

## **ICPC ZetaSquad Notebook (2019-20)**

**<< Ahmed Zaheer Dadarkar, Devansh Singh Rathore, Rakesh Kumar >>**

***Indian Institute of Technology, Palakkad***

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## 1. GRAPH :

### a. All Topological Orderings (Kahn algorithm similarity): $O((n + m) \cdot n)$

```
vector<vector<int>> > a; vector<int> topo;
int n, indeg[N];
void all_t_orders() {
    bool foundIndegZero = false;
    for(int u = 0; u < n; u++) {
        if(indeg[u] == 0) {
            foundIndegZero = true;
            indeg[u]--;
            for(int v : a[u]) indeg[v]--;
            topo.push_back(u);

            all_t_orders();

            indeg[u]++;
            for(int v : a[u]) indeg[v]++;
            topo.pop_back();
        }
    }
    if(!foundIndegZero) { printVec(topo); // We have a topo ordering }
}
```

### b. Bipartite Check: $O(n + m)$

```
vector<vector<int>> > a;
int color[N];
bool isBipartite;

void dfs(int u) {
    for(int v : a[u]) {
        if(color[v] == -1) {
            color[v] = 1 - color[u];
            dfs(v);
        }
        else if(color[v] == color[u]) isBipartite = false;
    }
}

void bip_check()
{
```

```

    isBipartite = true;
    for(int u = 0; u < n; u++) {
        if(color[u] == -1) { color[u] = 0; dfs(u); }
    }
}

```

**c. Cycle check / Backedge detection in an undirected gph:  $O(n + m)$**

```

vector<vector<int>> > a;
bool vis[N];
bool cycleFound;

void dfs(int u, int parent) {
    vis[u] = true;
    for(int v : a[u]) {
        if(!vis[v]) dfs(v, u); // tree edge
        else if(v != parent) cycleFound = true; // (u, v) is a backedge
    }
}

void cycle_det() {
    cycleFound = false;
    for(int u = 0; u < n; u++) {
        if(!vis[u]) dfs(u, -1);
    }
}

```

**d. Cycle check / Backedge detection in a directed gph:  $O(n + m)$**

```

vector<vector<int>> > a;
int state[N];
bool cycleFound;

void dfs(int u) {
    state[u] = 1;
    for(int v : a[u]) {
        if(state[v] == 0) dfs(v); // Tree Edge
        else if(state[v] == 1) cycleFound = true; // Back Edge
        // state value of 2 means Forward or Cross edge
    }
    state[u] = 2;
}

```

```

void cycle_det() {
    cycleFound = false;
    for(int u = 0; u < n; u++) if(state[u] == 0) dfs(u);
}

```

**e. Articulation points and Bridges, Undirected:  $O(n + m)$**

```

vector<vector<int>> a; vector<pair<int, int>> br;
int dfs_low[N], dfs_num[N], numChildren, ct;
bool art_p[N];

void dfs(int u, int parent) {
    dfs_num[u] = dfs_low[u] = ct++;
    for(int v : a[u]) {
        if(dfs_num[v] == -1) {
            if(parent == -1) numChildren++;
            dfs(v);
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
            if(parent != -1 && dfs_low[v] >= dfs_num[u]) art_pt[u] = true;
            if(dfs_low[v] > dfs_num[u]) br.push_back({u, v});
        }
        else if(v != parent) dfs_low[u] = min(dfs_low[u], dfs_num[v]);
    }
}

void findArtBr() {
    ct = 0;
    for(int u = 0; u < n; u++) {
        if(dfs_num[u] == -1) {
            numChildren = 0;
            dfs(u, -1);
            if(numChildren > 1) art_pt[u] = true;
        }
    }
}

```

**f. Finding SCC - Tarjan:  $O(n + m)$**

```

vector<vector<int>> a;
int dfs_low[N], dfs_num[N], state[N], scc[N], scc_no;
stack<int> s;

void dfs(int u) {

