Tarea 2 Punto 4

Antonio Vargas y Thomás Gomez

February 11, 2024

1 Resolver analíticamente la ecuación diferencial no lineal:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u^q \tag{1}$$

con

$$t \in [0, 10] \tag{2}$$

La solución exacta es: $u(t) = e^t$ para $q = 1$ y $u(t) = (t(1-q)+1)^{\frac{1}{1-q}}$ para $q < 1$ y $t(1-q)+1 > 0$.

Para $q = 1$:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u \tag{3}$$

$$\frac{\mathrm{d}u}{u} = \mathrm{d}t \tag{4}$$

Integrando a ambos lados se obtiene:

$$ln(u) = t + C \tag{5}$$

$$u(t) = e^{t+C} = Ae^t = e^t \tag{6}$$

Para $q < 1$:

$$\frac{\mathrm{d}u}{u^q} = \mathrm{d}t \tag{7}$$

$$\int \frac{\mathrm{d}u}{u^q} = \int \mathrm{d}t \tag{8}$$

$$\frac{u^{1-q}}{1-q} = t + C \tag{9}$$

$$u^{1-q} = (t + C)(1 - q) \tag{10}$$

$$u^{1-q} = (t(1-q)+1) \tag{11}$$

$$u^{\frac{1-q}{1-q}} = (t(1-q)+1)^{\frac{1}{1-q}} \tag{12}$$

$$u(t) = (t(1-q)+1)^{\frac{1}{1-q}} \tag{13}$$