether estimated coefficients are identical for both samples, s_1 and s_2, with null-hypothesis: $\phi_{i,s_1} = \phi_{i,s_2}, \quad \forall i$, *i.e.,* no structural break.

Figure A.10
Chow-tests 01:1984-10:2019

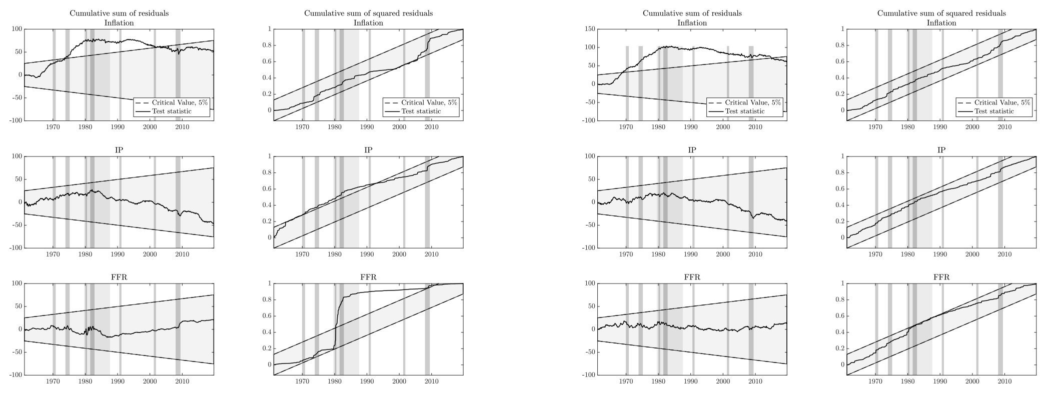
(a) Gaussian residuals (b) GARCH(1,1)-residuals



1 The Chow-test splits the sample *01:1984-10:2019* into two subsamples and conducts a regression (with identical specification) on both subsamples, here we use the FA(3)-VAR(3), to check whether estimated coefficients are identical for both samples, s_1 and s_2, with null-hypothesis: $\phi_{i,s_1} = \phi_{i,s_2}, \quad \forall i$, *i.e.,* no structural break.

Figure A.11
Cusum-tests, observables 01:1959-10:2019

(a) Gaussian residuals (b) GARCH(1,1)-residuals



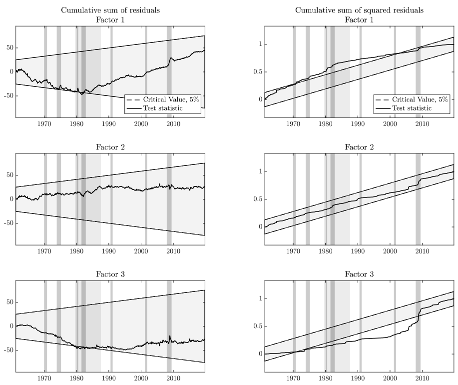
1 The Cusum-test tests the stability of a model by assuming typically, white noise residuals of the model. By taking the cumulative sum we transform the residuals to a persistent random-walk process, which we expect to be increasing in variance proportional to t, if the cumulative sum show behavior inconsistent with our expectations it counts as evidence against our H_0 of model stability.

2 Three-factor, three-lag DFM-FAVAR in $\boldsymbol{Y} = \{\Delta \log IP,\ \Delta FFR,\ \Delta \log CPI\}$.

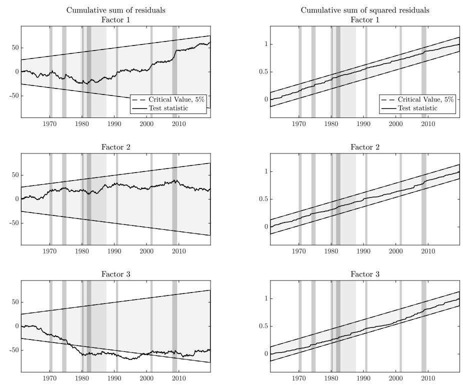
3 The two different tests are known for their respective implications. Structural instability in the Cusum-test, are likely to be a break in mean. Whereas instability in the Cusum-square is likely a break in standard deviation.

