11. Adaptive quadrature explained using `quadtx.m`.

Numerical Analysis E2021

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Motivation

Let $f : [a, b] \to \mathbb{R}$. We seek to compute the value

$$\int_a^b f(x)dx.$$

- The integrand $f(x)$ may be known only at certain points, such as obtained by sampling.
- A formula for the integrand may be known, but it may be difficult or impossible to find an antiderivative that is an elementary function.
  - *Example:* $\exp -x^2$.
- It may be easier to compute a numerical approximation than to compute the antiderivative.

Adaptive quadrature

Exploits quadrature rules combined with the additive nature of integrals

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

If we can approximate each of the two integrals on the right to within a specified tolerance, then the sum gives us the desired result. If not, we can recursively apply the additive property to each of the intervals $[a,c]$ and $[c,b]$.

Adaptive quadrature with `quadtx`

Textbook function that is a simplified version of `quad`. *One serious defect:* it is possible to try to evaluate integrals that do not exist.

- Evaluates the integrand $f(x)$ three times to give the first, unextrapolated, Simpson's rule estimate.
- A recursive subfunction, `quadtxstep`, is then called to complete the computation.
- Each recursive call of `quadtxstep` combines three previously computed function values with two more to obtain the two Simpson's approximations for a particular interval.
- If their difference is small enough, they are combined to return the extrapolated approximation for that interval.
- If their difference is larger than the tolerance, the recursion proceeds on each of the two half intervals.

MATLAB demo of exercise 6.6.