TUTORIAL 8

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Q1.

Given -

G is strongly connected digraph
P is transition probability matrix of a random walk on G with n vertices

To show -

π is strictly positive

Proof by contradiction -

Let's assume π is not strictly positive.
i.e. there exists i ∈V(G), s.t. $\pi_i = 0$. ----- Statement 1

Since π is a stationary distribution
π P = π

So, for all x ∈V(G), $\sum_{k=1}^{n} P_{kx}\pi_k = \pi_x$

So, for x = i, $\pi_i = \sum_{k=1}^{n} P_{ki}\pi_k$ -----Equation 1

Since LHS = 0, for RHS to be zero to satisfy the equation,
π_k = 0 for all k where P_{ki} is non-zero. (There exists non-zero P_{ki} values since the graph is strongly connected) ------- Statement 2

For statement 1 to be true statement 2 needs to be true as well.

Now we use the equation 1 on all the 'k' s in statement 2. ------let us name this process Recursive step 1

We will get the corresponding $\pi_{k'}$ = 0 where k' are the points which are connected to the points k. ----- Observation 1

If we repeat the recursive step 1 (at most n times) we will eventually visit all the vertices in the graph G (since G is strongly connected).

And by using observation 1 repeatedly we can say that $\pi_a = 0$ for all a

But this false because $\sum_{a=1}^{n} \pi_a$ =1 since π is a probability vector.

Which means our assumption that π is not strictly positive is wrong.

Hence the π is strictly positive. Hence Proved.

Q2.

Given -

X is geometric random variable with mean p i.e.

$\forall$ k $\in N^+$, P[X = k] = $(1 - p)^{k-1}$ p.

To show -

E[X] = 1/p

Proof -

E[X] = $\sum_k k\ x\ E[X = k]$

= 1 x E[X = 1] + 2 x E[X = 2] + ….

= 1 x $(1 - p)^0$ p + 2 x $(1 - p)$ p + …. --1

(1-p) E[X] = 1 x $(1 - p)^1$ p + 2 x $(1 - p)^2$ p + …. --2

From (1) - (2):

E[X] - E[X] + p E[X] = p + p (1 - p) + p $(1 - p)^2$+ …

Therefore, p E[X] = p (1 / (1 - (1 - p)))

= p (1 / p)

= 1

Therefore E[X] = 1 / p.

Hence Proved.

Q3.

1)

Let suppose it has a stationary distribution R.

So

Rj = R(j-1) ½ + R(j+1) ½

= $(½)^{n}$(sigma of nth neighbours of j)

Since sigma of Rj for j belong to Z = 1 (prob. vector)

Therefore Rj <= $(1/2)^{n}$ and n -> infinity

So Rj -> 0 as n -> infinity, for every j in Z.

Which is not possible as sigma Rj = 1.

For every j in Z

Q3

1) alternate solution

If we expand P_{ij} we will get an equation like $\pi_i = \pi_{i+1}$ for all i from which we can say that pi is uniform.

The from that we can say that π_i= 1/n but here n is infinity.

So π_i=0. For all i.

This does not satisfy the condition sigma(π_i)=1

From this we can say that this random walk has no stationary distribution.