Linear Programming

Typical Linear Programming Problems

BINGHAMTON
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Linear Programming (LP)

- A linear programming (LP) problem is an optimization problem for which
  - The objective function must be linear (of the decision variables)
  - Every constraint must be a linear equation or inequality
  - A sign restriction is associated with each variable. For any variable x_i, $x_i \geq 0$ or x_i is unrestricted in sign (URS)

Example

- A company manufactures two types of products (P1 and P2)

Price and Nutrients	Unit Contribution		Availability
	P1	P2	
Raw Material Cost ($1000)	2	2	
Labor Cost ($1000)	5	4	
Machining Time (hr)	2	1	9 hr/day
Packaging Time (hr)	1	2	9 hr/day
Selling Price ($1000)	10	8	

Problem Formulation

- Decision variables

  x_1 = units of P1 produced each day

  x_2 = units of P2 produced each day

  Assume we are allowed to make fractional numbers of P1 and P2

- Objective Function
  - Maximizing daily profit (Revenue-cost)

$$z = (10x_1+8x_2) - [(2+5)x_1 + (2+4)x_2] = 3x_1+2x_2 \quad \text{(in \$1000)}$$

- Constraints
  - No more than 9 hours of machining time: $2x_1 + x_2 \leq 9$
  - No more than 9 hours of packaging hours: $x_1 + 2x_2 \leq 9$
  - Sign restrictions: $x_1, x_2 \geq 0$

Problem Formulation

$$\max \; z = 3x_1 + 2x_2 \quad \text{(in \$1000)}$$
$$\text{s.t.} \quad 2x_1 + x_2 \le 9$$
$$x_1 + 2x_2 \le 9$$
$$x_1, x_2 \ge 0$$

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SUB

Converting a Minimization Problem to a Maximization Problem

$$\min \ z = 8x_1 - 3x_2$$

$$\text{s.t.} \quad x_1 + 4x_2 \le 12$$

$$3x_1 + x_2 \le 8$$

$$x_1, x_2 \ge 0$$

$$\Downarrow$$

$$\max \ y = -z = -8x_1 + 3x_2$$

$$\text{s.t.} \quad x_1 + 4x_2 \le 12$$

$$3x_1 + x_2 \le 8$$

$$x_1, x_2 \ge 0$$