TEX Cheatsheet for proof boxes

Reduced from Paul Taylor's paper

3 September 2014

1 Miscellaneous logical notations

This is how the basic signs are written:

`\elim\forall` $\forall\mathcal{E}$ `\intro\land` $\land\mathcal{I}$

Default TEX logical connectives and quantifiers:

`\lor` $\lor$ `\land` $\land$ `\lnot` $\neg$ `\forall` $\forall$ `\exists` $\exists$

Two ways of writing implication:

`A\implies B` $A \to B$ *versus* `A\implic B` $A \to B$

2 Cheat sheet

2.1 Here you should write the ex. number you are solving

1	$P \land Q$	premise
2	P	$\land\mathcal{E}$

2.2

1	P	premise
2	Q	assumption
3	P	(1)
4	$Q \to P$	$\to\mathcal{I}$

2.3

1	P	premise
2	Q	assumption
3	$P \land Q$	$\land\mathcal{I}(1, 2)$
4	$Q \to (P \land Q)$	$\to\mathcal{I}$

2.4

1	$P \to (Q \to R)$	premise
2	$P \to Q$	assumption
3	P	assumption
4	Q	$\to\mathcal{E}(2,3)$
5	$Q \to R$	$\to\mathcal{E}(1,3)$
6	R	$\to\mathcal{E}(5,4)$
7	$P \to R$	$\to\mathcal{I}$
8	$(P \to Q) \to (P \to R)$	$\to\mathcal{I}$

2.5

1	$P \to (Q \to R)$	premise
2	$P \wedge Q$	assumption
3	P	$\wedge\mathcal{E}1(2)$
4	$Q \to R$	$\to\mathcal{E}(1,3)$
5	Q	$\wedge\mathcal{E}2(2)$
6	R	$\to\mathcal{E}(4,5)$
7	$P \wedge Q \to R$	$\to\mathcal{I}$

2.6

1	$P \wedge Q \to R$	premise
2	P	assumption
3	Q	assumption
4	$P \wedge Q$	$\wedge\mathcal{I}(2,3)$
5	R	$\to\mathcal{E}(1,4)$
6	$Q \to R$	$\to\mathcal{I}$
7	$P \to (Q \to R)$	$\to\mathcal{I}$

2.7

1	$P \to Q$	premise
2	$\neg Q$	
3	P	assumption
4	Q	$\to\mathcal{E}(1,3)$
5	$\bot$	$\neg\mathcal{E}(2,4)$
6	$\neg P$	$\neg\mathcal{I}$

2.8

1	$\neg P$	premise
2	P	assumption
3	$\neg Q$	assumption
4	$\neg P \wedge P$	$\wedge\mathcal{I}(1,2)$
5	$\bot$	$\neg\mathcal{E}(1,2)$
6	$\neg\neg Q$	$\neg\mathcal{I}$
7	Q	$\neg\neg$
8	$P \to Q$	$\to\mathcal{I}$

2.9

1	$P \to Q$	premise
2	$\neg Q$	assumption
3	P	assumption
4	Q	$\to\mathcal{E}(1,3)$
5	$\bot$	$\neg\mathcal{E}(2,4)$
6	$\neg P$	$\neg\mathcal{I}$
7	$\neg Q \to \neg P$	$\to\mathcal{I}$

2.10

1	$P \to Q$	premise
2	$\neg\neg P$	assumption
3	P	$\neg\neg$
4	$\neg Q$	assumption
5	Q	$\to\mathcal{E}(1,3)$
6	$Q \wedge \neg Q$	$\wedge\mathcal{I}(5,4)$
7	$\bot$	$\neg\mathcal{E}$
8	$\neg\neg Q$	$\neg\mathcal{I}$
9	$\neg\neg P \to \neg\neg Q$	$\to\mathcal{I}$

2.11

1	$P \vee \neg P$			premise
2	P	assumption	$\neg P$	assumption
3	$Q \to P$	by (e)	$P \to Q$	by (k)
4	$(P \to Q) \vee (Q \to P)$	$\vee\mathcal{I}$	$(P \to Q) \vee (P \to Q)$	$\vee\mathcal{I}$
5	$(P \to Q) \vee (P \to Q)$			$\vee\mathcal{E}$

