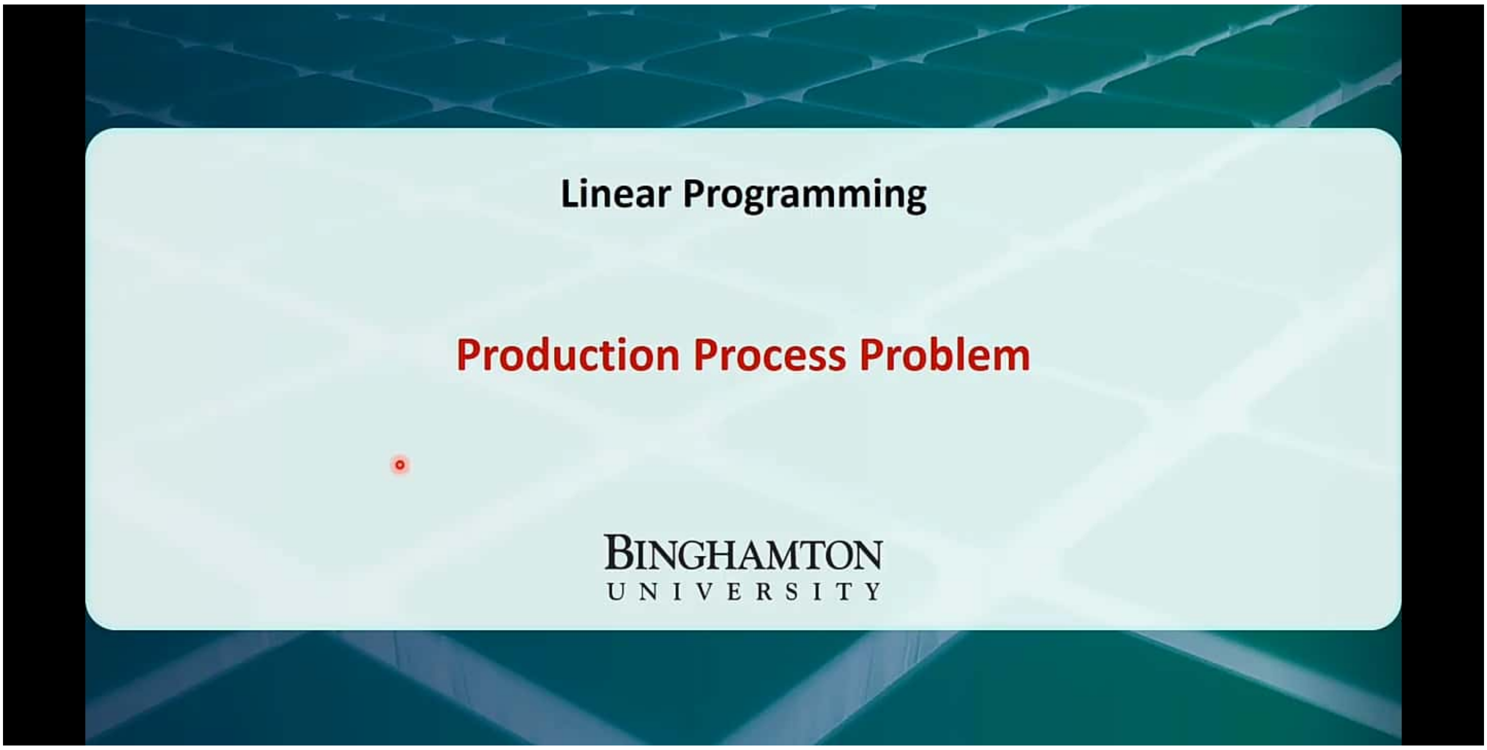
Linear Programming
Production Process Problem
BINGHAMTON
UNIVERSITY

## Production Process Model

- Weekly upper bounds of resources
  - Raw Materials (RM): 2000 units/week
  - Labor: 6000 hr/week
- Four types of products
  - Product 1 (P1), Product 2 (P2)
  - Luxury Product 1 (L1), Luxury Product 2 (L2)
- Prices
  - RM $3/unit
  - P1: $7/unit, P2: $6/unit
  - L1: $17/unit, L2: $16/unit

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \ and$$

$$1\ P_1 \xrightarrow[\ ]{\substack{\$5 \\ 3hr}} 1\ L_1$$

$$1\ P_2 \xrightarrow[\ ]{\substack{\$4 \\ 2hr}} 1\ L_2$$

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## Initial LP Formulation

$$x_{RM} = units\ of\ RM\ purchased\ weekly$$
$$x_{P1} = units\ of\ P_1\ sold\ weekly$$
$$x_{P2} = units\ of\ P_2\ sold\ annually$$
$$x_{L1} = units\ of\ L_1\ sold\ annually$$
$$x_{L2} = units\ of\ L_2\ sold\ annually$$

$$\max z = 7x_{P1} + 6x_{P2} + 17x_{L1} + 16x_{L2}$$
$$-(3x_{RM} + 5x_{L1} + 4x_{L2})$$

s.t. $x_{RM} \leq 2000$

$$x_{RM} + (3x_{L1} + 2x_{L2}) \leq 6000$$
$$x_* \geq 0\ (* = RM, P1, P2, L1, L2)$$

