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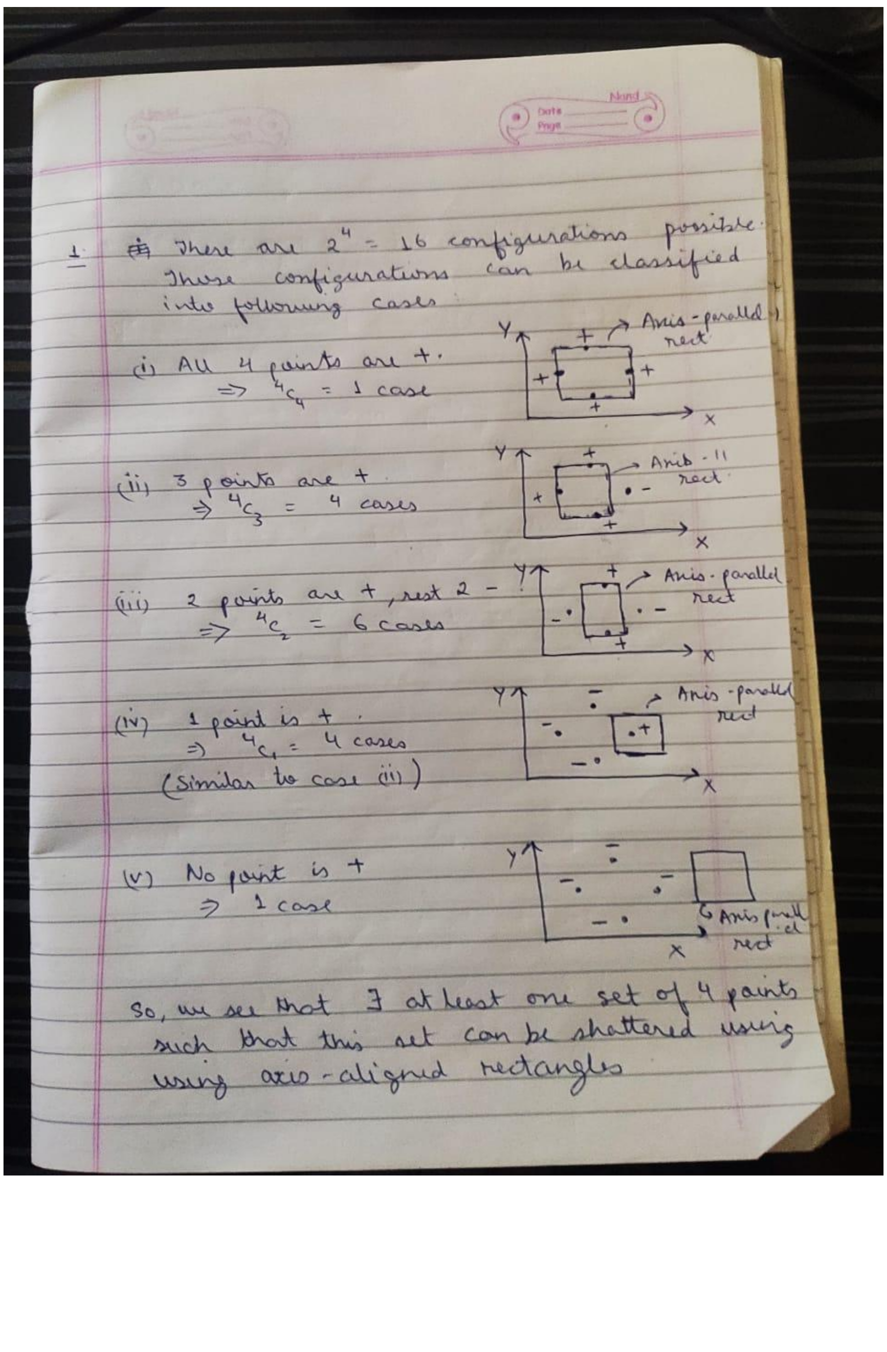
Answer 1

We are supposed to show that the VC dimension of the axis aligned rectangle in R^2 is 4.
There are two steps in this proof :

