Linear Programming

Graphically Solving LP Problems

BINGHAMTON
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Optimal Solution

- For a max (or min) problem, an optimal solution to an LP is a point in the feasible region with the largest (or smallest) objective function value

Graphical Method for Solving LP

Step 1. Find the feasible region

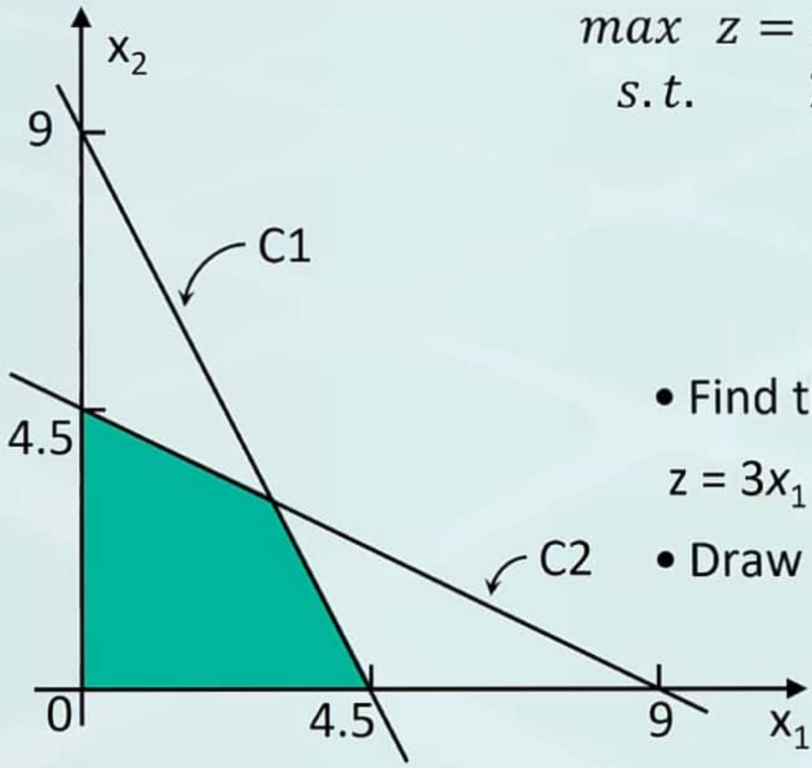
Step 2. Find the slope *s* of the objective function

Step 3. Draw a line with slope *s* that intersects with the feasible region

Step 4. Move the line in parallel towards the direction that increases (for max problems) or decreases (for min problems) the objective function value

Step 5. The last intersection point(s) before leaving the feasible region will be the optimal solution(s)

Feasible Region and Objective Function Slope



$$\max \quad z = 3x_1 + 2x_2$$
$$s.t. \quad 2x_1 + x_2 \le 9 \text{ (C1)}$$
$$x_1 + 2x_2 \le 9 \text{ (C2)}$$
$$x_i \ge 0 \ (i = 1,2)$$

- Find the slope

$z = 3x_1 + 2x_2 \rightarrow x_2 = z/2 - 3x_1/2 \rightarrow s = -3/2$

- Draw a line with s

Feasible Region and Objective Function Slope

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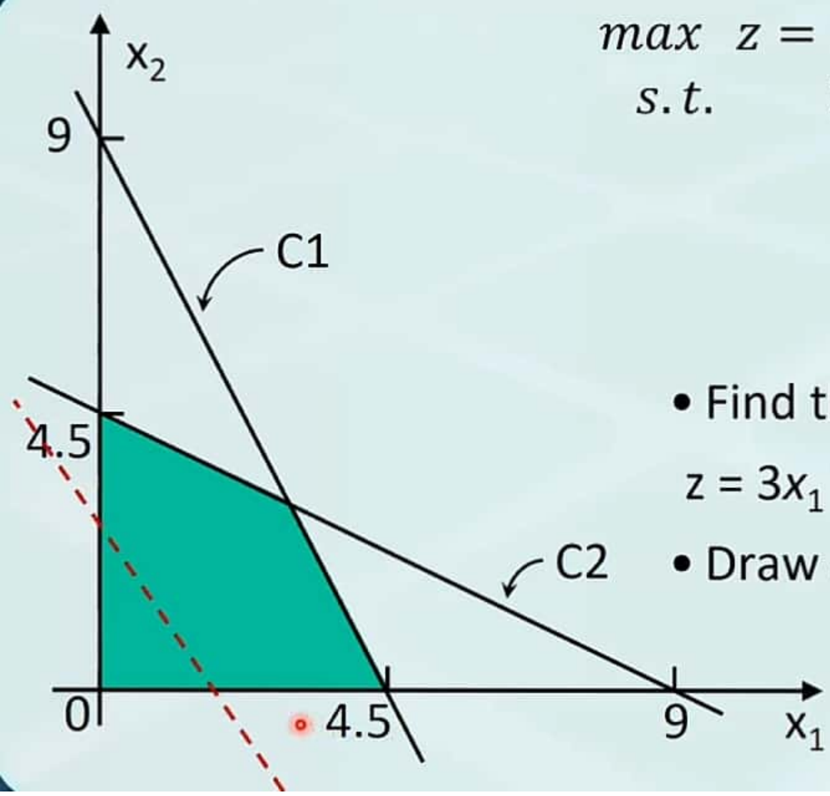

