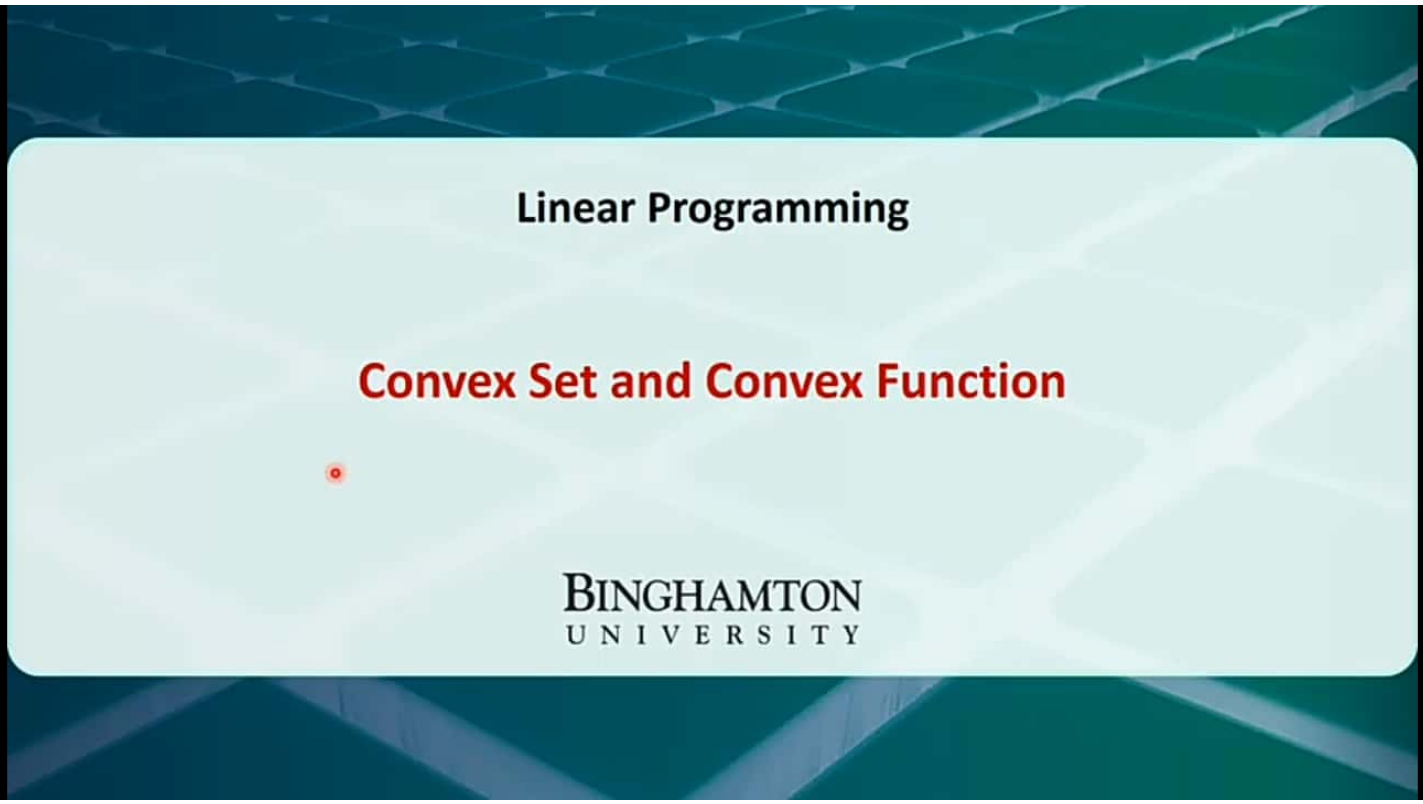
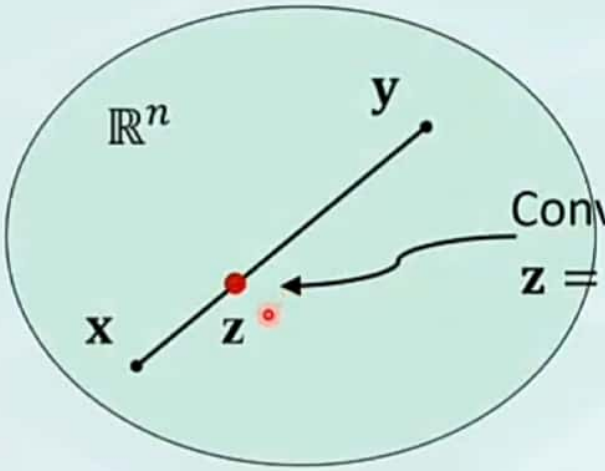


