GLS and FGLS

Econometrics

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Introduction

Last lecture we saw some of the issues that can arise if the classical assumptions on OLS are not valid.

In particular, we saw that OLS stops being efficient or even unbiased.

Today we will focus on the failing of the homoskedasticity and no autocorrelation assumptions.

We will show that OLS stops being efficient and that tests using the OLS covariance matrices are unreliable.

Then, we will show how we can deal with these problems.

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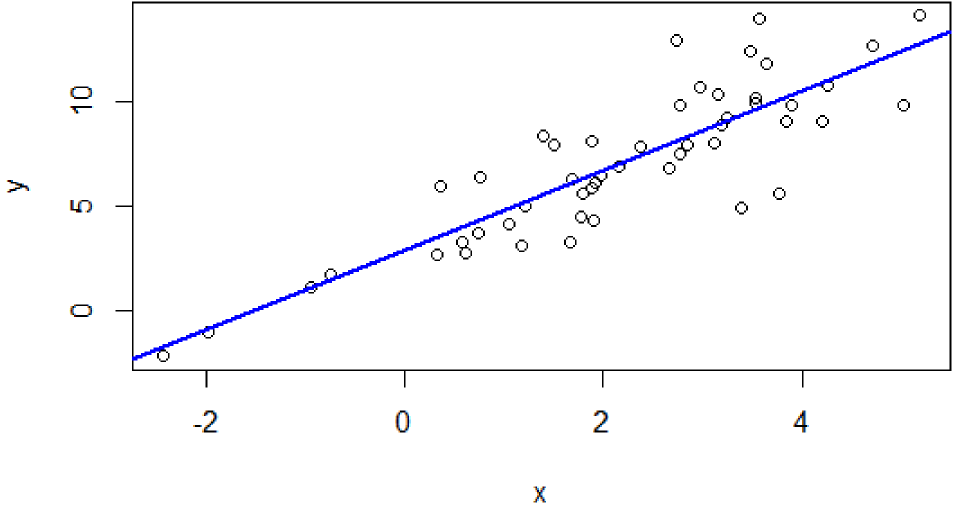
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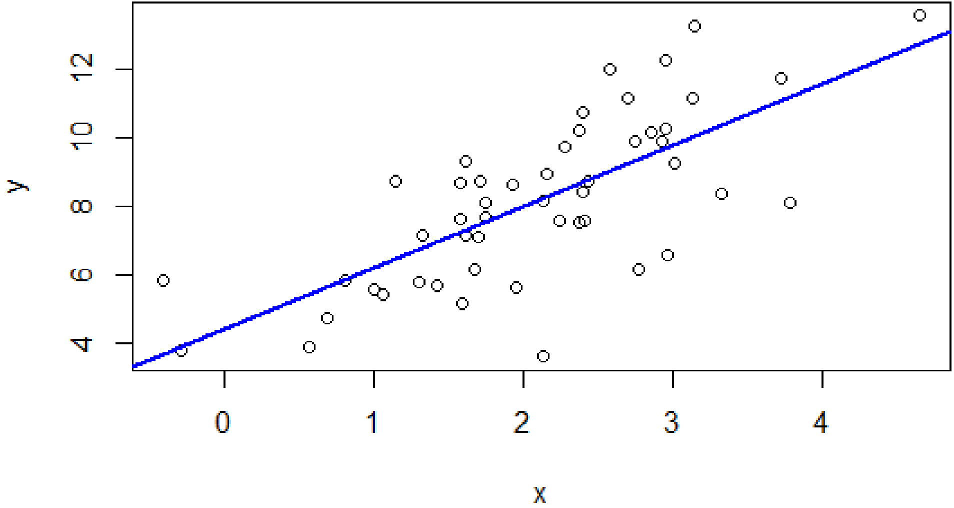

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Heteroskedasticity and autocorrelation

Under heteroskedasticity and/or autocorrelation, the model we want to estimate can be written as

$$Y = X\beta + U, \qquad E[UU^T] = \Omega.$$

If Ω is diagonal with nonconstant elements, the errors are heteroskedastic.