- Find the slope

$z = 3x_1 + 2x_2 \rightarrow x_2 = z/2 - 3x_1/2 \rightarrow s = -3/2$

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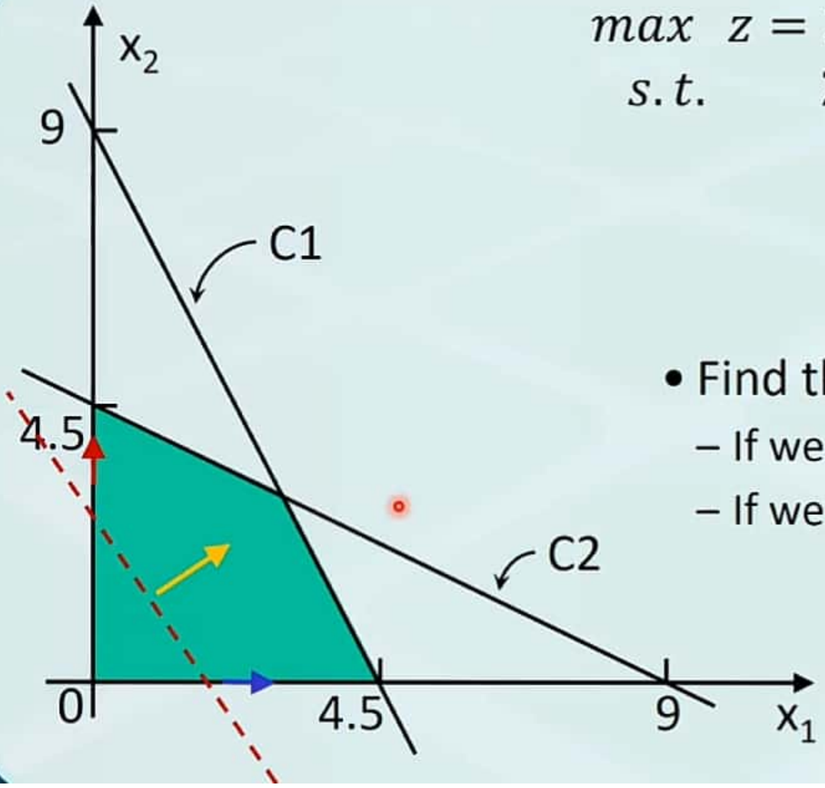
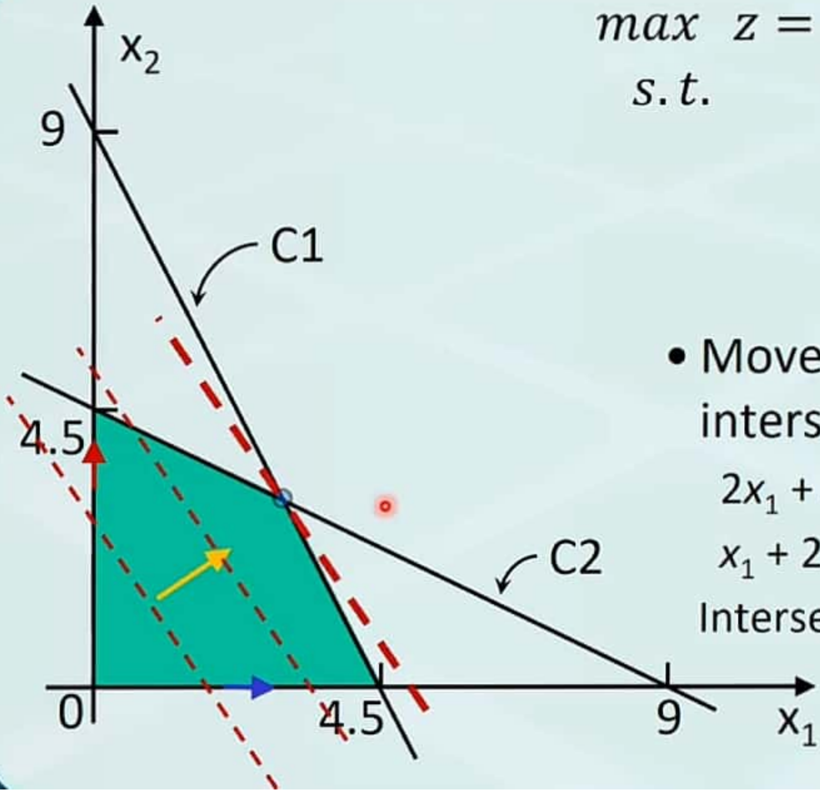
The Moving Direction