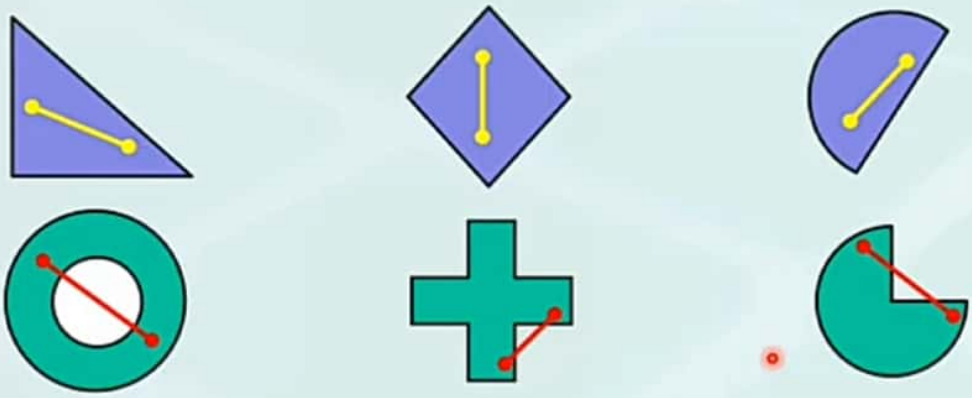
Convex Combination

- Given two points $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, a convex combination of them is any point of the form $\mathbf{z} = \lambda\mathbf{x} + (1-\lambda)\mathbf{y}, \lambda \in [0,1]$



Convex Set

- A set of $\mathbf{S} \subseteq \mathbb{R}^n$ is a convex set if it contains all convex combinations of any two points within it
- Graphically: A set of points **S** is a convex set if the line segment joining any two points in **S** is wholly contained in **S**



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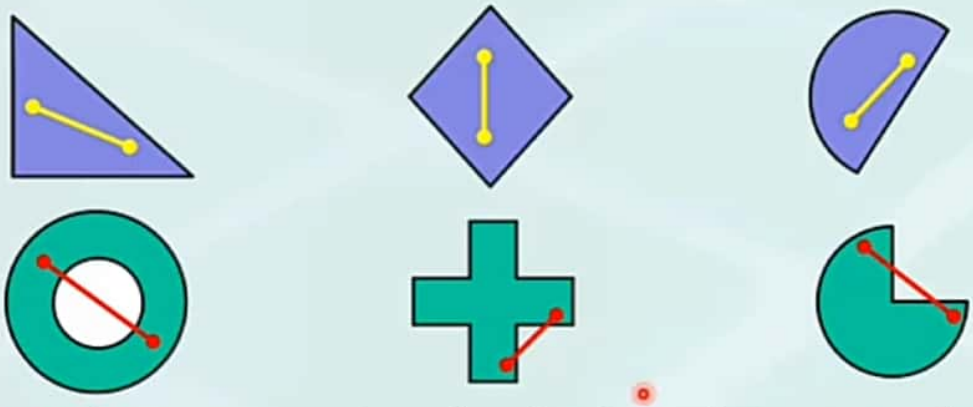


- Special case: An empty set and a single-point set are both convex

Is the Intersection of Two Convex Sets Convex?

- If the intersection is empty, or consists of a single point, it is true by definition
- Otherwise, take any two points A, B in the intersection. The line AB joining these points must also lie wholly within their intersection
- Therefore, the intersection is a convex set

Is the Intersection of Two Convex Sets Convex?

