

Training Large Language Models (LLMs)

Motivation: Why LLM Training Exists

- Traditional ML paradigm:
 - One model per task
 - Spam detection → separate model
 - Sentiment analysis → separate model
- Key observation:
 - These tasks are **not disjoint**
 - All require **language understanding**

Transfer Learning (Core Idea)

- Train a model once on massive data
 - Reuse learned knowledge across tasks
 - Fine-tune instead of training from scratch
 - **Fundamental paradigm behind LLMs**
-

Two-Stage Training Framework

1. Pre-training (Most Expensive Stage)

- Goal:
 - Learn structure of **language + code**
- Model type:
 - Text-to-text
 - Mostly **decoder-only Transformers** ($\approx 90\%$)
- Objective:
 - **Next-token prediction**
- Trained on **unlabeled web-scale data**

$$\mathcal{L}_{\text{pretrain}} = - \sum_{t=1}^T \log P(x_t \mid x_1, \dots, x_{t-1})$$

2. Tuning (Adaptation Stage)

- Start from pre-trained weights
- Adapt model to specific tasks:
 - Spam detection
 - Sentiment analysis
 - Classification

- Chat / instruction following
 - Much cheaper than pre-training
-

Pre-training Data

Common Data Sources

- **Common Crawl**
 - ~3 billion web pages per month
 - Wikipedia
 - Reddit conversations
 - Social media text
 - GitHub repositories (code)
 - Stack Overflow
 - Technical forums and blogs
-

Scale of Training Data

- Measured in **number of tokens**
- Typical scale:
 - Hundreds of billions → trillions → tens of trillions

Examples

- **GPT-3** → ~300B tokens
 - **LLaMA-3** → ~15T tokens
-

Compute Notations (Very Important)

1. FLOPs — Floating Point Operations

- Measures **total computation required**
- Used to estimate **training cost**

$$\text{Training FLOPs} \approx 6 \times N_{\text{params}} \times N_{\text{tokens}}$$

Where:

- (N_{params}) = number of model parameters
- (N_{tokens}) = number of training tokens

Order of magnitude

$$\text{LLM Training} \approx 10^{25} \text{ FLOPs}$$

- Exact value depends on architecture:

- Dense models
 - MoE models (lower active FLOPs)
-

2. FLOPS — Floating Point Operations per Second

- Measures **hardware speed**
- Indicates how fast GPUs/TPUs can execute FLOPs

$$\text{Training Time} = \frac{\text{Total FLOPs}}{\text{FLOPS}}$$

⚠ Important

- FLOPs = total work
 - FLOPS = speed
 - Papers may mix notation → rely on context
-

Why Pre-training Is So Expensive

- Trillions of training tokens
- Hundreds of billions of parameters
- Long training runs
- Large GPU clusters

Architecture impact

- Dense models → all parameters active
 - MoE models → partial activation → lower compute per step
-

Key Takeaways

- LLM training is built on **transfer learning**
- Pre-training dominates cost and compute
- Data scale is **trillions of tokens**
- Compute understanding requires:
 - **FLOPs** → total work
 - **FLOPS** → hardware speed

Scaling Laws for Language Models

Why Scaling Laws Matter

- Pre-training is extremely expensive:
 - Huge models
 - Massive datasets

- Limited compute budgets
- Key question:

How does performance change as we scale **model size**, **data size**, and **compute**?

Scaling Laws (Kaplan et al., 2020)

Paper: *Scaling Laws for Neural Language Models*

Experimental Setup

- Trained many Transformer-based LMs
- Varied:
 - Model size (number of parameters)
 - Dataset size (number of tokens)
 - Compute budget
- Objective:
 - **Next-token prediction loss**

Key Findings

- Performance improves predictably with:
 - Larger models
 - More data
 - More compute
- Loss follows a **power-law relationship** with scale

$$\mathcal{L} \propto N^{-\alpha}$$

Where:

- (N) = model size, data size, or compute
 - (α) = scaling exponent
-

Sample Efficiency

- Larger models are **more sample efficient**
 - Meaning:
 - For the same number of training tokens,
 - Larger models achieve **lower loss**
 - Result:
 - Bigger models learn faster **per token**
-