$$\begin{aligned} \max\ z &= 3x_1 + 2x_2 \\ s.t.\quad & 2x_1 + x_2 \le 9 \text{ (C1)} \\ & x_1 + 2x_2 \le 9 \text{ (C2)} \\ & x_i \ge 0\ (i = 1,2) \end{aligned}$$

- Find the direction
  - If we increase x1, what happens to z?
  - If we increase x2, what happens to z?

Unique Optimal Solution



$$\max \ z = 3x_1 + 2x_2$$

$$s.t. \quad 2x_1 + x_2 \le 9 \text{ (C1)}$$

$$x_1 + 2x_2 \le 9 \text{ (C2)}$$

$$x_i \ge 0 \ (i = 1,2)$$

- Move the line and find the last intersection point(s)

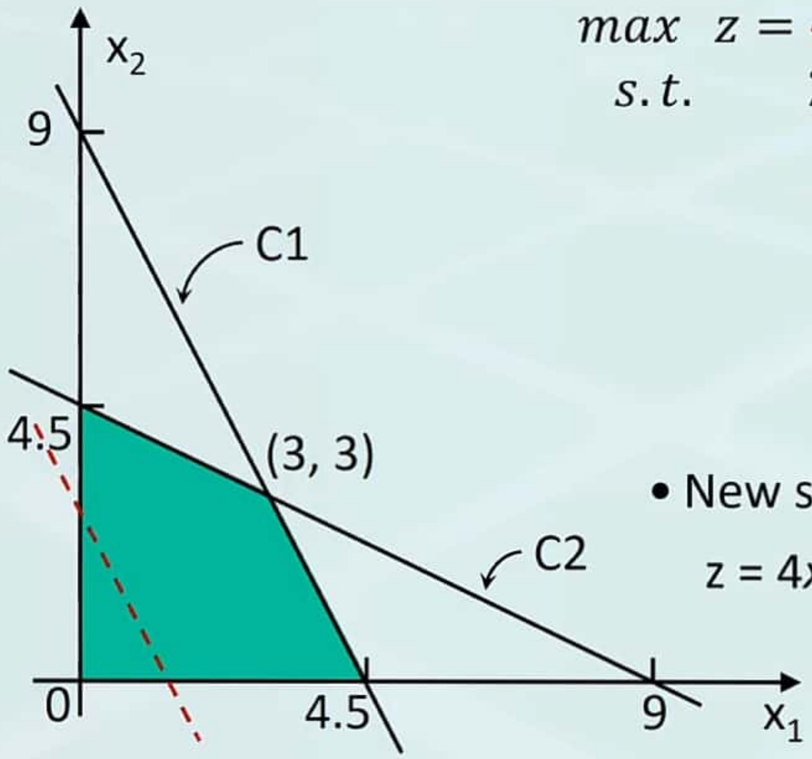
$2x_1 + x_2 = 9$

$x_1 + 2x_2 = 9$

Intersection (3, 3) → z*=15

Infinite Number of Optimal Solutions

$$\max \; z = 4x_1 + 2x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \le 9 \; (C1)$$
$$x_1 + 2x_2 \le 9 \; (C2)$$
$$x_i \ge 0 \; (i = 1,2)$$

