Instrumental Variables Estimation

Econometrics

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Introduction

We have talked about some scenarios where the assumption on exogenous regressors cannot be expected to hold true.

As we argued, this means that OLS is inconsistent and biased.

Today we will study a method that can alleviate these problems.

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Instrumental Variables Estimation

Suppose our model has K endogenous regressors.

Assume we can find K variables W that satisfy:

i) They are related to the regressors

$$\operatorname*{plim}_{n\to\infty} \frac{1}{n} W^T X = S_{WX},$$

where S_{WX} is a finite $K \times K$ matrix with rank K.

ii) They are not related to the errors

$$\operatorname*{plim}_{n\to\infty} \frac{1}{n} W^T U = 0.$$

We call W **instruments** and we say they are **relevant** [i)] and **valid** [ii)].

Instrumental Variables Estimation

Under the assumptions before,

$$W^T Y = W^T X\beta + W^T U,$$

and thus,

$$\text{plim}\, W^T Y = \text{plim}\, W^T X\beta + \text{plim}\, W^T U.$$

This suggests the **instrumental variable estimator** given by

$$\hat{\beta}_{IV} = (W^T X)^{-1} W^T Y,$$

which is consistent given the above.

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Instrumental Variables Estimation

The (asymptotic) variance of IV is given by

$$Var(\text{plim}\,\hat{\beta}_{IV}) = \sigma^2 S_{WX}^{-1} S_{WW} (S_{WX}^T)^{-1} = \hat{\sigma}^2 \underset{n\to\infty}{\text{plim}} (n^{-1} X^T P_W X)^{-1},$$

where we have assumed that W is well behaved so that S_{WW} exists.

Thus, we estimate its variance by

$$\widehat{Var}(\hat{\beta}_{IV}) = \hat{\sigma}^2 (W^T X)^{-1} W^T W (X^T W)^{-1},$$

with

$$\hat{\sigma}^2 = \frac{(Y - X\hat{\beta}_{IV})^T (Y - X\hat{\beta}_{IV})}{n}.$$

Instrumental Variables Estimation

The distribution of IV can be computed by

$$\sqrt{n}(\hat{\beta}_{IV} - \beta) = \left(\frac{W^T X}{n}\right)^{-1} \frac{1}{\sqrt{n}} W^T U.$$

Using the CLT given the relevance and validity of the instruments,

$$\hat{\beta} \sim N\left(\beta, \frac{\sigma^2}{n} S_{WX}^{-1} S_{WW} (S_{WX}^T)^{-1}\right).$$

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Generalized IV Estimator

Suppose now that W has L columns with $L > K$; that is, we have more instruments than regressors.

We say that the system is **overidentified**.

Note that $W^T X$ is no longer a squared matrix; hence, the IV estimator cannot be used.

We use the extra moment conditions to find K linear combinations of the instruments in the subspace generated by the instruments.

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Generalized IV Estimator

Thus, the generalized instrumental variable estimator is given by

$$\hat{\beta}_{GIV} = (X^T P_W X)^{-1} X^T P_W Y,$$

with covariance matrix estimated by

$$\widehat{Var}(\hat{\beta}_{GIV}) = \hat{\sigma}^2 (X^T P_W X)^{-1},$$

where $\hat{\sigma}^2$ is analogous to before.

Two-Stage Least Squares

Recall from OLS that given that P_W is a projection matrix, we can define $\hat{X} = P_W X$, as the fitted values from a regression of X on W.

Then, $\hat{\beta}_{GIV}$ can be computed as the OLS estimator from the equation

$$Y = \hat{X}\beta + V.$$

This shows that $\hat{\beta}_{GIV}$ can be computed by **two stage least squares**. In the first stage we compute OLS of the regressors on the instruments, and then the fitted values on the regressand.

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Test for Relevance
Weak Instruments

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Recall that to use IV we must find valid instruments, that means relevant and exogenous regressors.

Non-relevant instruments, those not (significantly) correlated with the regressors are called **weak instruments**.

Weak instruments can have perverse effects on estimation: misleading standard errors, finite sample distribution deviates substantially from the Normal, and even biased estimates.

Thus, we should always try to avoid having weak instruments.

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Test for Relevance

Weak Instruments

We can use the first stage from the two-stage least squares estimator to obtain some evidence regarding the relevance of the instruments.

The first stage of 2SLS estimates

$$X = W\delta + V.$$

This suggests that we can check if the instruments are related to the regressors by looking at the δ coefficients.

If the instruments explain the regressors relatively well, we can take that as an indication of instrument relevance.

A simple rule-of-thumb is that we do not have to worry of having weak instruments if the F statistic is greater than 10.

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Test for Instruments' Validity

The second condition to verify is that the instruments are valid; that is, that they are not related to the errors.

Thus, we would like to test the condition

$$E[W^T U] = 0.$$

In practice, we need more instruments than regressors to be able to test this assumption.

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Test for Instruments' Validity

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Sargan's test for valid instruments proposes to estimate the following equation by OLS

$$\hat{U} = W\alpha + V,$$

and compute the statistic

$$\mathcal{J} = nR^2,$$

where n is the sample size and R^2 is the uncentered coefficient of determination.

The statistic follows a chi-squared distribution with $L - K$ degrees of freedom.

This test is also called of **overidentifying restrictions**.

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Test for Exogeneity

Another important assumption that we could check is whether we actually need to use IV at all. That is, we will like to test if

$$E[X^T U] = 0$$

Hausman's test (or Durbin-Wu-Hausman as called in the book) relies on the fact that under exogenous regressors, OLS is BLUE.

Thus, the test proposes to check if whether $\hat{\beta}_{OLS}$ is significantly different from $\hat{\beta}_{IV}$.

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Test for Exogeneity

The statistic for the Hausman test is given by

$$H = (\hat{\beta}_{IV} - \hat{\beta}_{OLS})^T (Var(\hat{\beta}_{IV} - \hat{\beta}_{OLS}))^{-1} (\hat{\beta}_{IV} - \hat{\beta}_{OLS}),$$

where everything is known besides $Var(\hat{\beta}_{IV} - \hat{\beta}_{OLS})$.

Hausman showed that the covariance between an efficient estimator and its difference from an inefficient estimator is zero.

Hausman result is more general, but in this particular case it means that

$$Var(\hat{\beta}_{IV} - \hat{\beta}_{OLS}) = Var(\hat{\beta}_{IV}) - Var(\hat{\beta}_{OLS}).$$

The statistic follows a chi-square distribution with degrees of freedom given by the number of possibly endogenous regressors.

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Hypothesis Tests

Tests for significance of the parameters follow the same steps used up to this point.

We estimate the model and compute either t-tests for univariate tests or the multivariate versions and divide by an estimate of the variance.

If there is evidence of autocorrelation or heteroskedasticity, it is recommended to use robust matrices or bootstrap.

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Examples

Instrumental variables is a really important development in Econometrics.

Most of the formal applied research is concerned with arguing that the regressors are exogenous, or to find valid instruments.

Several variables have been proposed as instruments:

- education of the parents for ability
- dummy on whether country was a colony for quality of institutions
- lags of variables for time series regressions
- energy consumption for GDP

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Summing Up

- We have studied a method to obtain consistent estimates when the regressors are endogenous.
- The method relies on finding instruments, variables related to the regressors but not to the errors.
- The first stage of 2SLS can be used to test for their relevance.
- If we have more instruments than endogenous regressors we can test for their validity.
- Weak instruments can have perverse effects on the estimation.
- Assuming we found good instruments, a Hausman test allows us to check if the regressors are indeed exogenous.

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