```

```

    dfs_num[u] = dfs_low[u] = ct++;
    state[u] = 1;
    s.push(s);
    for(int v : a[u]) {
        if(state[v] == 0) dfs(u);
        if(state[v] == 1) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
    }
    if(dfs_low[u] == dfs_num[u]) {
        int v = -1;
        while(v != u) {
            v = s.top();
            s.pop();
            state[v] = 2;
            scc[v] = scc_no;
        }
        scc_no++;
    }
}

void findSCC() {
    for(int u = 0; u < n; u++) if(state[u] == 0) dfs(u);
}

```

**g. Prim's Algorithm MST:  $O(m \cdot \log(n))$**

```

priority_queue<pair<int, int>, vector<pair<int, int> >, greater<pair<int, int> > > pq;
vector<vector<pair<int, int> > > a; // first: weight, second: vertex
bool taken[N];

```

```

void printMST() {
    int mst_wt = 0;
    pq.push({0, 0});
    while(!pq.empty()) {
        pair<int, int> u = pq.top();
        pq.pop();
        if(taken[u.second]) continue;
        taken[u.second] = true;
        mst_cost += u.first;
        for(pair v : a[u.second]) if(!taken[v.second]) pq.push(v);
    }
}

```

**h. Floyd Warshall with path information:  $O(n^3)$** 

```

int n, adj[N][N], dist[N][N], p[N][N];
// adj is initialized with INF for edges which don't exist
void FW() {
    int k, i, j;
    for(i = 0; i < n; i++) {
        for(j = 0; j < n; j++) {
            dist[i][j] = adj[i][j];
            if(adj[i][j] != INF) p[i][j] = i;
        }
    }
    for(k = 0; k < n; k++) {
        for(i = 0; i < n; i++) {
            for(j = 0; j < n; j++) {
                if(dist[i][k] + dist[k][j] < dist[i][j])
                {
                    dist[i][j] = dist[i][k] + dist[k][j];
                    p[i][j] = p[k][j];
                }
            }
        }
    }
}

void printPath(int s, int v) {
    if(v == s) {cout<<v<<" ";return; }
    printPath(s, p[s][v]); cout<<v<<" ";
}

```

**i. Finding Diameter in a Tree:  $O(n)$** 

```

vector<vector<pair<int, int> > > a;
int n, dist[N];
void dfs(int u) {
    for(pair<int, int> v : a[u]) {
        if(dist[v.second] == -1) { dist[v.second] = dist[u] + 1; dfs(v.second); }
    }
}

void findDiameterTree(){
    resetDist(); dfs(0);
    int s = 0, t = 0;
    for(int i = 1; i < n; i++) if(dist[i] > dist[s]) s = i;
    resetDist(); dfs(s);
    for(int i = 1; i < n; i++) if(dist[i] > dist[t]) t = i;
    int dia = dist[t];
}

```

```
}

```

**j. Edmonds Karp:  $O(n \cdot (m^2) + n^2)$**

```
vector<vector<int>> > a;
queue<int> q;
int s, t, f, n, res[N][N];
bool vis[N];

void augment(int v, int minEdge) {
    if(v == s) { f = minEdge; return; }
    augment(p[v], min(minEdge, res[p[v]][v]));
    res[p[v]][v] -= f;
    res[v][p[v]] += f;
}

void EdmondsKarp() {
    int mf = 0;
    while(true) {
        f = 0;
        resetBFS();
        vis[s] = true; q.push(s);
        while(!q.empty()) {
            int u = q.front();
            q.pop();
            if(u == t) { augment(t, INF); break; }
            for(int v : a[u]) {
                if(res[u][v] > 0 && !vis[v]) {
                    vis[v] = true;
                    q.push(v);
                }
            }
            if(f == 0) break;
            mf += f;
        }
        cout<<mf<<"\n";
    }
}
```

