Project Update: Bio-Inspired Quadruped

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December 5, 2015

1 Goals

1. To create a kinematic model of the quadruped.
2. Get an expression for the Center of Mass of the quadruped in terms of the 8 joint angles.
3. Find the range of values of joint angles for which the quadruped is stable when it is moving one leg only.

Since the model is kinematic, there is no requirement to consider the situation where more than one leg is moving since this initial control strategy assumes zero accelerations.

2 Mathematical Framework

To simplify the derivation of the expression for the center of mass, homogeneous transformation matrices are used.

The homogeneous transformation matrix shown below represents a translation followed by a rotation about the y axis. The subscripts represent the base and target frames. $T_{i,j}$ represents a transformation from frame i to frame j.

$$T_{i,j} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & dx \\ 0 & 1 & 0 & dy \\ \sin\theta & 0 & \cos\theta & dz \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Multiple transformations of this type can be represented by multiplying several transformation matrices together in cascade.

3 Kinematic modelling

The kinematic model of the quadruped was developed in Mathematica. Transformation matrices from the body link to the joints and the center of masses of all other 8 links were constructed. Figure 1 shows a visualization of the transformation frames. Each separate coordinate frame is represented by a cube.

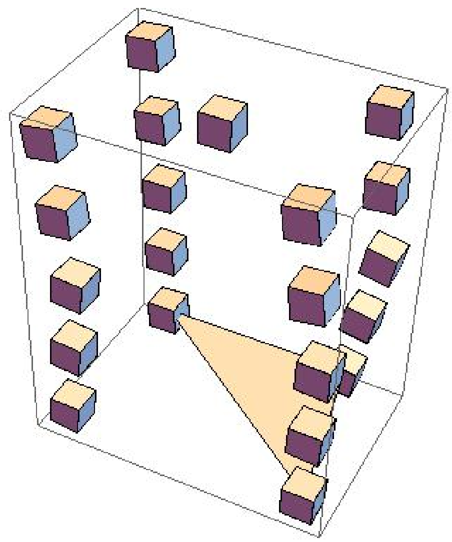


Figure 1: Kinematic Visualization of the transformation frames.

The cube at the center of the body and the links represent the centers of masses of these objects. Figure 2 shows the different frames of reference for which transformation frames were calculated. Below are the transformation marices for the various frames on one leg. Refer to the caption of Figure 2 to see the frames that the indices of the matrices are referring to.

$$T_{LSJ,0} = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & \frac{1}{2}(2d-l) \\ 0 & 1 & 0 & \frac{1}{2}(-2d-w) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T_{1,LSJ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{l_1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{LEJ,1} = \begin{pmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) & -\frac{l_1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T_{2,LEJ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{l_2}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{LH,2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{l_2}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.1 Finding the Center of Mass

The center of mass of the whole system in the base frame (assume base frame to the the frame attached to the body) can be found using the standard weighted sum of the centers of masses of the links and body. The center of mass transformation matrix for each link is found by multiplying together the appropriate transformation matrices. For example, the center of mass of link 2 marked with the number two in Figure 2 can be calculated using the formula.

$$T_{2,0} = T_{LSJ,0} T_{1,LSJ} T_{LEJ,1} T_{2,LEJ} \tag{2}$$

Using this idea the transformation matrices for the centers of masses of all 8 links were found. Below are the transformation matrices for the frames 1, 3, 5 and 7. The other matrices are too large to display.

$$T_{1,0} = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & \frac{1}{2}(2d-l) + \frac{1}{2}\sin(\theta_1) l_1 \\ 0 & 1 & 0 & \frac{1}{2}(-2d-w) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & -\frac{1}{2}\cos(\theta_1) l_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{3,0} = \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & \frac{1}{2}(2d-l) - \frac{1}{2}\sin(\theta_3) l_3 \\ 0 & 1 & 0 & \frac{1}{2}(2d+w) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -\frac{1}{2}\cos(\theta_3) l_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{5,0} = \begin{pmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & \frac{1}{2}(l-2d) - \frac{1}{2}\sin(\theta_5) l_5 \\ 0 & 1 & 0 & \frac{1}{2}(2d+w) \\ -\sin(\theta_5) & 0 & \cos(\theta_5) & -\frac{1}{2}\cos(\theta_5) l_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{7,0} = \begin{pmatrix} \cos(\theta_7) & 0 & -\sin(\theta_7) & \frac{1}{2}(l-2d) + \frac{1}{2}\sin(\theta_7) l_7 \\ 0 & 1 & 0 & \frac{1}{2}(-2d-w) \\ \sin(\theta_7) & 0 & \cos(\theta_7) & -\frac{1}{2}\cos(\theta_7) l_7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Using the transformation matrices $T_{i,0}$ the center of mass of the whole body of the quadruped can be calculated using the formula:

$$C.O.M = \frac{\sum_{i=0}^{8} \left\{ m_i T_{i,0} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}}{\sum_{j=0}^{8} m_j} \tag{3}$$

In Figure 2 the sphere shows the position of the center of mass of the whole robot.

