Preface

Gilles, June 2021

List of symbols

$M \pitchfork N$	Transverse intersection
$\langle \cdot, \cdot \rangle$	Riemannian metric on a manifold or, Inner product on space of critical points $\langle c, d \rangle = \delta_{cd}$
$N \cdot N'$	Intersection number of two manifolds
A	A $\mathbb{Z}$-module, i.e. an abelian group
B^n	Closed disk of dimension n
$C_k(f, \mathbb{Z})$	Free module over $\mathbb{Z}$ generated by index k critical points of f, i.e. the space of formal sums of index k critical points
$C_k(f, \mathbb{Z}_2)$	Vector space over $\mathbb{Z}_2$ generated by the index k critical points of the Morse function f
$\operatorname{codim} N$	Codimension of N
$\dim N$	Dimension of N
$\operatorname{Crit}_k f$	Critical points of f of index k
$\operatorname{Crit} f$	Critical points of f
$C^\infty(M, N)$	Smooth maps from M to N
$\partial_{X,k}$	Morse differential associated to pseudo-gradient X
$[\partial_k]$	Matrix of the Morse differential $\partial_k : C_k \to C_{k-1}$
D^n	Open disk of dimension n
$\operatorname{grad} f$	Gradient of f, i.e. $(df)^\sharp$
$HM(M; \mathbb{Z}_2)$	Morse homology of a manifold M with coefficients in $\mathbb{Z}_2$
$HM(M; \mathbb{Z})$	Morse homology of a manifold M with coefficients in $\mathbb{Z}$
$HM(C_\bullet(f), \partial_X)$	Morse homology of Morse function f and pseudo-gradient X
$H_k(M, N)$	Singular homology of M relative to N
$H_k(M; \mathbb{Z})$	Singular homology of M over $\mathbb{Z}$
$H_k(M; \mathbb{Z}_2)$	Singular homology M over $\mathbb{Z}_2$

$\operatorname{Ind} a$	Index of critical point a
$\mathcal{L}(p,q)$	Moduli space of unbroken trajectories between p and q, i.e. $\mathcal{M}(p,q)/\mathbb{R}$, where $\mathbb{R}$ acts by time translations
$\overline{\mathcal{L}}(p,q)$	Space of broken and unbroken trajectories between p and q, the compactification of $\mathcal{L}(p,q)$
M	A smooth manifold
$\mathcal{M}(p,q)$	Set of all points on trajectories following a pseudo-gradient from p to q, $W^u(p) \pitchfork W^s(q)$
$N_X(p,q)$	Signed number of trajectories of X connecting p to q
$n_X(p,q)$	Number of trajectories of X connecting p to q
$\pi_k(M)$	Homotopy group of a manifold
$r_0(A)$	Free rank of a $\mathbb{Z}$-module, i.e. $\dim_{\mathbb{Q}} A \otimes \mathbb{Q}$
$r_p(A)$	p-torsion rank of a $\mathbb{Z}$-module, i.e. cardinality of a maximal set of independent elements of order p^k for some k
$r_t(A)$	Total torsion rank of a $\mathbb{Z}$-module, i.e. $\sum r_t$
$r(A)$	Total rank of a $\mathbb{Z}$-module, i.e. $r_t(A) + r_0(A)$
$S^s(p)$	Stable sphere associated to a critical point p, alternatively called the belt sphere
$S^u(p)$	Unstable sphere associated to a critical point p, alternatively called the attachment sphere
S^n	Sphere of dimension n
$W^s(p)$	Stable manifold of a critical point p
$W^u(p)$	Unstable manifold of a critical point p
$\overline{W}^u(p)$	Compactification of the unstable manifold associated to a critical point p
X	Pseudo-gradient vector field

Contents

CHAPTER ONE

Preamble

Definition 1.1 (Test)**.** This is a test definition.

$$R = 2B \int x \, dt$$

Corollary 1.2 (Test)**.** Test autoref: Definition 1.1, nameref: Test, cref: definition 1.1

Proposition 1.3 (Test)**.** Test

Theorem 1.4 (Test)**.** Test

Example 1.5 (Test)**.** Test

Nonexample 1.6 (NoTest)**.** NoTest

Remark 1.7 (Test)**.** Test

Proof (Test)**.** Test

Proof (Test)**.** Test □

Exercise (Test)**.** Test

Note (Test)**.** Test

Notation (Test)**.** Test

TODO. TEst