**k. Euler's Path:  $O(m)$**

```
list cyc; // we need list for fast insertion in the middle
void EulerTour(list::iterator i, int u) {
    for (int j = 0; j < (int)AdjList[u].size(); j++) {
```

```

        ii v = AdjList[u][j];
        if (v.second) { // if this edge can still be used/not removed
            v.second = 0; // make the weight of this edge to be 0 ('removed')
            for (int k = 0; k < (int)AdjList[v.first].size(); k++) {
                ii uu = AdjList[v.first][k]; // remove bi-directional edge
                if (uu.first == u && uu.second) { uu.second = 0; break; }
            }
            EulerTour(cyc.insert(i, u), v.first);
        }
    }
}
//inside int main()
    cyc.clear();
    EulerTour(cyc.begin(), A); // cyc contains an Euler tour starting at A
    for (list::iterator it = cyc.begin(); it != cyc.end(); it++)
        printf("%d\n", *it); //Euler tour

```

#### I. MCBM: $O(n*m)$

//but Hopcroft Karp's algorithm that can solve the MCBM problem in  $O(\sqrt{V}E)$

vi match, vis; // global variables

```

int Aug(int l) { // return 1 if an augmenting path is found
    if (vis[l]) return 0; // return 0 otherwise
    vis[l] = 1;
    for (int j = 0; j < (int)AdjList[l].size(); j++) {
        int r = AdjList[l][j]; // edge weight not needed -> vector AdjList
        if (match[r] == -1 || Aug(match[r])) {
            match[r] = l; return 1; // found 1 matching
        }
    }
    return 0; // no matching
}

```

```

// inside int main()
    // build unweighted bipartite graph with directed edge left->right set
    int MCBM = 0;
    match.assign(V, -1); // V is the number of vertices in bipartite graph
    for (int l = 0; l < n; l++) { // n = size of the left set
        vis.assign(n, 0); // reset before each recursion
        MCBM += Aug(l);
    }
    printf("Found %d matchings\n", MCBM);

```



**m. TSP:  $O((n^2) \cdot (2^n))$** 

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
struct tsp {
```

```
    using ll = long long;
```

```
    static const int N = 17;
```

```
    const ll inf = (ll) 1e18;
```

```
    ll dp[N][1 << N];
```

```
    int par[N][1 << N], start[N][1 << N], wgt[N][N];
```

```
    int n;
```

```
    ll solve() {
```

```
        // for (int k = 0; k < n; ++k) //Floyd-Warshall for shortest dist wgts (undirected)
```

```
        // for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)
```

```
        // wgt[j][i] = wgt[i][j] = min(wgt[i][j], wgt[i][k] + wgt[k][j]);
```

```
        for (int mask = 0; mask < (1 << n); ++mask)
```

```
            for (int i = 0; i < n; ++i)
```

```
                dp[i][mask] = inf;
```

```
        for (int i = 0; i < n; ++i)
```

```
            dp[i][1 << i] = 0, par[i][1 << i] = -1, start[i][1 << i] = i;
```

```
        for (int mask = 1; mask < (1 << n); ++mask) {
```

```
            for (int pos = 0; pos < n; ++pos)
```

```
            {
```

```
                if (!(mask & (1 << pos))) continue;
```

```
                ll mn_val = inf;
```

```
                for (int prv = 0; prv < n; ++prv) {
```

```
                    if (pos == prv || !(mask & (1 << prv)) || !wgt[prv][pos]) continue;
```

```
                    ll guess = dp[prv][mask-(1<<pos)] + wgt[prv][pos];
```

```
                    if (mn_val > guess) {
```

```
                        mn_val = guess;
```

```
                        par[pos][mask] = prv;
```

```
                        start[pos][mask] = start[prv][mask-(1 << pos)];
```

```
                    }}
```

```
                dp[pos][mask] = min(dp[pos][mask], mn_val);
```

```

    }}

    ll mn_cycle = inf;
    int pos = -1, mask = (1 << n) - 1;
    for (int i = 0; i < n; ++i) {
        ll go = dp[i][mask] + wgt[i][start[i][mask]];
        if (mn_cycle > go) {
            mn_cycle = go;
            pos = i;
        }
    }

    vector<int> cycle;
    while (pos != -1) {
        cycle.push_back(pos);
        int tmp = pos;
        pos = par[pos][mask];
        mask -= (1 << tmp);
    }
    for (int v : cycle) { cerr << v << ' '; } cerr << '\n';
    return mn_cycle;
}
};

```

#### n. 2-SAT

