

Foundations of Data Science & Machine Learning

Tutorial 02

March 5, 2021

Question 1. Rewrite the perceptron learning algorithm so that it works directly on any two sets of points G and B which are separated by a hyperplane (not necessarily passing through the origin). Give an upper bound on the number of updates in terms of parameters like R and δ of the G and B .

Definition 1. A function $K : (\mathbb{R}^n \times \mathbb{R}^n) \rightarrow \mathbb{R}$ is called a *kernel* if there exists an inner product space V and a function $\phi : \mathbb{R}^n \rightarrow V$ such that

$$\forall x, y \in \mathbb{R}^n, K(x, y) = \langle \phi(x), \phi(y) \rangle.$$

Question 2. Show that the function $K : (\mathbb{R}^n \times \mathbb{R}^n) \rightarrow \mathbb{R}$ given by

$$K(x, y) = (1 + \langle x, y \rangle)^d$$

is a kernel for every degree $d \in \mathbb{N}$.

Hints. Show that the constant function c for any $c > 0$ and $\langle x, y \rangle$ are kernels (easy). Then show that for any two kernels K_1 and K_2 , the functions $K_1 + K_2$ and $K_1 K_2$ are kernels. Then show that the given Kernel K can be generated from 1 and $\langle x, y \rangle$ using additions and multiplications.