3. Integrador de Adams - Moulton. Demuestre la fórmula de iteración para tres y cuatro puntos.

$$y_{n+1} = y_n + \frac{h}{12}\left(5f_{n+1} + 8f_n - f_{n-1}\right)$$

$$y_{n+1} = y_n + \frac{h}{24}\left(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right)$$

Para 3 puntos: $\Omega = \{(t_{n-1}, f_{n-1}), (t_n, f_n), (t_{n+1}, f_{n+1})\}$

$$P_2(t) = \frac{(t-t_n)(t-t_{n+1})}{(t_{n-1}-t_n)(t_{n-1}-t_{n+1})} f_{n-1} + \frac{(t-t_{n-1})(t-t_{n+1})}{(t_n-t_{n-1})(t_n-t_{n+1})} f_n + \dots$$

$$+ \frac{(t-t_{n-1})(t-t_n)}{(t_{n+1}-t_{n-1})(t_{n+1}-t_n)} f_{n+1} \qquad h = t_k - t_{k-1}, \quad k = n-1, n, n+1 \dots$$

$$P_2(t) = \frac{(t-t_n)(t-t_{n+1})}{2h^2} f_{n-1} + \frac{(t-t_{n-1})(t-t_{n+1})}{-h^2} f_n + \frac{(t-t_{n-1})(t-t_n)}{2h^2} f_{n+1}$$

$$y_{n+1} = y_n + \frac{f_{n-1}}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_n)(t-t_{n+1})\,dt - \frac{f_n}{h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-1})(t-t_{n+1})\,dt + \dots$$

$$+ \frac{f_{n+1}}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-1})(t-t_n)\,dt \qquad \begin{matrix} t_{n+1} = t_n + h \\ t_{n-1} = t_n - h \end{matrix} \; ; \; \begin{matrix} u = t - t_n \\ du = dt \end{matrix} \; ; \; \begin{matrix} u(t_n) = 0 \\ u(t_{n+1}) = h \end{matrix}$$

$$y_{n+1} = y_n + \frac{f_{n-1}}{2h^2}\int_0^h (u)(u-h)\,du - \frac{f_n}{h^2}\int_0^h (u+h)(u-h)\,du + \frac{f_{n+1}}{2h^2}\int_0^h (u+h)(u)\,du$$

$$y_{n+1} = y_n + \frac{f_{n-1}}{2h^2}\left[-\frac{1}{6}h^3\right] - \frac{f_n}{h^2}\left[-\frac{2}{3}h^3\right] + \frac{f_{n+1}}{2h^2}\left[\frac{5}{6}h^3\right]$$

$$y_{n+1} = y_n + \frac{h}{12}\left[5f_{n+1} + 8f_n - f_{n-1}\right]$$

Para 4 puntos: $\Omega = \{(t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n), (t_{n+1}, f_{n+1})\}$

$$P_3(t) = \frac{(t-t_{n-1})(t-t_n)(t-t_{n+1})}{(t_{n-2}-t_{n-1})(t_{n-2}-t_n)(t_{n-2}-t_{n+1})} f_{n-2} + \dots \longrightarrow -6h^3 \qquad t_{n-2} = t_n - 2h$$

$$\dots + \frac{(t-t_{n-2})(t-t_n)(t-t_{n+1})}{(t_{n-1}-t_{n-2})(t_{n-1}-t_n)(t_{n-1}-t_{n+1})} f_{n-1} + \dots \longrightarrow 2h^3 \qquad \begin{matrix} u = t - t_n \\ du = dt \end{matrix}$$

$$\dots + \frac{(t-t_{n-2})(t-t_{n-1})(t-t_{n+1})}{(t_n-t_{n-2})(t_n-t_{n-1})(t_n-t_{n+1})} f_n + \dots \longrightarrow -2h^3$$

$$\dots + \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{(t_{n+1}-t_{n-2})(t_{n+1}-t_{n-1})(t_{n+1}-t_n)} f_{n+1}. \longrightarrow 6h^3$$

$$y_{n+1} = y_n - \frac{f_{n-2}}{6h^3}\int_0^h (u+h)(u)(u-h)\,du + \frac{f_{n-1}}{2h^3}\int_0^h (u+2h)(u)(u-h)\,du \dots$$

$$- \frac{f_n}{2h^3}\int_0^h (u+2h)(u+h)(u-h)\,du + \frac{f_{n+1}}{6h^3}\int_0^h (u+2h)(u+h)(u)\,du$$

$$y_{n+1} = y_n - \frac{f_{n-2}}{6h^3}\left[-\frac{1}{4}h^4\right] + \frac{f_{n-1}}{2h^3}\left[-\frac{5}{12}h^4\right] - \frac{f_n}{2h^3}\left[-\frac{19}{12}h^4\right] + \frac{f_{n+1}}{6h^3}\left[\frac{9}{4}h^4\right]$$

$$y_{n+1} = y_n + \frac{h}{24}\left[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right]$$