```

int n;
vector<vector<int>>> g, gt;
vector<bool> used;
vector<int> order, comp;
vector<bool> assignment;
void dfs1(int v) {
    used[v] = true;
    for (int u : g[v]) {
        if (!used[u])
            dfs1(u);
    }
    order.push_back(v);
}
void dfs2(int v, int cl) {
    comp[v] = cl;
    for (int u : gt[v]) {

```

```

        if (comp[u] == -1)
            dfs2(u, cl);
    }
}
bool solve_2SAT() {
    used.assign(n, false);
    for (int i = 0; i < n; ++i) {
        if (!used[i])
            dfs1(i);
    }
    comp.assign(n, -1);
    for (int i = 0, j = 0; i < n; ++i) {
        int v = order[n - i - 1];
        if (comp[v] == -1)
            dfs2(v, j++);
    }
    assignment.assign(n / 2, false);
    for (int i = 0; i < n; i += 2) {
        if (comp[i] == comp[i + 1])
            return false;
        assignment[i / 2] = comp[i] > comp[i + 1];
    }
    return true; }

```

**o. Heavy Light Decomposition**

```

vector<int> parent, depth, heavy, head, pos;
int cur_pos;
int dfs(int v, vector<vector<int>> const& adj) {
    int size = 1;
    int max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;
            int c_size = dfs(c, adj);
            size += c_size;
            if (c_size > max_c_size)
                max_c_size = c_size, heavy[v] = c;
        }
    }
}

```

```

    return size;
}
int decompose(int v, int h, vector<vector<int>> const& adj) {
    head[v] = h, pos[v] = cur_pos++;
    if (heavy[v] != -1)
        decompose(heavy[v], h, adj);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c, adj);
    }
}
void init(vector<vector<int>> const& adj) {
    int n = adj.size();
    parent = vector<int>(n);
    depth = vector<int>(n);
    heavy = vector<int>(n, -1);
    head = vector<int>(n);
    pos = vector<int>(n);
    cur_pos = 0;

    dfs(0, adj);
    decompose(0, 0, adj);
}
int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max = segment_tree_query(pos[head[b]], pos[b]);
        res = max(res, cur_heavy_path_max);
    }
    if (depth[a] > depth[b])
        swap(a, b);
    int last_heavy_path_max = segment_tree_query(pos[a], pos[b]);
    res = max(res, last_heavy_path_max);
    return res;
}

```

#### p. Binary Lifting

```

vector<int> tin, tout;
vector<vector<int>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool is_ancestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v)
{
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

```

## 2. STRINGS:

### a. Knuth-Morris-Pratt's (KMP) Algorithm: $O(n + m)$

```
void kmpPreprocess_Search(string T, string P, int n, int m)
```

```

{
    //Preprocess =====>
    // b = back table, n = length of T, m = length of P
    int i=0, j=-1, b[n+100];
    b[0]=-1;
    while(i<m) {
        while(j>=0 && P[i]!=P[j]) j=b[j];
        i++;j++;
        b[i]=j;
    }

    //Search =====>
    i=0;j=0;
    while(i<n) {
        while(j>=0 && T[i]!=P[j]) j=b[j];
        i++;j++;
        if(j==m) {
            cout<<"P found at index "<<i-j<<" in T\n";
            j=b[j];
        }
    }
}

void kmp(string T, string P) { //T is main string in which we search for string P
    int n=T.length(), m=P.length();
    kmpPreprocess_Search(T,P,n,m);
}

```

**b. Z - function:  $O(n)$**