Fixed Compute Problem

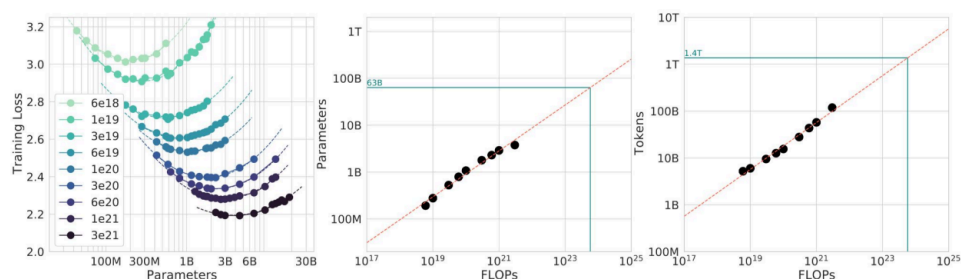
- Compute is **not unlimited**
- Given a fixed compute budget:
 - How big should the model be?
 - How much data should we train on?

Chinchilla Law (DeepMind, 2022)

Core Insight

- Many large models (e.g. GPT-3) were **undertrained**
- Optimal training requires **more data**, not just larger models

Chinchilla law.



Optimal Compute Allocation

$$N_{\text{tokens}} \approx 20 \times N_{\text{parameters}}$$

Where:

- ($N_{\text{parameters}}$) = number of model parameters
- (N_{tokens}) = number of training tokens

Interpretation

- If model is too large and data too small → **undertrained**
- If data is large but model is too small → **capacity-limited**
- Optimal point balances **model size + data size**

Chinchilla law.

| Parameters | FLOPs | FLOPs (in <i>Gopher</i> unit) | Tokens |
|-------------|----------|-------------------------------|----------------|
| 400 Million | 1.92e+19 | 1/29, 968 | 8.0 Billion |
| 1 Billion | 1.21e+20 | 1/4, 761 | 20.2 Billion |
| 10 Billion | 1.23e+22 | 1/46 | 205.1 Billion |
| 67 Billion | 5.76e+23 | 1 | 1.5 Trillion |
| 175 Billion | 3.85e+24 | 6.7 | 3.7 Trillion |
| 280 Billion | 9.90e+24 | 17.2 | 5.9 Trillion |
| 520 Billion | 3.43e+25 | 59.5 | 11.0 Trillion |
| 1 Trillion | 1.27e+26 | 221.3 | 21.2 Trillion |
| 10 Trillion | 1.30e+28 | 22515.9 | 216.2 Trillion |

Example: GPT-3

- Parameters: ~175B
- Tokens: ~300B
- According to Chinchilla:
 - Should have used ~**3.5T tokens**
- Conclusion:
 - GPT-3 was **compute-inefficient**

| Model | Pretraining size (# tokens) |
|---------|-----------------------------|
| GPT-3 | 300 billion |
| LLaMA 3 | 15 trillion |

Architecture Assumptions

- Scaling laws assume:
 - Transformer-based
 - Decoder-only
- Architecture choice matters **less** than:
 - Model size
 - Data size
 - Compute budget

Challenges of Pre-training

1. Cost

- Training cost:
 - Millions → tens of millions → hundreds of millions USD
- Requires:
 - Large GPU clusters
 - Long training runs

2. Environmental Impact

- High energy consumption
 - Carbon footprint increasingly reported in papers
-

3. Knowledge Cutoff

- Model only learns from data **available before training ends**
- Cannot natively know:
 - New events
 - New research
 - New facts

✦ Knowledge Cutoff Date

- Explicitly mentioned in model cards
- Example:
 - GPT-5 → Knowledge cutoff: *September 30*

The screenshot shows the GPT-5 model card interface. At the top, it says "GPT-5" with a "Default" dropdown and a "Compare" button. Below this, it states "The best model for coding and agentic tasks across domains". A "Try in Playground" button is also present. The interface is divided into sections for REASONING (Higher), SPEED (Medium), PRICE (\$1.25 · \$10), INPUT (Text, image), and OUTPUT (Text). Below these, it mentions "GPT-5 is our flagship model for coding, reasoning, and agentic tasks across domains. Learn more in our [GPT-5 usage guide](#)." On the right, it lists features: "400,000 context window", "128,000 max output tokens", "Sep 30, 2024 knowledge cutoff" (highlighted with a red box), and "Reasoning token support".

