



Faculty of Science



# Project Related Discussion

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September 2014 PMPH Lecture Notes



- 1 Code Structure
- 2 CPU parallelization
- 3 Code Transformations
  - Reasoning About Parallelism: Privatization & Array Expansion
  - Creating CUDA Kernels via Loop Distribution
  - Various Optimizations, e.g., Coalesced Memory
- 4 Project Code: A Bit More Complex Due to a Seq Loop



# Datasets

- Small: OUTER=16, NUM\_X=32, NUM\_Y=256, NUM\_T=90
- Medium: OUTER=32, NUM\_X=47, NUM\_Y=181, NUM\_T=93
- Large: OUTER=128, NUM\_X=256, NUM\_Y=256, NUM\_T=128

Target primarily the *large* dataset, in which the map-like parallel dimensions (OUTER and (NUM\_X or NUM\_Y)) offer enough parallelism to fully utilize the hardware.

As time permits, have a go to parallelizing all three parallel dimensions, i.e., TRIDAG can be re-written based on segmented scans. The target for this is the *small* dataset, which, otherwise, does not has enough parallelism to fully utilize the hardware.

Make sure to optimize global-memory accesses such that they are either *coalesced*, or efficiently cached (if applicable). This is paramount for achieving good performance!



# Code Structure

## Code Entry Point

```

void run_OrigCPU(...) {
    REAL strike;
    PrivGlobs globs(numX,numY,numT);
    for(int i=0; i<outer; ++ i) {
        strike = 0.001*i;
        res[i] = value( globs,s0,strike,t,
                        alpha,nu,  beta,
                        numX, numY, numT );
    }
}

REAL value( ... ) {
    initGrid(s0,alpha,nu,t,
             numX,numY,numT,
             globs);
    initOperator(globs.myX,
                 globs.myDxx);
    initOperator(globs.myY,
                 globs.myDyy);
    setPayoff(strike, globs);
    for(int i=numT-2;i>=0;--i){
        updateParams(i,alpha,beta,
                     nu,globs);
        rollback(i, globs);
    }
    return globs.myResult[globs.myXindex]
           [globs.myYindex];
}

```

# Loop Nests

## Loop Nests

```

rollback( ... ) {
    vector<vector<REAL> > u(numY, vector<REAL>(numX));    // [numY] [numX]
    vector<vector<REAL> > v(numX, vector<REAL>(numY));    // [numX] [numY]
    vector<REAL> a(numZ), b(numZ), c(numZ), y(numZ);    // [max(numX,numY)]
    vector<REAL> yy(numZ); // temporary used in tridag    // [max(numX,numY)]
    for(i=0;i<numX;i++) {
        for(j=0;j<numY;j++) {
            u[j][i] = dtInv*globals.myResult[i][j];
        } } .....
    // implicit y
    for(i=0;i<numX;i++) {
        for(j=0;j<numY;j++) { // here a, b, c should have size [numY]
            a[j] = - 0.5*(0.5*globals.myVarY[i][j]*globals.myDyy[j][0]);
            b[j] = dtInv - 0.5*(0.5*globals.myVarY[i][j]*globals.myDyy[j][1]);
            c[j] = - 0.5*(0.5*globals.myVarY[i][j]*globals.myDyy[j][2]);
        }
        for(j=0;j<numY;j++)
            y[j] = dtInv*u[j][i] - 0.5*v[i][j];
        // here yy should have size [numY]
        tridag(a,b,c,y,numY,globals.myResult[i],yy);
    } }

```



# How To Parallelize

- summarize accesses inter-procedurally. For each loop what does it write and what does it read?
- Within each loop: are all reads covered by writes executed within the same iteration? If so then privatization solves those dependencies!
- For CUDA: do array expansion instead of privatization.
- Decide for each loop whether it can or cannot be parallelize.
- Use loop distribution to create perfect nests, which will become later your CUDA kernels.
- Use loop interchange and/or matrix transposition to obtain coalesced access to global memory.



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# CPU Parallelization

In function `run_OrigCPU` move the declaration of `strike` and `globs` inside the loop, and parallelize the loop via an `OPENMP` pragma:

## Parallelizing the Outermost Loop Via OpenMP

```
#pragma omp parallel for default(shared) schedule(static)
  for( unsigned i = 0; i < outer; ++ i ) {
    REAL strike;
    PrivGlobs      globs(numX, numY, numT);

    strike = 0.001*i;
    res[i] = value( globs, s0, strike, t,
                    alpha, nu,      beta,
                    numX,  numY,  numT );
  }
```

Explain why this is safe in the report!

(For example if you do NOT move the declarations inside the loop and still parallelize the loop, the execution will NOT validate).