2.12

1	$P \vee Q$			premise
2	$(\neg P) \wedge (\neg Q)$			assumption
3	P	assumption	Q	assumption
4	$\neg P$	$\wedge\mathcal{E}1(2)$	$\neg Q$	$\wedge\mathcal{E}2(2)$
5	$\perp$	$\neg\mathcal{E}(4,3)$	$\perp$	$\neg\mathcal{E}(3,4)$
6	$\perp$			$\vee\mathcal{E}(1)$
7	$\neg((\neg P) \wedge (\neg Q))$			$\neg\mathcal{I}$

2.13 We will not require anything that is this complicated

$\forall x :$	1	$\forall x'.x' \prec x \to \phi(x')$	induction hypothesis
	2	$\vdots$	
	3	$u \prec x$	various terms u
	4	$\phi(u)$	$\forall\mathcal{E}(1,3)$
	5	$\vdots$	
	6	$\phi(x)$	the property
	7	$\forall x.(\forall x'.x' \prec x \to \phi(x')) \to \phi(x)$	$\forall\mathcal{I}$
	8	$\forall x.\phi(x)$	$\prec$-induction for ϕ

1	$\phi(0)$	z
2	$\forall n.\phi(n) \to \phi(n+1)$	s
3	$\phi(0) \to \phi(1)$	$\forall\mathcal{E}(2)$
4	$\phi(1)$	$\to\mathcal{E}(3,1)$
5	$\phi(1) \to \phi(2)$	$\forall\mathcal{E}(2)$
6	$\phi(2)$	$\to\mathcal{E}(5,4)$
7	$\phi(2) \to \phi(3)$	$\forall\mathcal{E}(2)$
8	$\phi(3)$	$\to\mathcal{E}(7,6)$

	1	$\forall x.(\forall x'.x' < x \to \phi(x')) \to \phi(x)$	hypothesis
	2	$\psi(y) = \forall x.(fx = y) \to \phi(x)$	definition
$\forall y :$	3	$\forall y'.y' \prec y \to \psi(y')$	assumption
$\forall x :$	4	$fx = y$	assumption
$\forall x' :$	5	$x' < x$	assumption
	6	$fx' \prec y$	monotonicity
	7	$\psi(fx')$	$\forall\mathcal{E}(3)$
	8	$\phi(x')$	$\forall\mathcal{E}(\text{def } 2, 4)$
	9	$\forall x'.x' < x \to \phi(x')$	$\forall\mathcal{I}$
	10	$\phi(x)$	$\forall\mathcal{E}(1)$
	11	$\forall x.(fx = y) \to \phi(x)$	$\forall\mathcal{I}$
	12	$\psi(y)$	def(2)
	13	$\forall y.(\forall y'.y' \prec y \to \psi(y')) \to \psi(y)$	$\forall\mathcal{I}$
	14	$\forall y.\psi(y)$	$(Y, \prec)$-induction

	1	$\forall y.[\forall y'.y' \epsilon\, y \to \phi(y')] \to \phi(y)$	
$\forall x :$	2	$\forall x'.x' \prec x \to \phi(fx')$	assumption
$\forall y' :$	3	$y' \epsilon\, fx$	assumption
	4	$\exists x'.x' \prec x \wedge y' = fx'$	surj on pred
$\exists x' :$	5	$x' \prec x$	assumption
	6	$y' = fx'$	
	7	$\phi(fx')$	$\forall\mathcal{E}(2)$
	8	$\phi(y')$	substitution
	9	$\phi(y')$	$\exists\mathcal{E}(4)$
	10	$\forall y'.y' \epsilon\, fx \to \phi(y')$	$\forall\mathcal{I}$
	11	$\phi(fx)$	$\forall\mathcal{E}(1, y := fx)$
	12	$\forall x.[\forall x'.x' \prec x \to \phi(fx')] \to \phi(fx)$	$\forall\mathcal{I}$