```

vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
}

```

```
return z; }
```

**c. Trie:  $O(n \cdot \log(n))$**

```
#define MAX_N 100010    // second approach:  $O(n \log n)$ 
char T[MAX_N];         // the input string, up to 100K characters
int n;                 // the length of input string
int RA[MAX_N], tempRA[MAX_N];    // rank array and temporary rank array
int SA[MAX_N], tempSA[MAX_N];    // suffix array and temporary suffix array
int c[MAX_N];          // for counting/radix sort

void countingSort(int k) {    //  $O(n)$ 
    int i, sum, maxi = max(300, n);    // up to 255 ASCII chars or length of n
    memset(c, 0, sizeof c);           // clear frequency table
    for (i = 0; i < n; i++)            // count the frequency of each integer rank
        c[i + k < n ? RA[i + k] : 0]++;
    for (i = sum = 0; i < maxi; i++) { int t = c[i]; c[i] = sum; sum += t; }
    for (i = 0; i < n; i++)            // shuffle the suffix array if necessary
        tempSA[c[SA[i]+k < n ? RA[SA[i]+k] : 0]++] = SA[i];
    for (i = 0; i < n; i++)            // update the suffix array SA
        SA[i] = tempSA[i];
}

void constructSA() {          // this version can go up to 100000 characters
    int i, k, r;
    for (i = 0; i < n; i++) RA[i] = T[i];    // initial rankings
    for (i = 0; i < n; i++) SA[i] = i;        // initial SA: {0, 1, 2, ..., n-1}
    for (k = 1; k < n; k <= 1) { // repeat sorting process  $\log n$  times
        countingSort(k);    // actually radix sort: sort based on the second item
        countingSort(0);    // then (stable) sort based on the first item
        tempRA[SA[0]] = r = 0;    // re-ranking; start from rank  $r = 0$ 
        for (i = 1; i < n; i++)    // compare adjacent suffixes
            tempRA[SA[i]] = (RA[SA[i]] == RA[SA[i-1]] && RA[SA[i]+k]
            == RA[SA[i-1]+k]) ? r : ++r;
        // if same pair => same rank r; otherwise, increase r
        for (i = 0; i < n; i++) RA[i] = tempRA[i];    // update the rank array RA
        if (RA[SA[n-1]] == n-1) break;    // nice optimization trick
    }
}

int main() {
    n = (int)strlen(gets(T));    // input T as per normal, without the '$'
```

```

    T[n++] = '$';           // add terminating character
    constructSA();
    for (int i = 0; i < n; i++) printf("%2d\t%s\n", SA[i], T + SA[i]);
} // return 0;

```

### **Applications of Suffix Array ⇒**

**String Matching:  $O(m \cdot \log(n)) \rightarrow$**

```

vector<int> sort_cyclic_shifts(string const& s) {
    int n = s.size();
    const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++)
        cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }

    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)

```



```

        p[--cnt[c[pn[i]]]] = pn[i];
    cn[p[0]] = 0;
    classes = 1;
    for (int i = 1; i < n; i++) {
        pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
        pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h)) % n]};
        if (cur != prev)
            ++classes;
        cn[p[i]] = classes - 1;
    }
    c.swap(cn);
}
return p;
}

```

### **Finding the Longest Common Prefix: $O(n) \rightarrow$**

```

vector<int> lcp_construction(string const& s, vector<int> const& p) {
    int n = s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++)
        rank[p[i]] = i;

