Tarea 2 Punto 3

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1 Resolver analíticamente la ecuación diferencial de Riccati:

$$x^3y' = x^4y^2 - 2x^2y - 1 \tag{1}$$

Una solución particular está dada por: $y_1 = x^{-2}$. Encuentre numéricamente la solución usando alguno de los métodos vistos en clase con la condición inicial $y(\sqrt{2}) = 0$.

$$y' = \frac{x^4y^2}{x^3} - \frac{2x^2y}{x^3} - \frac{1}{x^3} = xy^2 - \frac{2y}{x} - \frac{1}{x^3} \tag{2}$$

$$y_1' = -2x^{-3} \tag{3}$$

$$y = y_1 + u^{-1} = x^{-2} + u^{-1} \tag{4}$$

$$y' = -2x^{-3} - u^{-2}u' \tag{5}$$

Entonces:

$$x^3(-2x^{-3} - u^{-2}u') = x^4(x^{-2} + u^{-1})^2 - 2x^2(x^{-2} + u^{-1}) - 1 \tag{6}$$

$$-2 - u^{-2}u'x^3 = x^4(x^{-4} + 2x^{-2}u^{-1} + u^{-2}) - 2 - u^{-1}x^2 - 1 \tag{7}$$

$$-u^{-2}u'x^3 = 1 + 2x^2u^{-1} + u^{-2}x^4 - 2u^{-1}x^2 - 1 \tag{8}$$

$$-u^{-2}u'x^3 = u^{-2}x^4 \tag{9}$$

$$u' = -x \tag{10}$$

Entonces:

$$u = \int -x dx = -\frac{x^2}{2} + C \tag{11}$$

$$y = x^{-2} + \left(-\frac{x^2}{2} + C\right)^{-1} \tag{12}$$

$$y = x^{-2} + \frac{1}{C - \frac{x^2}{2}} \tag{13}$$

Usando $y(\sqrt{2}) = 0$:

$$0 = (\sqrt{2})^{-2} + \frac{1}{C - \frac{\sqrt{2}^2}{2}} \tag{14}$$

$$0 = \frac{1}{2} + \frac{1}{C - 1} \tag{15}$$

Se tiene entonces que:

$$C = -1 \tag{16}$$

Y la solución de la ecuación diferencial es la siguiente:

$$y = \frac{1}{x^2} + \frac{1}{-1 - \frac{x^2}{2}} \tag{17}$$

$$y = \frac{1}{x^2} - \frac{2}{2 + x^2} \tag{18}$$

$$y = \frac{2 - x^2}{2x^2 + x^4} \tag{19}$$