# Additional Constraints

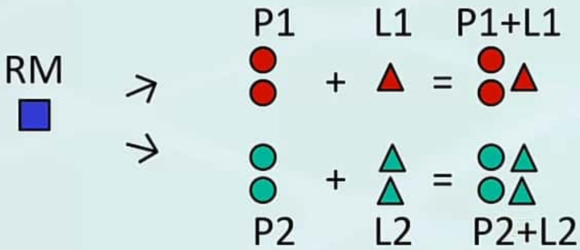

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \quad and$$

$$1\ P_1 \overset{\$5}{\xrightarrow{3hr}} 1\ L_1$$

$$1\ P_2 \overset{\$4}{\xrightarrow{2hr}} 1\ L_2$$

## Additional Constraints

$$x_{P1} + x_{L1} = 3x_{RM}$$




$$x_{P2} + x_{L2} = 4x_{RM}$$

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases}\ and$$

$$1\ P_1 \xrightarrow[\ ]{\$5 \atop 3hr} 1\ L_1$$

$$1\ P_2 \xrightarrow[\ ]{\$4 \atop 2hr} 1\ L_2$$

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## Correct LP Formulation

$$\max z = 7x_{P1} + 6x_{P2} + 17x_{L1} + 16x_{L2}$$
$$-(3x_{RM} + 5x_{L1} + 4x_{L2})$$

s.t.

$$x_{RM} \leq 2000$$
$$x_{RM} + 3x_{L1} + 2x_{L2} \leq 6000$$
$$x_{P1} + x_{L1} - 3x_{RM} = 0$$
$$x_{P2} + x_{L2} - 4x_{RM} = 0$$
$$x_* \geq 0 \ (* = RM, P1, P2, L1, L2)$$

Optimal solution:

$x_{RM} = 2000, \quad x_{P1} = 6000, \quad x_{P2} = 6000$

$x_{L1} = 0, \quad x_{L2} = 2000, \quad z^* = \$96{,}000$

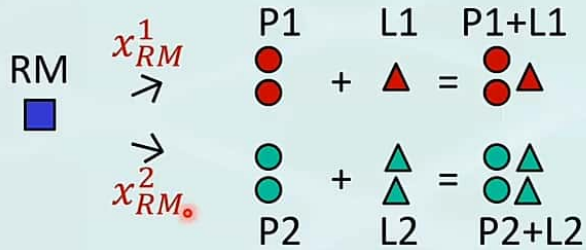
## What If?

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \textbf{or}$$

$$1\ P_1 \xrightarrow[\ ]{\$5 \\ 3hr} 1\ L_1$$

$$1\ P_2 \xrightarrow[\ ]{\$4 \\ 2hr} 1\ L_2$$

## What If?




$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \textbf{or}$$

\$5

$$1\ P_1 \xrightarrow{3hr} 1\ L_1$$

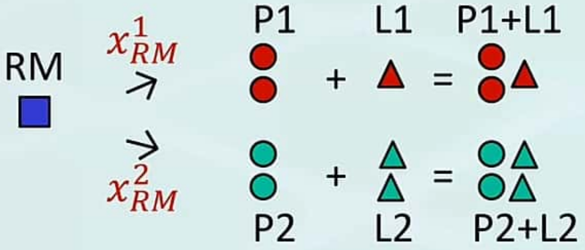
\$4

$$1\ P_2 \xrightarrow{2hr} 1\ L_2$$

## What If?

$$x_{P1} + x_{L1} = 3x_{RM}^{1}$$

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \textbf{or}$$




$$1\ P_1 \xrightarrow[3hr]{\$5} 1\ L_1$$

$$1\ P_2 \xrightarrow[2hr]{\$4} 1\ L_2$$

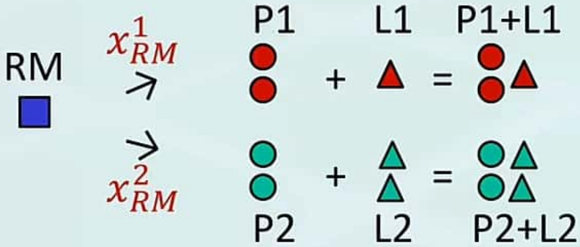
$$x_{P2} + x_{L2} = 4x_{RM}^{2}$$

$$x_{RM}^{1} + x_{RM}^{2} = x_{RM}$$

## What If?

$$x_{P1} + x_{L1} = 3x_{RM}^1$$

$$1\ RM \xrightarrow{1hr} \begin{cases} 3\ P_1 \\ 4\ P_2 \end{cases} \textbf{or}$$




$$1\ P_1 \xrightarrow[3hr]{\$5} 1\ L_1$$

$$1\ P_2 \xrightarrow[2hr]{\$4} 1\ L_2$$

$$x_{P2} + x_{L2} = 4x_{RM}^2$$

$$x_{RM}^1 + x_{RM}^2 = x_{RM} \quad \Rightarrow \quad \frac{x_{P1} + x_{L1}}{3} + \frac{x_{P2} + x_{L2}}{4} = x_{RM}$$

# Correct LP Formulation

$$\max z = 7x_{P1} + 6x_{P2} + 17x_{L1} + 16x_{L2}$$
$$-(3x_{RM} + 5x_{L1} + 4x_{L2})$$

s.t.

$$x_{RM} \leq 2000$$
$$x_{RM} + 3x_{L1} + 2x_{L2} \leq 6000$$
$$\frac{x_{P1} + x_{L1}}{3} + \frac{x_{P2} + x_{L2}}{4} = x_{RM}$$
$$x_* \geq 0 \ (* = RM, P1, P2, L1, L2)$$

Optimal solution:

$x_{RM} = 2000,\quad x_{P1} = 0,\quad x_{P2} = 6000$

$x_{L1} = 0,\quad x_{L2} = 2000,\quad z^* = \$54{,}000$