Illustration_FWL

Econometrics

Demeaned Series

We generate the data, both regressor and regressand with mean different than zero.

```
T = 100
x = rnorm(T, mean=2.5,sd=1)
eps = rnorm(T, mean=0,sd=1)
y = -1+3*x+eps
```

First, we estimate the model including the constant using raw data.

```
lm.raw = lm(y~x)
summary(lm.raw)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.28128 -0.48999  0.07645  0.58283  2.44686
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.9863     0.2970  -3.321  0.00126 **
## x             3.0558     0.1054  28.993  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.044 on 98 degrees of freedom
## Multiple R-squared:  0.8956, Adjusted R-squared:  0.8945
## F-statistic: 840.6 on 1 and 98 DF,  p-value: < 2.2e-16
```

Then, we demean the series and estimate the model on these data without the constant.

```
xm = x-mean(x)
ym = y-mean(y)

lm.dem = lm(ym~xm-1)
summary(lm.dem)
```

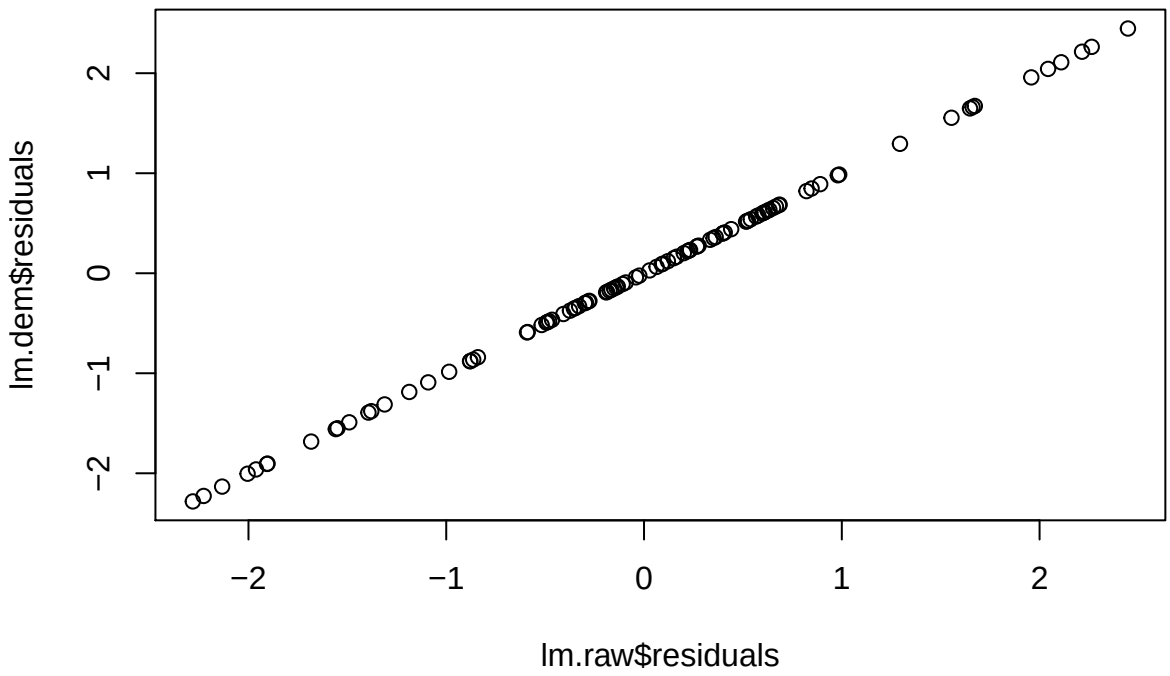
```
##
## Call:
## lm(formula = ym ~ xm - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.28128 -0.48999  0.07645  0.58283  2.44686
##
## Coefficients:
##    Estimate Std. Error t value Pr(>|t|)
## xm   3.0558     0.1049   29.14   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.039 on 99 degrees of freedom
## Multiple R-squared:  0.8956, Adjusted R-squared:  0.8945
## F-statistic: 849.2 on 1 and 99 DF,  p-value: < 2.2e-16
```

The results show that the estimator for the parameter of the regressor is numerically the same in both regressions.

