
Preface

This is my personal script for the lecture Riemannian and Lorentzian Geometry in the winter term of 25/26 at the University of Hamburg. The script mostly follows the lecture of [Prof. Dr. Melanie Graf](#) with occasional bits adapted from the available literature. The layout is a personal adaption of [Gilles Casel's](#) layout. We will adapt most notations from ?? and ?? and use the Einstein summation convention throughout. The lecture first aims to fill some gaps often left in undergraduate differential geometry lectures, mainly the theorem of Hopf and Rinow. After that, we continue with some notions inherent to Lorentzian geometry before focussing our attention again on the (semi-)Riemannian case. Later, we will use Jacobi fields and do some comparison geometry.

Rasmus Raschke, December 2025

List of symbols

| | |
|--------------------|--|
| $M \pitchfork N$ | Transverse intersection |
| $\Gamma(M)$ | Space of smooth sections $\sigma : M \rightarrow TM$ |
| $\Gamma_\gamma(M)$ | Space of smooth sections |
| $\mathfrak{X}(M)$ | Space of vector fields on M |

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CHAPTER ONE

Preamble

Definition 1.1 (Test). This is a test definition.

$$R = 2B \int x dt$$

Corollary 1.2 (Test). Test autoref: Definition 1.1, nameref: Test, cref: definition 1.1

Proposition 1.3 (Test). Test

Theorem 1.4 (Test). Test

Example 1.5 (Test). Test

Nonexample 1.6 (NoTest). NoTest

Remark 1.7 (Test). Test

Proof (Test). Test

Proof (Test). Test □

Exercise (Test). Test

Note (Test). Test

Notation (Test). Test

TODO. TEst