

Punto 3 Métodos computacionales II

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En primera instancia note que el operador de laplace en coordenadas cilíndricas es:

$$\nabla^2 u = \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \varphi^2} \quad (1)$$

aproximando las derivadas mediante diferencias finitas (para la primera derivada se utiliza la diferencia hacia atrás y para las segundas derivadas se utiliza la fórmula habitual) y teniendo en cuenta la ecuación de onda ($\partial_{tt} u = c \nabla^2 u$), se tiene:

$$\begin{aligned} \frac{u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1}}{\Delta t^2} &= \alpha^2 \left(\frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\rho^2 \Delta \varphi^2} + \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta \rho^2} \right. \\ &\quad \left. + \frac{u_{i,j}^n - u_{i-1,j}^n}{\rho \Delta \rho} \right) \end{aligned} \quad (2)$$

Defina $\lambda := \frac{\Delta \rho}{\Delta \varphi}$, $\nu := \frac{\alpha \Delta t}{\Delta \rho}$

Se obtiene:

$$\begin{aligned} u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1} &= \alpha^2 \Delta t^2 \left(\frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\rho^2 \Delta \varphi^2} \right. \\ &\quad \left. + \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta \rho^2} + \frac{u_{i,j}^n - u_{i-1,j}^n}{\rho \Delta \rho} \right) \\ &= \frac{\alpha^2 \Delta t^2}{\Delta \rho^2} \left(\frac{\Delta \rho^2}{\rho^2 \Delta \varphi^2} \cdot (u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n) + u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n \right. \\ &\quad \left. + \frac{\Delta \rho (u_{i,j}^n - u_{i-1,j}^n)}{\rho} \right) \\ &= \nu^2 \left(\left[\frac{\lambda}{\rho} \right]^2 (u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n) + u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n + \frac{\Delta \rho}{\rho} [u_{i,j}^n - u_{i-1,j}^n] \right) \end{aligned} \quad (3)$$

Por lo tanto:

$$u_{i,j}^{n+1} = \nu^2 \left(\left[\frac{\lambda}{\rho} \right]^2 (u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n) + u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n + \frac{\Delta \rho}{\rho} [u_{i,j}^n - u_{i-1,j}^n] \right) + 2u_{i,j}^n - u_{i,j}^{n-1} \quad (4)$$

□