3.2 Finding the Stable Region

The position of the center of mass changes as a function of all the joint angles. Assume that only one of the four legs of the robot is moving. For the robot to be statically stable, the center of mass has to be above the triangle formed by the tips of the three feet that are

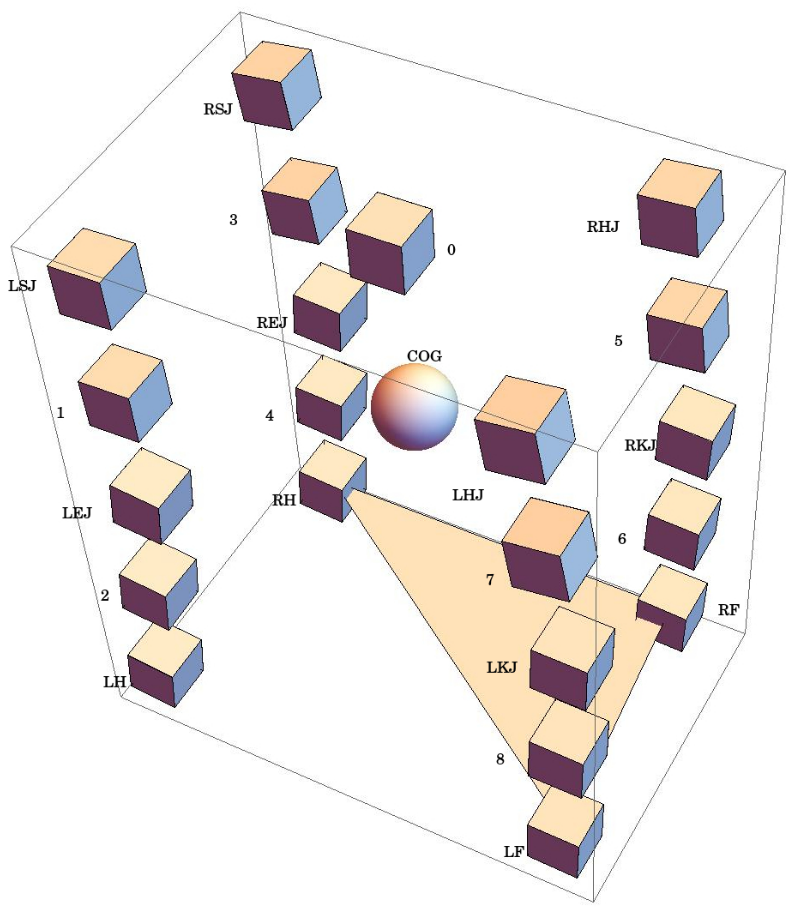


Figure 2: Kinematic Visualization Showing the Position of the Center of Mass. The numbers 0 to 8 represent the centers of mass of the links and the body of the quadruped.

LSJ - Left Shoulder Joint	LEJ - Left Elbow Joint	LH - Left Hand
RSJ - Right Shoulder Joint	REJ - Right Elbow Joint	RH - Right Hand
RHJ - Right Hip Joint	RKJ - Right Knee Joint	RF - Right Foot
LHJ - Left Hip Joint	LKJ - Left Knee Joint	LF - Left Foot

not moving. To check if the center of mass satisfies this condition, the following procedure was used.

1. Project the center of mass (P) on to the plane formed by the tips of the three feet with coordinates P_1, P_2, P_3.
2. Construct three new triangles PP_1P_2, PP_2P_3 and PP_1P_3.
3. Calculate the sum of the areas of these three triangles.
4. If the area of the triangle $P_1P_2P_3$ is equal to the sum of the areas of the three new triangles, the center of mass is inside the triangle $P_1P_2P_3$.

