regression_task

April 3, 2019

1 Data discription

1. **FIRMCOST** - The measure of the firm's risk management cost effectiveness, defined as total property and casualty premiums and uninsured losses as a percentage of total assets.
2. **ASSUME** - Per occurrence retention amount as a percentage of total assets.
3. **SIZELOG** - Logarithm of total assets.
4. **INDCOST** - A measure of the firm's industry risk.
5. **CENTRAL** - A measure of the importance of the local managers in choosing the amount of risk to be retained.
6. **SOPH** - A measure of the degree of importance in using analytical tools.

In [3]:
```
import pandas as pd
import numpy as np
```

2 Display data statistics

In [4]:
```
def corr(df, col_names):
    '''
        compute correlation matrix
    '''
    rs = np.random.RandomState(0)
    df = pd.DataFrame(rs.rand(len(col_names), len(col_names)), columns=col_names)
    corr = df.corr()
    corr = corr.style.background_gradient(cmap='coolwarm').set_precision(2)

    return corr
```

In [5]:
```
# define path to RiskSurvey data
PATH_TO_DATA = 'regression-data/data-actuarial/RiskSurvey.csv'

# creating dataframe
df = pd.read_csv(PATH_TO_DATA)
df = df.drop('CAP', axis=1)
```

```
In [6]: # display data
        df.head()
```

```
Out[6]:    FIRMCOST  ASSUME  SIZELOG  INDCOST  CENTRAL  SOPH
        0      3.29    0.29     9.55     0.32        1    25
        1      9.31    0.89     8.04     0.33        2    24
        2      4.07    1.67     7.90     0.34        2    15
        3      6.94    1.21     8.10     0.34        1    16
        4      5.35    0.28     7.74     0.09        3    18
```

```
In [7]: # data description
        df.describe()
```

```
Out[7]:         FIRMCOST     ASSUME    SIZELOG    INDCOST    CENTRAL       SOPH
        count  73.000000  73.000000  73.000000  73.000000  73.000000  73.000000
        mean   10.973288   2.573562   8.331918   0.418356   2.246575  21.191781
        std    16.158611   8.444978   0.963378   0.216243   1.255884   5.303713
        min     0.200000   0.000000   5.270000   0.090000   1.000000   5.000000
        25%     3.510000   0.240000   7.650000   0.330000   1.000000  18.000000
        50%     6.080000   0.510000   8.270000   0.340000   2.000000  23.000000
        75%    12.710000   1.670000   8.950000   0.500000   3.000000  25.000000
        max    97.550003  61.820000  10.600000   1.220000   5.000000  31.000000
```

3 Show correlation between predictors

```
In [8]: # build correlation matrix of predictors
        corr(df.loc[:, 'FIRMCOST':'SOPH'], ['FIRMCOST', 'ASSUME', 'SIZELOG', 'INDCOST', 'CENTRA
```

```
Out[8]: <pandas.io.formats.style.Styler at 0x7f2a8000b208>
```

4 Fit linear model

```
In [9]: def fit_linear_model(df):
            input_matrix = df.copy()
            input_matrix['INTERCEPT'] = pd.Series(1, index=input_matrix.index)

            X = input_matrix.loc[:, 'ASSUME':'INTERCEPT'].values
            y = input_matrix['FIRMCOST'].values

            coefs = np.dot(np.dot(np.linalg.inv(np.dot(X.transpose(), X)),
                                  X.transpose()), y)

            return coefs

        # split dataset on train and test 60:40, due to low number of examples
        X_train, X_test = df[:40], df[40:]
        # compute coefficient for linear model
        coefs = fit_linear_model(X_train)
        coefs
```

```
Out[9]: array([ 1.65692943e-01, -2.06567267e+00,  3.21444851e+01, -2.46195547e-02,
               -1.91112242e-01,  1.82712436e+01])
```