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# Reasoning About Parallelism: Privatization

## Parallelizing the Outermost Loop Via Privatization

```
float A[N];  
for(int i=0;i<M;i++){ //seq  
    for(int j=0;j<N;j++){  
        A[j] = ...  
    }  
    ...  
    for(int j=0;j<N;j++){  
        ... = A[j]  
    }  
}
```

```
for(int i=0;i<M;i++){ //par  
    float A[N];  
    for(int j=0;j<N;j++){  
        A[j] = ...  
    }  
    ...  
    for(int j=0;j<N;j++){  
        ... = A[j]  
    }  
}
```

- The outermost loop of index  $i$  is NOT parallel as it is, because all its iterations write and read all indices of array  $A$ .
- However, the iteration reads (is covered by) what was written in the same iteration, a.k.a., array  $A$  can be privatized.
- Privatization can be achieved by moving the declaration of  $A$  inside the outermost loop (each iteration works with its own private version of  $A$ ).



# Array Expansion

## Semantically Equivalent: Privatized vs Expanded A

```
for(int i=0;i<M;i++){ //par
    float A[N];
    for(int j=0;j<N;j++){
        A[j] = ...
    }
    ...
    for(int j=0;j<N;j++){
        ... = A[j]
    }
}
```

```
float A[M, N];
for(int i=0;i<M;i++){ //par
    for(int j=0;j<N;j++){
        A[i,j] = ...
    }
    ...
    for(int j=0;j<N;j++){
        ... = A[i,j]
    }
}
```

- In CUDA it is preferable that all memory is allocated before the kernel starts, hence making array A local would not work.
- Instead, expand array A with an extra (outermost) dimension, whose size is the count of the outermost loop.
- Now iteration i has exclusive access, i.e., writes to and reads from, row i of expanded array A.
- The two versions of code below are **semantically equivalent!**



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# Create CUDA Kernels via Loop Distribution

- **Theorem:** A parallel loop can be distributed across its statements (guaranteed that its dependency graph does not have cycles).
- CUDA kernels are obtained by distributing the outer loop around the inner loops (in order to improve the degree of parallelism).

Degree of parallelism M

vs.

Degree of parallelism:  $M*N$

```
float A[M, N];
for(int i=0;i<M;i++){ //par
    for(int j=0;j<N;j++){
        A[i,j] = ...
    }
    ...
    for(int j=0;j<N;j++){
        ... = A[i,j]
    }
}
```

```
float A[M, N];
for(int i=0;i<M;i++){ //par
    for(int j=0;j<N;j++){ //par
        A[i,j] = ...; // 2D CUDA kernel
    } }
for(int i=0;i<M;i++){ //par
    ...
}
for(int i=0;i<M;i++){ //par
    for(int j=0;j<N;j++){ //par
        ... = A[i,j]; // 2D CUDA Kernel
    } }
```

# Inline Simple Expression vs Array Expansion

- Loop distribution requires array expansion of the local variables.
- If the local variable is a simple scalar expression it is better to inline that expression rather than creating an array for it.
- Use your better judgment when to distribute and when to inline, i.e., do not create too many arrays (tradeoff between redundant computation AND extra memory & global accesses)

Inline Scalar Variables	Rather Than	Array Expansion
<pre>float A[M, N]; for(int i=0;i&lt;M;i++){ //par     int tmp = i*i;     for(int j=0;j&lt;N;j++){         A[i,j] = ... * tmp;     } } // inline scalar exp float A[M, N]; for(int i=0;i&lt;M;i++) //par     for(int j=0;j&lt;N;j++){//par         A[i,j] = ... * ((float)i*i);     } }</pre>		<pre>// Systematic distribution will create // Many Arrays, and Many access to Global // Memory. (It might be cheaper to do some // ← redundant computation instead). float tmps[M]; float A[M, N]; for(int i=0;i&lt;M;i++) //par     tmps[i] = (float)(i*i); for(int i=0;i&lt;M;i++) //par     for(int j=0;j&lt;N;j++){//par         A[i,j] = ... * tmps[i];     } }</pre>

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# Optimising CUDA Kernel (Memory Coalescing)

- After creating the CUDA Kernels, one might also want to optimise them, for example
- Coalesced Access to global memory may be obtained via loop interchange or (segmented) matrix transposition.

## Coalesced Access via Loop Interchange or Matrix Transposition

```

float A[M, N];
for(int j=0;j<N;j++){ //par
    for(int i=0;i<M;i++){ //par
        A[i,j] = ... // uncoalesced access
    } } ↓    loop interchange    ↓

for(int i=0;i<M;i++){ //par
    for(int j=0;j<N;j++){ //par
        A[i,j] = ... // coalesced access
    } }

// Fixing uncoalesced accesses via
// matrix transposition
float Atr[N, M];
for(int j=0;j<N;j++){ //par
    for(int i=0;i<M;i++){ //par
        Atr[j,i] = ... // coalesced
    } }
float A[M,N];
A=transpose(Atr); //Atr[j,i]≡A[i,j]

```

Note that applying loop interchange may make some uncoalesced accesses coalesced, but it might also make other (originally) coalesced accesses uncoalesced. In those cases use TRANSPOSITION.





