Opgave 1

Opgave 2

a

Since we have the definition $\Delta a(n) = a(n+1) - a(n)$ (2.42) and $(-1)^n$ is alternating 1 (even) and -1 (odd), we see that for an odd n we get $\Delta a(n) = 1 - (-1) = 2$ and for en even n we get $\Delta a(n) = -1 - 1 = -2$ so we simply get

$$a(n) = -2(-1)^n$$

To define g we simply find the inverse of Δa, which is

$$g(n) = \frac{1}{-2(-1)^n}$$

Where $\Delta g = -\frac{1}{2} - \frac{1}{2} = -1$ when n is odd and $\Delta g = \frac{1}{2} + \frac{1}{2} = 1$ when n is even.

b

dsa

c

Using the formula from (b) and using that $\Delta k = 1$, we get the following

$$\begin{aligned}
\sum (-1)^k k \ \delta k &= -\frac{1}{2}((-1)^k k + \sum (-1)^k \ \delta k \\
&= -\frac{1}{2}[(-1)^k k + \frac{(-1)^k}{-2}]_0^{n+1} \\
&= -\frac{1}{2}[(-1)^{n+1}(n+1) + \frac{(-1)^{n+1}}{-2} - \frac{(-1)^0}{-2}] \\
&= -\frac{1}{2}[(-1)^{n+1}(n+1) - \frac{(-1)^{n+1}}{2} + \frac{1}{2}] \\
&= -\frac{(-1)^{n+1}(n+1)}{2} + \frac{(-1)^{n+1} - 1}{4} \\
&= \frac{(-1)^n(n+1)}{2} + \frac{(-1)^{n+1} - 1}{4}
\end{aligned}$$

For the second summation, we use the same approach with $\Delta k^2 = 2k+1$, so

$$\begin{aligned}
\sum(-1)^k k^2 \ \delta k &= -\frac{1}{2}((-1)^k k^2 + \sum(-1)^k(2k+1) \ \delta k \\
&= -\frac{1}{2}[(-1)^k k^2 + 2\sum(-1)^k k + \sum(-1)^k]_0^{n+1} \\
&= -(-1)^n(n+1) - \frac{(-1)^{n+1}-1}{2} - \frac{1}{2}[(-1)^k k^2 + \sum(-1)^k]_0^{n+1} \\
&= -(-1)^n(n+1) - \frac{(-1)^{n+1}-1}{2} - \frac{1}{2}[(-1)^k k^2 + \frac{(-1)^k+1}{2}]_0^{n+1} \\
&= -(-1)^n(n+1) - \frac{(-1)^{n+1}-1}{2} - \frac{(-1)^{n+1}(n+1)^2}{2} - \frac{(-1)^{n+1}+1}{4} + \frac{1}{2} \\
&= -(-1)^n(n+1) - \frac{(-1)^{n+1}-1}{2} - \frac{(-1)^{n+1}(n+1)^2}{2} - \frac{(-1)^{n+1}+1}{4} + \frac{1}{2}
\end{aligned}$$