5 Get predictions

```
In [10]: # get results on test dataset
         def linear_model_predict(coefficients, to_predict):
             pred = to_predict.apply(
                 lambda x: sum(list(map(
                     lambda k: k[0]*k[1], zip(np.append(x.values[1:], 1), coefficients))))
                 , axis=1)

             return pred

         # comparison
         y_predictions = linear_model_predict(coefs, X_test)
         comp = y_predictions.to_frame(name='predictions')
         comp['actual'] = X_test['FIRMCOST']
         comp['residual'] = comp['actual'] - comp['predictions']
         comp
```

Out[10]:

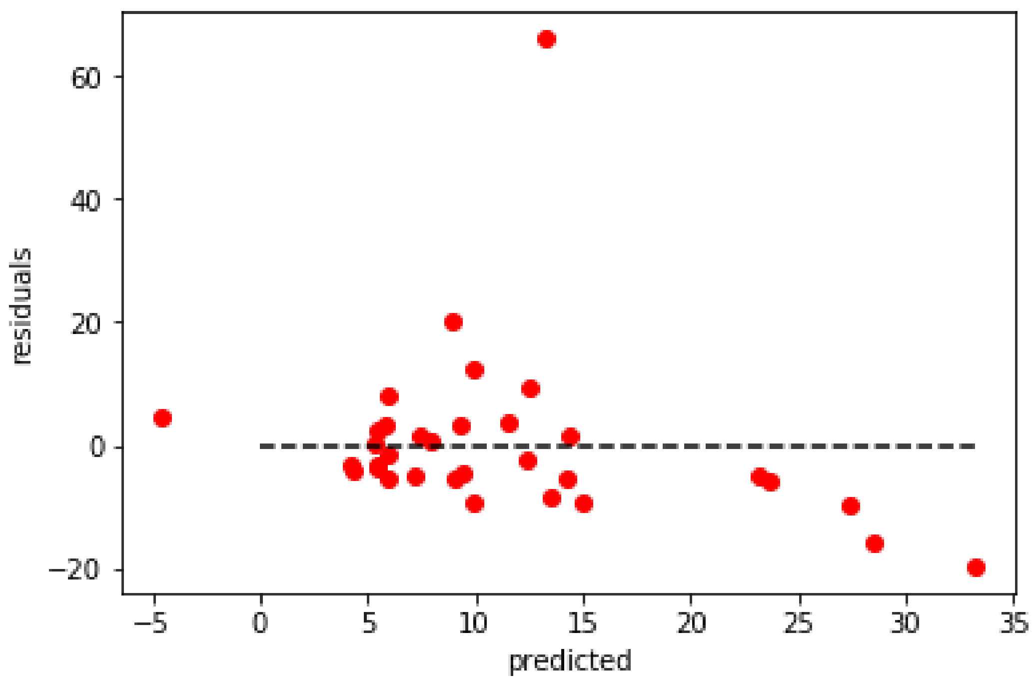
	predictions	actual	residual
40	7.160693	2.160000	-5.000693
41	4.355577	0.360000	-3.995577
42	5.532433	7.830000	2.297567
43	9.434665	5.090000	-4.344665
44	-4.519594	0.200000	4.719594
45	7.408182	8.850000	1.441818
46	9.943768	0.760000	-9.183768
47	5.365878	5.710000	0.344122
48	27.372334	17.530001	-9.842334
49	5.918854	14.000000	8.081146
50	5.524667	2.060000	-3.464667
51	4.203710	0.930000	-3.273710
52	12.420589	10.000000	-2.420589
53	14.960786	5.820000	-9.140786
54	5.799707	9.130000	3.330293
55	14.295559	9.000000	-5.295559
56	28.505357	12.610000	-15.895358
57	5.473248	2.150000	-3.323248
58	9.991330	22.219999	12.228669
59	9.269259	12.710000	3.440741
60	14.418950	15.970000	1.551050
61	5.914935	4.320000	-1.594934
62	7.990755	8.490000	0.499245
63	13.591023	5.250000	-8.341023
64	23.189775	18.330000	-4.859775

```
65    12.571876  21.719999   9.148123
66     5.913659   0.400000  -5.513659
67     9.121543   3.700000  -5.421543
68    11.491456  15.000000   3.508544
69    23.757871  18.000000  -5.757871
70     8.911315  29.120001  20.208686
71    13.261827  79.300003  66.038176
72    33.217764  13.570000 -19.647764
```

