

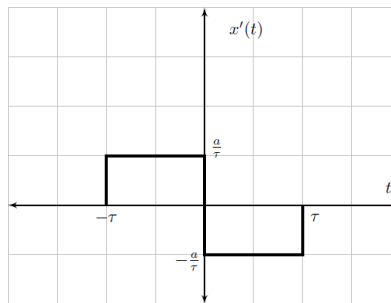
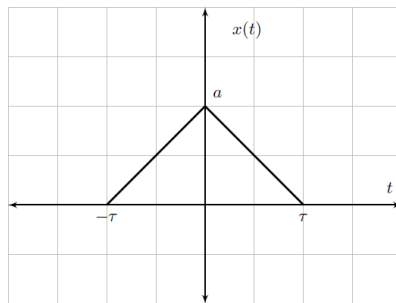
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## Práctica #7. Transformada de Fourier.

### Respuestas

- 1) a)  $\frac{2j}{\omega} [\text{sa}(\omega) + 4 \text{sa}(4\omega) - 3 \text{sa}(3\omega) - 2 \cos(2\omega)]$
- b)  $\frac{2j}{\omega} [\text{sa}(\omega) - 2 \text{sa}(2\omega) + \cos(4\omega)]$
- c)  $2 \text{sa}(2\omega) \left( \frac{\omega-j}{\omega} \right) + \frac{j}{\omega} e^{-j2\omega}$
- d)  $\frac{4 \cos(3\omega)}{\omega^2} \left[ \sin^2 \left( \frac{\omega}{2} \right) + \omega \sin(\omega) \right] + \frac{2j}{\omega^2} \cos(3\omega) \left[ \omega \cos(\omega) - 2 \cos \left( \frac{\omega}{2} \right) \sin \left( \frac{\omega}{2} \right) \right]$
- e)  $3 \text{sa}(3\omega) - 3 \text{sa}(\omega) + \frac{2 \cos(2\omega)}{\omega^2} - \frac{2 \cos(3\omega)}{\omega^2}$
- f)  $\frac{2}{\omega^2} [\cos(3\omega) - \cos(2\omega)] + 16 \text{sa}(4\omega) - 2 \text{sa}(2\omega)$
- g)  $\frac{4}{\omega} \cos(2\omega) \sin^2 \left( \frac{\omega}{2} \right) + 2 \text{sa}(\omega) [\cos(2\omega) - 1] + j \frac{2 \cos(2\omega)}{\omega} \left[ \cos(\omega) - \cos \left( \frac{\omega}{2} \right) \text{sa} \left( \frac{\omega}{2} \right) \right]$
- h)  $\frac{2j}{\omega} [1 - 2 \text{sa}(2\omega)]$

2)  $\frac{2a}{a^2 + \omega^2}$



3)

$$X'(j\omega) = -\frac{j2a}{\omega\tau} [\cos(\omega\tau) - 1]$$

$$X(j\omega) = -\frac{2a}{\omega^2\tau} [\cos(\omega\tau) - 1]$$

4)  $a|T|X(j\omega T)e^{-j\omega t_0}$

5)  $X_D(j\omega) = 2 \cos(\omega t_0) X(j\omega), \quad X_D(j\omega) = 2j \sin(\omega t_0) X(j\omega)$

$$X_D(j\omega) = \text{sa} \left( \frac{\omega}{2} \right), \quad X_D(j\omega) = -j \frac{2}{\omega} \cos \left( \frac{\omega}{2} \right)$$

6)  $-j\frac{T}{\omega} \text{sa}\left(\frac{\omega T}{2}\right) + T\pi\delta(\omega)$

