Oversætter - Week 1

1 - DFA Minimisation

We are given a DFA that we wish to minimize.
We split it into 2 sets, $G1$ with states F and $G2$ with states $S \setminus F$.

$$G_1 = \{3\}$$
$$G_2 = \{1, 2, 4, 5, 6, 7, 8\}$$

We check if the groups are consistent. We see that $G1$ is consistent since it consists of only 1 element. However $G2$ might not be.

G2	0	1
1	G2	G2
2	G2	G1
4	G1	G2
5	G2	G2
6	G1	G2
7	G2	G2
8	G2	G1

We see that it's not consistent so we split it into maximal consistent subgroups, so that

$$G_1 = \{3\}$$
$$G_3 = \{1, 5, 7\}$$
$$G_4 = \{2, 8\}$$
$$G_5 = \{4, 6\}$$

The procedure[1] is repeated for $G3$ we get

G3	0	1
1	G4	G5
5	G4	G5
7	G3	G3

[1] Algorithm 1.4

So we get

$$G_1 = \{3\}$$
$$G_4 = \{2, 8\}$$
$$G_5 = \{4, 6\}$$
$$G_6 = \{1, 5\}$$
$$G_7 = \{7\}$$

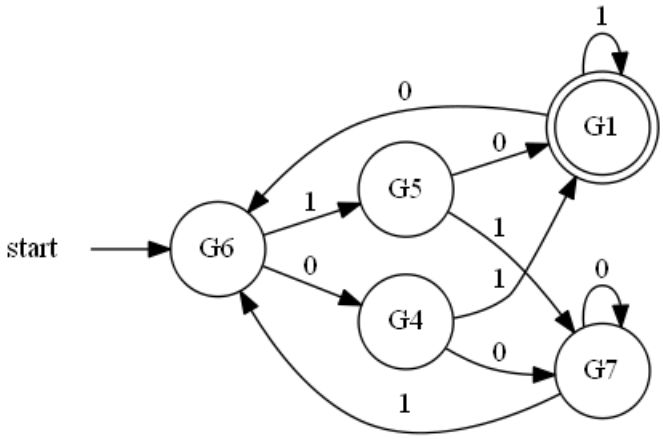
and using Algorithm 1.4 on $G4, G5$ and $G6$ we get

G4	0	1
2	G7	G1
8	G7	G1

G5	0	1
4	G1	G7
6	G1	G7

G6	0	1
1	G4	G5
5	G4	G5

The last 2 are singletons and therefore all are consistent.
The minimized DFA is shown below drawn with the program Graphviz.



2 - Backtracking Automaton

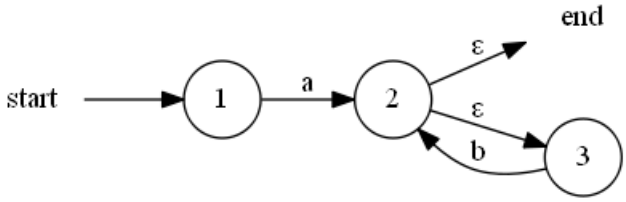
We are given three regular expressions

$$a_1 = ab^*$$
$$a_2 = a^*b$$
$$a_3 = (ab)^*c$$

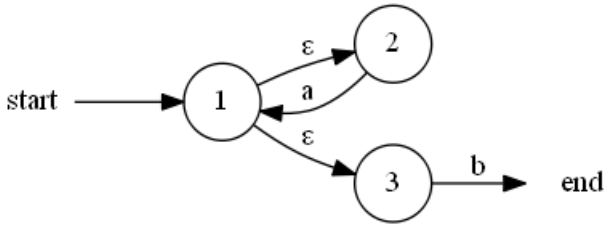
a

Give an NFA for each expression and construct a combined NFA.