6 Residual plot

In [11]:
```
import matplotlib.pyplot as plt

plt.plot(comp['predictions'], comp['residual'], 'ro', np.linspace(0, max(comp['predic
plt.xlabel('predicted')
plt.ylabel('residuals')
plt.show()
```

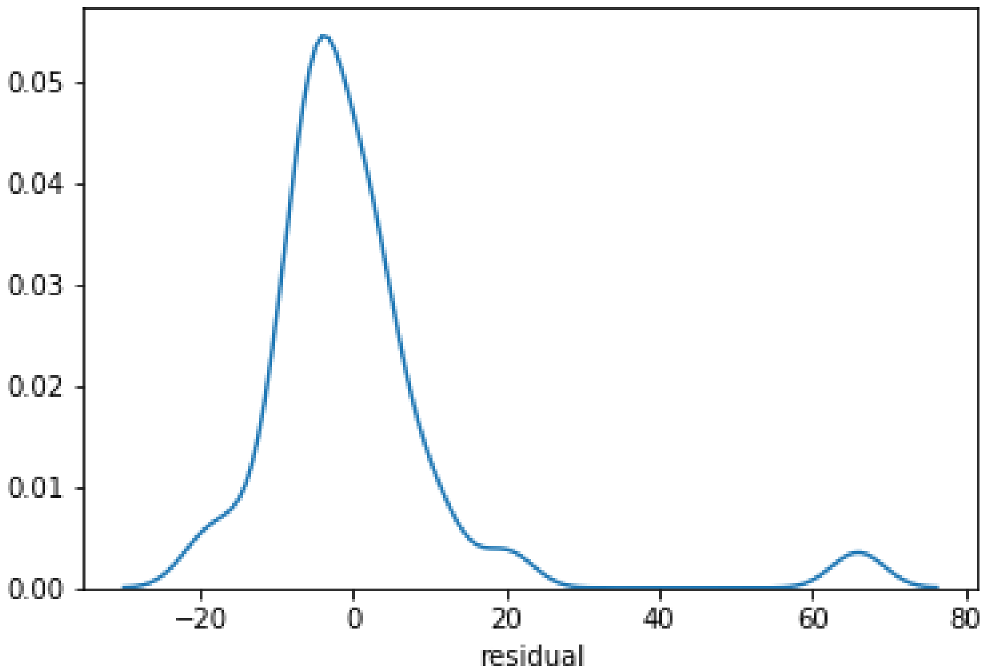


There is no visible pattern but we have several outliers.

In [12]:
```
# density plot
import seaborn as sns
sns.distplot(comp['residual'], hist=False)
```

```
/home/daniil/anaconda3/lib/python3.6/site-packages/scipy/stats/stats.py:1713: FutureWarning: U
  return np.add.reduce(sorted[indexer] * weights, axis=axis) / sumval
```

```
Out[12]: <matplotlib.axes._subplots.AxesSubplot at 0x7f2a68844ac8>
```



7 Compute R^2

```
In [13]: def r_squared(actual, predicted):
             mean = sum(actual)/len(actual)
             RSS = sum(list(map(lambda x: (x[0] - x[1])**2, zip(predicted, actual))))
             TSS = sum(list(map(lambda x: (x - mean)**2, actual)))

             return 1 - RSS/TSS

         score = r_squared(comp['actual'].tolist(), comp['predictions'].tolist())
         score
```

```
Out[13]: 0.016972853550193512
```

R^2 of the model equals to **0.016**, so the model fits a bit better than a horizontal hyperplane. Due to high correlation between predictors and rank of X is less than p + 1 (5 < 6), so we can assume that ridge regression can be a better choise for our problem.