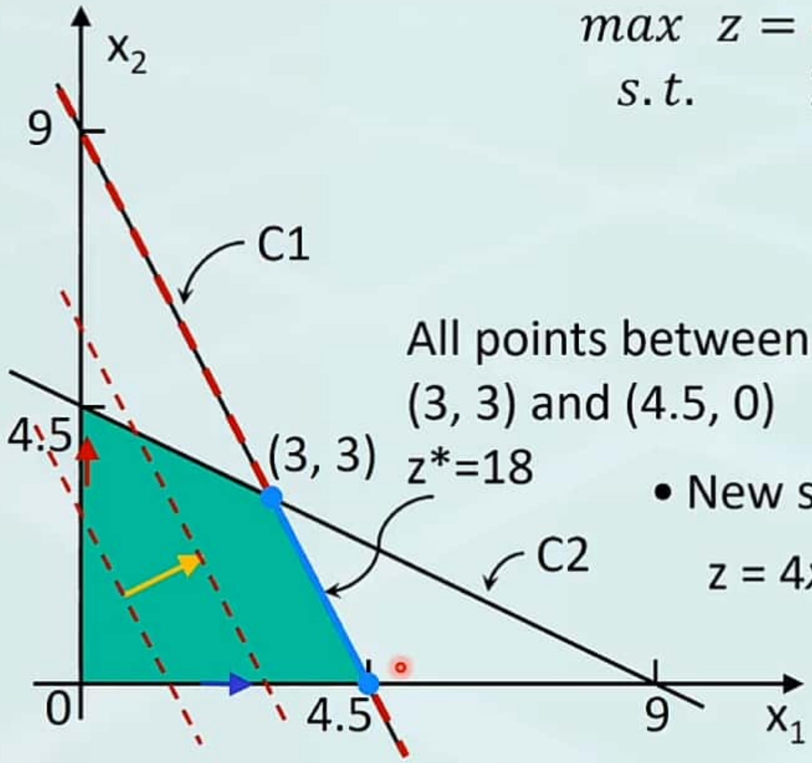


- New slope

$z = 4x_1 + 2x_2 \rightarrow x_2 = z/2 - 2x_1 \rightarrow s = -2$

Infinite Number of Optimal Solutions

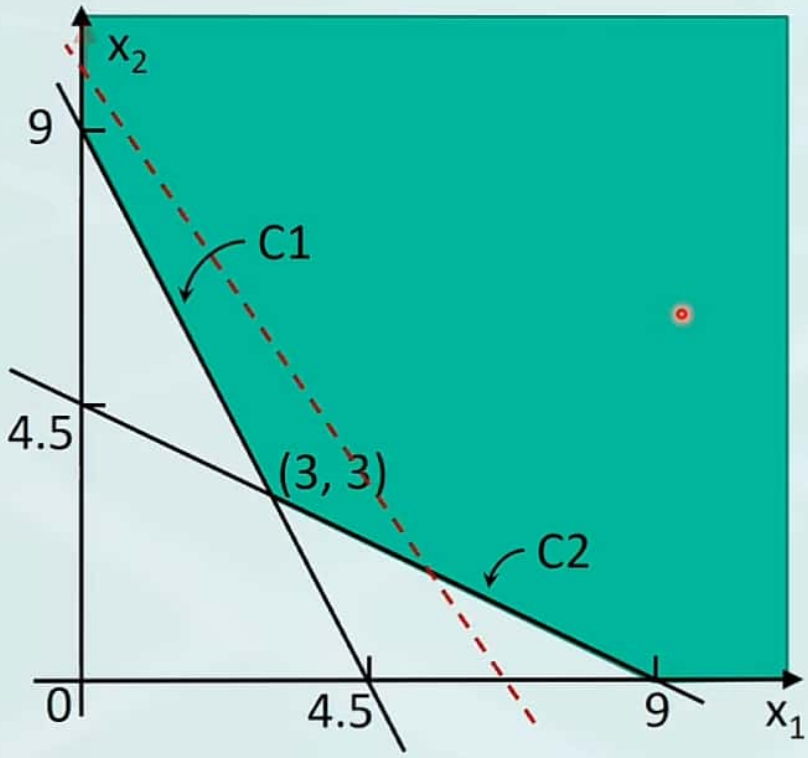
$$\begin{aligned} max\quad z &= \mathbf{4}x_1 + 2x_2 \\ s.t.\quad 2x_1 &+ x_2 \le 9\ \text{(C1)} \\ x_1 &+ 2x_2 \le 9\ \text{(C2)} \\ x_i &\ge 0\ (i = 1,2) \end{aligned}$$



