Exercise 3

2) Use 11 to verify that
$$W_t^{(1)}$$
, and $W_t^{(2)} = B_t^{(2)}$

Satisfy $\langle W_t^{(1)}, W_t^{(2)} \rangle_t = \rho t$
 $\langle V_{1-\rho^{1}} B^{(1)} + \rho B^{(2)}, B^{(2)} \rangle_t = \langle V_{1-\rho^{1}} B^{(1)}, B^{(2)} \rangle_t + \langle \rho B^{(2)}, B^{(2)} \rangle_t$

$$= \sqrt{1-\rho^2} \langle B^{(2)}, B^{(2)} \rangle_{t} + \rho \langle B^{(2)}, B^{(2)} \rangle_{t} = 0 + \rho t,$$
where we used that $B^{(1)}$ and $B^{(2)}$ were independent, and $\langle B^{(2)}, B^{(2)} \rangle_{t} = \langle B^{(2)}, \rangle_{t} = t.$

Exercise 4:

$$\frac{dS_{t}}{S_{t}} = \mu dt + V_{t} dw_{t}^{(1)}, S_{t} \ge 0$$

$$dV_{t} = (\alpha - \lambda V_{t}) dt + \sigma_{t} V_{t} dw_{t}^{(2)}, V_{0} > 0, \lambda, \alpha, \sigma_{t} \ge 0$$

$$ewritten as:$$

Can be written as:

$$dY_{\epsilon} = b(Y_{\epsilon})d\epsilon + \sigma(Y_{\epsilon})dB_{\epsilon}, Y_{o} = (S_{o}, V_{o})^{T}, Y_{\epsilon} = \begin{bmatrix} S_{\epsilon} \\ V_{\epsilon} \end{bmatrix}$$

First we rewrite (3)

$$\frac{dS_{t}}{S_{t}} = \mu dt + \sqrt{V_{t}} d(\sqrt{1-p^{2}} \beta_{t}^{(1)} + \rho \beta_{t}^{(2)})$$

$$= \mu dt + \sqrt{1-p^{2}} V_{t}^{2} d\beta_{t}^{(1)} + \sqrt{V_{t}} \rho d\beta_{t}^{(2)}$$

$$dV_{\xi} = (x - \lambda V_{\xi})dt + \sigma \sqrt{V_{\xi}}dB_{\xi}^{(2)}$$

$$= > b(x) = \begin{bmatrix} \mu \times_{1} \\ x - \lambda \times_{2} \end{bmatrix}$$

$$= (x - \lambda V_{\xi})dt + \sigma \sqrt{V_{\xi}}dB_{\xi}^{(2)}$$

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 $\langle V_{1-\rho^2}, B^{(1)} + \rho B^{(2)}, B^{(2)} \rangle_t = \langle V_{1-\rho^2}, B^{(1)}, B^{(2)} \rangle_t + \langle \rho B^{(2)}, B^{(2)} \rangle_t$
 $= V_{1-\rho^2} \langle B^{(1)}, B^{(2)} \rangle_t + \rho \langle B^{(2)}, B^{(2)} \rangle_t = 0 + \rho t$, where we used that $B^{(1)}$ and $B^{(2)}$ were independent, and $\langle B^{(2)}, B^{(2)} \rangle_t = \langle B^{(2)}_t \rangle_t = t$.

Exercise 4:

Find functions $b: R^2 \to R^2$, $\sigma: R^2 \to R^{2\times 2}$ such that

Find functions
$$b: \mathbb{R}^2 \to \mathbb{R}^2$$
, $\sigma: \mathbb{R}^2 \to \mathbb{R}^{2\times 2}$ such that

$$\frac{dS_t}{S_t} = \mu dt + VV_t dW_t^{(1)}, S_0 \ge 0$$

$$dV_t = (x - \lambda V_t) dt + \sigma_t VV_t dW_t^{(2)}, V_0 > 0, \lambda, \alpha, \sigma_t \ge 0$$
Can be written as:
$$(x - \lambda V_t) dt + \sigma_t (Y_t) dS_t, \quad Y_t = (S_0, V_0)^T Y_t S_t$$

d Y = b(Y)dt + o(Y)dBt, Y = (So, Vo), Y= [Se]

First we rewrite (3)

$$\frac{dS_{t}}{S_{t}} = \mu dt + \sqrt{V_{t}} d(\sqrt{1-P^{2}} \beta_{t}^{(1)} + P \beta_{t}^{(2)})$$

$$= \mu dt + \sqrt{(1-P)} V_{t}^{1} d\beta_{t}^{(1)} + \sqrt{V_{t}^{1}} P d\beta_{t}^{(2)}$$

$$dV_{\xi} = (\alpha - \lambda V_{\xi})dt + \sigma \sqrt{V_{\xi}}dB_{\xi}^{(2)}$$

$$= > b(x) = \begin{bmatrix} \mu \times_{1} \\ x - \lambda \times_{2} \end{bmatrix}$$

$$= (\alpha - \lambda V_{\xi})dt + \sigma \sqrt{V_{\xi}}dB_{\xi}^{(2)}$$

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