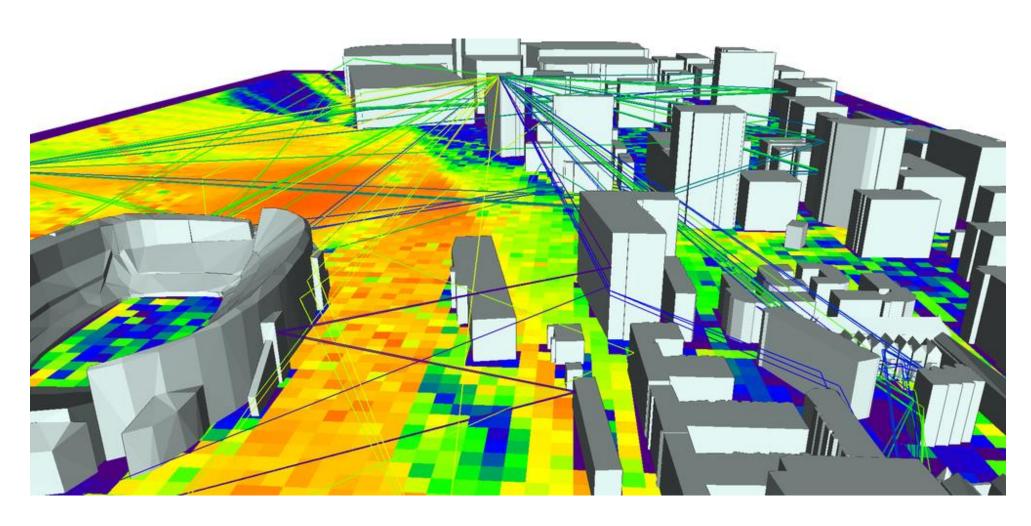
## Dual-Kernel Online Reconstruction of Power Maps

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## Power map

An indication of the average received power from one or several base station(BSs)



### Motivation



- Site-specific modeling rather than a statistical modeling (Hata, COST-231)
- More accurate pathloss values
- Network coverage analysis
- Proactive resource allocation

### Problem statement

#### **Scenario:**

- Users are randomly distributed
- Each BS has a coverage area
- The coverage area is obtained by setting a threshold on the pathloss value
- One user can be in the coverage area of several BSs

#### **Measurements:**

Average received signal power at each user location from assigned BSs

#### **Objective:**

• Build a power map for each BS: for some input coordinates, produce power values

### Problem formulation

#### Naturally falls into the regression task

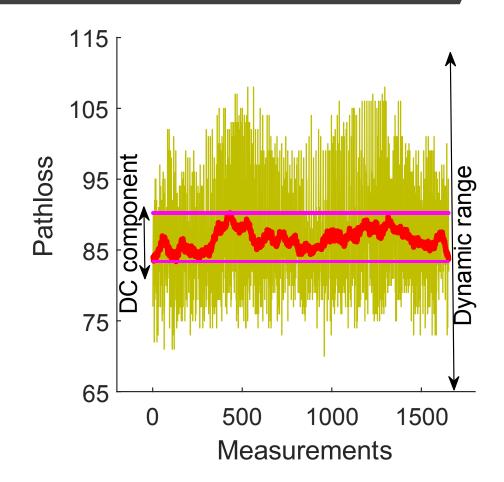
- Averaged received power is given for some locations (training set)
- Learn a function that connects the locations to the power levels
- Evaluate the performance of the learned function in the test set

#### Regression task can be categorized

- Online regression
   Process measurement one by one (simpler and faster)
- Batch regression
   Process measurements altogether (slower but more accurate)

#### Contributions

- Pathloss or average received power is well bounded, let's say between 50 and 160 dB
- Majority of users have pathloss in order of 60-120 dB
   The center of this interval is called the DC component
- Extract the DC component from the pathloss values and find the regression function for the rest
- Advantages:
  - Faster convergence
  - Better accuracy in terms of std and MSE



# How can we choose the regression function?

- Define a similarity measure or kernel function between measurements
- Build the regression function based on the combination of kernel basis