4. Knowledge Editing Is Hard

- Updating weights post-training can:
 - Break other knowledge
 - Cause regressions
- No clean, safe way to edit knowledge in-place

5. Memorization & Plagiarism

- Risk:
 - Model reproduces training data verbatim
- Especially problematic for:
 - Private data
 - Copyrighted content

Key Takeaways

- Scaling improves performance predictably
- Bigger models are more sample efficient
- Compute is the real bottleneck
- **Chinchilla Law:**

- Data matters as much as parameters
- Pre-training introduces:
 - Cost
 - Environmental concerns
 - Knowledge cutoff limitations

How Are Large Language Models Trained in Practice?

Why Training Is Hard

- LLMs have:
 - Billions → hundreds of billions of parameters
 - Trillions of training tokens
 - Training involves **massive matrix multiplications**
 - Hardware that excels at this:
 - **GPUs** (or TPUs at Google)
-

Training Pipeline of an LLM

1. Model Initialization

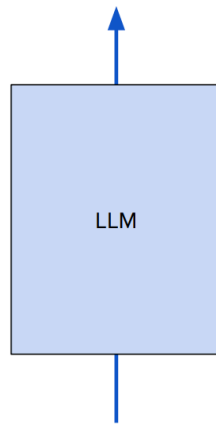
- Decoder-only Transformer
 - Parameters:
 - $(10^9) \rightarrow (10^{11+})$
 - Randomly initialized before training
-

Core Training Steps

1 Forward Pass

- Input:
 - Batch of token sequences
- Operations:
 - Embedding lookup
 - Self-attention
 - Feedforward layers
- Output:
 - Next-token probability distribution
- Compute **training loss**

$$\mathcal{L} = - \sum_{t=1}^T \log P(x_t \mid x_1, \dots, x_{t-1})$$



Forward pass. Compute loss.

Activations. Needed to compute the loss.

$$\mathcal{L}(\theta_t)$$

Function of model size, batch size, context length

Activations (Very Important)

- Activations = intermediate layer outputs
- Must be **stored in memory**
- Needed for backpropagation

Activation memory depends on:

- Model size
- Batch size
- Context length
- Self-attention complexity

$$\text{Attention Complexity} = O(N^2)$$

Where:

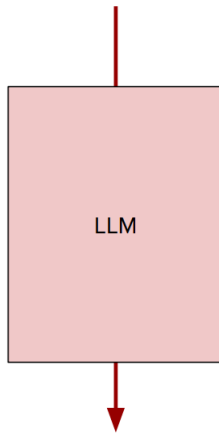
- (N) = sequence length

2 Backward Pass (Backpropagation)

- Compute gradients of loss w.r.t. parameters

$$\nabla_{\theta} \mathcal{L}$$

- Gradients must be stored in memory



Backwards pass. Compute gradients.

Gradients. Needed for weights update.

$$\nabla \mathcal{L}(\theta_t)$$

3 Weight Update

- Update parameters using an optimizer

Adam Optimizer (Common)

Maintains:

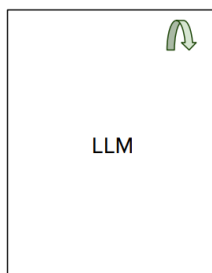
- First moment (mean of gradients)
- Second moment (mean of squared gradients)

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

- These optimizer states also consume **memory**

Weights update. Update model parameters.



Optimizer state. Updates the weights.

$$\theta_{t+1} \leftarrow \theta_t - \alpha \frac{m_t}{\sqrt{v_t} + \epsilon}$$

Adam optimizer

with: $m_{t+1} \leftarrow \beta_1 m_t + (1 - \beta_1) \nabla \mathcal{L}(\theta_t)$
 $v_{t+1} \leftarrow \beta_2 v_t + (1 - \beta_2) (\nabla \mathcal{L}(\theta_t))^2$

What Must Be Stored in GPU Memory?

For **each parameter**:

- Model weights

- Gradients
- Optimizer states (Adam: 2 extra tensors)

For **each layer & token**:

- Activations

👉 Memory grows very fast.

