Numerical analysis 2021
Exam questions

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General remarks

First, the exam will be held in Danish, unless you ask for it to be held in English.

The questions below are given in English, in order to avoid problems with Danish terminology in numerical analysis.

Most exam questions are detailed and are narrowly focused. During the exam questions on related topics may be asked. Some of them involve presentation of a solution to a problem, either from Moler or posed in the exam question.

Almost all questions require use of MATLAB during the examination. Many of them require use of files accompanying Moler.

The exam is a digital exam using MS Teams. The procedure is the same as previous digital exams. We give the main points relevant for this course.

1) Log on to MS Teams and go to the general channel for the exam. Do this well in advance of the time indicated in the schedule for the start of your exam.

2) You will be invited to the channel 'Eksamenslokale' when your exam starts. You are assigned an exam question randomly.

3) Have prepared a short presentation on the topic of the questiton, in the form of slides and MATLAB demos. Concentrate on the essentials. Both examiners know all the definitions. We will ask if a definition needs to be presented, etc. The presentation should last 8–10 minutes. After that the examiners will ask questions related to your presentation, but also other topics related to the exam question or general results from the course.

4) An alternative to slides is emulation of a blackboard. It can be done with an iPad or other tablet, or you can use a mobile phone. In the latter case you have to ask for an invitation for your phone. You can rig up a usable blackboard by putting you phone on top of a pile of books and get video of you writing on a piece of paper. If you want to use this option be sure to try it out with your group beforehand, so technical problems are eliminated before the exam.

5) At the end of the exam the examiners leave and decide on the grade (pass/fail). You stay on the channel 'Eksamenslokale'. The examiners return and give you your grade.

Exam questions

1. Floating point arithmetic. Present your solution to Exercise 1.34 in Moler. Discuss aspects of the representation of real numbers using floating point arithmetic (based on the IEEE754-standard).

2. Overflow and underflow. Explain these concepts in the context of floating point arithmetic (based on the IEEE754-standard). Present your solution to Exercise 4.11 in Moler.

3. *LU*-factorization of a matrix. Presentation of the algorithm. Discussion of pivoting and pivoting strategies. Give examples using `lugui`.

4. Error analysis for solutions to systems of linear equations. The condition number and its role in the analysis.

5. The condition number of a matrix. Definitions and properties. Present your solution to Exercise 2.18 and discuss it.

6. Polynomial interpolation. General results. Comparison of polynomial interpolation and piecewise polynomial interpolation. Use `interpgui` to give examples.

7. Piecewise cubic polynomial interpolation. Present results on piecewise cubic interpolation. Use either `pchiptx` or `splinetx` to discuss implementation of the algorithms.

8. Zero finding for real functions of one variable. Present the three methods, bisection, secant and IQI, and discuss advantages and disadvantages of the three methods.

9. Zero finding for real functions of one variable. Present Newton's method and discuss its advantages and disadvantages. Present your solution to Exercise 4.3 in Moler.

10. Basic quadrature rules. Composite quadrature rule. Order of a quadrature rule.

11. Adaptive quadrature explained using `quadtx.m`.

12. Quadrature. Present your solution to Exercise 6.4.

13. Error estimates for quadrature rules.

14. Present some single step methods for solving an IVP for a system of ODE. Compare the methods presented.

15. Error analysis for ODE IVP solvers.

16. Examples of stiff ODEs and discussion of methods of solution. Discussion of the difference between stiff and non-stiff ODEs.

17. Event handling for ODEs. Use computation of the period in the Lotka-Volterra system in Exercise 7.15 to illustrate the method.

18. Pseudo-random number generators. Examples of generators based on multiplicative congruence relations.

19. Pseudo-random number generators. Examples using `randgui`.

20. Pseudo-random number. Explain how to obtain normally distributed pseudo-random numbers.