

# Exercises Week 11

## Econometrics

1. You are going to show that the GMM estimator is equivalent to 3SLS. That is, show that

$$(P_W X_\bullet)^T (\Sigma^{-1} \otimes I) = X_\bullet (\Sigma^{-1} \otimes P_W),$$

where  $P_W X_\bullet$  is a block-diagonal matrix with blocks given by  $P_W X_i$ .

Hint: Write  $P_W X_\bullet$  as a Kronecker product.

2. **Exercises 12.29-30 in ETM:** Consider the demand-supply model

$$q_t = \beta_{1,1} + \beta_{2,1}x_{t2} + \beta_{3,1}x_{t3} + \gamma_{21}p_t + u_{t1},$$

$$q_t = \beta_{1,2} + \beta_{4,2}x_{t4} + \beta_{5,2}x_{t5} + \gamma_{22}p_t + u_{t2},$$

where  $q_t$  is the log of quantity,  $p_t$  is the log of price,  $x_{t2}$  is the log of income,  $x_{t3}$  is a dummy variable that accounts for regular demand shifts, and  $x_{t4}$  and  $x_{t5}$  are the prices of inputs. Thus the first equation is a demand function and the second equation is a supply function.

- (a) For this model, precisely what is the vector  $\beta_\bullet$  defined in the lecture?
  - (b) How many overidentifying restrictions are there for each equation?
  - (c) The file *demand\_supply.csv* contains 120 observations generated by the model. Estimate this model by 2SLS.
  - (d) Test the overidentifying restrictions for each equation.
  - (e) Now estimate the model using 3SLS.
3. The dataset *smoke* in the *wooldridge* package contains information on smoking habits and other variables for a random sample of single adults from the United States. It contains 807 observations in 10 variables:

-*educ*: years of schooling

-*cigpric*: state cig. price, cents/pack

-*white*: 1 if white

-*age*: in years

-*income*: annual income

-*cigs*: cigs. smoked per day

-*restaurn*: 1 if restaurant smk. restrictions

-*lincome*: log(income)

-*agesq*: age squared

-*lcigpric*: log(cigprice)

We use the data to estimate a demand function for daily cigarette consumption.

- (a) A model to estimate the effects of smoking on annual income (perhaps through lost work days due to illness, or productivity effects) is

$$\log(\text{income}) = \beta_0 + \beta_1 \text{cigs} + \beta_2 \text{educ} + \beta_3 \text{age} + \beta_4 \text{age}^2 + u_1.$$

How do you interpret  $\beta_1$ ?

- (b) To reflect the fact that cigarette consumption might be jointly determined with income, a demand for cigarettes equation is

$$\text{cigs} = \gamma_0 + \gamma_1 \log(\text{income}) + \gamma_2 \text{educ} + \gamma_3 \text{age} + \gamma_4 \text{age}^2 + \gamma_5 \log(\text{cigpric}) + \gamma_6 \text{restaurn} + u_2.$$

Assuming these are exogenous to the individual, what signs would you expect for  $\gamma_5$  and  $\gamma_6$ ?

- (c) Estimate the income equation by OLS and discuss the estimate of  $\beta_1$ .
- (d) Estimate the reduced form for *cigs*. (Recall that this entails regressing *cigs* on all exogenous variables.) Are *log(cigpric)* and *restaurn* significant in the reduced form?
- (e) Now, estimate the income equation by 2SLS. Discuss how the estimate of  $\beta_1$  compares with the OLS estimate.
- (f) Do you think that cigarette prices and restaurant smoking restrictions are exogenous in the income equation?