If Ω is not diagonal, then the errors are autocorrelated.

The errors do not satisfy the assumptions for the Gauss-Markov theorem so OLS is not efficient. Moreover, tests based on the OLS estimated variance, $s^2(X^TX)^{-1}$, will be misleading.

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Heteroskedasticity and autocorrelation

Under heteroskedasticity and/or autocorrelation, the variance of the estimator is given by

$$Var(\hat{\beta}) = (X^T X)^{-1} X^T \Omega X (X^T X)^{-1}.$$

This form of covariance matrix is often called a **sandwich covariance matrix**.

Inefficient estimators typically have sandwich covariance matrices.

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Heteroskedasticity-Consistent Covariance Matrices

Under heteroskedasticity, the matrix Ω is diagonal with t-th element equal to w_t^2.

If we know the w_t^2, we can evaluate the sandwich covariance matrix, or even better, we can obtain efficient estimates.

In the general case, we do not know w_t^2 and we have to estimate them.

Note that there are n different values for w_t^2; thus, we cannot in general consistently estimate the whole Ω matrix.

Nonetheless, we are mainly interested in estimating $X^T \Omega X$, a $k \times k$ symmetric matrix.

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Heteroskedasticity-Consistent Covariance Matrices

White showed that $X^T \Omega X$ can be estimated consistently by $\frac{1}{n}\sum_{t=1} \hat{u}_t^2 x_{ti} x_{tj}$, which suggests estimators for the variance given by:

- $$HCCME = HC_0 = (X^T X)^{-1} X^T diag(\hat{u}_t^2) X (X^T X)^{-1}.$$

- $$HC_1 = (X^T X)^{-1} X^T diag(n/(n-k)\hat{u}_t^2) X (X^T X)^{-1}.$$

- $$HC_2 = (X^T X)^{-1} X^T diag(\hat{u}_t^2/(1-h_t)) X (X^T X)^{-1},$$

  where $h_t = (P_X)_{tt}$.

- $$HC_3 = (X^T X)^{-1} X^T diag(\hat{u}_t^2/(1-h_t)^2) X (X^T X)^{-1}.$$

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Heteroskedasticity and Autocorrelation Consistent

Under autocorrelation, the extension to HCCME would be to estimate the off-diagonal elements of Ω, w_{ts}, with $\hat{u}_t \hat{u}_s$.

Analogous to the heteroskedastic case, we cannot consistently estimate all correlations, we have to confine our attention to processes exhibiting temporally limited dependence.

Thus, we cut the computation of w_{ts} at some maximum value $p = t - s$. Typically, we make the **lag truncation parameter**, p, grow at rate $n^{1/4}$.

Heteroskedasticity and Autocorrelation Consistent

Let $\Sigma = X^T \Omega X$, we write

$$\Sigma = \Gamma(0) + \sum_{j=1}^{p} \omega_j \left(\Gamma(j) + \Gamma^T(j) \right),$$

where $\Gamma(j) = \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j} X_t^T X_{t-j}$, and ω_j is a weight given to lag j.

- Hansen-White used $\omega = 1$, but the resulting matrix is not guaranteed to be positive definite.
- **Newey-West** suggested a Bartlett kernel for the weights.
- Alternatively, a Parzen kernel or Quadratic Spectral kernel can be used.
- Rules of thumb for the lag truncation parameter are $p = 0.75 n^{1/3}$ and $p = 4(n/100)^{2/9}$.

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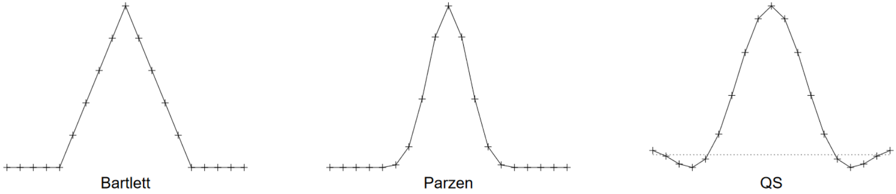

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Generalized Least Squares

To obtain efficient estimates, we look to transform the model so that it satisfies the Gauss-Markov assumptions.