Figure A.12
Cusum-tests, factors 01:1959-10:2019

(a) Gaussian residuals



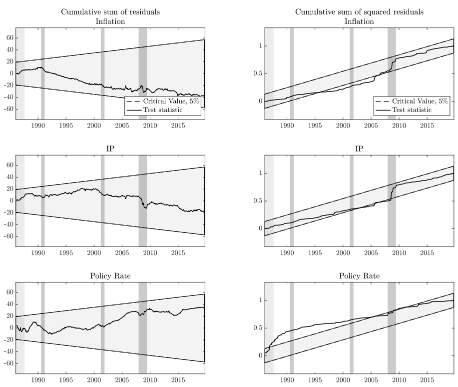
(b) GARCH(1,1)-residuals



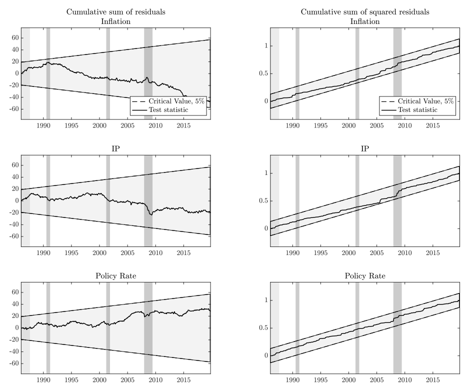
1. See figure A.11 note.

Figure A.13
Cusum-tests, observables: 01:1984-10:2019

(a) Gaussian residuals



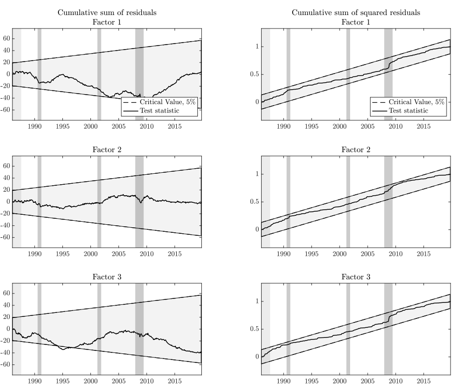
(b) GARCH(1,1)-residuals



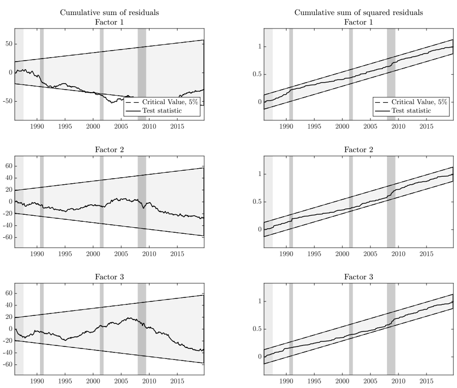
1. See figure A.11 note.

Figure A.14
Cusum-tests, factors: 01:1984-10:2019

(a) Gaussian residuals



(b) GARCH(1,1)-residuals



1. See figure A.11 note.

Table A.7

Part of total variance explained by a single factor R^2, Dynamic factor model

Factor 1		*Factor 2*		*Factor 3*		*IP*		*FFR*		*CPI*	
PAYEMS	0.758	*CUSR0000SA0L5*	0.750	*CUSR0000SAC*	0.711	*IP**	1	*FFR**	1	*CPI**	1
IPMANSICS	0.718	*CUSR0000SA0L2*	0.750	*CUSR0000SA0L2*	0.710	*IPMANSICS*	0.828	*TB3MS*	0.540	*M2REAL*	0.445
USGOOD	0.711	*CUSR0000SAC*	0.749	*CPITRNSL*	0.702	*IPMAT*	0.816	*TB6MS*	0.531	*ACOGNO*	0.343
CUMFNS	0.647	*CPIULFSL*	0.736	*DNDGRG3M086SBEA*	0.698	*IPFPNSS*	0.809	*CP3Mx*	0.490	*CUSR0000SA0L2*	0.288
SRVPRD	0.614	*DNDGRG3M086SBEA*	0.728	*CPIULFSL*	0.698	*IPFINAL*	0.744	*GS1*	0.450	*CUSR0000SA0L5*	0.283
MANEMP	0.594	*CPITRNSL*	0.724	*CUSR0000SA0L5*	0.698	*CUMFNS*	0.739	*T1YFFM*	0.206	*CPIULFSL*	0.282
IPFPNSS	0.594	*PCEPI*	0.553	*PERMITW*	0.547	*IPDMAT*	0.561	*GS5*	0.165	*CUSR0000SAC*	0.264
DMANEMP	0.591	*WPSFD49502*	0.542	*HOUST*	0.537	*IPBUSEQ*	0.561	*TB6SMFFM*	0.161	*CPITRNSL*	0.260
USWTRADE	0.576	*WPSFD49207*	0.532	*HOUSTW*	0.536	*IPCONGD*	0.541	*USWTRADE*	0.157	*DNDGRG3M086SBEA*	0.256
USTPU	0.545	*PERMITW*	0.530	*PERMIT*	0.520	*IPDCONGD*	0.357	*USGOOD*	0.152	*PCEPI*	0.234

1 R^2 from the regressions $x_t = \boldsymbol{\lambda}_i \boldsymbol{F}_i$, (contemporaneous and 2 additional lags of the i'th factor on x_t) .

