TUTORIAL-3

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Question 1.
A set S ⊂ R n is convex if for any two points x, y ∈ S and any λ ∈ [0, 1], the point z = λx + (1 − λ)y lies in S. Show that if S ⊂ R n is convex, then

(a) for any 3 points x1, x2, x3 ∈ S and any λ1, λ2, λ3 ∈ [0, 1] such that λ1 + λ2 + λ3 = 1, the point z = λ1x1 + λ2x2 + λ3x3 lies in S.

(b) for any k points x1, . . . , xk ∈ S, k ≥ 2, and any λ1, . . . , λk ∈ [0, 1] such that Pk i=1 λi = 1, the point z = Pk i=1 λixi lies in S.

Sol:- Base case k = 2, it is true by convex property .
Assume true for k-1 points,

z = lambda1 * x1 + lambda2 * x2 + lambda3 * x3 + ... lambda(k-1)*x(k-1) belongs to S

where summation(lambda_i) = 1

Now consider a point xk which belongs to S
Now according to convex property,

lambdak * (xk) + (lambdak - 1) * (z) belongs to S

Expand z

lambdak *(xk) + (lambdak-1) * (lambda1 * x1 + ...).

Now sum coefficients

lambdak + (lambdak-1) *(lambda1 + ...) (for k-1 points summation lambda_i = 1 is true, induction step)

lamdbak + (lambdak-1) * 1 = 1.

Hence shown

Question 2.
Let $X \subset R^n$ be the set of (corner) vertices of the hypercube $\{0, 1\}^n$. That is, $X = \{(x_1, . . . , x_n) : x_i = 0 \text{ or } 1\}$. We say that a point $x = (x_1, . . . , x_n) \in X$ is good if the last 1 in x is at an even position. That is, $\max\{i : x_i = 1\} = 0 \pmod 2$. The remaining points of X are bad. Are the good and bad points defined above linearly separable. Prove your answer.

YES, they are linearly separable.

Let the set of good points be $G \subseteq X$, and the set of bad points be $B \subseteq X$.

We can take a special weight vector **a** = $(-e^1, e^2, -e^3, ...)$, since we need to give exponential weightage to the $\max\{i : x_i = 1\}$ term.

Subclaim: Here we can see that

$<a,x> < 0$ for bad points

$<a.x> > 0$ for good points.

Proof of sub-claim:
We also have to show that $e^n > \Sigma(e^{(i)})$ where $i \in [n-1]$.

Sum of GP for (n) terms = $(1 - r^n)/(1 - r)$
$= (e^{(n)} - 1)/(e - 1)$

(n+1)th term in GP = $e^{(n+1)}$ > Sum of GP for (n) terms

So for Good points i.e. $x \in G$, $<a,x> > 0$
And for Bad points i.e. $x \in B$, $<a,x> < 0$

Hence they are linearly separable by hyperplane represented by (a, b) where
a = $(-e^1, e^2, -e^3, ...)$, **b** = 0

Question 3. Find a vector space V and an embedding function $\varphi : R^2 \to V$ such that the resulting kernel K on $R^2 \times R^2$ is the function $K(x, y) = (1 + <x, y>)^2$.

$\phi: R^2 -> V$

$$K(x,y) = (1 + <x,y>)^2$$
$$= 1 + (<x,y>)^2 + 2(<x,y>)$$

$$= 1 + 2(x1)(y1) + 2(x2)(y2) + (x1.y1)^2 + (x2.y2)^2 + 2(x1.x2)(y1.y2)$$
$$= <\phi(x), \phi(y)>$$

Therefore, ϕ(x) = ϕ((x1, x2)) = (1, $\sqrt{2}x1$, $\sqrt{2}x2$, (x1)^2, (x2)^2, $\sqrt{2}(x1)$(x2))

V : R^6