

## Problem 1)

$$a) \text{ identity: } 1 \times \eta \quad \left\{ \begin{array}{l} i \times -1 = -i \\ (i)^2 = -1 \end{array} \right. \quad \left\{ \begin{array}{l} i \times -i = 1 \end{array} \right.$$

The table for  $G$ :

	1	i	-1	-i
1	1	i	-1	-i
i	i	-1	-i	1
-1	-1	-i	1	i
-i	-i	1	i	-1

b) we will justify 5 axioms:

closure: for all  $a, b \in G$ ,  $a \times b$  must be in  $G$  under multiplication.

Proof: every entry inside the grid is either 1, i, or -i, and since no new elements are there, the set is closed under multiplication.

Associative:  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ 

Proof: multiplication of complex numbers is known to be associative, so associative holds.



1 identity :  $\exists e \in G$  st  $ax$  and  $ex = a$  and  
 2  $a \times e = a$ , for  $\forall a$ .

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4 for all row and column for 1;

5

$$6 \quad 1 \times 1 = 1$$

$$7 \quad 1 \times i = i$$

$$8 \quad 1 \times -1 = -1$$

$$9 \quad 1 \times -i = -i$$

— since multiplying by 1, leaves  
 everything the same, 1 is the  
 identity element,

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11 Inverse :  $\forall a \in G, \exists b \in G$  st  $a \cdot b = 1$

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13 The inverse of 1 is  $1 \cdot 1 = 1$

$$14 \quad i \quad i \cdot i = 1$$

$$15 \quad -1 \quad -1 \cdot -1 = 1$$

$$16 \quad -i \quad -i \cdot i = 1$$

17 every element has a corresponding inverse in the  
 18 set, so it is satisfied.

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20 So commutativity:  $a \times b = b \times a$  for  $a, b \in G$

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22 table is symmetrical across the main diagonal  
 23 so the operation is commutative

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