8 Estimating covariance matrix of˜

In [14]:
```
# Variance of
score = r_squared(comp['actual'].tolist(), comp['predictions'].tolist())
X = X_train.copy()
X['INTERCEPT'] = pd.Series(1, index=X.index)
y = X['FIRMCOST'].values
X = X.loc[:, 'ASSUME':'INTERCEPT'].values
H = np.dot(np.dot(X, np.linalg.inv(np.dot(X.transpose(), X))), X.transpose())
# estimating ˜2
sigma2 = np.dot(np.dot(y.transpose(), (np.identity(H.shape[0]) - H)), y)/(X.shape[0] -
var = np.dot(np.linalg.inv(np.dot(X.transpose(), X)), sigma2)
pd.DataFrame(var)
```

Out[14]:

	0	1	2	3	4	5
0	1.264688	0.530214	-2.308232	0.044273	-0.052752	-4.163613
1	0.530214	12.687781	0.411645	1.493031	0.760806	-125.394404
2	-2.308232	0.411645	150.220666	5.600334	-1.571776	-41.155763
3	0.044273	1.493031	5.600334	6.343226	-0.295370	-22.117418
4	-0.052752	0.760806	-1.571776	-0.295370	0.333641	-11.916516
5	-4.163613	-125.394404	-41.155763	-22.117418	-11.916516	1367.329784

9 Ridge regression

In [15]:
```
from sklearn.preprocessing import StandardScaler

def fit_ridge_regression(x, y, alpha=1):
    X = x.copy()
    X = np.array([np.append(step, 1) for step in X])

    return np.dot(np.dot(np.linalg.inv((np.dot(X.transpose(), X) +
                  np.dot(alpha, np.identity(X.shape[1])))),
                                          X.transpose()), y)

def ridge_reg_predict(coefficients, to_predict):
    pred = to_predict.apply(
        lambda x: sum(list(map(
            lambda k: k[0]*k[1], zip(np.append(x.values, 1), coefficients))))
        , axis=1)

    return pred

# Standarize data
scaler = StandardScaler()
X_train_rg, y_train = X_train.loc[:, 'ASSUME':'SOPH'].values, X_train['FIRMCOST'].valu
X_train_std = scaler.fit_transform(X_train_rg)

# getting most sutable alpha
```

```
alphas, output = np.linspace(0, 10, 1000), []
for alpha in alphas:
    ridge_coefs = fit_ridge_regression(X_train_std, y_train, alpha=alpha)
    ridge_pred = ridge_reg_predict(ridge_coefs, pd.DataFrame(X_train_std))
    output.append(r_squared(y_train, ridge_pred))

best_alpha = alphas[output.index(max(output))]
ridge_coefs = fit_ridge_regression(X_train_std, y_train, alpha=best_alpha)
ridge_pred = ridge_reg_predict(ridge_coefs, pd.DataFrame(scaler.transform(X_test.loc[
y_test = X_test['FIRMCOST'].values
r_squared(y_test, ridge_pred)
```

Out[15]: 0.016972853550185962

R^2 score of ridge regression is the same as R^2 of pure linear model. Due to most sutable alpha, which is **0.0**. So, ridge regression doesn't improve perfomance.

