Classical Magnetic Systems

On the behaviour of macroscropic magnetized moments in dissipative environments

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Preface

This document contains our work regarding the novel spin revolution effect, first described by PD Dr. Elena Vedmedenko and Prof. Roland Wiesendanger[**Vedmedenko**]. The goal of this project is to further the understanding of the presented systems by investigating certain mathematical aspects of the system's description. Our hope is to procure either analytically solvable equations of motions or prove that such solutions do, in fact, not exist. We start by introducing the necessary mathematical principles rooted in the theory of manifolds, variational calculus and ordinary differential equations. From there on, we investigate various realizations of the spin revolution effect.

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List of symbols

$M \pitchfork N$	Transverse intersection

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CHAPTER ONE

Mathematical Introduction

We start with the necessary mathematical theory. If the reader already feels comfortable with the theory of smooth manifolds, variational calculus on aforementioned spaces, (Lie) groups as well as autonomous differential equations, they may skip this chapter. We assume basic knowledge of linear algebra and calculus.

1.1 Group Theory

We need some fundamental group theory.

Definition 1.1 (Group)**.** A **Group** is a pair $(G, \circ)$ consisting of a set G and an operation $\circ$ such that the following axioms are satisfied:

$\forall a, b, c \in G : a \circ (b \circ c) = (a \circ b) \circ c$

(G12) $\forall a \in G \; \exists a^{-1} \in G : a \circ a^{-1} = 1$

(G3) $\exists e \in G \; \forall a \in G : e \circ a = a$

If the operation is commutative, we call the group **abelian**.

One should think of groups in terms of symmetries: A symmetry of an abstract object can be thought of as a collection of operations leaving said object invariant.

There are plenty of examples of groups important in physics:

Example 1.2. Let $V \cong \mathbb{R}^n$ be a real vector space endowed with a scalar product

$$\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}.$$

We call an endomorphism

$$T : V \to V$$

orthogonal if for all $v, w \in V$ the scalar product is preserved:

$$\langle T(v), T(w) \rangle = \langle v, w \rangle.$$

Define the *orthogonal Group* $O(n)$ of dimension n as set of all orthogonal endomorphisms on V with group multiplication given by composition. Note that this group is not abelian. In the same setting, we define the *special orthogonal group* as

$$SO(n) := \{T \in O(n) \mid \det T = 1\} \leq O(n). \tag{1.1}$$

This group is also often called *rotation group* in physics. We will use this group heavily later on.

Note that this coordinate-free definition reduces to the usual definition by real $n \times n$-matrices if we choose any basis of V and represent the endomorphism in this basis.

1.2 Configuration Manifolds and Lie groups

Sometimes, we need special charts centered at a point $p \in M$. This means that $\phi(p) = 0$.

Definition 1.3 (Smooth Manifold)**.** A **smooth** n**-manifold** is a topological space M such that:

M has the Hausdorff property.

2. M is second-countable.
3. M is locally euclidean of class $\mathcal{C}^\infty$: For every $p \in M$ there is an open neighbourhood UM of p and a $\mathcal{C}^\infty$-diffeomorphism $\phi : U \to V$ such that $V\mathbb{R}^n$ is open. We call (ϕ, U) a **chart** on M.

We will be working exclusively in this smooth category to ensure the existence of smooth k-forms on any given manifold. Furthermore, we use the Einstein summation convention:

1.3 Variational Calculus

1.4 Ordinary Differential Equations

CHAPTER TWO

Magnetic Incline System