

Stream: $\sigma = \langle a_1, a_2, \dots, a_m \rangle$, token $a_i \in U = [n]$.

Goal: process σ using space s st $s \ll m \wedge s \ll n$, $s = O(\min(m, n))$, pref $s = O(\log m + \log n)$.

Sometimes best $s = \text{polylog}(\min(m, n))$, $\text{polylog} = O((\log(g(n)))^c)$ for $c > 0$.

Streams can only be access in sequence.

Multiplicative condition: Let $A(\sigma)$ be random stream alg A on σ : Let ϕ be target function. A is an ϵ, δ -approx alg of ϕ if

$$\Pr\left[\left|\frac{A(\sigma)}{\phi(\sigma)} - 1\right| > \epsilon\right] \leq \delta$$

To strong a condition when $\phi(\sigma)$ has values close to zero.

Additive-approx: (ϵ, δ) -additive-approx-alg A of ϕ if

$$\Pr[|A(\sigma) - \phi(\sigma)|] \leq \sigma$$

Often instrested in statistical properties of multiset in σ . Let vector $\vec{F} = (f_1, \dots, f_n)^T$, $f_j = |\{i : a_i = j\}|$. So $\phi(\vec{F})$ target. \vec{F} frequency vector.

Turnstile model: $\sigma : [n] \times \{-L, \dots, L\}$ so $f_j = \sum_{\{i : a_i = j\}} l_i$. Redefined m to max number tokens in multiset $\|\vec{F}\|_1 \leq m$.

Strict turnstile model: $\vec{F} \geq 0$

Cash register model: $\forall i. l_i > 0$

1 Frequency Problem

Majority problem: if $\exists j : f_j > m/2$, then output j else null. Frequency problem: For some k, output $\{j : f_j > m/k\}$

Frequency-estimateion problem: For stream σ produce structure that can estimate \hat{f}_a for freq f_a for $a \in [n]$.

Misra-Gries Alg: Takes param k (same as freq problem). Maintain associative array, fx bbt

Process j:

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If j in keys(A) then
  A[j] <- A[j]+1
else if |keys(A)| < k -1 then
  A[j] <- 1
else
  foreach l in keys(A)
    A[l] <- A[l] - 1
    if A[l] = 0 then remove l
output:  $\hat{f}_a = A[a]$  if keys(a) else 0

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When $A[j]$ decr then we decr k-1 other counters \Rightarrow decr witnessed (applied to) k tokens $\Rightarrow \leq m/k$ decrs as $|\sigma| = m$, therefore $\hat{f}_j \leq f_j - m/k$.