

Problem 1)

a) identity : $i \times i$ ($i \times -1 = -i$
 $(i)^2 = -1$ ($i \times i = 1$

The table for G :

1	1	-1	-i
1	1	i	-1
i	i	1	-i
-1	-1	-i	i
-i	-i	1	i

b) we will justify 5 axioms :

closure: for all $a, b \in G$, $a \times b$ must be in G

Proof: every entry inside the grid is either 1, i, -1 or -i, and since no new elements are there, the set is closed under multiplication.

Associative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Proof: multiplication of complex numbers is known to be associative, so associative holds.

1 identity: $\exists e \in G$ st or and ex $a = a$ and
2 $a \times c = a$, for $\forall a$.

3

4 for all row and column for 1;

5

6 $1 \times 1 = 1$

7 $1 \times i = i$

8 $1 \times -1 = -1$

9 $1 \times -i = -i$

since multiplying by 1, leaves
everything the same, 1 is the
identity element.

10

11 Inverse: for $\forall a \in G, \exists b \in G$ st, $a \cdot b = 1$

12

13 The inverse of 1 is $1 \cdot 1 = 1$

14 $i \cdot i = 1$

15 $-1 \cdot -1 = 1$

16 $-i \cdot i = 1$

17 every element has a corresponding inverse in the
18 set, so it is satisfied.

19

20 So commutativity: $a \times b = b \times a$ for $a, b \in G$

21

22 Table is symmetrical across the main diagonal
so the operation is commutative

23

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