Example: kernel as the correlation between average received power in the location of users

$$\kappa(x_1, x_2) = e^{-\frac{|x_1 - x_2|}{2\sigma^2}}$$

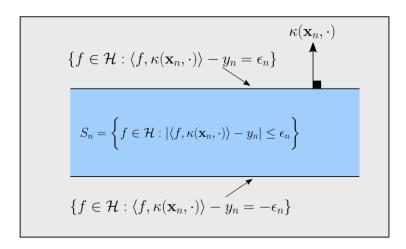
Then the optimum regressor function is

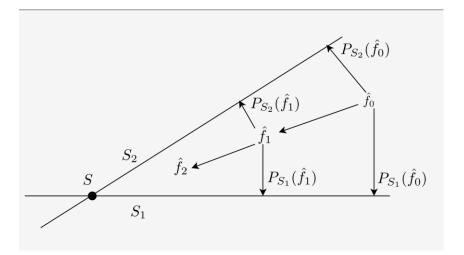
$$f^{\star}(x) = \sum_{n} \alpha_n e^{-\frac{|x-x_n|}{2\sigma^2}}$$

Each term in summation models the effect of one measurement and called a dictionary element

Adaptive projected sub-gradient method (APSM) An online regression tool

- Create a set of functions that can explain a specific measurement
- Find the intersection of these sets, or functions that can explain all or most of the measurements data
  - Use projection to a convex set to find the intersection





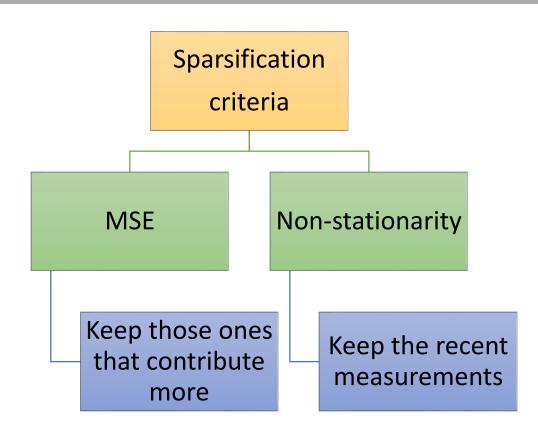
## Regularization

Regression is a convex optimization problem

- In a convex optimization problem, regularization is Including the function norm in the objective
- In APSM algorithm regularization is Projection over norm ball

Projection over norm ball can also be seen as the forgetting factor

## Sparsification The act of selecting some measurements and throwing away the rest



## Sparsification threshold

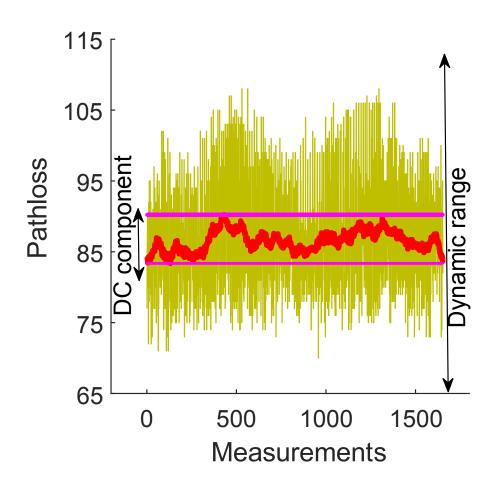
- Projection on the norm ball takes care of nonstationarity
- A threshold is defined:
  - To keep the most effective dictionary elements (in MSE sense)
  - The larger the dictionary size, the better the MSE error
  - The bigger the threshold, the more sparse the dictionary

$$P_{B[0,\delta]}(f) = \begin{cases} f & ||f|| \le \delta \\ f/||f|| & ||f|| > \delta. \end{cases}$$

$$\mathcal{J}_{n} = \left\{ j \mid \frac{\left(\alpha_{n,j}\right)^{2}}{\sum_{\ell \in \mathcal{L}_{n}} \left(\alpha_{n,\ell}\right)^{2}} > \tau \right\}$$

$$f^{\star}(x) = \sum_{n} \alpha_n e^{-\frac{|x-x_n|}{2\sigma^2}}$$

## Dual-kernel approach: One kernel for DC part and another for varying part



$$\kappa(x_1, x_2) = \exp\left(-\frac{\|x_1 - x_2\|}{D_{DC}}\right) + \exp\left(-\frac{\|x_1 - x_2\|}{D_{Var}}\right)$$