7)  $\frac{T+j\omega T^2}{1+2j\omega T-\omega^2 T^2+\omega_0^2 T^2}$

8)  $x(t-t_0)$

9)  $|T| \text{sa}\left(\frac{\pi t}{T}\right)$

10)  $X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$

11)  $X(j\omega) = \sum_{n=-K}^K e^{j\omega n T}$

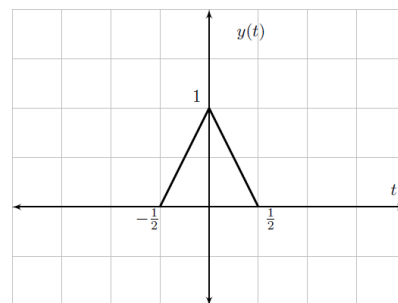
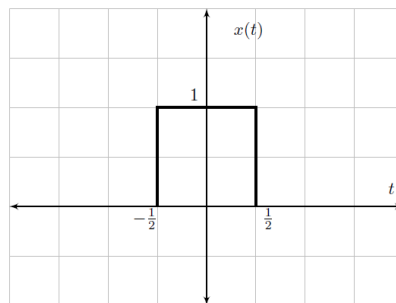
12) Ambas relaciones son correctas.

13)  $\sin\left(\frac{\omega\pi}{a}\right) \left[\frac{2\omega^2+2\omega a-2a^2}{\omega^3-\omega a^2}\right]$

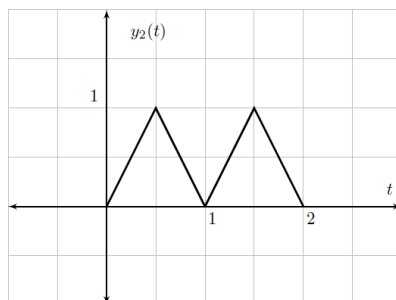
14)  $\frac{1}{4}X(j\omega-j2\omega_0) + X(j\omega_0) + \frac{1}{4}X(j\omega+j2\omega_0)$

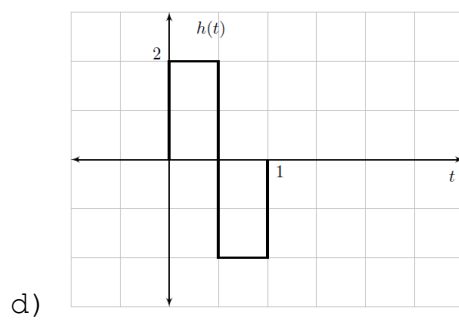
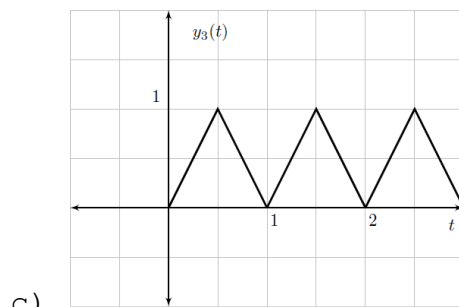
15)  $f_{s_{min}} = 20 \text{ kHz}$

16) a)



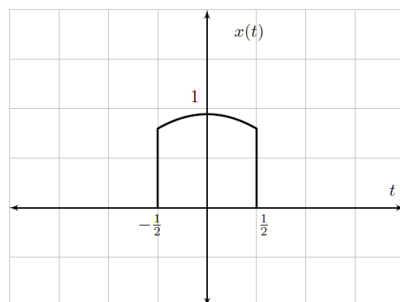
b)



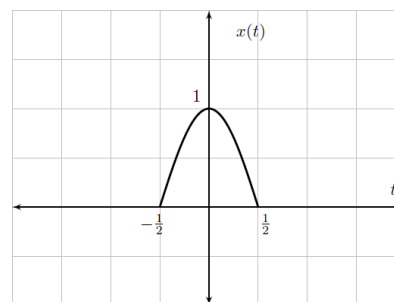


17)

