```
> restart: with(LinearAlgebra):
> GausElim:=proc(n,M)
    local a,i,j,k,l,z;
    l:=0;
    for k from 1 to n-1 do
      for i from k+1 to n do
        l:=l+1;
        z:=(M[i,k])/(M[k,k]);
        M[i,k]:=z*(m);
        for j from k+1 to n do
          M[i,j]:=(M[i,j]-(z*M[k,j]));
        end do;
      end do;
    end do;
    print(M);
  end proc:

> GausElimPiv:=proc(n,M)
    local i,j,k,p,s,M2,x,y,z;
    for i from 1 to n do
      s[i]:=0;
      p[i]:=i;
      for j from 1 to n do
        if abs(M[i,j]) > s[i] then s[i]:=abs(M[i,j]); end if;
      end do;
    end do;
    for k from 1 to n-1 do
      z:=0;
      for i from k to n do
        if M[i,k]/s[i] > z then z:=M[i,k]/s[i]; y:=i end if;
      end do;
      x:=p[k];
      p[k]:=p[y];
      p[y]:=x;
      for i from k+1 to n do
        z:=(M[p[i],k])/(M[p[k],k]);
        M[p[i],k]:=z*(m);
        for j from k+1 to n do
          M[p[i],j]:=(M[p[i],j]-(z*M[p[k],j]));
        end do;
      end do;
    end do;
    M2:=Matrix(n);
    for i from 1 to n do
      M2[i,p[i]]:=1;
    end do;
    print(M2);
    print(M);
  end proc:

> a:=<<-1,2,3>|<1,2,3>|<-4,0,2>>:
> b:=<<1,2,0>|<6,1,2>|<0,0,1>>:
> c:=<<-1,1,0,3>|<1,0,1,0>|<0,3,-1,1>|<-3,1,-1,2>>:
> d:=<<6,12,3,-6>|<-2,-8,-13,4>|<2,4,3,2>|<4,10,3,-18>>:
> e:=<<1,4,8,2>|<0,-9,16,3>|<2,2,6,2>|<1,1,5,1>>:
```

```
> 4.3.1b)
```

```
> GausElim(3,b);
```

$$\begin{bmatrix} 1 & 6 & 0 \\ 2\,m & -11 & 0 \\ 0 & -\frac{2}{11}\,m & 1 \end{bmatrix} \tag{1}$$

```
> ReducedRowEchelonForm(<b|<3,1,1>>);
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{11} \\ 0 & 1 & 0 & \frac{5}{11} \\ 0 & 0 & 1 & \frac{1}{11} \end{bmatrix} \tag{2}$$

```
> GausElimPiv(3,b);
```

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\,m & \frac{11}{4}\,m & -\frac{11}{4} \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \tag{3}$$

```
> 4.3.1c)
```

```
> GausElim(4,c);
```

$$\begin{bmatrix} -1 & 1 & 0 & -3 \\ -m & 1 & 3 & -2 \\ 0 & m & -4 & 1 \\ -3\,m & 3\,m & 2\,m & -3 \end{bmatrix} \tag{4}$$

```
> ReducedRowEchelonForm(<c|<4,0,3,1>>);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{5}$$

```
> GausElimPiv(4,c);
```

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{3}m & m & \frac{4}{3} & -\frac{4}{3} \\ \frac{1}{3}m & 0 & 2m & 3 \\ 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 \end{bmatrix} \quad (6)$$

```
> 4.3.1d)
> GausElim(4,d);
```

$$\begin{bmatrix} 6 & -2 & 2 & 4 \\ 2m & -4 & 0 & 2 \\ \frac{1}{2}m & 3m & 2 & -5 \\ -m & -\frac{1}{2}m & 2m & -3 \end{bmatrix} \quad (7)$$

```
> ReducedRowEchelonForm(<d|<0,-10,-39,-16>>);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (8)$$

```
> GausElimPiv(4,d);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & 2 & 4 \\ 2m & -2m & \frac{4}{13}m & -\frac{6}{13} \\ \frac{1}{2}m & -6m & 26 & -83 \\ -m & 2 & 4 & -14 \end{bmatrix} \quad (9)$$

```
> 4.3.1e)
> GausElim(4,e);
```

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 4\,m & -9 & -6 & -3 \\ 8\,m & -\frac{16}{9}\,m & -\frac{62}{3} & -\frac{25}{3} \\ 2\,m & -\frac{1}{3}\,m & \frac{6}{31}\,m & -\frac{12}{31} \end{bmatrix} \tag{10}$$

```
> ReducedRowEchelonForm(<e|<2,14,-3,0>>);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{11}$$

```
> GausElimPiv(4,e);
```

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\,m & -\frac{3}{2} & 1 & \frac{1}{2} \\ 2\,m & 10\,m & -12 & -6 \\ 4\,m & -\frac{8}{3}\,m & -\frac{1}{18}\,m & 2 \\ 2 & 3 & 2 & 1 \end{bmatrix} \tag{12}$$