```
> restart: with(LinearAlgebra):
> GausElim:=proc(n,M)
    local a,i,j,k,l,z;
    l:=0;
    for k from 1 to n-1 do
      for i from k+1 to n do
        l:=l+1;
        z:=(M[i,k])/(M[k,k]);
        M[i,k]:=z*(m);
        for j from k+1 to n+1 do
          M[i,j]:=(M[i,j]-(z*M[k,j]));
        end do;
      end do;
    end do;
    print(M);
  end proc:


> GausElimPiv:=proc(n,M)
    local i,j,k,p,s,M2,x,y,z;
    for i from 1 to n do
      s[i]:=0;
      p[i]:=i;
      for j from 1 to n do
        if abs(M[i,j]) > s[i] then s[i]:=abs(M[i,j]); end if;
      end do;
    end do;
    for k from 1 to n-1 do
      z:=0;
      for i from k to n do
        if M[i,k]/s[i] > z then z:=M[i,k]/s[i]; y:=i end if;
      end do;
      x:=p[k];
      p[k]:=p[y];
      p[y]:=x;
      for i from k+1 to n do
        z:=(M[p[i],k])/(M[p[k],k]);
        M[p[i],k]:=z*(m);
        for j from k+1 to n+1 do
          M[p[i],j]:=(M[p[i],j]-(z*M[p[k],j]));
        end do;
      end do;
    end do;
    M2:=Matrix(n);
    for i from 1 to n do
      M2[i,p[i]]:=1;
    end do;
    print(M2);
    print(M);
  end proc:

> backSub := proc(n, A, B)
    local x,i,j;
    x := Vector(n):
    for i from n to 1 by -1 do
      x(i) := (B(i)-add(A(i,j)*x(j),j=i+1..n))/A(i,i):
    end do:
```

```
    return x:
    end proc:
```

> `a:=<<-1,2,3>|<1,2,3>|<-4,0,2>>:`
> `b:=<<1,2,0>|<6,1,2>|<0,0,1>|<3,1,1>>:`
> `c:=<<-1,1,0,3>|<1,0,1,0>|<0,3,-1,1>|<-3,1,-1,2>|<4,0,3,1>>:`
> `d:=<<6,12,3,-6>|<-2,-8,-13,4>|<2,4,3,2>|<4,10,3,-18>|<0,-10,-39,`
  `-16>>:`
> `e:=<<1,4,8,2>|<0,-9,16,3>|<2,2,6,2>|<1,1,5,1>|<2,14,-3,0>>:`

> `4.3.1b)`

> `GausElim(3,b);`

$$\begin{bmatrix} 1 & 6 & 0 & 3 \\ 2\,m & -11 & 0 & -5 \\ 0 & -\dfrac{2}{11}\,m & 1 & \dfrac{1}{11} \end{bmatrix} \qquad (1)$$

> `U:=<<1,0,0>|<6,-11,0>|<0,0,1>>:`
> `backSub(3,U,<3,-5,1/11>);`

$$\begin{bmatrix} \dfrac{3}{11} \\ \dfrac{5}{11} \\ \dfrac{1}{11} \end{bmatrix} \qquad (2)$$

> `GausElimPiv(3,b);`

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{1}{2}\,m & \dfrac{11}{4}\,m & -\dfrac{11}{4} & -\dfrac{1}{4} \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \qquad (3)$$

> `U2:=<<2,0,0>|<1,2,0>|<0,1,-11/4>>:`
> `backSub(3,U2,<1,1,-1/4>);`

$$\begin{bmatrix} \dfrac{3}{11} \\ \dfrac{5}{11} \\ \dfrac{1}{11} \end{bmatrix} \qquad (4)$$

```
> 4.3.1c)
> GausElim(4,c);
```

$$\begin{bmatrix} -1 & 1 & 0 & -3 & 4 \\ -m & 1 & 3 & -2 & 4 \\ 0 & m & -4 & 1 & -1 \\ -3\,m & 3\,m & 2\,m & -3 & 3 \end{bmatrix}$$