- New slope

$z = 4x_1 + 2x_2 \rightarrow x_2 = z/2 - 2x_1 \rightarrow s = -2$

Unbounded



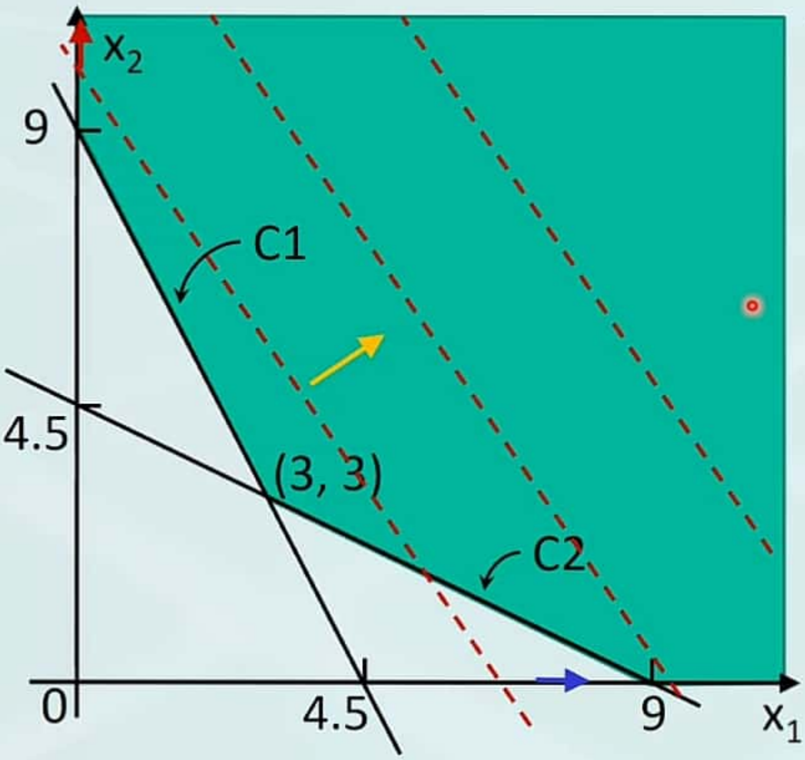
$$\max \quad z = 3x_1 + 2x_2$$

$$s.t. \quad 2x_1 + x_2 \geq 9 \text{ (C1)}$$

$$x_1 + 2x_2 \geq 9 \text{ (C2)}$$

$$x_i \geq 0 \ (i = 1,2)$$

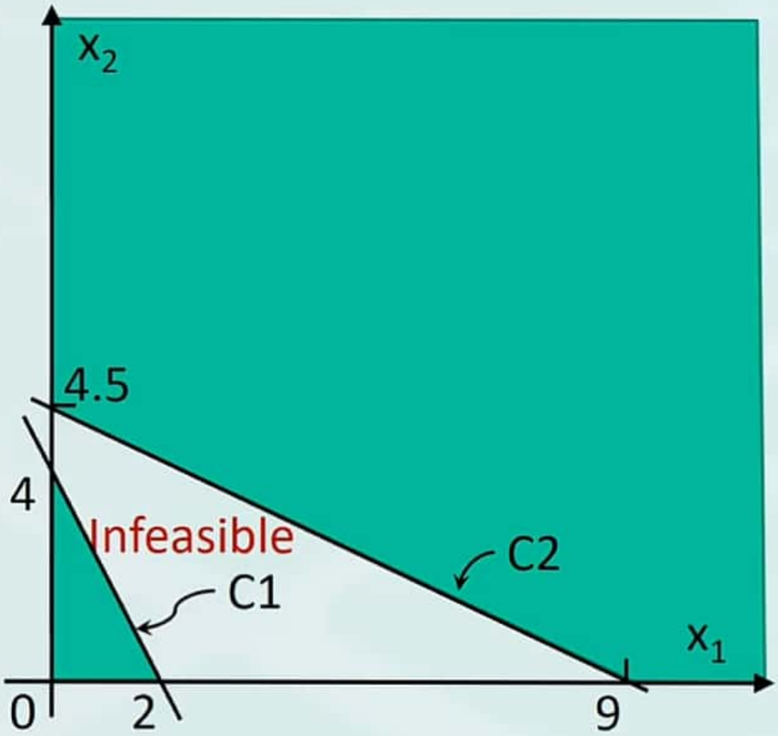
Unbounded



$$\max \quad z = 3x_1 + 2x_2$$

$$\begin{aligned} s.t. \quad & 2x_1 + x_2 \geq 9 \text{ (C1)} \\ & x_1 + 2x_2 \geq 9 \text{ (C2)} \\ & x_i \geq 0 \ (i = 1,2) \end{aligned}$$

Infeasible - No Optimal Solution



$$\max \quad z = 3x_1 + 2x_2$$
$$s.t. \quad 2x_1 + x_2 \le 4 \text{ (C1)}$$
$$x_1 + 2x_2 \ge 9 \text{ (C2)}$$
$$x_i \ge 0 \ (i = 1,2)$$