

# Foundations of Data Science & Machine Learning

## Tutotial 02

March 5, 2021

**Question 1.** Rewrite the perceptron learning algorithm so that it works directly on any two sets of points  $G$  and  $B$  which are separated by a hyperplane (not necessarily passing through the origin). Give an upper bound on the number of updates in terms of parameters like  $R$  and  $\delta$  of the  $G$  and  $B$ .

**Definition 1.** A function  $K : (\mathbb{R}^n \times \mathbb{R}^n) \rightarrow \mathbb{R}$  is called a *kernel* if there exists an inner product space  $V$  and a function  $\phi : \mathbb{R}^n \rightarrow V$  such that

$$\forall x, y \in \mathbb{R}^n, K(x, y) = \langle \phi(x), \phi(y) \rangle.$$

**Question 2.** Show that the function  $K : (\mathbb{R}^n \times \mathbb{R}^n) \rightarrow \mathbb{R}$  given by

$$K(x, y) = (1 + \langle x, y \rangle)^d$$

is a kernel for every degree  $d \in \mathbb{N}$ .

*Hints.* Show that the constant function  $c$  for any  $c > 0$  and  $\langle x, y \rangle$  are kernels (easy). Then show that for any two kernels  $K_1$  and  $K_2$ , the functions  $K_1 + K_2$  and  $K_1 K_2$  are kernels. Then show that the given Kernel  $K$  can be generated from 1 and  $\langle x, y \rangle$  using additions and multiplications.