Let P be the center of mass of the robot and P_1, P_2 and P_3 be the points that form the triangle. The vector normal to the triangle $P_1P_2P_3$ is:

$$\hat{n} = \frac{(P_3 - P_2) \times (P_3 - P_1)}{||(P_3 - P_2) \times (P_3 - P_1)||} \tag{4}$$

The projected point on the plane P' can be calculated using the formula:

$$P' = P - (P_1 \cdot \hat{n} - P \cdot \hat{n})\hat{n} \tag{5}$$

Consider the projection of the center of mass on the plane with the triangle. Three new triangles are formed between the new point and considering two of each of the the three remaining points. If the projected point is inside the triangular region the sum of the areas of the three new triangles should be the same as the area of the original triangle. If the projected point is outside the triangle, the sum of the areas of the new triangles will be greater than the area of the original triangle. This condition can be checked using by verifying if the point P satisfies the following condition:

$$||(P_3-P')\times(P_2-P')||+||(P_2-P')\times(P_1-P')||+||(P_3-P')\times(P_1-P')|| = ||(P_3-P_1)\times(P_3-P_2)|| \tag{6}$$

Under this condition and the assumption that the robot is moving only one leg while keeping all other legs stationary, the range of values of the two link angles in each leg for which the robot was computed and plotted.

4 Walking

This section tries to solve the problem of quadruped locomotion using the framework of static stability explored in the previous section. Each of the quadruped's legs is moved forward one at a time in such a way that the base and knee angles only go through stable areas in the configuration space.

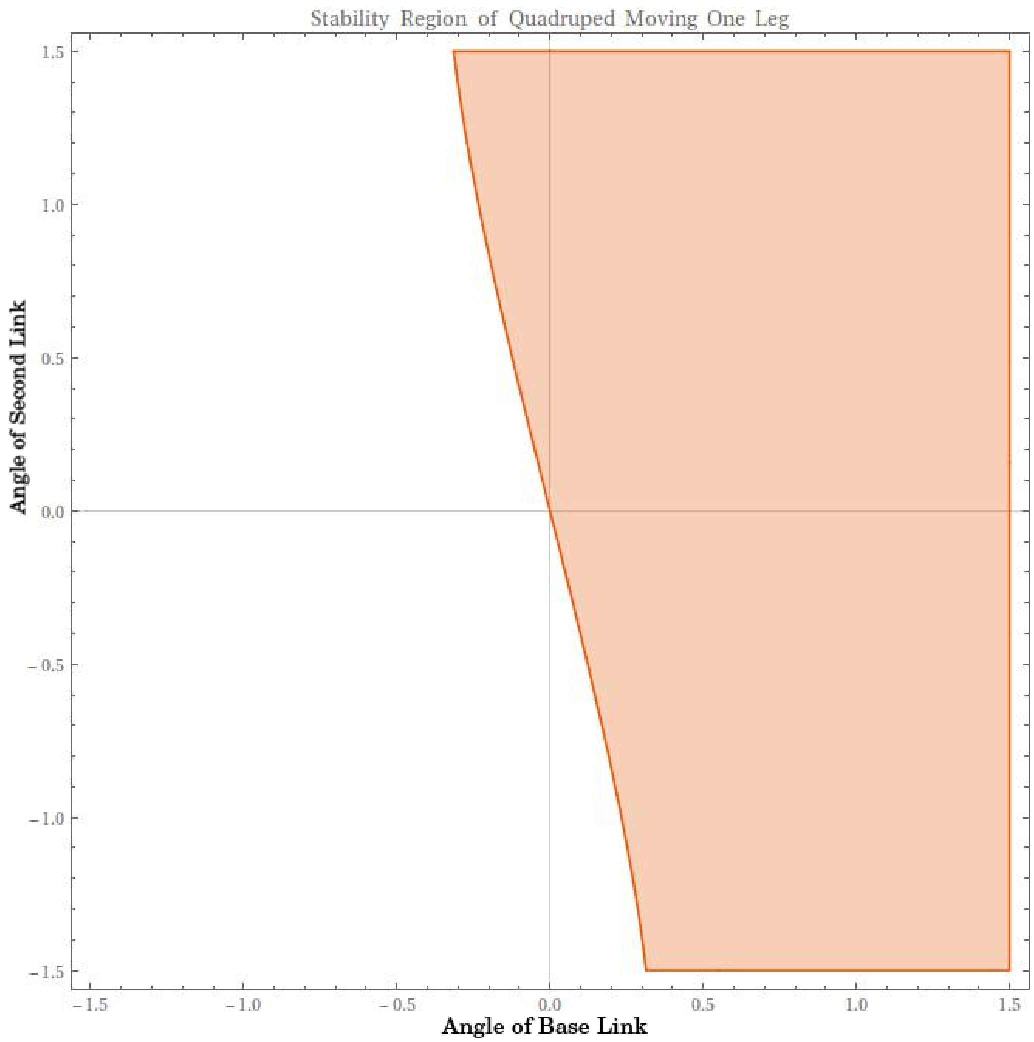
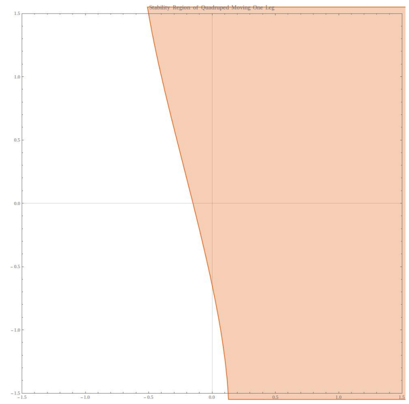
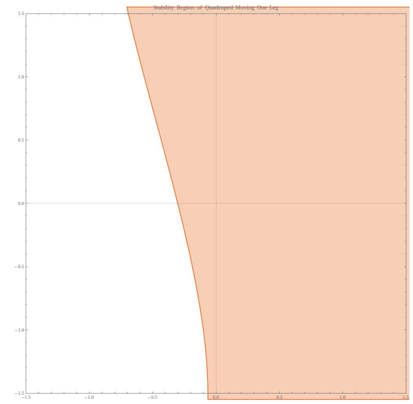