Given that Ω is a covariance matrix, it is positive definite. Thus, we can find a, not necessarily unique, matrix P such that

$$\Omega^{-1} = PP^T.$$

Define $Y^* = P^T Y$, $X^* = P^T X$, and $U^* = P^T U$, so that the transformed model is

$$Y^* = X^* \beta + U^*.$$

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The OLS estimator from the transformed regression is called the **generalized least squares**, **GLS**, estimator. It is given by

$$\tilde{\beta}_{GLS} = ((X^*)^T X^*)^{-1}(X^*)^T Y^*.$$

Note that the transformed error term satisfies the Gauss-Markov theorem's assumptions.

The variance of the OLS estimator from the transformed model is

$$Var(\hat{\beta}) = ((X^*)^T X^*)^{-1}.$$

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We can show directly that GLS is more efficient than OLS.

Similar to last lecture, we prove this by showing that $Var^{-1}(\tilde{\beta}_{GLS}) - Var^{-1}(\hat{\beta}_{OLS})$ is a positive semidefinite matrix.

Thus,

$$Var^{-1}(\tilde{\beta}_{GLS}) - Var^{-1}(\hat{\beta}_{OLS}) = X^T\Omega^{-1}X - X^TX(X^T\Omega X)^{-1}X^TX,$$

which is a positive semidefinite matrix.

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We have obtained OLS estimates without knowing the variance of the error terms.

Thus, it should be possible to obtain GLS estimates without knowing all about Ω.

Suppose that $\Omega = \sigma^2 \Delta$, where Δ is known to us, but σ^2 is unknown. Then if we replace Ω by Δ we will obtain the estimate

$$(X^T \Delta X)^{-1} X^T \Delta Y = (X^T \Omega X)^{-1} X^T \Omega Y = \hat{\beta}_{GLS},$$

thus the GLS estimates will be the same whether we use Ω or Δ.

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If σ^2 is known, we can use the true covariance matrix for the estimators.

Otherwise, we can use our usual variance estimator

$$Var(\hat{\beta}_{GLS}) = s^2(X^T\Delta^{-1}X)^{-1},$$

where s^2 is the usual OLS estimate of the error variance from the transformed regression.

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Weighted Least Squares

Under just heteroskedasticity, the matrix Ω is diagonal with t-th element equal to w_t^2.

Thus, P is a diagonal matrix with w_t^{-1} in the diagonal. Hence, a typical observation of the transformed model can be written as

$$w_t^{-1} Y_t = w_t^{-1} X_t \beta + w_t^{-1} u_t.$$

This special case of GLS is called **weighted least squares**.

The weight given to each observation is inversely proportional to its variance.

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Weighted Least Squares

As argued before, Ω can be written as $\Omega = \sigma^2\Delta$, and sometimes we can infer Δ from the experimental design.

For example, if the data were obtained by grouping data on different numbers of individual units.

Suppose that the error terms for the ungrouped data have constant variance, but that observation t is the average of N_t observations. In this case, $w_t = N_t^{-1/2}$.

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Feasible Generalized Least Squares

In practice Ω is not known, which makes it impossible to compute GLS estimates.

However, in many cases it is reasonable to suppose that it depends in a known way on a vector of unknown parameters. Thus, it may be possible to estimate it consistently.

This type of procedure is called **feasible generalized least squares**, **FGLS**.

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Suppose we want to estimate the model

$$Y_t = X_t\beta + U_t, \qquad E[U_t^2] = \exp(Z_t\gamma),$$

where β, γ are parameters to be estimated.

In order to obtain consistent estimators for γ, we must first obtain consistent estimators of U_t, which we obtain by computing a consistent estimator of β, typically by OLS.

We can then run the auxiliary regression

$$\log \hat{U}_t^2 = Z_t\gamma + \nu_t,$$

and estimate it by OLS to obtain estimates of γ.

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Once we have obtained $\hat{\gamma}$, we can compute

$$\hat{w}_t^2 = \left(\exp(Z_t\hat{\gamma})\right)^{1/2},$$

for all t.