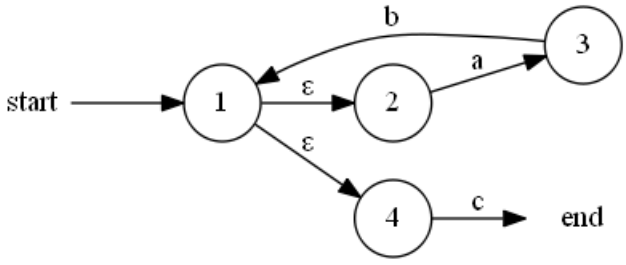
For ab^*



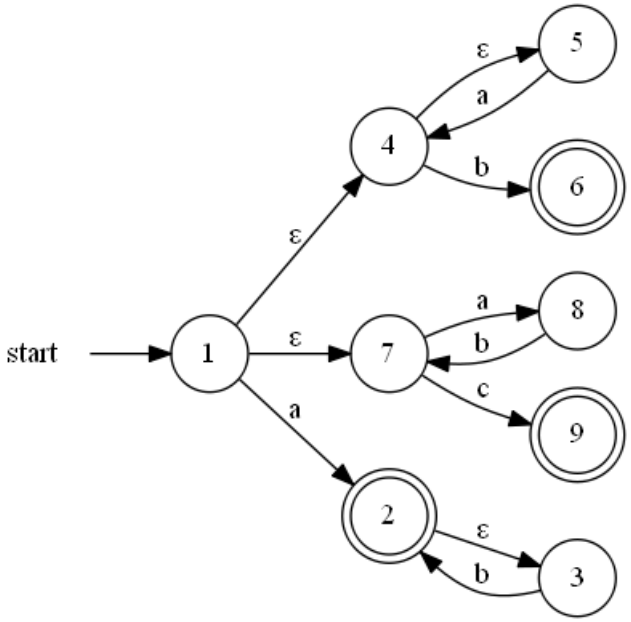
for a^*b



for $(ab)^*c$



Below is the combined NFA with superfluous epsilon transitions removed.



b

We wish to convert this NFA to DFA by subset construction.
We begin by adding all epsilon transitions we can reach from state 1 to s'_0 and then call $move$[2] on this set.

$$s'_0 = \{1, 4, 5, 7\}$$

$$\begin{aligned} s'_1 &= move(s'_0, a) \\ &= \{2, 3, 4, 5, 8\} \end{aligned}$$

$$\begin{aligned} s'_2 &= move(s'_0, b) \\ &= \{6\} \end{aligned}$$

$$\begin{aligned} s'_3 &= move(s'_0, c) \\ &= \{9\} \end{aligned}$$

[2] Algorithm 1.3

Proceeding with the function *move* on s'_1

$$\begin{aligned} s'_4 &= move(s'_1, a) \\ &= \{4, 5\} \end{aligned}$$

$$\begin{aligned} s'_5 &= move(s'_1, b) \\ &= \{2, 3, 6, 7\} \end{aligned}$$

$$\begin{aligned} s'_6 &= move(s'_1, c) \\ &= \{\} \end{aligned}$$

And for s'_2 and s'_3 it is easy to see that all sets coming from them are empty sets.
We use *move* on s'_4

$$\begin{aligned} s'_6 &= move(s'_4, a) \\ &= \{4, 5\} \end{aligned}$$

$$\begin{aligned} s'_7 &= move(s'_4, b) \\ &= \{6\} \end{aligned}$$

$$\begin{aligned} s'_8 &= move(s'_4, c) \\ &= \{\} \end{aligned}$$

Since s'_7 is the same this set is not included but s'_4 points to itself and that s'_8 is the same as s'_2, so we have no new sets and $S' = \{s'_0, s'_1, s'_2, s'_3, s'_4\}$.
For s'_5 we get

