CS5014 Foundations of Data Science & Machine Learning

Quiz 01

March 10, 2021 — 9.10 - 9.40 AM

Instructions

1. This is a **webcam proctored** exam. Please adjust your seating so that **your face, hands, answer book and the mobile phone that you will use to scan the sheets are always in the camera view**. Do not leave your seat or talk to anyone during the exam.

2. Write your answer on plain paper with your **name and roll number on the first sheet.**

3. This is a **closed book** exam. Do not refer to any books, notes, the Internet or any other person during the exam.

4. You can take **maximum 5 minutes after the exam to scan** the sheets into a **single PDF** file and upload to Moodle. Submissions made after 9:45 AM will be evaluated only if there is a genuine reason for the delay.

Questions

Question 1. A set $S \subset \mathbb{R}^n$ is *convex* if for any two points $x, y \in S$ and any $\lambda \in [0, 1]$, the point $z = \lambda x + (1 - \lambda)y$ lies in S. Show that if $S \subset \mathbb{R}^n$ is convex, then

(a) for any 3 points $x_1, x_2, x_3 \in S$ and any $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$ such that $\lambda_1 + \lambda_2 + \lambda_3 = 1$, the point $z = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$ lies in S.

(b) for any k points $x_1, \ldots, x_k \in S$, $k \geq 2$, and any $\lambda_1, \ldots, \lambda_k \in [0, 1]$ such that $\sum_{i=1}^{k} \lambda_i = 1$, the point $z = \sum_{i=1}^{k} \lambda_i x_i$ lies in S.

Question 2. Let $X \subset \mathbb{R}^n$ be the set of (corner) vertices of the hypercube $\{0, 1\}^n$. That is, $X = \{(x_1, \ldots, x_n) : \ x_i = 0 \text{ or } 1\}$. We say that a point $x = (x_1, \ldots, x_n) \in X$ is good if the last 1 in x is at an even position. That is, $\max\{i : \ x_i = 1\} = 0 \pmod 2$. The remaining points of X are bad. Are the good and bad points defined above linearly separable. Prove your answer.

Question 3. Find a vector space V and an embedding function $\phi : \mathbb{R}^2 \to V$ such that the resulting kernel K on $\mathbb{R}^2 \times \mathbb{R}^2$ is the function $K(x, y) = (1 + \langle x, y \rangle)^2$.

Total points: $3 \times 10 = 30$.