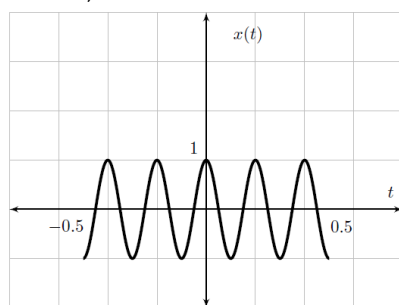
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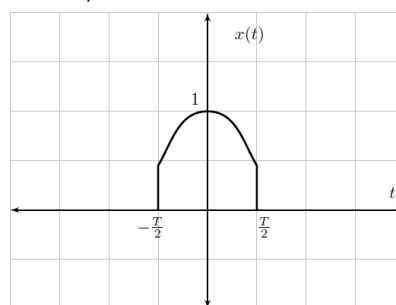
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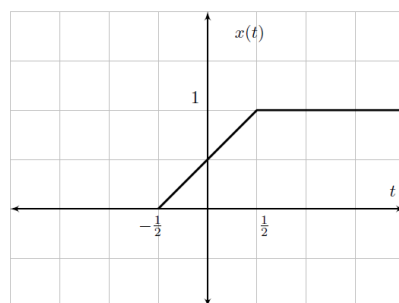
c)



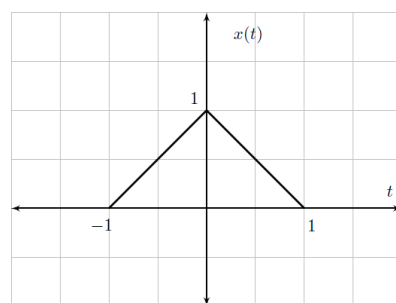
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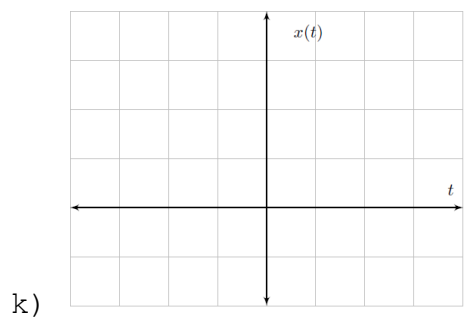
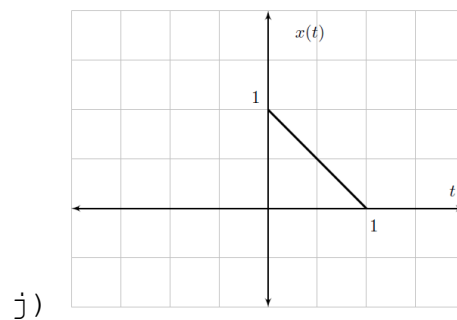
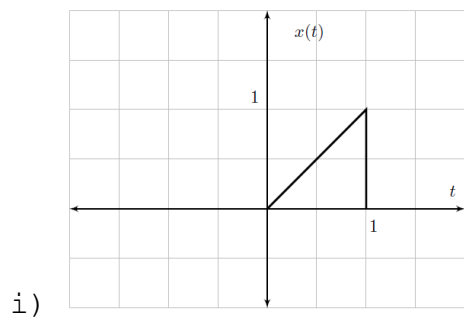
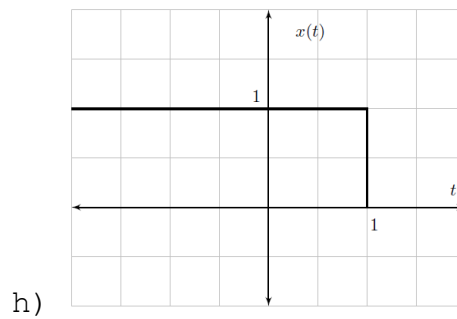
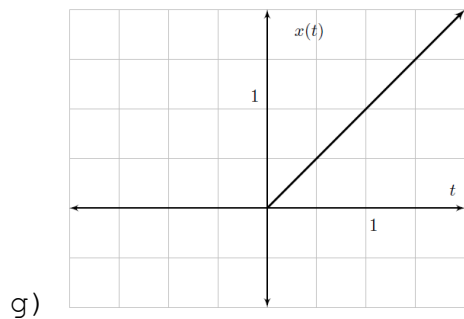


e)



f)



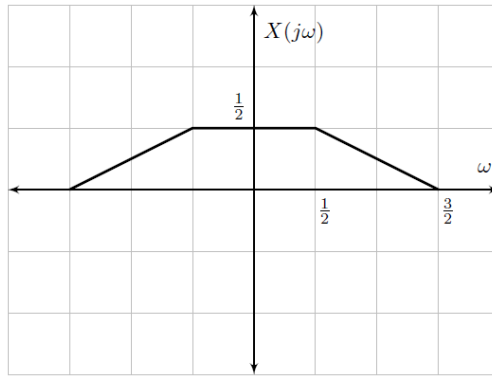


18) Si es correcto.