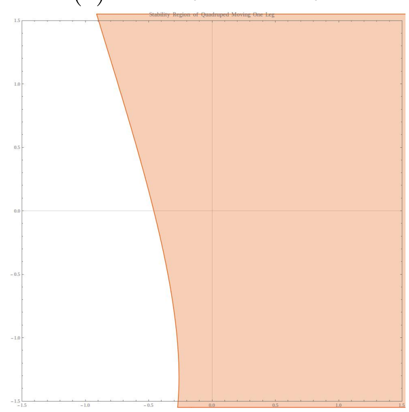
Figure 3: Visualization of the range of values of the two link angles of the front right limb for which the robot is stable. The angles are in radians. All other link angles are zero.



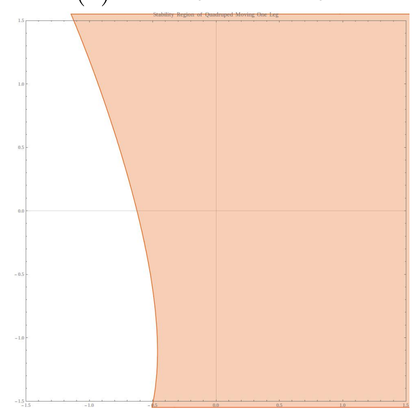
(a) Base: 0.1 Knee: 0.2



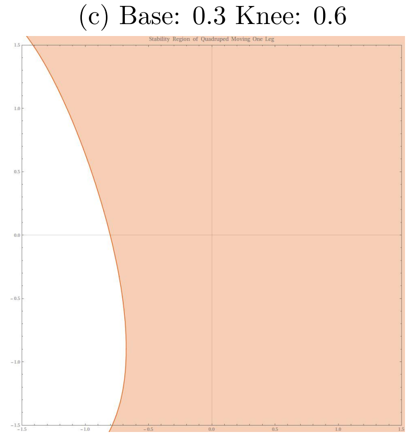
(b) Base: 0.2 Knee: 0.4



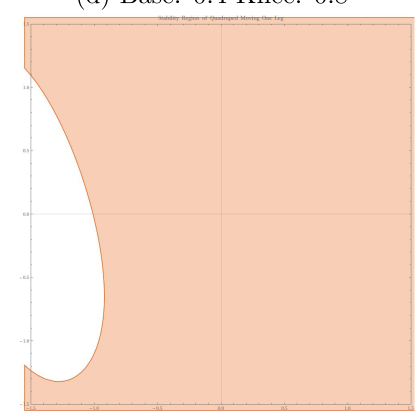
(c) Base: 0.3 Knee: 0.6



(d) Base: 0.4 Knee: 0.8



(e) Base: 0.5 Knee: 1.0



(f) Base: 0.6 Knee: 1.2

Figure 4: The images (a) to (f) show the range of values of the base and knee angles for which the quadruped is stable for different configurations of the stationary limbs. In (a) all the base angles of the three limbs are 0.1 radians and the knee angles are 0.2 radians. The horizontal and vertical axes represent base and link angles respectively. Both axes show angles ranging from -1.5 radians to +1.5 radians.

Assume that the the quadruped is initially in a configuration where all the base angles are 0.6 radians and all the knee angles are at 1.2 radians. The robot starts the walk sequence by lifting the front right foot, swinging it forward and placing the foot back on the ground slightly in front of its initial position. The problem of finding the correct position to put down the foot can be solved using the visualization in Figure 5.

