$$e^{-r(T-t)} \boxed{\mathbb{E}_{\mathbb{Q}}\left[\int_0^T S_r\,dr \,\middle|\, \mathcal{F}_t\right]}$$

$t \leq T$

$$\int_0^T S_r\,dr = \underbrace{\int_0^t S_r\,dr}_{\in \mathcal{F}_t} + \int_t^T S_r\,dr$$

$$= \int_0^t S_r\,dr$$

$$+ \boxed{\mathbb{E}_{\mathbb{Q}}\left[\int_t^T S_r\,dr \,\middle|\, \mathcal{F}_t\right]}$$

$\forall \quad r \geq t$

$$S_r = S_t \boxed{\exp\left(\left(r - \frac{\sigma^2}{2}\right)(r-t) + \sigma\left(B_r^{\mathbb{Q}} - B_t^{\mathbb{Q}}\right)\right)}$$

$$\int_t^T S_t \cdot dr$$

$$\boxed{= S_t \int_t^T \cdot dr}$$

$$\boxed{\mathbb{E}_Q\left[\int_t^T S_r \, dr \,\middle|\, \mathcal{F}_t\right]}$$

$$= \mathbb{E}_Q\left[S_t \int_t^T \cdot dr \,\middle|\, \mathcal{F}_t\right]$$

$$= S_t \, \mathbb{E}_Q\left[\int_t^T \cdot dr\right]$$

$$\boxed{\exp\left(\left(r-\frac{\sigma^2}{2}\right)(r-t)+\sigma\left(B_r^{\mathbb{Q}}-B_t^{\mathbb{Q}}\right)\right)}$$

$\underline{B_r^{\mathbb{Q}}-B_t^{\mathbb{Q}}} \perp \underline{\mathcal{F}_t}$