##urimar-opg-1##

$$F(t,x) := e^{-r(T-t)} \mathbb{E}_Q[\phi(X(x,t,T))],$$

$$X(x,t,T) := x e^{(r-\bar{\sigma}^2/2)(T-t)+\bar{\sigma}\sqrt{T-t}Z}, \quad Z \sim N(0,1)$$

$$\xi := (S_T)^2$$

$$F(t,S_t) := e^{-r(T-t)} \mathbb{E}_Q[X(S_t,t,T)^2]$$

$$= e^{-r(T-t)} \mathbb{E}_Q\left[\left(S_t e^{(r-\bar{\sigma}^2/2)(T-t)+\bar{\sigma}\sqrt{T-t}Z}\right)^2\right]$$

$$= e^{-r(T-t)} \mathbb{E}_Q\left[S_t^2 e^{2(r-\bar{\sigma}^2/2)(T-t)} e^{2\bar{\sigma}\sqrt{T-t}Z}\right]$$

$$= e^{-r(T-t)} S_t^2 e^{2(r-\bar{\sigma}^2/2)(T-t)} \mathbb{E}_Q\left[e^{2\bar{\sigma}\sqrt{T-t}Z}\right]$$

$$= e^{-r(T-t)} S_t^2 e^{2(r-\bar{\sigma}^2/2)(T-t)} \quad e^{2(T-t)\bar{\sigma}^2}$$

$$= S_t^2 e^{r(T-t)+\bar{\sigma}^2(T-t)}$$

$$= S_t e^{(r+\bar{\sigma}^2)(T-t)}$$