



1. The connection of the road network is specified as a matrix  $M$  (filename is “road.txt”). There are 10 sites indexed as  $(0, 1, \dots, 9)$ . Entry  $M(i, j)$  denotes the length of the road between site  $i$  and  $j$  in  $kms$  (the roads are uni-directional). The roads are uni-directional (i.e., there could be a road from  $A$  to  $B$ , but not from  $B$  to  $A$ ). If the entry is 0, it means no connection. Vehicles depart from a source and then take a path (filename “vehicle.txt”). The time of departure (in minutes) is given in filename “time.txt”,  $n^{\text{th}}$  vehicle departs at time instance  $n$ . The speed (in kmph) of the vehicle in a road is given by the formula  $e^{0.5x}/(1 + e^{0.5x}) + 15/(1 + e^{0.5x})$ , where  $x$  is the number of vehicles ahead in a given road. If there are  $n$  vehicles in the road, then for the first vehicle  $x = 0$ , second vehicle  $x = 1$ , and for the last vehicle  $x = n - 1$ . The time taken to travel the road is  $\text{dist}/\text{speed}$ . Implement a function that takes time as input and reports the position of various vehicles at that given time. [20]

2. There is an under-powered car stuck in the bottom of a 1-dim valley. It needs to find its way to the top. The car has three actions namely  $A = \{-1, 0, +1\}$  which means accelerate backward, no acceleration and accelerate forward, respectively. A chosen action is applied for a fixed duration of time, after which a new action is chosen. The ranges for position and velocity are  $[-1.2, 0.5]$  and  $[-0.07, 0.07]$ , respectively. The car is needs to reach the top on the right, i.e., position of 0.5. The dynamics is according to the equations:

$$v_{n+1} = v_n + 0.001a_n - 0.0025 \cos(3p_n)$$

$$p_{n+1} = p_n + v_n$$

- (a) Implement a function which takes in the current position and action and outputs the position and velocity at the next step. [25]
- (b) Implement a random agent which at each time instant chooses one of the three actions uniformly at random. Plot  $\{(p_n, v_n), n \geq 1\}$  produced by the random agent. Take the initial position to be  $-0.5$ , and initial velocity to be 0. [15]
- (c) Implement another agent which accelerates in the direction of the current velocity. Plot  $\{(p_n, v_n), n \geq 1\}$  produced by this agent. Take the initial position to be  $-0.5$ , and initial velocity to be 0. [10]
3. Read the text file “speeches.txt”
- (a) Compute the next word probabilities. [10]
- (b) Based on the next word probabilities, write down a code that will produce random text of length 5000. [10]
- (c) Compute the next word probability from the previous task and show that it converges to the next word probabilities calculated from the original text. [10]