$$\begin{aligned} s'_6 &= move(s'_5, a) \\ &= \{8\} \end{aligned}$$

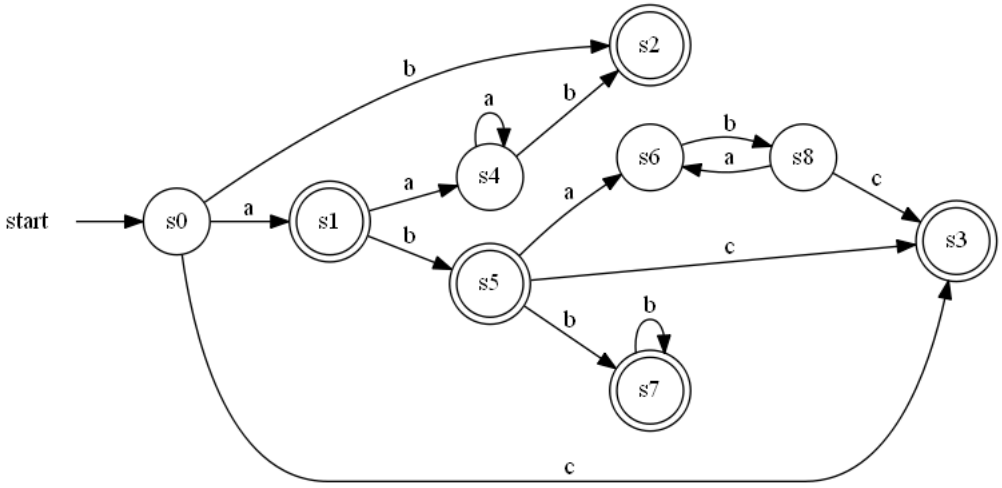
$$\begin{aligned} s'_7 &= move(s'_5, b) \\ &= \{2, 3\} \end{aligned}$$

$$\begin{aligned} s'_8 &= move(s'_5, c) \\ &= \{9\} \end{aligned}$$

The new sets are s'_6 and s'_7 and s'_8 is the same as s'_3. We can tell that s'_7 will point to itself if $move(s'_7, b)$ is called and the other sets are empty. s'_6 will make a new set if $move(s'_6, b)$ is called so

$$s'_8 = \{7\}$$

Which can point back to s'_6
So $S' = \{s'_1, s'_2, s'_3, s'_4, s'_5, s'_6, s'_7, s'_8\}$ and the constructed DFA is seen below



c

Describe the transitions and backtracking used when the combined DFA analyzes the input 'abababacca'.

It starts in s'_0
Goes to s'_1 (a), goes to s'_5 (b), goes to s'_6 (a), goes to s'_8 (b), goes to s'_6 (a)
And it cannot take a 'c' route and s'_6 is not an accepting state, so it backtracks to the last accepting state which is 'ab'

The input is then 'abacca'.
It goes to s'_1 (a), goes to s'_5 (b), goes to s'_6 (a).
And fails at 'c', and backtracks to the last accepting state which is 'ab'.

The input is then 'acca'.
It goes to s'_1 (a).
And there is no 'c' route but it is in an accepting state.

The input is then 'cca'.
It goes to s_3' (a).
And there is no 'c' route but it is in an accepting state.

The input is then 'ca'.
It goes to s_3' (a).
And there is no 'a' route but it is in an accepting state.

The input is then 'a'.
It goes to s_1' (a).
Which is the end and it is in an accepting state.

So it analyzes it to be 'ab', 'ab', 'a', 'c', 'c', 'a'.

Mosmllex Scanner Construction

The output for the scanner is: 'ab', 'ab', 'a', 'c', 'c', 'a'.
This is the same that was got in (2c).

More on Regular Languages and Tokenisation

a

Using the alphabet of decimal digits, give regular expressions describing the following languages

i: Numbers divisible by 5.

$$[0-9]^*[05]$$

ii: Numbers in which digit 5 occurs exactly three times.

$$[0-46-9]^*5[0-46-9]^*5[0-46-9]^*5[0-46-9]^*$$

b

Are the following languages over the alphabet of decimal digits regular? Give short convincing reasons for your answers.

i: Numbers which contain digit '1' exactly as many times as digit '2'.

This language is not regular since it can contain infinite 1's and infinite 2's and you cannot match on a number that potentially can be infinitely long.

ii: Numbers N<1000000 which contain digit '1' exactly as often as digit '2'.

This language is regular since it has a finite amount of numbers (assuming these are natural numbers) so it is possible to match on a number.