19)  $y(t) = te^{-at}u(t)$

20) 
$$y(t) = \begin{cases} \frac{\sin(\omega_c t)}{\pi t} & \text{si } \omega_c \leq \omega_i \\ \frac{\sin(\omega_i t)}{\pi t} & \text{si } \omega_i \leq \omega_c \end{cases}$$

21) 
$$R(j\omega) = \frac{1}{2}S(j(\omega - \omega_0)) + \frac{1}{2}S(j(\omega + \omega_0))$$



22)

23) a)  $\frac{e^{-j\omega}}{2+j\omega}$

b)  $\frac{4e^{-j\omega}}{4+\omega^2}$

24) a)  $2 \cos(\omega)$

b)  $-2j \sin(2\omega)$

25) a)  $\frac{\pi}{j} \left[ e^{j\frac{\pi}{4}} \delta(\omega - 2\pi) - e^{-j\frac{\pi}{4}} \delta(\omega + 2\pi) \right]$

b)  $2\pi\delta(\omega) + \pi \left[ e^{j\frac{\pi}{8}} \delta(\omega - 6\pi) + e^{-j\frac{\pi}{8}} \delta(\omega + 6\pi) \right]$

26) a)  $1 + \cos(4\pi t)$

b)  $-\frac{4j \sin^2(t)}{\pi t}$

27)  $x(t) = -\frac{2 \sin\left(3\left(t-\frac{3}{2}\right)\right)}{\pi\left(t-\frac{3}{2}\right)}, \quad t = \frac{k\pi}{3} + \frac{3}{2}$  para  $k$  enteros distintos de 0

28) a)  $X_1(j\omega) = 2X(-j\omega) \cos(\omega)$

b)  $X_2(j\omega) = \frac{1}{3} e^{-j2\omega} X\left(j\frac{\omega}{3}\right)$

c)  $X_3(j\omega) = -\omega^2 e^{j\omega} X(j\omega)$

29) a) ninguno, ninguno

b) imaginario, impar

c) imaginario, ninguno

d) real, par

30) a)  $\frac{2 \sin\left(\frac{\omega}{2}\right)}{j\omega^2} + \pi\delta(\omega)$

b)  $\frac{2 \sin(\frac{\omega}{2})}{j\omega^2}$

31) a)  $\frac{\sin(\omega)}{j\omega^2} - \frac{e^{-j\omega}}{j\omega}$

b)  $\frac{\sin(\omega)}{\omega}$

c)  $\frac{\sin(\omega)}{j\omega^2} - \frac{\cos(\omega)}{j\omega}$

32) a)  $X(j\omega) = \begin{cases} \frac{j}{2\pi} & (-2 \leq \omega < 0) \\ -\frac{j}{2\pi} & (0 \leq \omega < 2) \\ 0 & \text{con otro valor} \end{cases}$

b)  $A = \frac{1}{2\pi^3}$

33)  $A = \frac{1}{3}, \quad B = 3$

34) a)  $-\frac{4j\omega}{(1+\omega^2)^2}$

b)  $j2\pi\omega e^{-|\omega|}$

35) a) No

b) Sí

c) Sí

36)  $x(t) = \sqrt{12}[e^{-t} - e^{-2t}]u(t)$

37)  $x(t) = 2te^{-|t|}u(t)$

38) a)  $g(t) = \pi \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{k\pi}{4}\right)$

b)  $X(j\omega) = \begin{cases} 4 & |\omega| \leq 1 \\ 0 & 1 < |\omega| \leq 4 \end{cases}$

