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CSCE A351
Homework #1

1.)
   a.) $\{x \in N \mid 10 > x > 2\}$
   b.) $\{x \mid \varnothing \in x\}$
   c.) {x | x is the string “ab” with any number of additional “ab” strings concatenated to it}
2.)
   a.) Theorem: if n is an integer and n^2 is even, then n is even.

Proof: We will prove the theorem by contradiction. Assume that if n is an integer and n^2 is even, then n is odd. Let’s set n^2 = 16. This means that n = 4. n^2 is even, and n is an integer, but n is not odd. This means the statement “if n is an integer and n^2 is even, then n is odd” is false. Thus, our theorem is true.

3.) Theorem: All binary trees of height n, in which all interior nodes (i.e. non-leaves) must have 2 children, have at least n+1 leaves.

Proof: We will prove the theorem by induction. Let’s start with a binary tree of height = 0, the basis of the induction. This tree is composed of one node, the root node, which is a leaf node. This means that the tree of height 0 has 1 leaf. The theorem holds.

For the induction step, let the tree height (n) >= 0. Let’s assume that the theorem works for trees with height n. We need to prove # of leaves >= n+1.

If a binary tree is composed of non-leaf nodes with 2 children each, and leaf nodes, then it is a full binary tree, by definition.

If a binary tree is full, then the number of leaf nodes it is composed of is equal to 2^(the height of the binary tree), by definition (e.g.) Full binary tree height = 5, number of leaf nodes = 2^5 = 32

(i.e.) # of leaf nodes = 2^height(n)

Now we can prove this theorem.

# of leaves = 2^n+1 = 2^n * 2
2^n * 2 >= n+1 for all n >= 0

Theorem proved.

4.) Key: State “P” = Purgatory

a.)

	1	0
->A	B	P
B	B	C
*C	B	C
P	P	P

b.)

	1	0
->A	B	A
B	C	B
C	D	C
*D	D	D

c.)

	1	0
->*A	B	B
*B	C	C
*C	D	D
*D	E	E
*E	F	F
*F	G	G
G	G	G

d.)

	1	0
->A	B	B
B	C	C
C	P	D
*D	D	D

P	P	P

e.)

	1	0
->*A	B	A
*B	C	A
*C	C	P
P	P	P

5.) A.)

	The entire alphabet excluding: {h,t,p,c,o,m,e,d,u,r,g,n,v} + all numbers > 255 and < 0 + all special characters excluding “.” and “@”	Whitespace	h	t	p	c	o	m	e	d	u	r	g	n	v	Any # 0-255	“.”	@
->A	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	N	Z
B	A	B	C	A	A	A	A	A	A	A	A	A	A	A	A	G	A	Z
C: Potential start of http	A	B	A	D	A	A	A	A	A	A	A	A	A	A	A	A	A	Z
D	A	B	A	E	A	A	A	A	A	A	A	A	A	A	A	A	A	Z
E	A	B	A	A	F	A	A	A	A	A	A	A	A	A	A	A	A	Z
*F: All following input, led by an “http”, until the next white space, is a url	F	B	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
G: Potential start of IPA	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	H	Z
H	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	I	A	Z
I	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	J	Z
J	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	K	A	Z
K	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	A	L	Z
L	A	B	A	A	A	A	A	A	A	A	A	A	A	A	A	M	A	Z
*M: All following input, led by the previous 4 numbers and 3 “.”s, until the next white space, is a url	M	B	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
N: Potential URL	A	B	A	A	A	O	O	A	E	A	A	A	O	O	A	G	A	Z
O	A	B	A	A	A	A	P	A	P	P	A	P	A	A	A	A	A	Z
P	A	B	A	Q	A	A	A	Q	A	A	Q	A	Q	A	Q	A	A	Z
Q	A	R	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	Z
*R: All input between the past 2 whitespaces	A	B	C	A	A	A	A	A	A	A	A	A	A	A	A	G	A	Z

is a url																		
Z	Z	B	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z

B.) Separate file