5. The condition number of a matrix. Definitions and properties. Present your solution to Exercise 2.18 and discuss it.

Numerical Analysis E2021

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Motivation

We wish to analyse errors for when we compute solutions to linear equations. Thus, we introduce the condition number

$$\kappa(A) = \|A^{-1}\|\|A\| \tag{1}$$

satisfying

$$\begin{aligned} \kappa(cA) &= \kappa(A), \quad c \neq 0 \\ \kappa(A^{-1}) &= \kappa(A) \\ \kappa(AB) &\leq \kappa(A)\kappa(B) \\ \kappa(Q) &= 1 \\ \kappa(QA) &= \kappa(AQ) = 1 \end{aligned} \tag{2}$$

for orthogonal Q.

Interpretations

κ is a measure of "how" singular a matrix is. If it is not too much larger than one, we can calculate the inverse with good accuracy.

Can also be used in analysis of round-off errors introduced during Gaussian elimination.

Is also used in the following relative error estimate for solving a system $Ax = b$, where $(A + E)x_c = b$

$$\frac{\|x_t - x_c\|}{\|x_t\|} \leq \frac{\kappa(A)\|E\|/\|A\|}{1 - \kappa(A)\|E\|/\|A\|} \tag{3}$$

MATLAB

MATLAB demo for poorly conditioned system.

MATLAB demo of exercise 2.18.