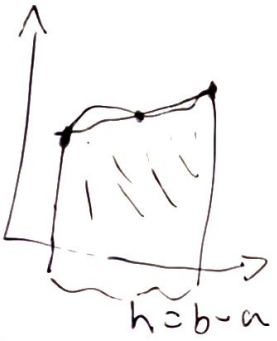
Kvadratur

Så få evalueringar som möjligt

$\int_a^b f(x)\,dx$ ← Estimera

Adaptiv kvadratur

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$



$M = h\,f\left(\frac{a+b}{2}\right)$ (mittpunkt.) ⇐ $O(h^2)$ 2. gradspolynom

$T = h\,\frac{f(a)+f(b)}{2}$ (trapez) ← $O(h^2)$

$S = \int_0^1 x^2\,dx = \frac{1}{3}$, $M = \frac{1}{4}$, $T = \frac{1}{2}$

fel: $S - M = \frac{1}{12}$, $S - T = -\frac{1}{6}$

$-2(S-M) = S-T$

$-2S + 2M = S - T$

$2M + T = 3S$

$S = \frac{2}{3}M + \frac{1}{3}T = \frac{h}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$

⇑ Simpsons regel

$O(h^4)$

$$S_2 = \frac{h}{12}\left(f(a) + 4f\left(\frac{a+b}{4}\right) + 2f\left(\frac{a+b}{2}\right) + 4f\left(3\frac{a+b}{4}\right) + f(b)\right)$$

$$Q - S = 16(Q - S_2) \Leftrightarrow Q = S_2 + (S_2 - S)/15$$

Betragt $\int_a^b f(x)\,dx$

Find S og S_2

hvis $S - S_2$ er stor

så indfør ydterligere, anvend

ovenstående på $S_a^{\frac{a+b}{2}}$ og $S_{\frac{a+b}{2}}^{b}$

ellers er svaret Q

bruger samme tolerance hele vejen

~~Diskret data~~

$a = x_1 < x_2 \dots < x_n = b$

$y_1 = f(x_1), \dots, y_n = f(x_n)$

$$T = \sum_{k=1}^{n-1} (x_{k+1} - x_k)\,\frac{y_k + y_{k+1}}{2}$$

$$\int_a^b p(x)\,dx = T - D$$

Hvis ækvidistante pkt.

$D = h^2 \frac{d_n - d_1}{12}$