

16. Examples of stiff ODEs and discussion of methods of solution. Discussion of the difference between stiff and non-stiff ODEs.

Numerical Analysis E2021

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- ▶ A subtle, difficult, and important concept in the numerical solution of ordinary differential equations.
- ▶ An ODE problem is stiff if the solution being sought varies slowly, but there are nearby solutions that vary rapidly, so the numerical method must take small steps to obtain satisfactory results.
- ▶ An efficiency issue. If we weren't concerned with how much time a computation takes, we wouldn't be concerned about stiffness.

What can be done about stiff problems?

Example of stiff ODE

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A ball that grows rapidly until it reaches a critical point, is expressed with the following model:

$$y' = y^2 - y^3, \quad y(0) = \delta, \quad 0 \leq t \leq 2/\delta.$$

The smaller the value of δ , the “stiffer” the problem becomes.

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Example of stiff ODE

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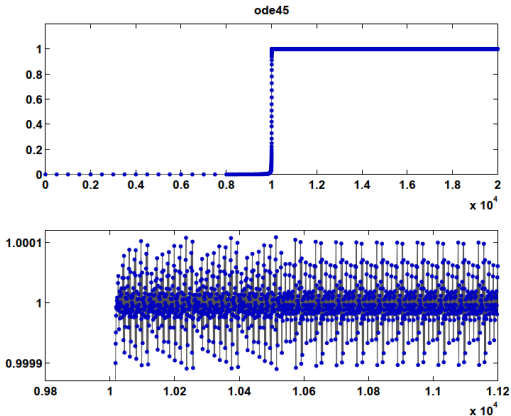


Figure: Stiff behaviour of ode45 with $\delta = 10^{-4}$.

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Example of stiff ODE

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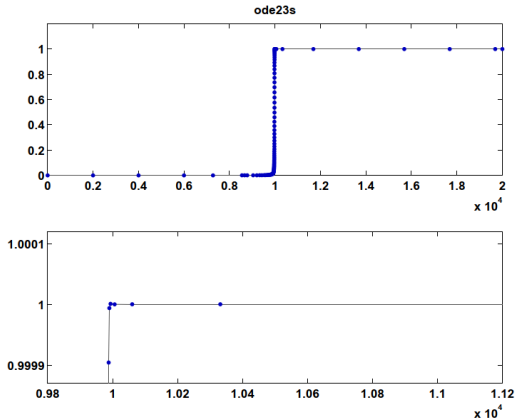


Figure: Stiff behaviour of `ode23s` with $\delta = 10^{-4}$.

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MATLAB demo

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Stiffness

MATLAB demo of exercise 7.6.

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