Moreover, we plot the residuals from both regressions to show that they are also numerically the same.

```
plot(lm.raw$residuals,lm.dem$residuals)
```



Groups of regressors

We generate the data, where the regressors are not orthogonal.

```
x1 = rnorm(T,mean=2,sd=1)
x2 = 0.2*x1+rnorm(T,mean=-1,sd=1.5)
eps = rnorm(T,0,0.5)

y = x1+x2+eps
```

We estimate the regression with raw data and both regressors

```
lm.both = lm(y~x1+x2)
summary(lm.both)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.05247 -0.38356  0.00163  0.40779  0.99058
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.09747    0.11389  -0.856    0.394
## x1           0.99938    0.04782  20.899   <2e-16 ***
## x2           1.01952    0.03479  29.302   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4952 on 97 degrees of freedom
## Multiple R-squared:  0.9425, Adjusted R-squared:  0.9413
## F-statistic: 794.3 on 2 and 97 DF,  p-value: < 2.2e-16
```

Then we clean the regressand and second regressor from the effect of the first regressor. To do so, we run both regressions and retrieve the residuals.

```
lm.x2 = lm(x2~x1)
x2.cl = lm.x2$residuals

lm.y = lm(y~x1)
y.cl = lm.y$residuals
```

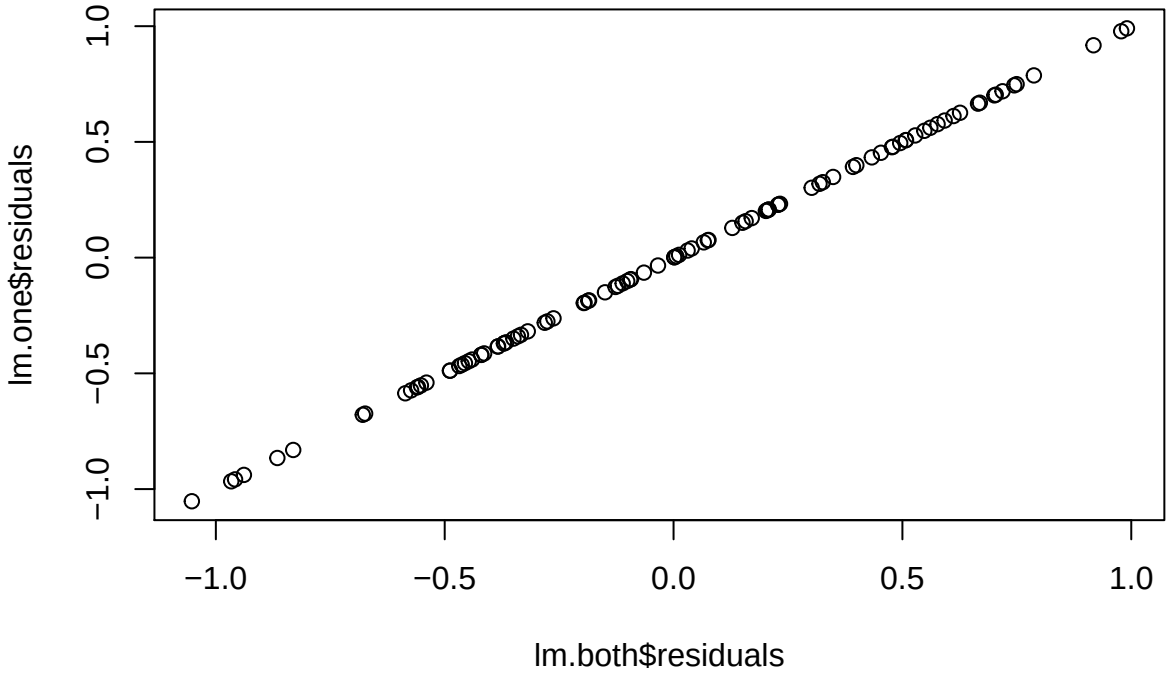
And we run the regressions on the "cleaned" data.

```
lm.one = lm(y.cl~x2.cl-1)
summary(lm.one)
```

```
##
## Call:
## lm(formula = y.cl ~ x2.cl - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.05247 -0.38356  0.00163  0.40779  0.99058
##
## Coefficients:
##       Estimate Std. Error t value Pr(>|t|)
## x2.cl  1.01952    0.03444    29.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4901 on 99 degrees of freedom
## Multiple R-squared:  0.8985, Adjusted R-squared:  0.8975
## F-statistic: 876.3 on 1 and 99 DF,  p-value: < 2.2e-16
```

Plotting the residuals.

```
plot(lm.both$residuals,lm.one$residuals)
```


lm.one$residuals
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
lm.both$residuals