2 The subset of the 10 variables loading heaviest on each factor

* By construction

Table A.8

Part of total variance explained by a single factor R^2, Principal Components

Factor 1		*Factor 2*		*Factor 3*		*IP*		*FFR*		*CPI*	
PAYEMS	0.735	*CUSR0000SA0L5*	0.535	*HOUST*	0.313	*IP**	1	*FFR**	1	*CPI**	1
USGOOD	0.686	*CUSR0000SA0L2*	0.533	*CUSR0000SA0L2*	0.313	*IPDMAT*	0.146	*T1YFFM*	0.128	*IPBUSEQ*	0.063
SRVPRD	0.593	*CPIULFSL*	0.523	*PERMITW*	0.310	*IPMAT*	0.121	*GS1*	0.108	*IPMAT*	0.039
MANEMP	0.569	*CUSR0000SAC*	0.519	*CPIULFSL*	0.309	*IPFPNSS*	0.099	*TB6SMFFM*	0.105	*AAAFFM*	0.031
DMANEMP	0.565	*DNDGRG3M086SBEA*	0.503	*CUSR0000SA0L5*	0.307	*IPFINAL*	0.096	*TB6MS*	0.098	*SRVPRD*	0.031
USWTRADE	0.543	*CPITRNSL*	0.490	*CUSR0000SAC*	0.304	*CUMFNS*	0.085	*TB3MS*	0.097	*EXCAUSx*	0.030
USTPU	0.535	*PCEPI*	0.447	*CPITRNSL*	0.303	*IPDMAT*	0.083	*USWTRADE*	0.084	*TB6SMFFM*	0.029
IPMANSICS	0.532	*WPSFD49502*	0.417	*HOUSTW*	0.299	*IPBUSEQ*	0.081	*USGOOD*	0.082	*ACOGNO*	0.029
USCONS	0..494	*WPSFD49207*	0.413	*DNDGRG3M086SBEA*	0.297	*IPCONGD*	0.081	*PAYEMS*	0.081	*BAAFFM*	0.029
IPFPNSS	0.454	*WPSID61*	0.408	*PERMIT*	0.289	*IPDCONGD*	0.080	*UEMP15OV*	0.077	*TB3SMFFM*	0.026

1 R^2 from the regressions $x_t = \lambda_i F_i$, (contemporaneous single factor on x_t) .

Code B

Note that only the central procedures are listed here. A complete toolbox including OoS-assesment and various hypothesis-tests is available as a repository on `https://github.com/RasmusJensen96/FAVAR`. Those listed here are used in the main-scripts to estimate the model and perform structural analysis.

TABLE B.1
Central codes

Function	*Purpose*
	Central Estimation functions
EstimateReducedFAVAR.m	Fits the FAVAR-model either by numerical ML (GARCH(1,1)-residuals) or analytical OLS (IID. Gaussian residuals)
FAVAR_Univ_GARCH.m	Single-equation numerical ML with GARCH(1,1)-residuals
Estimate_DCC.m	Full system estimation-routine of the Dynamic Conditional Correlations model by QML (Conditional Covariance)
FAVARirband.m	Generate impulse-responses for the estimated model; GARCH-residual based bootstrap, residual-based GARCH bootstrap, wild NP-bootstrap
FAVARfevd.m	Generate forecast-error variance decomposition of the panel X and observables Y to FAVAR-shocks
DFM_Extraction.m	Extracts factors from a state-space representation based on Doz et al. (2011).
NumFacsDFM_FAVAR.m	Hallin and Liška (2007)-procedure for optimal number of primitive factors.
	Prediction
RollingWindowFAVARComp.m	Rolling window pseudo out-of-sample forecasting h-periods ahead
	Graphics
FAVAR_Resp.m	Impulse Response plots for the observables Y (transformed to standardized level responses)
IRF_DFM.m	Impulse Response plots for the factor panel X (transformed to standardized level responses)
Plot_FAVARfilter.m	Plots the filtered mean of the FAVAR
PlotFilteredVolatility.m	Plots the filtered volatility of the FAVAR
PlotResidualFAVARGarch.m	Residuals and GARCH-filtered residuals
FAVAR_Cusum.m	Cusum- and Cusum-square plots of the FAVAR

