Foundations of Data Science & Machine Learning

Tutotial 08

April 30, 2021

Question 1. Let G be a strongly connected digraph and P be the transition probability matrix of a random walk on G. Show that the stationary distribution π for P is strictly positive, i.e., $\pi_x > 0$ for every vertex $x \in V(G)$.

Question 2. Let X be a geometric random variable with mean p, that is

$$\forall k \in \mathbb{N}^+, \ P[X = k] = (1-p)^{k-1} p.$$

Show that the expectation $E[X] = 1/p$.

Question 3. Let G be the two-sided *infinite*[1] path whose vertices are labelled by $\mathbb{Z}$ and all edge-weights 1. Consider the simple random walk on G. That is

$$p_{i,j} = \begin{cases} \frac{1}{2}, & j \in \{i-1, i+1\}, \\ 0, & \text{otherwise.} \end{cases}$$

1. Show that this random walk has no stationary distribution. (*Hint.* Think locally.)
2. Show that $h(0,1)$, the hitting time from 0 to 1, is ∞. (*Hint.* Is $h(i, i+1)$ the same for all i?)

[1] This is an out of syllabus question since I promised we will only study finite graphs. But this gives you a feel of how things start getting different when we make the leap to infinity.