Combinatorics
Assignment 1

Nikolaj Dybdahl Rathcke (Student ID: 74763954)

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Question 2

TODO

Question 3

Part (a)

TODO

Part (b)

TODO

Part (c)

TODO

Question 4

TODO

Question 5

TODO

Question 6

Let the ground set $E = \{1, 2, 3\}$. Let $M_1 = (E, \mathcal{I}(M_1))$ and $M_2 = (E, \mathcal{I}(M_2))$ be the matroids where:

$$\mathcal{I}(M_1) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}\}$$
$$\mathcal{I}(M_2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}\}$$

We can see that M_1 and M_2 are indeed matroids as they satisfy $(I1), (I2)$ and $(I3)$. However, if we look at $M_3 = (E, \mathcal{I}(M_3)) = (E, \mathcal{I}(M_1) \cap \mathcal{I}(M_2))$, it has following collection of subsets:

$$\mathcal{I}(M_3) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}\}$$

This is not a matroid as it does not satisfy $(I3)$. To see this, consider $I_1 = \{2\}$ and $I_2 = \{1, 3\}$. There exists no element $e \in I_2 - I_1 = \{1, 3\}$ such that $I_1 \cup \{e\} \in \mathcal{I}(M_3)$. That is, the subsets $\{1, 2\}$ and $\{2, 3\}$ do not exist in $\mathcal{I}(M_3)$.

Question 7

This is easiest shown with a counter-example. Consider the matrix:

$$A = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Where E is the set $\{1, 2, 3\}$ of column labels with the independent sets $\mathcal{I} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}.\{2, 3\}\}$. Now let $\mathcal{I}^- = \mathcal{I} \cup \{1, 2, 3\}$. This includes the dependent subset $\{1, 2, 3\}$, but it still satisfies $(I1), (I2)$ and $(I3)^-$.
It obviously satisfy $(I1)$ and $(I2)$ as I^- is the powerset of E, that is, all the possible subsets of E. Likewise, it is easy to see that the set $\mathcal{I}^-$ satisfies $(I3)^-$, since if we have two subsets I_1 and I_2 where $|I_1| < |I_2$, then you can add any element $e \in E$ (we are guaranteed there is one) to I_1 and that set will be in I^- as it is the powerset of E.