10 Applying Principal Component Analysis to reduce dimension of the space

In [16]:
```
import plotly.plotly as py

scaler = StandardScaler()
x_train, y_train = X_train.loc[:, 'ASSUME':'SOPH'].values, X_train['FIRMCOST'].values
x_test, y_test = X_test.loc[:, 'ASSUME':'SOPH'].values, X_test['FIRMCOST'].values
scaler.fit(x_train)

# rescale our data
x_train = scaler.transform(x_train)
x_test = scaler.transform(x_test)

# getting mean vector
mean_vec = np.mean(x_train, axis=0)
# build covariance matrix
cov_mat = np.dot((x_train - mean_vec).transpose(), (x_train - mean_vec))/(x_train.sha
# getting eigen values and corresponding vectors
eigen_values, eigen_vecs = np.linalg.eig(cov_mat)
eigen_pairs = [(np.abs(eigen_values[i]), eigen_vecs[:,i]) for i in range(len(eigen_va
eigen_pairs.sort(key=lambda x: x[0], reverse=True)

# compute variance explained by each principal component
var_exp = [val/sum(eigen_values)*100 for val in sorted(eigen_values, reverse=True)]
# cumulative sum of the elements
cum_var_exp = np.cumsum(var_exp)
# creat plot
trace1 = dict(
    type='bar',
```

```
        x=['PC{}'.format(i) for i in range(1, len(eigen_values) + 1)],
        y=var_exp,
        name='Individual'
    )

    trace2 = dict(
        type='scatter',
        x=['PC{}'.format(i) for i in range(1, len(eigen_values) + 1)],
        y=cum_var_exp,
        name='Cumulative'
    )

    data = [trace1, trace2]

    layout=dict(
        title='Explained variance by different principal components',
        yaxis=dict(
            title='Explained variance in percent'
        ),
        annotations=list([
            dict(
                x=1.16,
                y=1.05,
                xref='paper',
                yref='paper',
                text='Explained Variance',
                showarrow=False,
            )
        ])
    )

    fig = dict(data=data, layout=layout)
    py.iplot(fig, filename='selecting-principal-components')
```

```
High five! You successfully sent some data to your account on plotly. View your plot in your b
```

Out[16]: <plotly.tools.PlotlyDisplay object>

11 Projection onto the new Feature Space

In [24]:
```
# reducing the 5-dimensional feature space to a 4-dimensional feature subspace
# constructing of the projection matrix
def choose_n_comp(N):
    return np.hstack((eigen_pairs[i][1].reshape(len(eigen_values),1) for i in range(N)

num_of_comp = 4
matrix_w = choose_n_comp(num_of_comp)
```

```
    # transform our samples
    x_train_tr = np.dot(x_train, matrix_w)
    x_test_tr = np.dot(x_test, matrix_w)

    # train linear model
    coefs = fit_ridge_regression(x_train_tr, y_train, alpha=0)
    pred = ridge_reg_predict(coefs, pd.DataFrame(x_test_tr))
    r_squared(y_test, pred)
```

```
/home/daniil/.local/lib/python3.6/site-packages/ipykernel_launcher.py:4: FutureWarning:

arrays to stack must be passed as a "sequence" type such as list or tuple. Support for non-sequ
```

```
Out[24]: -0.24806993048618198
```

1 component, **36%** of variance explained, R^2 on linear model = **0.027** **2** components, **60%** of variance explained, R^2 on linear model = **-1.190** **3** components, **76%** of variance explained, R^2 on linear model = **-0.091** **4** components, **90%** of variance explained, R^2 on linear model = **-0.248** **5** components, **100%** of variance explained, R^2 on linear model = **0.0169**

So, optimal number of components is still 5. Better performace result with only one component we can consider as luck due to lack of samples in our dataset and impossibility to obtain balanced decision.

12 Variable Selection

12.0.1 CRITERION-BASED PROCEDURE (Adjusted R^2)

```
In [56]: import itertools

    # adjusted r2
    def adj_r_squared(actual, predicted, exp_var):
        r2 = r_squared(actual, predicted)
        return 1 - (1 - r2)*(len(actual) - 1)/(len(actual) - exp_var - 1)

    combinations = []

    #Looping over k = 1 to k = 5 components in X
    for k in range(2, x_train.shape[1] + 1):
        #Looping over all possible combinations
        for indices in itertools.combinations(range(len(x_train.transpose())), k):
            combo = np.array([x_train.transpose()[i] for i in indices]).transpose()
            reg_coefs = fit_ridge_regression(combo, y_train, alpha=0)
            pred = ridge_reg_predict(reg_coefs, pd.DataFrame(np.array([x_test.transpose()
            combinations.append([adj_r_squared(y_test, pred, k), indices])

    # sort combinations list by adjusted r2 in descending order
```

```
combinations = sorted(combinations, key=lambda x: x[0], reverse=True)
combinations
```