# You May Need Segmented Transpose

- Project uses three dimensional arrays, i.e., an array of matrices, and requires transposing each matrix (the two innermost dims).
- Nothing to be afraid of – this corresponds to a three dimensional CUDA kernel in which you write the matrix-transposition code for the innermost two dimensions. Pseudocode below:

## Segmented Transposition: Sequential and CUDA Kernel

```

float A[0, M, N];
for(int k=0;k<K;k++){ //par
    for(int i=0;i<M;i++){ //par
        for(int j=0;j<N;j++){ //par
            Atr[k,j,i] = A[k,i,j];
        } } }

__global__ void sgmMatTranspose( float* A,
                                float* trA, int rowsA, int colsA ) {
    __shared__ float tile[T][T+1];
    int gidz=blockIdx.z*blockDim.z*threadIdx.z;
    A+=gidz*rowsA*colsA; Atr+=gidz*rowsA*colsA;
    // follows code for matrix transp in x & y
    int tidz = threadIdx.z, tidy = threadIdx.y;
    int j=blockIdx.x*T+tidx,i=blockIdx.y*T+tidy;
    if( j < colsA && i < rowsA )
        tile[tidy][tidz] = A[i*colsA+j];
    __syncthreads();
    i=blockIdx.y*T+tidx; j=blockIdx.x*T+tidy;
    if( j < colsA && i < rowsA )
        trA[j*rowsA+i] = tile[tidx][tidy];
}

```

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# Sequential Loop in Between Parallel Loops

- Loop of index  $t$  is sequential because it reads the array `myResult[k,0:M-1,0:N-1]` produced by previous iteration  $t-1$ !
- Distribute the outermost loop across the two loop nests, then interchange to get loop of index  $t$  in the outermost position:

## Code after Array Expansion

```
float myResult[Outer, M, N];
for(int k=0;j<Outer;k++){ //par
    for(int i=0; i<M; i++) //par
        for(int j=0; j<N; j++) //par
            myResult[k,i,j] = ...;

    for(int t=0;t<T;t++){ //seq
        for(int i=0; i<M; i++) //par
            for(int j=0; j<N; j++) //par
                ... = ... myResult[k,i,j] ... ;
        for(int i=0; i<M; i++) //par
            for(int j=0; j<N; j++) //par
                myResult[k,i,j] = ...;
    }
}
```

## Get Seq Loop Outside

```
float myResult[Outer, M, N];
for(int k=0;j<Outer;k++){ //par
    for(int i=0; i<M; i++) //par
        for(int j=0; j<N; j++) //par
            myResult[k,i,j] = ...;
}
for(int t=0;t<T;t++){ //seq
    for(int k=0;j<Outer;k++){ //par
        for(int i=0; i<M; i++) //par
            for(int j=0; j<N; j++) //par
                ... = ... myResult[k,i,j] ... ;
        for(int i=0; i<M; i++) //par
            for(int j=0; j<N; j++) //par
                myResult[k,i,j] = ...;
    }
}
```

# Sequential Loop in Between Parallel Loops

- Finally, distribute again loop  $k$  against the two inner loop nests to create CUDA kernels! Then optimize coalescing, etc.!

After Distrib Kernels 2 & 3 are called inside a Sequential Loop

```
float myResult[Outer, M, N];
for(int k=0; k<Outer; k++){ //par
    for(int i=0; i<M; i++) //par
        for(int j=0; j<N; j++) //par
            myResult[k,i,j] = ...;
}
for(int t=0; t<T; t++){ //seq
    for(int k=0; k<Outer; k++){ //par
        for(int i=0; i<M; i++) //par
            for(int j=0; j<N; j++) //par
                ... = ... myResult[k,i,j] ... ;
    }
    for(int i=0; i<M; i++) //par
        for(int j=0; j<N; j++) //par
            myResult[k,i,j] = ...;
} }

float myResult[Outer, M, N];
for(int k=0; k<Outer; k++){ //Kernel1
    for(int i=0; i<M; i++) //Kernel1
        for(int j=0; j<N; j++) //Kernel1
            myResult[k,i,j] = ...;
}
for(int t=0; t<T; t++){ //seq
    for(int k=0; k<Outer; k++){ //Kernel2
        for(int i=0; i<M; i++) //Kernel2
            for(int j=0; j<N; j++) //Kernel2
                ... = ... myResult[k,i,j] ... ;
    }
    for(int k=0; k<Outer; k++){ //Kernel3
        for(int i=0; i<M; i++) //Kernel3
            for(int j=0; j<N; j++) //Kernel3
                myResult[k,i,j] = ...;
    }
} }
```