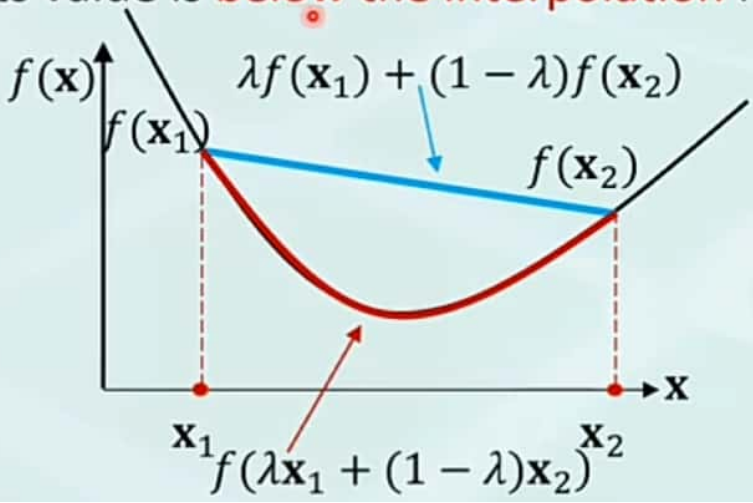
- If the intersection is empty, or consists of a single point, it is true by definition
- Otherwise, take any two points A, B in the intersection. The line AB joining these points must also lie wholly within their intersection
- Therefore, the intersection is a convex set
- The intersection of any number of convex sets is also convex

Convex Functions

- Let **S** be a convex set. The function $f(\mathbf{x})$: **S**→$\mathbb{R}$ is a convex function if for any two points $\mathbf{x}_1$, $\mathbf{x}_2$ in **S**

$$f(\lambda\mathbf{x_1} + (1-\lambda)\mathbf{x_2}) \leq \lambda f(\mathbf{x_1}) + (1-\lambda) f(\mathbf{x_2}),\ \lambda \in [0,1]$$

- f(x) is convex if its value is below the interpolation formed between any two points



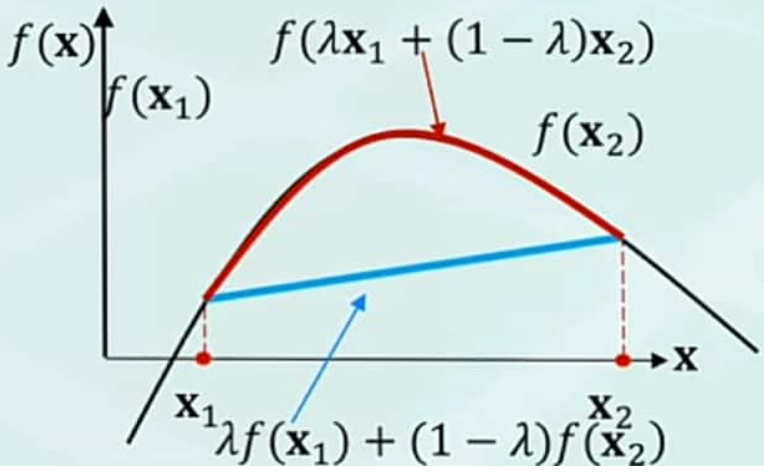
Is the Union of Two Convex Sets Convex?

