stykvis polynomiel interpolation

på intervallet $[x_k, x_{k+1}]$

vælges poly $p_k(x)$

stykvis lineær interpolation

se på stykvis kubisk interpol.

to eks.

(1) pchip, (2) splines

Hermite polynomial interpolation

special tilfælde

Givet x_0, x_1 (antag $x_0 < x_1$)

y_0, y_1

z_0, z_1

Der findes et polynomie af grad ≤ 3, så at

$$\left.\begin{aligned} p(x_j) &= y_j, \; j=0,1 \\ p'(x_j) &= z_j, \; j=0,1 \end{aligned}\right\} (*)$$

Det er entydigt bestemt

Bevis:

$$l_0(x) = \frac{x-x_1}{x_0-x_1} \qquad l_1(x) = \frac{x-x_0}{x_1-x_0}$$

$$H_j(x) = (l_j(x))^2(1-2l_j'(x_j)(x-x_j)) \quad j=0,1$$
$$K_j(x) = (l_j(x))^2(x-x_j)$$

påstand $\quad H_j(x_k) = \begin{cases} 1 & j=k \\ 0 & j\neq k \end{cases}$

$\quad H_j'(x_k) = 0 \quad j,k=0,1$

$$H_0(x_0) = (l_0(x_0))^2(1-2l_0'(x_0)(x-x_0))$$

$H_0(x_1) = 0$

$H_0'(x) = 2l_0(x)l_0'(x)(1-2l_0'(x)(x-x_0)) + (l_0(x))^2(-2l_0'(x_0))$

$H_0'(x_1) = 0$

$H_0'(x_0) = 2l_0'(x_0) + (-2l_0'(x_0)) = 0$

påstand

$K_j(x_k) = 0, \quad j,k=0,1$

$K_j'(x_k) = \begin{cases} 1 & j=k \\ 0 & j\neq k \end{cases}$

$K_0'(x) = 2l_0(x)\,l_0'(x)(x-x_0) + (l_0(x))^2$

Eksistens: $p(x) = y_0H_0(x) + z_0K_0(x) + y_1H_1(x) + z_1K_1(x)$

entydighed: antag $p(x)$ og $q(x)$ opfylder ※

$\quad r(x) = p(x) - q(x)$

$\quad$ heraf $\quad r(x_j) = 0, \; j=0,1, \quad r'(x_j) = 0 \quad j=0,1$

Iflg Rolles sætning eksisterer der ξ_i $x_0<\xi<x_1$ så $r'(\xi)=0$

Heraf $r'(x)=0$, $x\in\mathbb{R}$

Dvs. $r(x)=c$ $x\in\mathbb{R}$

Da $r(x_0)=0$, er $c=0$, dvs $p(x)=q(x)$

x_0, x_1, $h=x_1-x_0$, $s=x-x_0$ lokal var. $x_0\le x\le x_1$

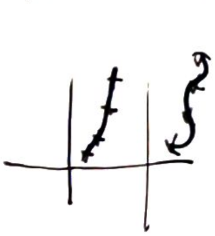
$$\delta_0=\frac{y_1-y_0}{h} \quad d_0=z_0,\ d_1=z_1$$

$$p(x)=\frac{3hs^2-2s^3}{h^3}y_1+\frac{h^3-3hs^2+2s^3}{h^3}y_0 + \frac{s^2(s-h)}{h^2}d_1+\frac{s(s-h)^2}{h^2}d_0$$

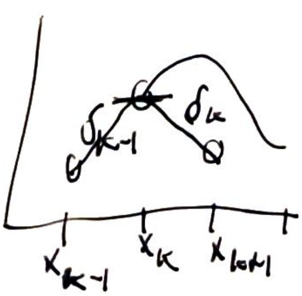
$$p(x_j)=y_j,\quad p'(x_j)=d_j,\quad j=0,1$$

pchip "shape preserving"

vores valg skal bevare "diskret monotonicitet" i datasæt



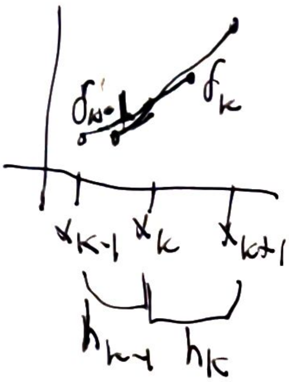
Vi skal specificere d_j



Tilfældet $\delta_{k-1}\cdot\delta_k\le 0$

vælg $d_k=0$

Tilfellet $\delta_{k-1} \cdot \delta_k > 0$



i tilfellet $h_{k-1} = h_k$

$$\frac{1}{d_k} = \frac{1}{2}\left(\frac{1}{\delta_{k-1}} + \frac{1}{\delta_k}\right)$$