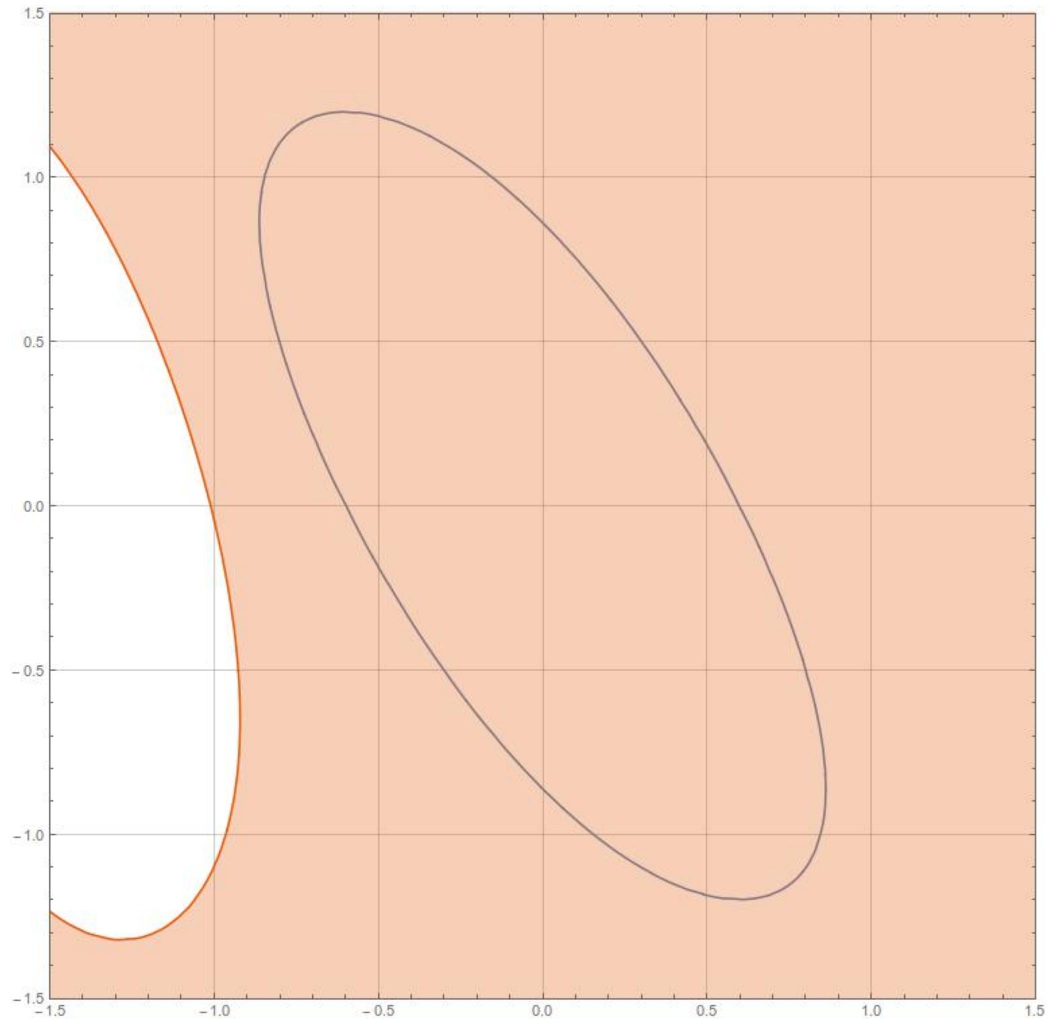


Figure 5: The horizontal axis represents the angle of the link attached to the body and the vertical axis represents the angle of link attached to the knee. Both axes range from -1.5 radians to +1.5 radians. The ellipse represents the range of values of the joint angles on the moving leg for which the foot touches the ground plane.

The ellipse represents the range of values of the joint angles on the moving leg for which the foot touches the ground plane. The ellipse is the set of points that satisfy the equation:

$$P_4 \cdot \hat{n} - P_3 \cdot \hat{n} = 0 \tag{7}$$

The initial angles of the moving limb will be a point on the ellipse. The angles of the limb when the foot is placed at a point in front of the initial position will be another point on the ellipse. The region inside the ellipse represents the range of angles for which the foot will collide with the ground. So the entire action of lifting the foot and putting it down while maintaining stability can be represented by a trajectory on the figure that starts at the initial point on the ellipse, avoids the unstable region, stays outside the ellipse, and ends on a second point on the ellipse. In Figure 5 negative angle displacements make the foot move in the "forward" direction (due to the way the angle conventions were selected). A typical trajectory would begin on a point on the ellipse in the second quadrant and end in the third quadrant.