GPU Memory Is Limited

Example: NVIDIA H100

- GPU Memory \approx **80 GB**
 - Sounds large **✗**
 - Insufficient for:
 - Full model
 - Gradients
 - Optimizer states
 - Activations
-

Memory Bottleneck Summary

| Component | Memory Cost |
|------------------|----------------|
| Parameters | High |
| Gradients | High |
| Optimizer states | Very high |
| Activations | Extremely high |

✂ **Memory, not compute, is often the bottleneck**

| Technical Specifications | | |
|--------------------------------|--|--|
| | H100 SXM | H100 NVL |
| FP64 | 34 teraFLOPS | 30 teraFLOPS |
| FP64 Tensor Core | 67 teraFLOPS | 60 teraFLOPS |
| FP32 | 67 teraFLOPS | 60 teraFLOPS |
| TF32 Tensor Core* | 989 teraFLOPS | 835 teraFLOPS |
| BFLOAT16 Tensor Core* | 1,979 teraFLOPS | 1,671 teraFLOPS |
| FP16 Tensor Core* | 1,979 teraFLOPS | 1,671 teraFLOPS |
| FP8 Tensor Core* | 3,958 teraFLOPS | 3,341 teraFLOPS |
| INT8 Tensor Core* | 3,958 TOPS | 3,341 TOPS |
| GPU Memory | 80GB | 94GB |
| GPU Memory Bandwidth | 3.35TB/s | 3.9TB/s |
| Decoders | 7 NVDEC 7 JPEG | 7 NVDEC 7 JPEG |
| Max Thermal Design Power (TDP) | Up to 700W (configurable) | 350-400W (configurable) |
| Multi-Instance GPUs | Up to 7 MIGs @ 10GB each | Up to 7 MIGs @ 12GB each |
| Form Factor | SXM | PCIe dual-slot air-cooled |
| Interconnect | NVIDIA NVLink™: 900GB/s PCIe Gen5: 128GB/s | NVIDIA NVLink: 600GB/s PCIe Gen5: 128GB/s |
| Server Options | NVIDIA HGX H100 Partner and NVIDIA-Certified Systems™ with 4 or 8 GPUs NVIDIA DGX H100 with 8 GPUs | Partner and NVIDIA-Certified Systems with 1-8 GPUs |
| NVIDIA Enterprise | Add-on | Included |

Key Problem

How do we train such large models if **one GPU cannot hold everything?**

Solution

- Use **multiple GPUs**
- **Distribute:**
 - Model
 - Data
 - Computation

➡ Leads to **distributed training strategies**

(Next section: Data Parallelism, Model Parallelism, Pipeline Parallelism, ZeRO)

Key Takeaways

- Training involves:
 - Forward pass
 - Backward pass
 - Weight updates
- Memory requirements dominate
- GPU memory is limited
- Single GPU is insufficient
- Distributed training is **mandatory** for LLMs

Distributed Training Strategies for LLMs

Large Language Models (LLMs) **cannot be trained on a single GPU** because:

- Model parameters are extremely large (billions to hundreds of billions)
- Activations scale with:
 - Batch size
 - Sequence length
 - Number of layers
- Optimizer states multiply memory usage

Hence, **distributed training across multiple GPUs is mandatory**.

1 Data Parallelism (DP)

Core Idea

- Each GPU holds a **full replica of the model**
 - Training data is **split across GPUs**
 - Each GPU performs:
 - Forward pass
 - Loss computation
 - Backward pass
-

Detailed Workflow

Let there be **N GPUs**.

1. Global batch size (B) is split:

$$B = \sum_{i=1}^N B_i$$

2. Each GPU computes gradients independently:

$$\nabla \mathcal{L}_i = \frac{\partial \mathcal{L}_i}{\partial \theta}$$

3. Gradients are **synchronized (All-Reduce)**:

$$\nabla \mathcal{L} = \frac{1}{N} \sum_{i=1}^N \nabla \mathcal{L}_i$$

4. Each GPU applies the **same weight update**

What Is Replicated on Each GPU

- Model parameters
- Activations
- Gradients
- Optimizer states (Adam moments)

➡ **Heavy memory duplication**

Advantages

- Simple conceptual model
 - Easy to scale batch size
 - Widely supported in frameworks (PyTorch DDP, Horovod)
-

Limitations

- ❌ Entire model **must fit on one GPU**
 - ❌ Memory grows linearly with model size
 - ❌ Communication overhead from gradient synchronization
 - ❌ Inefficient for very large models
-

ZeRO: Zero Redundancy Optimizer

Motivation

In standard Data Parallelism:

- Every GPU stores **identical copies** of:
 - Parameters
 - Gradients
 - Optimizer states

This leads to **massive memory waste**.

Core Idea

Shard (partition) training states across GPUs

instead of duplicating them.