    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k)
            k--;
    }
    return lcp;
}

```

### Finding the Longest Repeated Substring: $O(n) \rightarrow (UVa)$

If we have computed the Suffix Array in  $O(n \log n)$  and the LCP between consecutive suffixes in Suffix Array order in  $O(n)$ , then we can determine the length of the Longest Repeated Substring (LRS) of  $T$  in  $O(n)$ .

The length of the longest repeated substring is just the highest number in the LCP array.

In Table 6.5—left that corresponds to the Suffix Array and the LCP of  $T =$

‘GATAGACAS’, the highest number is 2 at index  $i=7$ . The first 2 characters of the corresponding suffix  $SA[7]$  (suffix 0) is ‘GA’. This is the longest repeated substring in  $T$ .

### Finding the Longest Common Substring: $O(n) \rightarrow (UVa)$

Without loss of generality, let’s consider the case with only two strings. We use the same example as in the Suffix Tree section earlier:  $T1 = \text{‘GATAGACAS’}$  and  $T2 = \text{‘CATA#’}$ .

To solve the LCS problem using Suffix Array, first we have to concatenate both strings (note that the terminating characters of both strings must be different) to produce  $T = \text{‘GATAGACA$CATA#’}$ . Then, we compute the Suffix and LCP array of  $T$  as shown in Figure 6.6.

Then, we go through consecutive suffixes in  $O(n)$ . If two consecutive suffixes belong to different owner (can be easily checked, for example we can test if suffix  $SA[i]$  belongs to  $T1$  by testing if  $SA[i] < \text{the length of } T1$ ), we look at the LCP array and see if the maximum LCP found so far can be increased. After one  $O(n)$  pass, we will be able to determine the Longest Common Substring. In Figure 6.6, this happens when  $i=7$ , as suffix  $SA[7] = \text{suffix } 1 = \text{‘ATAGACA$CATA#’}$  (owned by  $T1$ ) and its previous suffix  $SA[6] = \text{suffix } 10 = \text{‘ATA#’}$  (owned by  $T2$ ) have a common prefix of length 3 which is ‘ATA’. This is the LCS.

## 3. MATHEMATICS:

### a. Power: $O(\log(n))$

```
int power(int x, unsigned int y) {    //to calculate power(x, n)
    int temp;
    if( y == 0) return 1;

    temp = power(x, y/2);
    if (y%2 == 0) return temp*temp;
    else return x*temp*temp;
}
```

### b. Recurrence relations

Catalan Number  $\rightarrow \text{Cat}(m) = (2m * (2m - 1) / (m * (m + 1))) * \text{Cat}(m-1)$

Combination  $\rightarrow C(n+1,k)=C(n,k)+C(n,k-1)$

**c. Linear Diophantine Equation:  $O(\log(n))$**

```
int gcd(int a, int b, int &x, int &y) {
    if(a == 0) {
        x = 0; y = 1; return b;
    }
    int x1, y1;
    int g = gcd(a % b, b, x1, y1);
    x = y1 - (b / a) * x1; y = x1;
    return g;
}

void find_a_Solution(int a, int b, int c) {
    int x0, y0;
    int g = gcd(abs(a), abs(b), x0, y0);
    if(c % g) return;      // no solution

    x0 *= c / g;
    y0 *= c / g;
    if(a < 0) x0 = -x0;
    if(b < 0) y0 = -y0;
}
```

We can find all solutions this way: -

$$x = x0 + k * (b / g)$$

$$y = y0 - k * (a / g)$$

**d. Chinese Remainder Theorem**

**CODE** $\rightarrow$

```
for (int i = 0; i < k; ++i) {
    x[i] = a[i];
    for (int j = 0; j < i; ++j) {
        x[i] = r[j][i] * (x[i] - x[j]);
        x[i] = x[i] % p[i];
        if (x[i] < 0) x[i] += p[i];
    }
}
```

#### 4. POLYHASH:

```

int count_unique_substrings(string const& s) {
    int n = s.size();
    const int p = 31;
    const int m = 1e9 + 9;
    vector<long long> p_pow(n);
    p_pow[0] = 1;
    for (int i = 1; i < n; i++)
        p_pow[i] = (p_pow[i-1] * p) % m;

    vector<long long> h(n + 1, 0);
    for (int i = 0; i < n; i++)
        h[i+1] = (h[i] + (s[i] - 'a' + 1) * p_pow[i]) % m;
    int cnt = 0;
    for (int l = 1; l <= n; l++) {
        set<long long> hs;
        for (int i = 0; i <= n - l; i++) {
            long long cur_h = (h[i + l] + m - h[i]) % m;
            cur_h = (cur_h * p_pow[n-i-1]) % m;
            hs.insert(cur_h);
        }
        cnt += hs.size();
    }
    return cnt;
}

```

#### 5. DATA STRUCTURES AND LIBRARIES:

##### a. Segment Tree

