

Qpg. 1 use 3

$$\text{Var}(\hat{\beta}_{\text{GLS}}) - \text{Var}(\hat{\beta}_{\text{OLS}}) = X^T \Omega^{-1} X - X^T X (X^T \Omega X)^{-1} X^T X$$

$$= X^T P P^T X - X^T X (X^T P P^T X)^{-1} X^T X$$

$$= X^T P (I - P^{-1} X (X^T P^{-1} P^{-1} X)^{-1} X^T P^{-1}) P^T X$$

$$= \underbrace{X^T P}_{Z^T} \underbrace{(I - \bar{X}^T (\bar{X}^T \bar{X})^{-1} \bar{X})}_{M_Q} \underbrace{P^T X}_Z$$

$$= Z^T M_Q^T M_Q Z \geq 0$$