2. Integrador de Adams - Bashforth. Demuestre la fórmula de iteración para tres y cuatro puntos:

$$y_{n+1} = y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2})$$

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

Para tres puntos: $\Omega = \{(t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n)\}$

Tenemos que el polinomio interpolador está dado por:

$$P_2(t) = \frac{(t-t_{n-1})(t-t_n)}{(t_{n-2}-t_{n-1})(t_{n-2}-t_n)} f_{n-2} + \frac{(t-t_{n-2})(t-t_n)}{(t_{n-1}-t_{n-2})(t_{n-1}-t_n)} f_{n-1} + \dots$$

$$+ \frac{(t-t_{n-2})(t-t_{n-1})}{(t_n-t_{n-2})(t_n-t_{n-1})} f_n$$

$h = t_k - t_{k-1}$, entonces: $k = n-2, n-1, n, \dots$

$$P_2(t) = \frac{(t-t_{n-1})(t-t_n)}{2h^2} f_{n-2} + \frac{(t-t_{n-2})(t-t_n)}{-h^2} f_{n-1} + \frac{(t-t_{n-2})(t-t_{n-1})}{2h^2} f_n$$

$$y_{n+1} = y_n + \frac{f_{n-2}}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-1})(t-t_n)\,dt - \frac{f_{n-1}}{h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-2})(t-t_n)\,dt + \dots$$

$$+ \frac{f_n}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-2})(t-t_{n-1})\,dt$$

$$\left[\quad\right] \quad = \begin{matrix} t_{k-1} = t_k - h \\ t_{n-1} = t_n - h \end{matrix} \Rightarrow \begin{matrix} t_{n-2} = t_{n-1} - h \\ t_{n-2} = t_n - 2h \end{matrix}$$

$$\rightarrow y_{n+1} = y_n + \frac{f_{n-2}}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_n+h)(t-t_n)\,dt - \frac{f_{n-1}}{h^2}\int_{t_n}^{t_{n+1}} (t-t_n+2h)(t-t_n)\,dt + \dots$$

$$+ \frac{f_n}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_n+2h)(t-t_n+h)\,dt$$

$$\left[\begin{matrix} u = t - t_n & u(t_n) = t_n - t_n = 0 \\ du = dt & u(t_{n+1}) = t_{n+1} - t_n = h \end{matrix}\right.$$

$$= y_n + \frac{f_{n-2}}{2h^2}\int_0^h (u+h)(u)\,du - \frac{f_{n-1}}{h^2}\int_0^h (u+2h)(u)\,du + \frac{f_n}{2h^2}\int_0^h (u+2h)(u+h)\,du$$

$$= y_n + \frac{f_{n-2}}{2\cancel{h^2}}\left[\frac{5}{6}h^{\cancel{3}}\right] - \frac{f_{n-1}}{\cancel{h^2}}\left[\frac{4}{3}h^{\cancel{3}}\right] + \frac{f_n}{2\cancel{h^2}}\left[\frac{23}{6}h^{\cancel{3}}\right]$$

$$= y_n + \frac{h}{12}\left[23f_n - 16f_{n-1} + 5f_{n-2}\right]$$

Para cuatro puntos:

$$\Omega = \{(t_{n-3}, f_{n-3}), (t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n)\}$$

Tenemos que el polinomio interpolador está dado por:

$$P_3(t) = \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{(t_{n-3}-t_{n-2})(t_{n-3}-t_{n-1})(t_{n-3}-t_n)} f_{n-3} + \dots$$

$$+ \frac{(t-t_{n-3})(t-t_{n-1})(t-t_n)}{(t_{n-2}-t_{n-3})(t_{n-2}-t_{n-1})(t_{n-2}-t_n)} f_{n-2} + \dots$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_n)}{(t_{n-1}-t_{n-3})(t_{n-1}-t_{n-2})(t_{n-1}-t_n)} f_{n-1} + \dots$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_{n-1})}{(t_n-t_{n-3})(t_n-t_{n-2})(t_n-t_{n-1})} f_n$$

$h = t_k - t_{k-1}$, $k = n-3, n-2, n-1, n, \dots$ entonces:

$$P_3(t) = \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{-6h^3} f_{n-3} + \frac{(t-t_{n-3})(t-t_{n-1})(t-t_n)}{2h^3} f_{n-2} + \dots$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_n)}{-2h^3} f_{n-1} + \frac{(t-t_{n-3})(t-t_{n-2})(t-t_{n-1})}{6h^3} f_n$$

$(t_{n-3} = t_{n-2} - h) \longrightarrow (t_{n-3} = t_n - 3h)$

$$y_{n+1} = y_n + \frac{f_{n-3}}{6h^3}\int_{t_n}^{t_{n+1}} (t-t_n+2h)(t-t_n+h)(t-t_n)\,dt + \dots$$

$$+ \frac{f_{n-2}}{2h^3}\int_{t_n}^{t_{n+1}} (t-t_n+3h)(t-t_n+h)(t-t_n)\,dt - \frac{f_{n-1}}{2h^3}\int_{t_n}^{t_{n+1}} (t-t_n+3h)(t-t_n+2h)(t-t_n)\,dt + \dots$$

$$+ \frac{f_n}{6h^3}\int_{t_n}^{t_{n+1}} (t-t_n+3h)(t-t_n+2h)(t-t_n+h)\,dt$$

$u = t - t_n$ $\quad u(t_n) = 0$

$du = dt$ $\quad u(t_{n+1}) = h$

$$y_{n+1} = y_n - \frac{f_{n-3}}{6h^3}\int_0^h (u+2h)(u+h)(u)\,du + \frac{f_{n-2}}{2h^3}\int_0^h (u+3h)(u+h)(u)\,du + \dots$$

$$+ \left(-\frac{f_{n-1}}{2h^3}\right)\int_0^h (u+3h)(u+2h)(u)\,du + \frac{f_n}{6h^3}\int_0^h (u+3h)(u+2h)(u+h)\,du$$

$$y_{n+1} = y_n - \frac{f_{n-3}}{6h^3}\left[\frac{9}{4}h^4\right] + \frac{f_{n-2}}{2h^3}\left[\frac{37}{12}h^4\right] - \frac{f_{n-1}}{2h^3}\left[\frac{59}{12}h^4\right] + \frac{f_n}{6h^3}\left[\frac{55}{4}h^4\right]$$

$$y_{n+1} = y_n + \frac{h}{24}\left[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}\right]$$