```

struct SegTree {
    static const int MXN = int(2e6) + 10;
    int A[MXN];
    long long st[4 * MXN], lz[4 * MXN];
    inline int mid (int s, int e) { return s + ((e - s) >> 1); }
    void create (int s, int e, int x) {
        lz[x] = 0;
        if (s == e) {
            st[x] = A[s];

```

```

        return;
    }
    create (s, mid (s, e), x + x + 1);
    create (mid (s, e) + 1, e, x + x + 2);
    st[x] = st[x + x + 1] + st[x + x + 2];
}

void lzupd (int s, int e, int x) {
    if (lz[x] != 0) {
        st[x] += lz[x] * (e - s + 1);
        if (s != e) {
            lz[x + x + 1] += lz[x];
            lz[x + x + 2] += lz[x];
        }
        lz[x] = 0;
    }
}

void upd (int s, int e, int qs, int qe, int x, int k) {
    lzupd (s, e, x);
    if (e < qs || s > qe) return;
    if (s >= qs && e <= qe) {
        st[x] += k * (e - s + 1);
        if (s != e) {
            lz[x + x + 1] += k;
            lz[x + x + 2] += k;
        }
    }
    return;
}

upd (s, mid (s, e), qs, qe, x + x + 1, k);
upd (mid (s, e) + 1, e, qs, qe, x + x + 2, k);
st[x] = st[x + x + 1] + st[x + x + 2];
}

long long gets (int s, int e, int qs, int qe, int x) {
    lzupd (s, e, x);
    if (e < qs || s > qe) return 0;
    if (s >= qs && e <= qe) return st[x];
    long long lgets = gets (s, mid (s, e), qs, qe, x + x + 1);
    long long rgets = gets (mid (s, e) + 1, e, qs, qe, x + x + 2);
    return lgets + rgets;
}
};

```

**b. Fenwick tree:  $O(n)$** 

```

class FenwickTree {
    private: vi ft; // recall that vi is: typedef vector vi;
    public: FenwickTree(int n) { ft.assign(n + 1, 0); } // init n + 1 zeroes
    int rsq(int b) { // returns RSQ(1, b)
        int sum = 0;
        for (; b != LSONe(b)) sum += ft[b];
        return sum;
    } // note: LSONe(S) (S & (-S))
    int rsq(int a, int b) { // returns RSQ(a, b)
        return rsq(b) - (a == 1 ? 0 : rsq(a - 1));
    } // adjusts value of the k-th element by v (v can be +ve/inc or -ve/dec)
    void adjust(int k, int v) { // note: n = ft.size() - 1
        for (; k < (int)ft.size(); k += LSONe(k)) ft[k] += v; }
};

int main() {
    int f[] = { 2,4,5,5,6,6,6,7,7,8,9 }; // m = 11 scores
    FenwickTree ft(10); // declare a Fenwick Tree for range [1..10]
    // insert these scores manually one by one into an empty Fenwick Tree
    for (int i = 0; i < 11; i++) ft.adjust(f[i], 1); // this is  $O(k \log n)$ 
    printf("%d\n", ft.rsq(1, 1)); // 0 => ft[1] = 0
    printf("%d\n", ft.rsq(1, 2)); // 1 => ft[2] = 1
    printf("%d\n", ft.rsq(1, 6)); // 7 => ft[6] + ft[4] = 5 + 2 = 7
    printf("%d\n", ft.rsq(1, 10)); // 11 => ft[10] + ft[8] = 1 + 10 = 11
    printf("%d\n", ft.rsq(3, 6)); // 6 => rsq(1, 6) - rsq(1, 2) = 7 - 1
    ft.adjust(5, 2); // update demo
    printf("%d\n", ft.rsq(1, 10)); // now 13
} // return 0;

```

**c. Sparse Table**

