Tutorial - 1

Room - 5:
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Question 1.

Q.1) Proof: if $G, B \subset \mathbb{R}^n$ G and B are separable using an axis parallel hyperplane iff their bounding boxes are disjoint.

Part 1: G and B are separable using an axis parallel hyper plane $\rightarrow$ their bounding boxes are disjoint

let the axis parallel hyper plane lie $l :- \langle z, a \rangle = b$ which separates G and B

Since its an axis parallel plane

$\therefore \quad l = z_1 a_1 + z_2 a_2 + \dots + z_n a_n = b$

where $\exists i \leq n, i \geq 1$

s.t. $a_i = 0$ and $\forall j \neq i \; a_j = 1$, $1 \leq j \leq n$

and all the points in G be represented as

$g_1, g_2 \dots g_k, \dots$

all the points in B be represented as

$b_1, b_2 \dots b_m \dots$

$\because$ l seperates both G & B perpendicular at i^{th} axis.

$\therefore$ ~~...~~

$g_{ki} = i^{th}$ coordinate in g_k

$\therefore$ g_{ki} and b_{mi} $\forall k, \forall m$ are always at a distance and never intersect, which is required for bounding boxes to intersect,

Since bounding boxes can only intersect ~~...~~ iff $\forall i, 1 \leq i \leq n$ ~~vice versa~~

$\exists k$ s.t. $i_{min}(B) \leq g_{ki} \leq i_{max}(B)$

Part 2 : Bounding boxes of G and B are disjoint $\Rightarrow$ G and B are separable using hyperplane (axes ∥)

Since the bounding boxes of G, B

i.e. B_G and B_B are disjoint

and $G, B \in \mathbb{R}^n$

$\exists\ 1 \le i \le n$, s.t.

$\forall m \ (i_{min}(G) < b_{mi}$ && $i_{max}(G) < b_{mi})$

OR

$\forall m \ (i_{min}(G) > b_{mi}$ && $i_{max}(G) > b_{mi})$

$\therefore$ we can choose an intermediate i' s.t.

$\exists$ a hyperplane

$1.x_1 + 1.x_2 + \ldots 0.x_i + \ldots 1.x_n = i'$

which seperates G and B.

Using Part 1 and Part 2, we can prove the desired claim.

Question 2

H is the convex hull of a set ~~$S \subseteq R^n$~~ $S \subseteq R^n$.

~~Let~~ H is the minimal convex set containings S.

~~H is conv~~

Let $A = \bigcap_{S \subseteq B} B$, where B is convex.

we have to proove $H = A$.

~~$A \subseteq H$~~ we claim $H \subseteq A$, ~~if not then~~

A is the intersection of all convex sets which contains S then A itself is a convex set.

[$x, y \in A$ and $\alpha \in [0,1]$.

$x, y \in B$ for $\forall B$.

$\alpha x + (1-\alpha) y \in B$ for $\forall B \Rightarrow \alpha x + (1-\alpha) y \in A$]

Since H is the minimal convex set containing S. then $H \subseteq A$.

Now to proove $A \subseteq H$.