

CS5014 Foundations of Data Science & Machine Learning

Quiz 04

May 10, 2021 — 8.30 - 9.30 AM

Instructions

1. This is a **zoom proctored** exam. Please adjust your seating so that **your face, hands, answer book and the mobile phone that you will use to scan the sheets are always in the webcam view**. Do not leave your seat or talk to anyone during the exam.
2. Write your answer on plain paper with your **name and roll number on the first sheet**.
3. This is a **closed book** exam. Do not refer to any books, notes, the Internet or any other person during the exam.
4. You can take **maximum 5 minutes after the exam to scan** the sheets into a **single PDF** file and upload to Moodle. Submissions made after 9:35 AM will be evaluated only if there is a genuine reason for the delay.

Questions

Question 1. Let H be the 3-dimensional lattice $[n]^3$ with self loops at all the boundary vertices. All the non-loop edges have weight 1 while the self-loop at a boundary vertex v has weight $6 - d_v$, where d_v is the degree of v . Hence the total weight of all edges incident to any vertex is 6. Find the normalised conductance $\phi(H)$ of H .

Question 2. Let C_4 be the cycle on 4-vertices $\{0, 1, 2, 3\}$. Find the hitting time $h(0, 1)$, for the standard random walk on C_4 .

Hint. Search for symmetries.

Warning. It is recommended that you do this question at the end.

Question 3. Let A be a real symmetric square matrix. Find a relation between

1. Singular vectors and eigenvectors of A .
2. Singular values and eigenvalues of A .

Prove your answer.

Total points: $10 \times 3 = 30$.