12. Quadrature. Present your solution to Exercise 6.4.

Numerical Analysis E2021

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Motivation

Let $f : [a, b] \to \mathbb{R}$. We seek to compute the value

$$\int_a^b f(x)dx.$$

- The integrand $f(x)$ may be known only at certain points, such as obtained by sampling.
- A formula for the integrand may be known, but it may be difficult or impossible to find an antiderivative that is an elementary function.
  - *Example:* $\exp -x^2$.
- It may be easier to compute a numerical approximation than to compute the antiderivative.

Quadrature rules

Basic rules:

- **Midpoint rule** $M = hf\left(\frac{a+b}{2}\right)$,
- **Trapezoid rule** $T = h\frac{f(a)+f(b)}{2}$,
- **Simpson's rule** $S = \frac{h}{6}(f(a) + 4f(\frac{a+b}{2}) + f(b))$.

Composite Simpson's rule

Let $d = (a+c)/2$ and $e = (c+b)/2$. Apply Simpson's rule to each subinterval to obtain a quadrature rule over $[a,b]$:

$$S_2 = \frac{h}{12}(f(a) + 4f(d) + 2f(c) + f(e) + f(c)).$$

Both S and S_2 are of order 4, but the S_2 step size is half the S step size, so S_2 is roughly 2^4 times as accurate. Thus, a combination Q is obtained by solving

$$Q - S = 16(Q - S_2),$$

which yields

$$Q = S_2 + (S_2 - S)/15.$$

MATLAB demo of exercise 6.4.