Punto 1

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February 3, 2024

Muestre que los siguientes operadores diferenciales:

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} \tag{1}$$

y

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{2}$$

son consistentes. Usando $f(x) = x^2$ y $f(x) = sinx$.

1 $f(x) = x^2$:

$$f'(x^2) = 2x \tag{3}$$

$$f'(x^2) = \lim_{h\to 0} \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} \tag{4}$$

$$f'(x^2) = \lim_{h\to 0} \frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h} \tag{5}$$

$$f'(x^2) = \lim_{h\to 0} \frac{-x^2 - 4hx - 4h^2 + 4x^2 + 8hx + 4h^2 - 3x^2}{2h} \tag{6}$$

$$f'(x^2) = \lim_{h\to 0} \frac{4hx}{2h} \tag{7}$$

$$f'(x^2) = \lim_{h\to 0} 2x \tag{8}$$

$$f'(x^2) = 2x \tag{9}$$

Ahora:

$$f''(x^2) = 2 \tag{10}$$

$$f''(x^2) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{11}$$

$$f''(x^2) = \lim_{h \to 0} \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2} \tag{12}$$

$$f''(x^2) = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 2x^2 + x^2 - 2hx + h^2}{h^2} \tag{13}$$

$$f''(x^2) = \lim_{h \to 0} \frac{2h^2}{h^2} \tag{14}$$

$$f''(x^2) = \lim_{h \to 0} 2 \tag{15}$$

$$f''(x^2) = 2 \tag{16}$$

2 $f(x) = sinx$:

$$f'(sinx) = cosx \tag{17}$$

$$f'(sinx) = \lim_{h \to 0} \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} \tag{18}$$

$$f'(sinx) = \lim_{h \to 0} \frac{-sin(x+2h) + 4sin(x+h) - 3sin(x)}{2h} \tag{19}$$

Aplicando L'Hopital:

$$f'(sinx) = \lim_{h \to 0} \frac{-2cos(x+2h) + 4cos(x+h)}{2} \tag{20}$$

$$f'(sinx) = \frac{-2cos(x+2(0)) + 4cos(x+(0))}{2} \tag{21}$$

$$f'(sinx) = \frac{-2cos(x) + 4cos(x)}{2} \tag{22}$$

$$f'(sinx) = \frac{2cos(x)}{2} \tag{23}$$

$$f'(sinx) = cosx \tag{24}$$

Ahora:

$$f''(sinx) = -sinx \tag{25}$$

$$f''(sinx) = \lim_{h\to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{26}$$

$$f''(sinx) = \lim_{h\to 0} \frac{sin(x+h) - 2sin(x) + sin(x-h)}{h^2} \tag{27}$$

$$f''(sinx) = \lim_{h\to 0} \frac{sin(x)cos(h) + cos(x)sin(h) - 2sin(x) + sin(x)cos(h) - cos(x)sin(h)}{h^2} \tag{28}$$

$$f''(sinx) = \lim_{h\to 0} \frac{2sin(x)cos(h) - 2sin(x)}{h^2} \tag{29}$$

Aplicando L'Hopital 2 veces:

$$f''(sinx) = \lim_{h\to 0} \frac{-2sin(x)sin(h)}{2h} \tag{30}$$

$$f''(sinx) = \lim_{h\to 0} \frac{-2sin(x)cos(h)}{2} \tag{31}$$

$$f''(sinx) = -sin(x)cos(0) \tag{32}$$

$$f''(sinx) = -sinx \tag{33}$$

Observando los resultados obtenidos se muestra que los operadores diferenciales (1) y (2) son consistentes.