Concave Functions

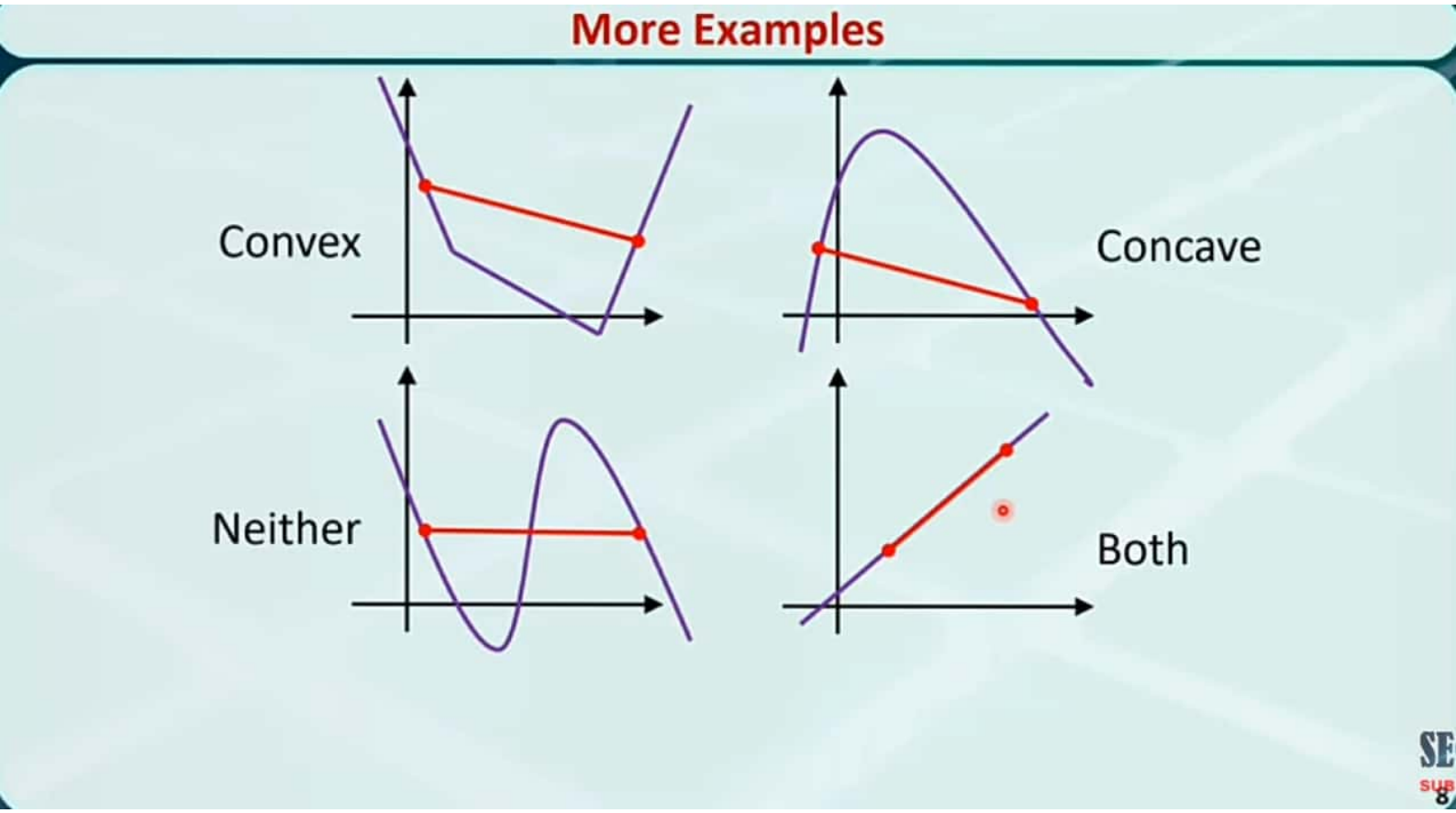
- Let **S** be a convex set. The function $f(\mathbf{x})$: **S**$\rightarrow\mathbb{R}$ is a concave function if for any two points $\mathbf{x}_1$, $\mathbf{x}_2$ in **S**

$$f(\lambda\mathbf{x_1} + (1-\lambda)\mathbf{x_2}) \geq \lambda f(\mathbf{x_1}) + (1-\lambda) f(\mathbf{x_2}),\ \lambda \in [0,1]$$

