Punto 3 Álgebra abstracta

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En primera instancia note que el operador de laplace en coordenadas cílindricas es:

$$\nabla^2 u = \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \varphi^2} \tag{1}$$

aproximando las derivadas mediante diferencias finitas(para la primera derivada se utiliza la diferencia hacia atrás y para las segundas derivadas se utiliza la fórmula habitual) y teniendo en cuenta la ecuación de onda ($\partial_{tt} u = c\nabla^2 u$),se tiene:

$$\begin{aligned}\frac{u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j,k}^{n-1}}{\Delta t^2} = \alpha^2(&\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\rho^2 \Delta\varphi^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta \rho^2} \\ &+ \frac{u_{i,j} - u_{i-1,j}}{\rho \Delta \rho})\end{aligned} \tag{2}$$

Defina $\lambda := \frac{\Delta \rho}{\Delta \varphi}, \nu := \frac{\alpha \Delta t}{\Delta \rho}$
Se obtiene:

$$\begin{aligned}u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1} &= \alpha^2 \Delta t^2 (\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\rho^2 \Delta \varphi^2} \\ &+ \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta \rho^2} + \frac{u_{i,j} - u_{i-1,j}}{\rho \Delta \rho}) \\ &= \frac{\alpha^2 \Delta t^2}{\Delta \rho^2}(\frac{\Delta \rho^2}{\rho^2 \Delta \varphi^2} \cdot (u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) + u_{i+1,j} - 2u_{i,j} + u_{i-1,j} \\ &+ \frac{\Delta \rho (u_{i,j} - u_{i-1,j})}{\rho}) \\ &= \nu^2([\frac{\lambda}{\rho}]^2 (u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) + u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + \frac{\Delta \rho}{\rho}[u_{i,j} - u_{i-1,j}])\end{aligned} \tag{3}$$

Por lo tanto:

$$u_{i,j}^{n+1} = \nu^2([\frac{\lambda}{\rho}]^2 (u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) + u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + \frac{\Delta \rho}{\rho}[u_{i,j} - u_{i-1,j}]) + 2u_{i,j}^n - u_{i,j}^{n-1} \tag{4}$$

$\square$