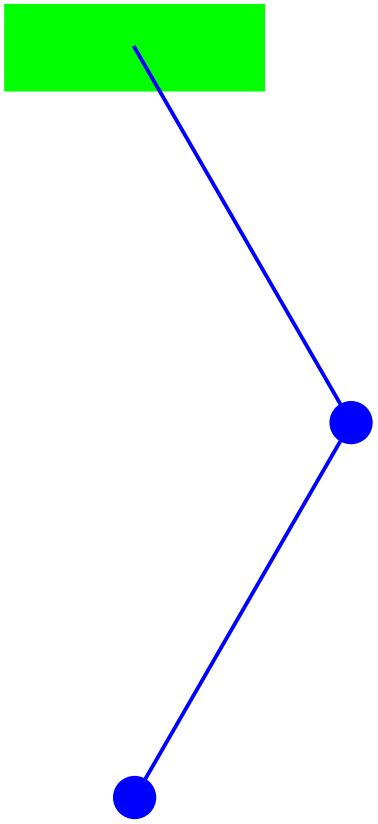
Modelling and Control of One Leg Hopper

Diagram

In[78]:=
```
Graphics[{Thick, Green, Rectangle[{-0.3, -0.1}, {0.3, 0.1}],
  Thick, Blue, Line[{{0, 0}, {Sin[Pi / 6], -Cos[Pi / 6]}}],
  Thick, Blue, Disk[{Sin[Pi / 6], -Cos[Pi / 6]}, 0.05],
  Thick, Blue, Line[{{Sin[Pi / 6], -Cos[Pi / 6]},
    {Sin[Pi / 6] + Sin[Pi / 6 - Pi / 3], -Cos[Pi / 6] - Cos[Pi / 6 - Pi / 3]}}],
  Disk[{Sin[Pi / 6] + Sin[Pi / 6 - Pi / 3], -Cos[Pi / 6] - Cos[Pi / 6 - Pi / 3]}, 0.05]}]
```

Out[78]=



Writing Down the Lagrangian

In[39]:=
```
ClearAll;
T[t_] = 1 / 2 M0 (x'[t]^2 + y'[t]^2) +
    1 / 2 M1 ((l1 θ1'[t] Cos[θ1[t]] + x'[t])^2 + (l1 θ1'[t] Sin[θ1[t]] + y'[t])^2) +
    1 / 2 M2 ((l2 θ2'[t] Cos[θ1[t] + θ2[t]] + l1 θ1'[t] Cos[θ1[t]] + x'[t])^2 +
       (l2 θ2'[t] Sin[θ1[t] + θ2[t]] + l1 θ1'[t] Sin[θ1[t]] + y'[t])^2);
V[t_] = M0 g y[t] + M1 g (y[t] - l1 Cos[θ1[t]]) +
    M2 (y[t] - l1 Cos[θ1[t]] - l2 Cos[θ1[t] + θ2[t]]);
L[t_] = T[t] - V[t];
```

Solving Dynamics Using Euler-Lagrange

Use the Euler-Lagrange equation to get the dynamics of each state variable.

In[45]:=
```
dyn = Simplify[
  {D[D[L[t], x'[t]], t] - D[L[t], x[t]], D[D[L[t], y'[t]], t] - D[L[t], y[t]],
   D[D[L[t], θ1'[t]], t] - D[L[t], θ1[t]], D[D[L[t], θ2'[t]], t] - D[L[t], θ2[t]]}]
```

Out[45]=
$$\{M0\, x''[t] + M1 \left(-l1 \sin[\theta1[t]]\, \theta1'[t]^2 + x''[t] + l1 \cos[\theta1[t]]\, \theta1''[t]\right) +$$
$$M2 \left(-l1 \sin[\theta1[t]]\, \theta1'[t]^2 - l2 \sin[\theta1[t] + \theta2[t]]\, \theta2'[t]\, (\theta1'[t] + \theta2'[t]) +$$
$$x''[t] + l1 \cos[\theta1[t]]\, \theta1''[t] + l2 \cos[\theta1[t] + \theta2[t]]\, \theta2''[t]\right),$$
$$g\, M0 + g\, M1 + M2 + M0\, y''[t] + M1 \left(l1 \cos[\theta1[t]]\, \theta1'[t]^2 + y''[t] + l1 \sin[\theta1[t]]\, \theta1''[t]\right) +$$
$$M2 \left(l1 \cos[\theta1[t]]\, \theta1'[t]^2 + l2 \cos[\theta1[t] + \theta2[t]]\, \theta2'[t]\, (\theta1'[t] + \theta2'[t]) +$$
$$y''[t] + l1 \sin[\theta1[t]]\, \theta1''[t] + l2 \sin[\theta1[t] + \theta2[t]]\, \theta2''[t]\right),$$
$$g\, l1\, M1 \sin[\theta1[t]] + l1\, M2 \sin[\theta1[t]] + l2\, M2 \sin[\theta1[t] + \theta2[t]] +$$
$$l2\, M2 \sin[\theta1[t] + \theta2[t]]\, x'[t]\, \theta2'[t] - l2\, M2 \cos[\theta1[t] + \theta2[t]]\, y'[t]\, \theta2'[t] -$$
$$l1\, l2\, M2 \sin[\theta2[t]]\, \theta2'[t]^2 + l1\, M1 \cos[\theta1[t]]\, x''[t] +$$
$$l1\, M2 \cos[\theta1[t]]\, x''[t] + l1\, M1 \sin[\theta1[t]]\, y''[t] + l1\, M2 \sin[\theta1[t]]\, y''[t] +$$
$$l1^2\, M1\, \theta1''[t] + l1^2\, M2\, \theta1''[t] + l1\, l2\, M2 \cos[\theta2[t]]\, \theta2''[t],$$
$$l2\, M2 \left(\sin[\theta1[t] + \theta2[t]] - \sin[\theta1[t] + \theta2[t]]\, x'[t]\, \theta1'[t] +\right.$$
$$\cos[\theta1[t] + \theta2[t]]\, y'[t]\, \theta1'[t] + \cos[\theta1[t] + \theta2[t]]\, x''[t] +$$
$$\left.\sin[\theta1[t] + \theta2[t]]\, y''[t] + l1 \cos[\theta2[t]]\, \theta1''[t] + l2\, \theta2''[t]\right)\}$$

Collecting Terms and Rewriting in Manipulator Expression Form

In[54]:=
```
Coefficient[dyn, θ1'[t]]
```

Out[54]=
$$\{-l2\, M2 \sin[\theta1[t] + \theta2[t]]\, \theta2'[t],\ l2\, M2 \cos[\theta1[t] + \theta2[t]]\, \theta2'[t],$$
$$0,\ l2\, M2 \left(-\sin[\theta1[t] + \theta2[t]]\, x'[t] + \cos[\theta1[t] + \theta2[t]]\, y'[t]\right)\}$$

In[56]:=
```
StreamPlot[{y, -9.8 Cos[x]}, {x, -10, 10}, {y, -10, 10}]
```

Out[56]=