

$$X_0 = I_g \otimes X, \quad \Sigma = \Sigma \otimes I_g$$

$$\Sigma = \sigma I_k$$

$$\hat{\beta}_{\text{GLS}} = (X^T \Sigma^{-1} X)^{-1} X^T \Sigma^{-1} Y$$

$$= (I_g \otimes X^T (\Sigma \otimes I)^{-1} I_g \otimes X)^{-1} I_g \otimes X^T (\Sigma \otimes I_g)^{-1} Y$$

$$= (I \otimes X^T \Sigma^{-1} \otimes I \quad I \otimes X)^{-1} I \otimes X^T \Sigma^{-1} \otimes I Y$$

$$= (\Sigma^{-1} \otimes X^T I \otimes X)^{-1} \Sigma^{-1} \otimes X^T Y$$

$$= (\Sigma^{-1} \otimes X^T X)^{-1} \Sigma^{-1} \otimes X^T Y$$

$$= \Sigma \otimes (X^T X)^{-1} \Sigma^{-1} \otimes X^T Y$$

$$= I \otimes (X^T X)^{-1} X^T Y$$

$$Y = \text{vec}(y_1, \dots, y_g)$$