```

int log[MAXN+1];
log[1] = 0;
for (int i = 2; i <= MAXN; i++) log[i] = log[i/2] + 1;

long long st[MAXN][K];
for (int i = 0; i < N; i++) st[i][0] = array[i];

for (int j = 1; j <= K; j++)

```

```

for (int i = 0; i + (1 << j) <= N; i++)
    st[i][j] = st[i][j-1] + st[i + (1 << (j - 1))][j - 1];

```

## 6. DP

### a. Divide and Conquer

```

vector<long long> dp_before(n), dp_cur(n);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr)
{
    if (l > r) return;
    int mid = (l + r) >> 1;
    pair<long long, int> best = {INF, -1};
    for (int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {dp_before[k] + C(k, mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}

```

### b. Paraquet

```

vector < vector<long long> > d;
void calc (int x = 0, int y = 0, int mask = 0, int next_mask = 0 {
    if (x == n) return;
    if (y >= m) d[x+1][next_mask] += d[x][mask];
    else
    {
        int my_mask = 1 << y;
        if (mask & my_mask)
            calc (x, y+1, mask, next_mask);
        else
        {
            calc (x, y+1, mask, next_mask | my_mask);
            if (y+1 < m && ! (mask & my_mask) && ! (mask & (my_mask << 1)))
                calc (x, y+2, mask, next_mask);
        }
    }
}
int main()

```

```

{
    cin >> n >> m;
    d.resize (n+1, vector<long long> (1<<m));
    d[0][0] = 1;
    for (int x=0; x<n; ++x) for (int mask=0; mask<(1<<m); ++mask) calc (x, 0, mask, 0);
    cout << d[n][0];
}

```

### c. Tree DP

```

list <int> *adj;
vector <long long> *suffix, *prefix;
long long *dp, *ans;
int n, m;
void dfs1(int u, int p) {
    prefix[u].push_back(1);
    int cnt = 0;
    long long prod = 1;
    for (int nxt : adj[u]) {
        if (nxt != p) {
            dfs1(nxt, u);
            prod = prod * (dp[nxt] + 1) % m;
            prefix[u].push_back(prefix[u][cnt] * (dp[nxt] + 1) % m);
        }
        else prefix[u].push_back(prefix[u][cnt]);
        cnt++;
    }
    cnt = 0;
    suffix[u].push_back(1);
    for (auto nxt = adj[u].rbegin(); nxt != adj[u].rend(); ++nxt) {
        if (*nxt != p) suffix[u].push_back(suffix[u][cnt] * (dp[*nxt] + 1) % m);
        else suffix[u].push_back(suffix[u][cnt]);
        cnt++;
    }
    dp[u] = prod;
}
void dfs2(int u, int p, long long val) {
    int cnt = 0;
    long long temp;
    for (int nxt : adj[u]) {

```



```

        if (nxt != p) {
            int len = suffix[u].size();
            temp = suffix[u][len - cnt - 2] * prefix[u][cnt] % m;
            temp = temp * val % m;
            ans[nxt] = dp[nxt] * (temp + 1) % m;
            //cout<<temp<<" "<<u<<" "<<nxt<<endl;
            dfs2(nxt, u, (temp + 1) % m);
        }
        cnt++;
    }
}

```

**d. Longest Increasing Subsequence:  $O(n \cdot \log n)$**

```

int lis(vector<int> const& a) {
    int n = a.size();
    const int INF = 1e9;
    vector<int> d(n+1, INF);
    d[0] = -INF;
    for (int i = 0; i < n; i++) {
        int j = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[j-1] < a[i] && a[i] < d[j]) d[j] = a[i];
    }
    int ans = 0;
    for (int i = 0; i <= n; i++) if (d[i] < INF) ans = i;
    return ans;
}

```

-----