Out[56]:

```
[[0.15736845008248512, (1, 3)],
 [0.13107789420863258, (1, 4)],
 [0.08440232852963192, (1, 3, 4)],
 [0.005256447227352745, (1, 2)],
 [-0.007745722688474066, (1, 2, 4)],
 [-0.02716435251750382, (1, 2, 3)],
 [-0.043712671980775, (1, 2, 3, 4)],
 [-0.08724039547125595, (0, 1, 2)],
 [-0.11427456702110295, (3, 4)],
 [-0.12351984229840496, (0, 1, 2, 4)],
 [-0.12465931938912012, (0, 1, 2, 3)],
 [-0.14968273946623256, (2, 3)],
 [-0.15524864967324192, (2, 4)],
 [-0.16506921060718693, (0, 1, 2, 3, 4)],
 [-0.1926115478503685, (2, 3, 4)],
 [-0.24696417611594135, (0, 2)],
 [-0.2816119797516208, (0, 1)],
 [-0.28567329009918163, (0, 2, 3)],
 [-0.3057600918280441, (0, 2, 4)],
 [-0.3205340505498979, (0, 1, 4)],
 [-0.33569006539354174, (0, 1, 3)],
 [-0.34885986079418907, (0, 2, 3, 4)],
 [-0.371973947569995, (0, 1, 3, 4)],
 [-0.5136209732785726, (0, 4)],
 [-0.5912135704717056, (0, 3, 4)],
 [-0.6474805992304404, (0, 3)]]
```

As we can see, best adjusted R^2 score we get when using **SIZELOG** and **CENTRAL** components.

In [71]:

```
best = combinations[0][1]
reg_coefs = fit_ridge_regression(np.array([x_train[:, i] for i in best]).transpose(),
pred = ridge_reg_predict(reg_coefs, pd.DataFrame(np.array([x_test[:, i] for i in best
r_squared(y_test, pred)
```

Out[71]: 0.2100329219523298

R^2 score = **0.21** for **SIZELOG** and **CENTRAL** features.

13 Perform F-test

In [83]:

```
def f_test(full, reduced, actual, dff, dfr):
    SSE_F = sum([(yi - yi_pred)**2 for yi, yi_pred in zip(actual, full)])
    SSE_R = sum([(yi - yi_pred)**2 for yi, yi_pred in zip(actual, reduced)])
    F_stat = ((SSE_R - SSE_F)/(dfr - dff))/(SSE_F/dff)
```

```
        return F_stat

    def p_value()
    # F-test between model with all 5 features and model with 2 features
    dff, dfr = len(comp['actual'].values) - 5 - 1, len(comp['actual'].values) - 2 - 1
    F_stat = f_test(comp['predictions'].values, pred, comp['actual'].values, dff, dfr)
    F_stat
```

```
Out[83]: -1.7675408272236814
```

As far as we can see, we can't reject the null hypothesis. Which tell us that model with 2 components is better choise.

```
In [86]: # F-test between model with 2 features and null hypothesis
         mean = sum(comp['actual'].values)/len(comp['actual'].values)
         F_stat_null = f_test(pred, [mean]*40, comp['actual'].values, 37, 39)
         F_stat_null
```

```
Out[86]: 4.918697454735734
```

But here, comparing with table value, we reject null hypothesis and confirm that model with two components performs better.

In conclusion, we choose linear model with **SIZELOG** and **CENTRAL** features and formula **y = sizelog*(-2.45)* + *central*(-1.27) + 10.21**. R^2 is 0.21, which is not really good, but is the best choise among other linear models based on our research.

```
In [ ]:
```