Finally, feasible GLS estimates for β, $\hat{\beta}_{FGLS}$, are obtained by weighted least squares using these weights.

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Iterated Feasible Generalized Least Squares

It is possible to iterate a feasible GLS procedure.

The estimator $\hat{\beta}_{FGLS}$ can be used to compute new set of residuals, which can then be used to obtain a second-round estimate of γ, which can be used to calculate second-round feasible GLS estimates, and so on.

The process can either be stopped after a predetermined number of rounds or continued until convergence is achieved.

In many cases, an iterated FGLS estimator will be the same as a maximum likelihood estimator assuming normally distributed errors.

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Test for Heteroskedasticity

In order to judge whether the OLS results are misleading because of heteroskedasticity, a number of alternative tests are available.

If these tests do not reject the null, there is no need to suspect OLS.

If rejections are found, we may consider the use of an FGLS estimator, the use of HCCME for the OLS estimator, or we may revise the specification of our model.

The idea for the tests for heteroskedasticity comes from the auxiliary regression

$$\log \hat{U}_t^2 = Z_t \gamma + \nu_t,$$

which shows that we expect $\hat{U}_t^2$ to depend on Z_t.

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Test for Heteroskedasticity

Breusch-Pagan suggested to test for heteroskedasticity by running the regression

$$\hat{U}_t^2 = \gamma_0 + Z_t\gamma + \nu_t,$$

and test using the statistic

$$LM = nR^2,$$

where n is the sample size and R^2 is the coefficient of determination of the regression.

The statistic follows (asymptotically) a chi-squared distribution with p degrees of freedom, where p is the number of elements in Z_t.

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Alternatively, **White** suggested to test for heteroskedasticity by running the regression

$$\hat{U}_t^2 = \gamma_0 + X_t\gamma_1 + X_t^T X_t\gamma_2 + \nu_t,$$

which includes all the regressors, their squares, and their cross-products, and test using the statistic

$$LM = nR^2,$$

where n is the sample size and R^2 is the coefficient of determination of the regression.

The statistic follows (asymptotically) a chi-squared distribution with p degrees of freedom, where p is the number of regressors besides the intercept.

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Test for Autocorrelation

The best know test for autocorrelation us due to **Durbin-Watson**.

They proposed to compute the following statistic, called the *DW* statistic,

$$d = \frac{\sum_{t=2}^{N}\left(\hat{U}_t - \hat{U}_{t-1}\right)^2}{\sum_{t=1}^{n}\hat{U}_t^2},$$

where $\hat{U}_t$ are the residuals.

- The value of d always lies between 0 and 4. A value of 2 indicates no autocorrelation.
- If d is much smaller than 2, this is an indication for positive autocorrelation.
- If d is much larger than 2, it indicates negative autocorrelation

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Test for Autocorrelation

The distribution of d depends not only upon the sample size n and the number of regressors, but also upon the actual values of them.

Thus, critical values cannot be tabulated for general use.

Nonetheless, upper and lower limits for the critical values of d that depend only upon sample size and number of regressors have been computed.

These values, d_L and d_U, were tabulated by Durbin-Watson and Savin-White.

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Thus, critical values cannot be tabulated for general use.

Nonetheless, upper and lower limits for the critical values of d that depend only upon sample size and number of regressors have been computed.

These values, d_L and d_U, were tabulated by Durbin-Watson and Savin-White.

An investigator will reject the null hypothesis if $d < d_L$, fail to reject if $d > d_U$, and come to no conclusion if $d_L < d < d_U$.

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Summing Up

- We have analysed the effect that heteroskedasticity and autocorrelation has on our estimators.
- They make OLS inefficient.
- Tests using the variance estimated by OLS are misleading.
- We can correct the tests by using HCCME or HAC matrices.
- If we know all about the form of the heteroskedasticity/ autocorrelation, we can use GLS to recover efficient estimates.
- If we only know their form up to some parameters, we can use FGLS.
- FGLS can be iterated to possibly improve its finite sample properties.
- We study some tests for heteroskedasticity and autocorrelation based on the residuals.

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