(5)

```
> U:=<<-1,0,0,0>|<1,1,0,0>|<0,3,4,0>|<-3,-2,1,-3>>:
> backSub(4,U,<4,4,-1,3>);
```

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

(6)

```
> GausElimPiv(4,c);
```

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\dfrac{1}{3}\,m & m & \dfrac{4}{3} & -\dfrac{4}{3} & \dfrac{4}{3} \\ \dfrac{1}{3}\,m & 0 & 2\,m & 3 & -3 \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{bmatrix}$$

(7)

```
> U2:=<<3,0,0,0>|<0,1,0,0>|<1,-1,4/3,0>|<2,-1,-4/3,3>>:
> backSub(4,U2,<1,3,4/3,-3>);
```

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

(8)

```
> 4.3.1d)
> GausElim(4,d);
```

(9)

$$\begin{bmatrix} 6 & -2 & 2 & 4 & 0 \\ 2\,m & -4 & 0 & 2 & -10 \\ \dfrac{1}{2}\,m & 3\,m & 2 & -5 & -9 \\ -m & -\dfrac{1}{2}\,m & 2\,m & -3 & -3 \end{bmatrix}$$

(9)

```
> U:=<<6,0,0,0>|<-2,-4,0,0>|<2,0,2,0>|<4,2,-5,-3>>:
> backSub(4,U,<0,-10,-9,-3>);
```

$$\begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$$

(10)

```
> GausElimPiv(4,d);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & 2 & 4 & 0 \\ 2\,m & -2\,m & \dfrac{4}{13}\,m & -\dfrac{6}{13} & -\dfrac{6}{13} \\ \dfrac{1}{2}\,m & -6\,m & 26 & -83 & -135 \\ -m & 2 & 4 & -14 & -16 \end{bmatrix}$$

(11)

```
> U2:=<<6,0,0,0>|<-2,2,0,0>|<2,4,26,0>|<4,-14,-83,-6/13>>:
> backSub(4,U2,<0,-16,-135,-6/13>);
```

$$\begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$$

(12)

```
> 4.3.1e)
> GausElim(4,e);
```

(13)

$$
\begin{bmatrix}
1 & 0 & 2 & 1 & 2 \\
4\,m & -9 & -6 & -3 & 6 \\
8\,m & -\dfrac{16}{9}\,m & -\dfrac{62}{3} & -\dfrac{25}{3} & -\dfrac{25}{3} \\
2\,m & -\dfrac{1}{3}\,m & \dfrac{6}{31}\,m & -\dfrac{12}{31} & -\dfrac{12}{31}
\end{bmatrix}
\tag{13}
$$

```
> U:=<<1,0,0,0>|<0,-9,0,0>|<2,6,-62/3,0>|<1,-3,-25/3,-12/31>>:
> backSub(4,U,<2,6,-25/3,-12/31>);
```

$$
\begin{bmatrix}
1 \\
-1 \\
0 \\
1
\end{bmatrix}
\tag{14}
$$

```
> GausElimPiv(4,e);
```

$$
\begin{bmatrix}
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
\dfrac{1}{2}\,m & -\dfrac{3}{2} & 1 & \dfrac{1}{2} & 2 \\
2\,m & 10\,m & -12 & -6 & -6 \\
4\,m & -\dfrac{8}{3}\,m & -\dfrac{1}{18}\,m & 2 & 2 \\
2 & 3 & 2 & 1 & 0
\end{bmatrix}
\tag{15}
$$

```
> U2:=<<2,0,0,0>|<3,-3/2,0,0>|<2,1,-12,0>|<1,1/2,-6,2>>:
> backSub(4,U2,<0,2,-6,2>);
```

$$
\begin{bmatrix}
1 \\
-1 \\
0 \\
1
\end{bmatrix}
\tag{16}
$$