

Foundations of Data Science & Machine Learning

Tutorial 05

March 26, 2021

Question 1. An axis aligned rectangle in \mathbb{R}^2 is a rectangle whose sides are parallel to the x or y axes of the plane. The hypothesis corresponding to an axis aligned rectangle R is the function $f_R : X \rightarrow \{+1, -1\}$ which assigns $+1$ to all points inside R (including the boundary) and -1 to the points outside of R . Let H be the hypothesis class defined as

$$H = \{f_R : R \text{ is an axis-aligned rectangle in } \mathbb{R}^2\}.$$

Show that the VC dimension of H is 4.

Question 2. Show that

1. Let $S = \{x_1, \dots, x_n\}$ be a collection of n vectors in \mathbb{R}^d which are linearly dependent. Moreover, the last coordinate is 1 for all the vectors. Show that S can be partitioned into two sets G and B such that the convex hulls of G and B have a non-empty intersection.
2. Show that any set S of $d + 2$ points in \mathbb{R}^d can be partitioned into two sets G and B such that the convex hulls of G and B have a non-empty intersection.
3. Show that the VC dimension of linear separators in \mathbb{R}^d is at most $d + 1$.