2015 solve

3(a) (KNN)

$\hat{Y}_1 = \frac{2+5}{2} = 3.5 \quad (x=1)$

$\hat{Y}_2 = \frac{1+5}{2} = 3 \quad (x=2)$

$\hat{Y}_3 = \frac{2+4}{2} = 3 \quad (x=3)$

$\hat{Y}_4 = \frac{\cancel{3}+5+5}{2} = 5.$

$\hat{Y}_5 = \frac{5+4}{2} = 4.5.$

$\therefore \hat{Y} = \{3.5, 3, 3, 5, 4.5\}$ (Ans)

3(b)

$$F\text{-stat} = \frac{\frac{TSS-RSS}{J}}{\frac{RSS}{n-J-1}}$$

$\bar{x} = \frac{1+2+3+4+5}{5} = 3$

$\bar{y} = \frac{6+7+4+3+2}{5} = 4.4$

$TSS = \Sigma(y_i - \bar{y})^2$

$= [(6-4.4)^2 + (7-4.4)^2 + (4-4.4)^2 + (3-4.4)^2 + (2-4.4)^2]$

$= 17.2.$

$RSS = \Sigma(y_i - \hat{y}_i)^2$

$= [(6-6.8)^2 + (7-5.6)^2 + (4-4.4)^2 + (3-3.2)^2 + (2-2)^2]$

$= 2.8.$

$J = 1, \quad n = 5$

$$\therefore F\text{-stat} = \frac{\frac{17.2-2.8}{1}}{\frac{2.8}{5-1-1}} = \frac{14.4}{\frac{14}{15}} = 15.43.$$ (Ans)

5(c)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$= \frac{1}{5}[2.8]$$

$$= 0.56.$$

6(b)

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
1	6	-2	1.6	4	-3.2
2	7	-1	2.6	1	-2.6
3	4	0	-0.4	0	0
4	3	1	-1.4	1	-1.4
5	2	2	-2.4	4	-4.8
$\bar{x}=3$	$\bar{y}=4.4$			$\Sigma=10$	$\Sigma=-12$

$$\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{-12}{10} = -1.2$$

$$\hat{\beta}_0 = 4.4 - \hat{\beta}_1 \times 3$$

$$= 4.4 + 1.2 \times 3$$

$$= 8$$

$$\boxed{\hat{y} = -1.2x + 8}$$

HTT 2 (1^{st} set)

1) Linear Regression Model:

$$\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_2 x_4$$

Here, p-value for $x_3 = 0.7$

which is > 0.05.

So, value of x_3 isn't significant.

$$\therefore \hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_2 x_4.$$

(Ans).

2)

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
-1	6	$\cancel{-2}$ -1	0.33	$\cancel{4}$ 1	$\cancel{-0.66}$ -0.33
2	7	$\cancel{-1}$ 0	1.33	$\cancel{1}$ 0	$\cancel{-1.33}$ 0
3	4	$\cancel{0}$ 1	-1.67	$\cancel{0}$ 1	$\cancel{0}$ -1.67
				$\Sigma = \cancel{5}\ 2$	$\Sigma = \cancel{-1.99}$ -2
$\bar{x} = 2$	$\bar{y} = 5.67$				

$\cancel{\beta_1 = \frac{-1.99}{5} = -0.398}$

$\cancel{\beta_0 = 5.67 - (-0.398 \times}$

$$\beta_1 = \frac{-2}{\cancel{5}\ 2} = \cancel{-0.4}\ -1$$

$$\beta_0 = 5.67 - (\cancel{-0.4}\ (-1) \times 2)$$

$$= 5.67 + 2$$

$$= 7.67.$$

$$\therefore \boxed{\hat{Y} = 7.67 - x.}$$

2^{nd} set

4) $\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_2 x_3 + \beta_5 x_1^2 + \beta_6 x_2^2 + \beta_7 x_3^2 + \beta_8 x_2^2 x_3^2$

5) $MSE = \frac{1}{5}\left[(6-6.8)^2 + (7-5.6)^2 + (4-4.4)^2 + (3-3.2)^2 + (2-2)^2\right]$

$= 0.56.$

$AIC = n \ln(MSE) + 2J$

$= 5 \ln(0.56) + 2 \times 1$

$= -0.9.$

$J = 1$

$n = 5$

$BIC = n \ln(MSE) + J \ln(n)$

$= -1.29.$