39) a) Falso

b) Verdadero

40)  $h(t) = \begin{cases} \frac{5}{4} & |t| < 1 \\ -\frac{|t|}{4} + \frac{3}{2} & 1 \leq |t| \leq 5 \\ -\frac{|t|}{8} + \frac{7}{8} & 5 < |t| < 7 \\ 0 & \text{con otro valor} \end{cases}$

41)  $h(t) = e^{-4t}u(t)$

42) a)  $\frac{1}{2} \left( \frac{1}{a-j\omega_0+j\omega} \right) + \frac{1}{2} \left( \frac{1}{a+j\omega_0+j\omega} \right)$

b)  $\frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2}$

c)  $\frac{2\sin(\omega)}{\omega} + \frac{2\omega\sin(\omega)}{\pi^2-\omega^2}$

d)  $\frac{1}{1-ae^{j\omega T}}$

e)  $\frac{\frac{1}{2j}}{(2-j4+j\omega)^2} - \frac{\frac{1}{2j}}{(2+j4+j\omega)^2}$

f) 
$$\begin{cases} e^{-j\omega} & |\omega| < \pi \\ \frac{1}{2\pi}(3\pi+\omega)e^{-j\omega} & -3\pi < \omega < -\pi \\ \frac{1}{2\pi}(3\pi+\omega)e^{-j\omega} & \pi < \omega < 3\pi \\ 0 & \text{en otro valor} \end{cases}$$

g)  $\frac{2j}{\omega} \left[ \cos(2\omega) - \frac{\sin(\omega)}{\omega} \right]$

h)  $\pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi) [2 + (-1)^k]$

i)  $\frac{1}{j\omega} + \frac{2e^{-j\omega}}{-\omega^2} - \frac{2e^{-j\omega}-2}{j\omega^3}$

43) a)  $x(t) = \begin{cases} e^{j2\pi t} & |t| < 3 \\ 0 & \text{en otro valor} \end{cases}$

b)  $x(t) = \frac{1}{2}e^{-j\frac{\pi}{3}}\delta(t-4) + \frac{1}{2}e^{j\frac{\pi}{3}}\delta(t+4)$

c)  $x(t) = \frac{1}{\pi} \left[ \frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2} \right]$

d)  $x(t) = \frac{2j}{\pi}\sin(t) + \frac{3}{\pi}\cos(2\pi t)$

e)  $x(t) = \frac{\cos(3t)}{j\pi t} + \frac{\sin(t)-\sin(2t)}{j\pi t^2}$

44) a)  $\frac{2-2e^{-1}\cos(\omega)-2\omega e^{-1}\sin(\omega)}{1+\omega^2}$

b)  $j \left[ \frac{-2\omega+2e^{-1}\sin(\omega)+2\omega e^{-1}\cos(\omega)}{1+\omega^2} \right]$

