Dual Propagation

Accelerating Contrastive Hebbian Learning with Dyadic Neurons

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Motivation

Contrastive hebbian learning (CHL) [1] and equilibrium propagation (EP) [2]

- Biological plausibility
- Neuromorphic computing

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A dyadic objective

$$\mathcal{L}_\alpha(\theta) := \min_{z^+} \max_{z^-} \alpha \ell(z_L^+) + (1-\alpha)\ell(z_L^-) \\ + \sum_{k=1}^{L} \tfrac{1}{\beta_k}\Big(G_k(z_k^+) - G_k(z_k^-) + (z_k^- - z_k^+)^\top W_{k-1}(\alpha z_{k-1}^+ + (1-\alpha) z_{k-1}^-)\Big)$$

- Linear units: $G_k = \|\cdot\|^2/2$
- ReLU units: $G_k = \|\cdot\|^2/2 + \imath_{\geq 0}(\cdot)$
- $\alpha \in [0, 1]$

Choice of α

- $\alpha = 1$ and $\alpha = 0$:
    - LPOM-like [3, 4] pure minimization objectives
    - First order gradient estimate
    - Slow iterative inference
- $\alpha = 1/2$:
    - Fast layer-wise closed form inference
    - Second order gradient estimate

$$
\begin{aligned}
\mathcal{L}_{\frac{1}{2}}(\theta) = \min_{z^+} \max_{z^-} & \tfrac{1}{2}\ell(z_L^+) + \tfrac{1}{2}\ell(z_L^-) \\
& + \sum_{k=1}^{L} \tfrac{1}{\beta_k}\Big(G_k(z_k^+) - G_k(z_k^-) + \tfrac{1}{2}(z_k^- - z_k^+)^\top W_{k-1}(z_{k-1}^+ + z_{k-1}^-)\Big)
\end{aligned}
$$

Connection to CHL and EP

Defining energy functions E_k as in CHL allows reformulating $\mathcal{L}_{\frac{1}{2}}$.

- $E_k(z_k, z_{k-1}) := G_k(z_k) - z_k^T W_{k-1} z_{k-1}$
- $\bar{z}_k := \frac{1}{2}(z_k^+ + z_k^-)$

$$\mathcal{L}_{\frac{1}{2}}(\theta) = \min_{z^+} \max_{z^-} \ell(z_L^+) + \ell(z_L^-) + \sum_{k=1}^{L} \frac{1}{\beta_k}\left(E(z_k^+, \bar{z}_{k-1}) - E(z_k^-, \bar{z}_{k-1})\right)$$

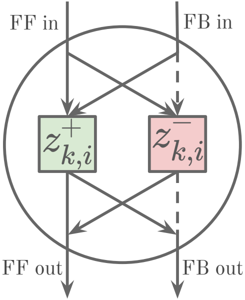
Similar to CHL and EP objectives but z_k^+ and z_k^- are inferred simultaneously and "tethered" via $\bar{z}_{k-1}$.

Inference

$$z_k^{\pm} \leftarrow f_k\left(\tfrac{1}{2}W_{k-1}(z_{k-1}^{+} + z_{k-1}^{-}) \pm \tfrac{\beta_k}{2\beta_{k+1}} W_k^{\top}(z_{k+1}^{+} - z_{k+1}^{-})\right)$$

- Neurons within layers are not coupled
- Fully optimize $\mathcal{L}_{\frac{1}{2}}$ with respect to a z_k^{+} and z_k^{-} in a single block-coordinate descent step!

Dyadic neurons



$$z_k^{\pm} \leftarrow f_k\left(\underbrace{\tfrac{1}{2}W_{k-1}(z_{k-1}^{+}+z_{k-1}^{-})}_{\text{FF in}} \pm \underbrace{\tfrac{\beta_k}{2\beta_{k+1}}W_k^{\top}(z_{k+1}^{+}-z_{k+1}^{-})}_{\text{FB in}}\right)$$

$$\text{FF out} = \frac{1}{2}(z_{k,i}^{+}+z_{k,i}^{-})$$

$$\text{FB out} = \frac{1}{2}(z_{k,i}^{+}-z_{k,i}^{-})$$

Biologically plausible?

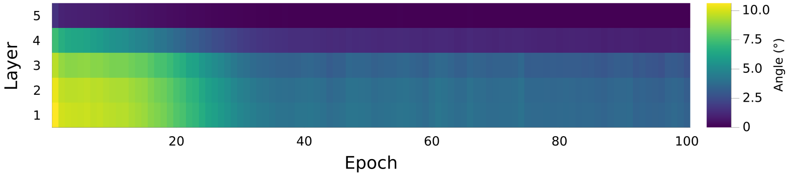
- Error signals depend on both FF and FB inputs.
- Similar issue in [5] resolved via inter-neurons.

Learning

- Fully local contrastive Hebbian gradient
- Second order gradient estimate
- Discounting not needed ($\beta_k = 1 \; \forall \; k$ employed in experiments)

$$\frac{\partial}{\partial W_{k-1}} \mathcal{L}_{\frac{1}{2}} = \frac{1}{2\beta_k} \left(z_k^- - z_k^+ \right) \left(z_{k-1} + z_{k-1} \right)^\top.$$

- Excellent gradient alignment between BP and DP



Experiments: MLP (MNIST)

- DP: cost-efficient (forwards + backwards)
- MS-DP: Multiple steps of inference and weight updates per datapoint
- L-DP: "Lazily" let old states persist providing potentially harmful feedback.
- P-DP: parallel neuron updates
- R-DP-100: Random update sequence

Method	BP	DP	MS-DP	L-DP	P-DP	R-DP-100
Test acc (%)	98.45 $\pm$0.04	98.43 $\pm$0.03	98.40 $\pm$0.02	98.42 $\pm$0.07	98.47 $\pm$0.04	98.48 $\pm$0.11

Experiments: VGG16

Method		BP	DP	KP-DP†	EP* [6]	DTP* [7]
CIFAR10	Top-1	92.26 ± 0.23	92.30 ± 0.11	91.84 ± 0.11	88.6 ± 0.2	89.38 ± 0.20
CIFAR100	Top-1	69.63 ± 0.24	69.57 ± 0.51	70.40 ± 0.25	61.6 ± 0.1	—
	Top-5	88.13 ± 0.22	88.36 ± 0.13	88.57 ± 0.15	86.0 ± 0.1	—
ImageNet32x32	Top-1	41.28 ± 0.19	41.48 ± 0.19	—	36.5 ± 0.3	36.81
	Top-5	64.89 ± 0.11	64.90 ± 0.13	—	60.8 ± 0.4	60.54

(†) Dual propagation with Kolen-Pollack learning of feedback weights.

(*) High computational costs limit EP (Laborieux 2022) and DTP (Ernoult 2022) to 5-7 layer VGG-like networks.

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Conclusion

- Unlike previous CHL methods DP computes errors across compartments rather than across time.
- The dyadic neuron model enables closed-form inference rules and a second order gradient estimate.
- Many viable update schemes including random and parallel updates.
- The efficient DP implementation matches back-propagation both in terms of accuracy and runtime.

Future work: Continuous inference and learning in a streaming setting.

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