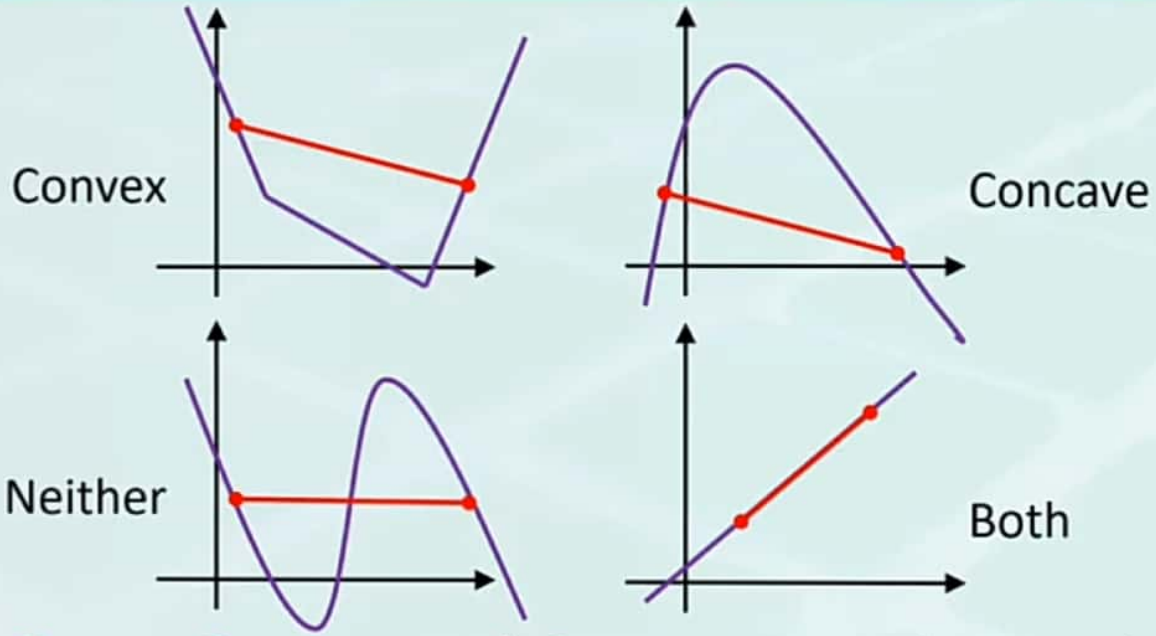
- f(x) is concave if its value is above the interpolation formed between any two points



More Examples

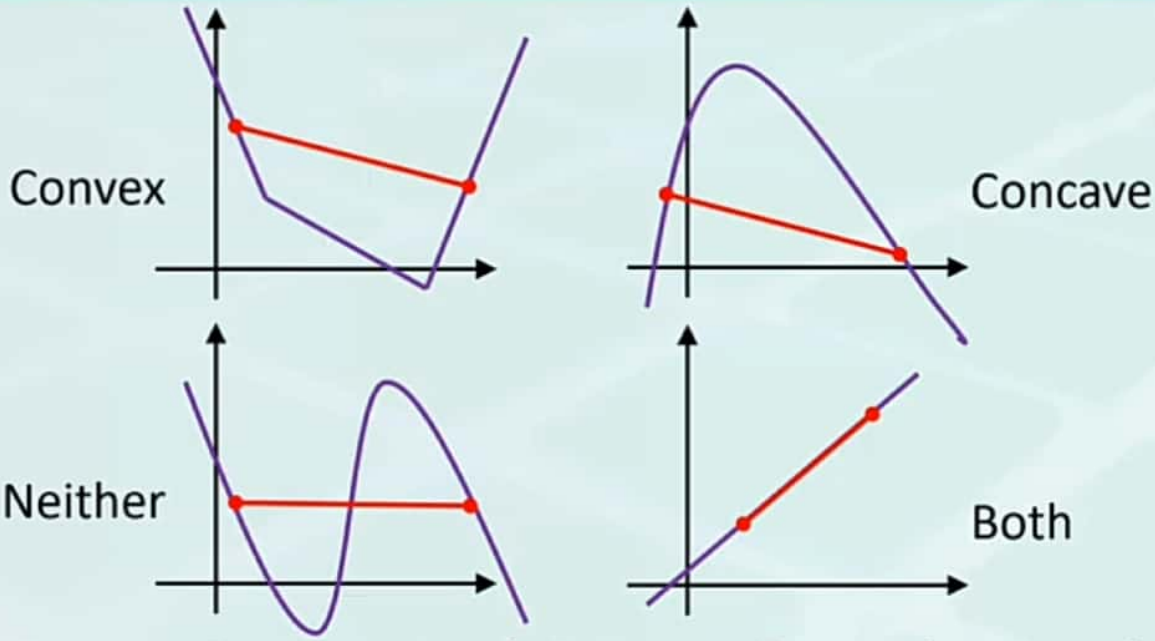


More Examples



Convex: Opens upward; Concave: Opens downward.

More Examples



Convex: Opens upward; Concave: Opens downward.

If $f(x)$ is convex in **S**, then $-f(x)$ is concave in **S**.