c)  $\frac{1+e^{j\omega}-e^{-1}(1+e^{-j\omega})}{1+j\omega}$

d)  $\frac{1-2e^{-1}e^{-j\omega}-j\omega e^{-1}e^{-j\omega}}{(1+j\omega)^2}$

45) a)  $x_1(t)$  y  $x_4(t)$

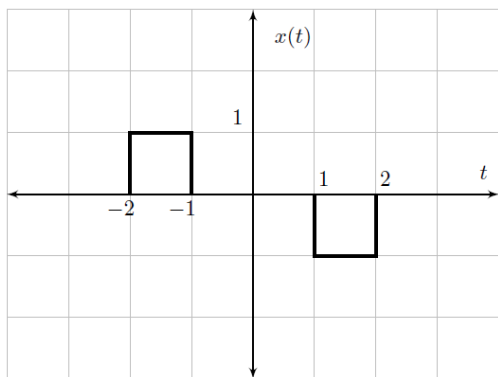
b)  $x_5(t)$  y  $x_6(t)$

c)  $x_1(t)$  y  $x_2(t)$

d) Todas excepto  $x_5(t)$

e)  $x_2(t)$ ,  $x_3(t)$ ,  $x_5(t)$  y  $x_6(t)$

f)  $x_1(t)$



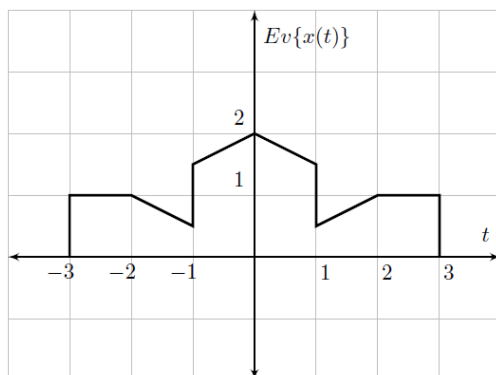
46) a)  $-\omega$

b) 7

c)  $4\pi$

d)  $7\pi$

e)  $26\pi$



f)

47) a)  $y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)$

b)  $y(t) = \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}te^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t) + \frac{1}{4}te^{-4t}u(t)$

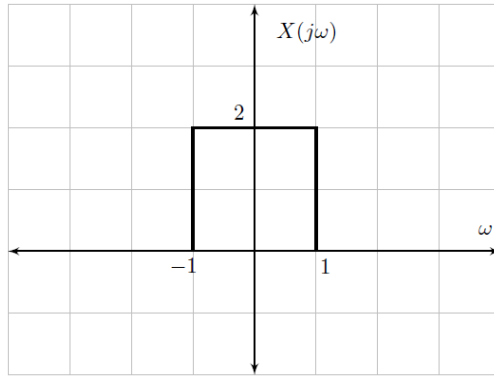
c)  $y(t) = \frac{1}{2}e^{-|t|}$

48) a)  $X(j\omega) = \frac{2 \sin(\frac{\omega}{2})}{\omega} [1 - e^{-j\omega}] e^{-j\frac{3\omega}{2}}$

b) Usando  $T = 2$ ,  $\frac{\sin(\frac{\pi k}{2})}{\pi k} [1 - e^{-j\pi k}] e^{-j\frac{3\pi k}{2}} = \begin{cases} -\frac{2j}{k\pi} & k \text{ impar} \\ 0 & k \text{ par} \\ 0 & k = 0 \end{cases}$

49) a)  $x(t) = \frac{\sin(2t)}{\pi t \cos(t)} = \frac{2 \sin(t)}{\pi t}$





b)

50) a)  $y_1(t) = y_2(t) = y_3(t) = \sin(t)$

b)  $h_4(t) = \frac{1}{2}[h_1(t) + h_2(t)]$

51) a)  $y_1(t) = 0$

b)  $y_2(t) = \frac{1}{2}\sin(3t - 3)$

c)  $y_3(t) = \frac{\sin(4t)}{\pi t}$

d)  $y_4(t) = \left(\frac{\sin(2(t-1))}{\pi(t-1)}\right)^2$

52) a)  $h(t) = e^{-2t}u(t) - e^{-4t}u(t)$

b)  $y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$

c)  $h(t) = 2\delta(t) - \sqrt{2}(1 + 2j)e^{-\frac{(1+j)t}{\sqrt{2}}}u(t) - \sqrt{2}(1 - 2j)e^{-\frac{(1-j)t}{\sqrt{2}}}u(t)$

53) a)  $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$

b)  $h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$

c)  $y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t)$

54) a)  $|H(j\omega)| = 1, \quad \angle H(j\omega) = -2 \tan^{-1}\left(\frac{\omega}{a}\right), \quad h(t) = -\delta(t) + 2ae^{-at}u(t)$

b)  $y(t) = \cos\left(\frac{t}{\sqrt{3}} - \frac{\pi}{3}\right) - \cos\left(t - \frac{\pi}{2}\right) + \cos\left(\sqrt{3}t - \frac{2\pi}{3}\right)$

55) a)  $H(j\omega) = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}$

b)  $h(t) = \frac{3}{2}[e^{-4t} + e^{-2t}]u(t)$

c)  $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t)$