1. We show that ∃ at least one set of 4 points such that we can shatter this set using axis aligned rectangles:



1. There are $2^4 = 16$ configurations possible. These configurations can be classified into following cases:

(i) All 4 points are +.
⇒ ${}^4C_4 = 1$ case

Axis-parallel rect

(ii) 3 points are +
⇒ ${}^4C_3 = 4$ cases

Axis-|| rect

(iii) 2 points are +, rest 2 -
⇒ ${}^4C_2 = 6$ cases

Axis-parallel rect

(iv) 1 point is +
⇒ ${}^4C_1 = 4$ cases
(Similar to case (ii))

Axis-parallel rect

(v) No point is +
⇒ 1 case

Axis parallel rect

So, we see that ∃ at least one set of 4 points such that this set can be shattered using using axis-aligned rectangles

2. We have to show that ∄ any set S of 5 points such that we can have an axis aligned rectangle for all binary functions over S.
    a. This can be shown by considering one particular assignment to the set of points where let us take the set S = $\{x_1, x_2, x_3, x_4, x_5\}$ where this set S can be arbitrary for generality the only restriction is $x_i \in R^2$.
    b. Now consider the leftmost point in set S (min x coordinate), rightmost point in set S (max x coordinate), topmost most point (max y coordinate), bottommost point (min y coordinate) and we assign all these points to +1 and the rest as -1.
    c. We will thus have at least 1 point with -1 classification and we cannot have any rectangle which can realize this particular binary function assignment since it would geometrically have to lie outside the rectangle of the flour +1 points but we have an assignment to the outermost points as +1.
    d. In case there is a tie as in two points are on the same margin of the rectangle then we can assign the point with the lesser index +1 and the other as -1
    e. Thus we have a binary function which cannot be realized using rectangles and thus we cannot have a set of 5 points.

Thus the max VC dimension is lesser than 5 and since we have already shown that the minimum VC dimension is 4 we can conclude from 1 and 2 the VC dimension of axis aligned rectangles is exactly 4.

Answer 2.

1. We know that since the n given vectors are linearly dependent $\Sigma a_i x_i = 0$ and we are also given that the last dimension of each of the n vectors is 1 so we can thus infer that
   $\Sigma a_i = 0$
   This means that few a_i are ≥ 0 and the rest are < 0.
   We can now construct the set G as the set of all vectors such that their coefficients (a_i) are non negative and the set of vectors whose coefficients are negative are to be contained in B.
   We now are required to show that the convex hull of these two sets are non empty
   We now know that when $x_i \in G$ and $x_j \in B$ then $\Sigma a_i x_i = \Sigma \text{-} a_j x_j$ by the linear dependency and thus if we normalize $b_i = a_i/\Sigma a_i$ where $i \in G$ and similarly $c_i = a_i/\Sigma a_i$ where $i \in B$ then we can also infer the following from the previous observations:

   $\Sigma b_i x_i = \Sigma \text{-} b_j x_j$ when $x_i \in G$ and $x_j \in B$ and since both sides are the convex combinations of G and B they are thus represen

2.

consider a set $X \in \mathbb{R}^{d+1}$ such that
$X = (x, 1)$, where $x \in \mathbb{R}^d$

Let us assume that the labeling function is s.t.:

$$f(X) = \begin{cases} 1 & \text{if } W^T X > 0 \\ -1 & \text{, else} \end{cases}$$

where $W^T = (w, b)$,
$w = (w_1, w_2, \cdots w_d)$, s.t. $w_i = y_i, i \in \{1, \ldots d\}$

Let us also assume that $d+2$ points can be shattered by perceptron.

Since $d+2$ points $\in \mathbb{R}^{d+1}$,
$\exists\, i$ s.t
$$X^{(i)} = \sum_{j \neq i} a_j \cdot X^{(j)}$$
, where atleast one $a_j \neq 0$

Let $S = \{ j \mid j \neq i, a_j \neq 0 \}$

$\forall j \in S$, we assign $x^{(j)}$ the label signum(a_j) and give $x(i)$ -1.

By our assumption W exists such that

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$f(x)$ should label $d+2$ points correctly.

so $\forall j \in S$, we have

$$a_j \cdot W^T x^{(j)} > 0$$

also, $W^T x^{(i)} \leq 0$

Again, $$W^T x^{(i)} = W^T \left(\sum_{j \neq i} a_j \cdot x^{(j)} \right)$$

$$= W^T \left(\sum a_j \cdot x^{(j)} \right)$$

$$> 0$$

Hence, our assumption is wrong.

$\therefore$ Cannot shatter $d+2$

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