Data C

Following the methodology of McCracken and Ng (2016), the transformation code are as follows:

1. Level/no transformation: x_t
2. First difference: Δx_t
3. Second difference: $\Delta^2 x_t$
4. Log-transformation: $\ln x_t$
5. First diff. of log: $\Delta \ln x_t$
6. Second diff. of log: $\Delta^2 \ln x_t$
7. First diff. of percentage change $\Delta\%$

Table C.1 Data Panel

MatLab Identifier	Transformation	FRED name	Description
		Observable factors	
6	5	INDPRO	IP Index
106	5	CPIAUCSL	CPI : All Items
78	2	FEDFUNDS	Effective Federal Funds Rate/Wu and Xia (2016)-shadow rate
		Panel for latent factors	
1	5	RPI	Real Personal Income

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
2	5	W875RX1	Real personal income ex transfer receipts
3	5	DPCERA3M086SBEA	Real personal consumption expenditures
4	5	CMRMTSPLx	Real Manu. and Trade Industries Sales
5	5	RETAILx	Retail and Food Services Sales
7	5	IPFPNSS	IP: Final Products and Nonindustrial Supplies
8	5	IPFINAL	IP: Final Products (Market Group)
9	5	IPCONGD	IP: Consumer Goods
10	5	IPDCONGD	IP: Durable Consumer Goods
11	5	IPNCONGD	IP: Nondurable Consumer Goods
12	5	IPBUSEQ	IP: Business Equipment
13	5	IPMAT	IP: Materials
14	5	IPDMAT	IP: Durable Materials
15	5	IPNMAT	IP: Nondurable Materials
16	5	IPMANSICS	IP: Manufacturing (SIC)
17	5	IPB51222s	IP: Residential Utilities
18	5	IPFUELS	IP: Fuels
19	2	CUMFNS	Capacity Utilization:
20	2	HWI	Help-Wanted Index for United States
21	2	HWIURATIO	Ratio of Help Wanted/No. Unemployed
22	5	CLF16OV	Civilian Labor Force
23	5	CE16OV	Civilian Employment

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
24	2	UNRATE	Civilian Unemployment Rate
25	2	UEMPMEAN	Average Duration of Unemployment (Weeks)
26	5	UEMPLT5	Civilians Unemployed - Less Than 5 Weeks
27	5	UEMP5TO14	Civilians Unemployed for 5-14 Weeks
28	5	UEMP15OV	Civilians Unemployed - 15 Weeks & Over
29	5	UEMP15T26	Civilians Unemployed for 15-26 Weeks
30	5	UEMP27OV	Civilians Unemployed for 27 Weeks and Over
31	5	CLAIMSx	Initial Claims
32	5	PAYEMS	All Employees: Total nonfarm
33	5	USGOOD	All Employees: Goods-Producing Industries
34	5	CES1021000001	All Employees: Mining and Logging: Mining
35	5	USCONS	All Employees: Construction
36	5	MANEMP	All Employees: Manufacturing
37	5	DMANEMP	All Employees: Durable goods
38	5	NDMANEMP	All Employees: Nondurable goods
39	5	SRVPRD	All Employees: Service-Providing Industries
40	5	USTPU	All Employees: Trade, Transportation & Utilities
41	5	USWTRADE	All Employees: Wholesale Trade

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
42	5	USTRADE	All Employees: Retail Trade
43	5	USFIRE	All Employees: Financial Activities
44	5	USGOVT	All Employees: Government
45	1	CES0600000007	Avg Weekly Hours : Goods-Producing
46	2	AWOTMAN	Avg Weekly Overtime Hours : Manufacturing
47	1	AWHMAN	Avg Weekly Hours : Manufacturing
48	4	HOUST	Housing Starts: Total New Privately Owned
49	4	HOUSTNE	Housing Starts, Northeast
50	4	HOUSTMW	Housing Starts, Midwest
51	4	HOUSTS	Housing Starts, South
52	4	HOUSTW	Housing Starts, West
53	4	PERMIT	New Private Housing Permits (SAAR)
54	4	PERMITNE	New Private Housing Permits, Northeast (SAAR)
55	4	PERMITMW	New Private Housing Permits, Midwest (SAAR)
56	4	PERMITS	New Private Housing Permits, South (SAAR)
57	4	PERMITW	New Private Housing Permits, West (SAAR)
58	5	ACOGNO	New Orders for Consumer Goods
59	5	AMDMNOx	New Orders for Durable Goods
60	5	ANDENOx	New Orders for Nondefense Capital Goods