### Numerical evaluation

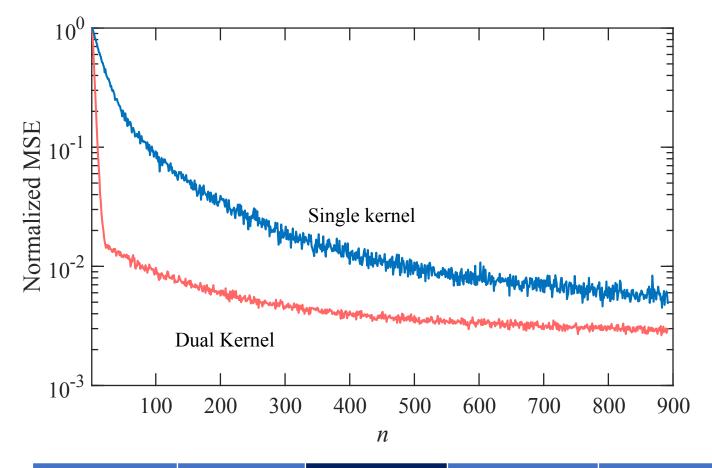
- Campus scenario:
  - High-resolution data (3m)
  - Measurement data in an 802.11 network
- Urban scenario:
  - Low-resolution data (50 m)
  - Ray-tracing data
- Available data samples are divided into:
  - 70% of learning data
  - 30% of test data

The dual-kernel regression is applied to 1000 random perturbations of available data samples, then averaged MSE and std are computed

## Campus scenario

#### Dual kernel advantages:

- Better convergence speed
- Better MSE error
- Performs close to the batch SVM
- Performs better than Multi-kernel approach

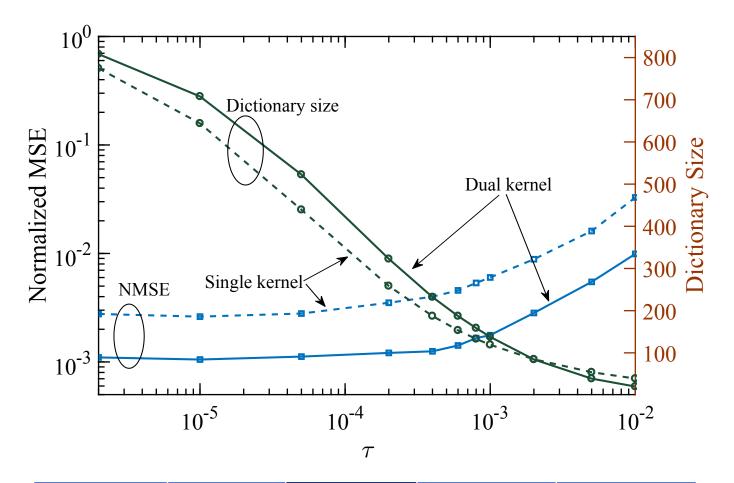


|            | Batch SVM | Online Dual<br>Kernel | Online Single<br>Kernel | Online<br>Multikernel |
|------------|-----------|-----------------------|-------------------------|-----------------------|
| NMSE ×10e3 | 1.7       | 2.7                   | 4.4                     | 6                     |
| std ×103   | 2.5       | 3.9                   | 11.2                    |                       |

## Urban scenario

#### Dual kernel approach:

- Has a smaller dictionary size for a given NMSE error
- Performs better than the single kernel and dual kernel approaches



|      |        | Batch<br>SVM | Online<br>Dual<br>Kernel | Online<br>Single<br>Kernel | Online<br>Multikernel |
|------|--------|--------------|--------------------------|----------------------------|-----------------------|
| NMSE | E 10e3 | .4           | 1.1                      | 2.8                        | 6                     |
| std  | 103    | 3.5          | 5                        | 20                         |                       |

#### Conclusion:

- Power maps are data-driven maps for received power level
- Power maps model site-specific behaviors rather than the statistical ones
- Power maps have a fairly stable DC component, which can be captured using DC kernel
- The online Dual kernel is an online regression tool that uses one kernel for DC component and another kernel for the varying part of power map
- The online Dual kernel outperforms other states of the art methods in terms of:
  - Convergence speed
  - Normalized MSE error



Questions

#### Define the solution set

1. For each measurement, define a solution set

$$S_n = \left\{ f \in \mathcal{H} : |y_n - \langle f, k(x_n, \cdot) \rangle| \le \epsilon \right\}$$

- Every point in Hilbert space is a function
- We assign a function (or a point in Hilbert space) to each user location (concept of feature projection)
- Computing the value of pathloss estimator function in a user location is equivalent to performing an inner product between their representatives in Hilbert space
- 2. Find the intersection or
  - Solution set for all but limited number of measurements
  - Solution set for pathloss estimator function

