Tutorial 9

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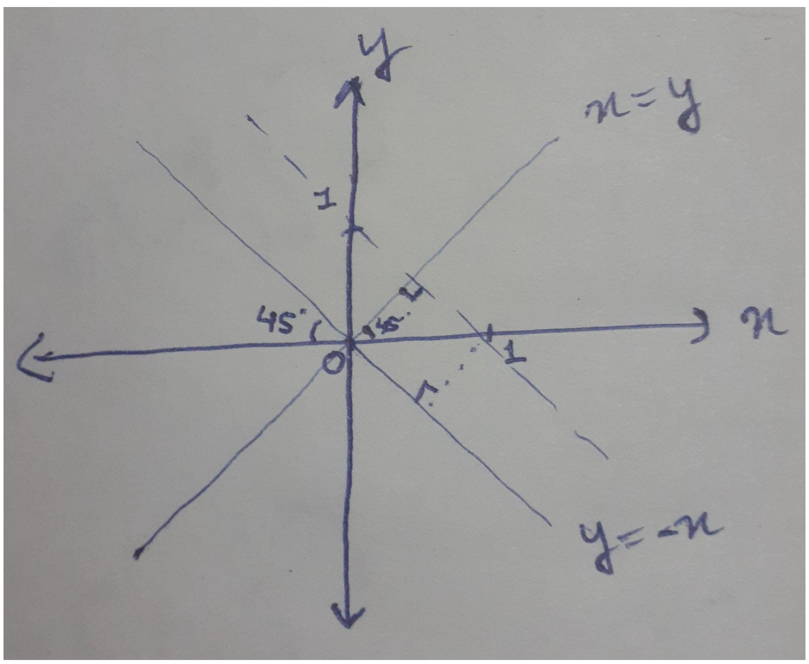
Q1.
1.



Since both the line (x = y) and (y = - x) are equidistant from both the points with a distance of ($1/\sqrt{2}$). Therefore both the lines are best fit lines. Hence no unique solution.

2.
Distance of a point(m,n) from a line(Ax + By + C = 0) is given by: (here C=0)

d = |Am + Bn| / $\sqrt{A^2 + B^2}$
d1 and d2 be the distance of a line from point 1 and point 2 respectively

Lets assume $\sqrt{A^2 + B^2}$ = 1

So we have to minimise d1 + d2

d1 + d2 = |B| + |2 A| (keeping the values of points given)

= 2 |A| + $\sqrt{1 - A^2}$

Since d1 + d2 >= 0
And assuming |A| = k is the final solution to minimise (d1 + d2).

So A = +k or A = -k
And B = +- $\sqrt{1 - k^2}$
So even if k =0, we have two solutions.

Hence we don't have a unique solution and there are multiple best fit lines for the given points.

Q2

2) $C_{p\times r} = A_{p\times q} B_{q\times r}$

we can write it as

$$C = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_q \\ | & & | \end{bmatrix} \begin{bmatrix} - & b_1 & - \\ & \vdots & \\ - & b_q & - \end{bmatrix} =$$

$$\sum_{i=1}^{q} \begin{bmatrix} | \\ a_i \\ | \end{bmatrix}_{p\times 1} \begin{bmatrix} - & b_i & - \end{bmatrix}_{1\times r}$$

Outer product of two coordinate vectors is a matrix. If the two vectors have dimensions p and r, then their outerproduct is an p×r matrix, here as in C.

Q3.

3) In general, the row vectors of a matrix M form an orthonormal set if and only if $M^T M = I$ (such a matrix is called orthogonal Matrix).

Thus by assumption, we have $A^T A = I$.

Let $B = A^T$.

then the row vectors of B is the ~~row~~ column vectors of A. Hence it suffices to show that

$$B^T B = I$$

Since $A^T A = I$, we know that A is invertible and $A^{-1} = A^T$

In particular, we have $A^T A = A A^T = I$

we have,

$$B^T B = (A^T)^T A^T = (A A^T)^T = (I)^T = I$$

Thus we obtain $B^T B = I$ and by general fact, the row vectors of B form an orthonormal set. Hence the column vectors of A form an orthonormal set.