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
61	5	AMDMUOx	Un...lled Orders for Durable Goods
62	5	BUSINVx	Total Business Inventories
63	2	ISRATIOx	Total Business: Inventories to Sales Ratio
64	6	M1SL	M1 Money Stock
65	6	M2SL	M2 Money Stock
66	5	M2REAL	Real M2 Money Stock
67	6	AMBSL	St. Louis Adjusted Monetary Base
68	6	TOTRESNS	Total Reserves of Depository Institutions
69	7	NONBORRES	Reserves Of Depository Institutions
70	6	BUSLOANS	Commercial and Industrial Loans
71	6	REALLN	Real Estate Loans at All Commercial Banks
72	6	NONREVSL	Total Nonrevolving Credit
73	2	CONSPI	Nonrevolving consumer credit to Personal Income
74	5	S&P 500	S&P's Common Stock Price Index: Composite
75	5	S&P: Indust	S&P's Common Stock Price Index: Industrials
76	2	S&P div yield	S&P's Composite Common Stock: Dividend Yield
77	5	S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio
79	2	CP3Mx	3-Month AA Financial Commercial Paper Rate
80	2	TB3MS	3-Month Treasury Bill:

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
81	2	TB6MS	6-Month Treasury Bill:
82	2	GS1	1-Year Treasury Rate
83	2	GS5	5-Year Treasury Rate
84	2	GS10	10-Year Treasury Rate
85	2	AAA	Moody's Seasoned Aaa Corporate Bond Yield
86	2	BAA	Moody's Seasoned Baa Corporate Bond Yield
87	1	COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS
88	1	TB3SMFFM	3-Month Treasury C Minus FEDFUNDS
89	1	TB6SMFFM	6-Month Treasury C Minus FEDFUNDS
90	1	T1YFFM	1-Year Treasury C Minus FEDFUNDS
91	1	T5YFFM	5-Year Treasury C Minus FEDFUNDS
92	1	T10YFFM	10-Year Treasury C Minus FEDFUNDS
93	1	AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS
94	1	BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS
95	5	TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies
96	5	EXSZUSx	Switzerland / U.S. Foreign Exchange Rate
97	5	EXJPUSx	Japan / U.S. Foreign Exchange Rate
98	5	EXUSUKx	U.S. / U.K. Foreign Exchange Rate

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
99	5	EXCAUSx	Canada / U.S. Foreign Exchange Rate
100	6	WPSFD49207	PPI: Finished Goods
101	6	WPSFD49502	PPI: Finished Consumer Goods
102	6	WPSID61	PPI: Intermediate Materials
103	6	WPSID62	PPI: Crude Materials
104	6	OILPRICEx	Crude Oil, spliced WTI and Cushing
105	6	PPICMM	PPI: Metals and metal products:
107	6	CPIAPPSL	CPI : Apparel
108	6	CPITRNSL	CPI : Transportation
109	6	CPIMEDSL	CPI : Medical Care
110	6	CUSR0000SAC	CPI : Commodities
111	6	CUSR0000SAD	CPI : Durables
112	6	CUSR0000SAS	CPI : Services
113	6	CPIULFSL	CPI : All Items Less Food
114	6	CUSR0000SA0L2	CPI : All items less shelter
115	6	CUSR0000SA0L5	CPI : All items less medical care
116	6	PCEPI	Personal Cons. Expend.: Chain Index
117	6	DDURRG3M086SBEA	Personal Cons. Exp. Durable goods
118	6	DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods
119	6	DSERRG3M086SBEA	Personal Cons. Exp: Services
120	6	CES0600000008	Avg Hourly Earnings : Goods-Producing

Table C.1 *continued*

MatLab Identifier	Transformation	FRED name	Description
121	6	CES2000000008	Avg Hourly Earnings : Construction
122	6	CES3000000008	Avg Hourly Earnings: Manufacturing
123	2	UMCSENTx	Consumer Sentiment Index
124	6	MZMSL	MZM Money Stock
125	6	DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding
126	6	DTCTHFNM	Total Consumer Loans and Leases Outstanding
127	6	INVEST	Securities in Bank Credit at All Commercial Banks
128	1	VXOCLSx	VXO