ZeRO Stage 1 — Optimizer State Partitioning

- Optimizer states (Adam moments) are sharded
- Parameters and gradients are still replicated

Memory per GPU:

$$\text{Params} + \text{Grads} + \frac{1}{N}(\text{Optimizer States})$$

 Memory reduction $\approx 2\times$

ZeRO Stage 2 — Gradient Partitioning

- Shard:
 - Optimizer states
 - Gradients
- Parameters still replicated

Memory per GPU:

$$\text{Params} + \frac{1}{N}(\text{Grads} + \text{Optimizer States})$$

 Memory reduction $\approx 4\times$

ZeRO Stage 3 — Parameter Partitioning

- Shard:
 - Parameters
 - Gradients
 - Optimizer states
- **No redundant copies**

Memory per GPU:

$$\frac{1}{N}(\text{Params} + \text{Grads} + \text{Optimizer States})$$

 Maximum memory savings

 Highest communication overhead

Communication Trade-off

- Higher ZeRO stage → more parameter gathering
 - Frequent all-gather and reduce-scatter operations
 - Slower step time but enables much larger models
-

2 Model Parallelism (MP)

Core Idea

Instead of splitting data, **split the model itself** across GPUs.

Used when:

- Model parameters cannot fit on a single GPU
 - Activation memory is too large
-

a) Tensor Parallelism

- Split **large matrix operations**
- Example: Linear layer

$$Y = XW$$

If ($W \in \mathbb{R}^{d \times d}$), split across GPUs:

- Column-wise or row-wise partition

Each GPU computes a partial result, then aggregates

Benefits:

- Reduces memory per GPU
- Reduces per-GPU compute

Used in:

- Megatron-LM
 - DeepSpeed
 - Most large LLMs
-

b) Pipeline Parallelism

- Split model **by layers**
- Each GPU handles a subset of layers

Example:

- GPU 1 → Layers 1–4

- GPU 2 → Layers 5–8
- GPU 3 → Layers 9–12

Activations are passed between GPUs.

Pipeline Challenges

- Pipeline bubbles (idle GPUs)
 - Requires micro-batching to improve utilization
 - Adds latency
-

c) Expert Parallelism (MoE Models)

- Used in **Mixture-of-Experts (MoE)** LLMs
- Each expert FFN placed on different GPUs
- Router sends tokens to experts dynamically

Only selected experts are active per token.

Used in:

- Switch Transformer
 - Mixtral
 - DeepSeek
 - GShard
-

Key Observation

- Data Parallelism → scales **data**
- ZeRO → removes **memory redundancy**
- Model Parallelism → splits **model computation**

These techniques are **composable** and almost always used together in practice.

Flash Attention (Exact Attention Optimization)

Flash Attention is an **exact attention algorithm** introduced in **2022 (Stanford)**. It **does not approximate attention** — it computes the *same result* as standard self-attention, but **much faster and with far less memory usage**.

The key idea is to **exploit GPU memory hierarchy**.

GPU Memory Hierarchy (Why Flash Attention Exists)

GPUs have **two very different memory types**:

1 HBM (High Bandwidth Memory)

- Large (\approx tens of GB, e.g., 80 GB on H100)
- Relatively **slow**
- Bandwidth: $\sim 1\text{--}3$ TB/s
- This is what we usually call *GPU memory*

2 SRAM (On-chip Memory)

- Very small (\approx tens of MB)
- **Extremely fast**
- Bandwidth: $\sim 10\text{--}20+$ TB/s
- Located next to compute units

⚠ Key bottleneck:

Modern GPUs are **memory-bound**, not compute-bound.

Most time is spent **reading/writing HBM**, not computing FLOPs.

Standard Self-Attention (Vanilla Implementation)

Self-attention formula:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right) V$$

Where:

- $(Q \in \mathbb{R}^{N \times d})$
 - $(K \in \mathbb{R}^{N \times d})$
 - $(V \in \mathbb{R}^{N \times d})$
 - (N) = sequence length
-

Naive Attention Memory Pattern

1. Load (Q, K) from **HBM**
2. Compute $(QK^T) \rightarrow$ store to **HBM**
3. Read (QK^T) from **HBM**
4. Apply softmax \rightarrow write to **HBM**
5. Read softmax result + (V) from **HBM**
6. Multiply \rightarrow write output to **HBM**

✗ **Multiple HBM reads/writes** ✗ Massive memory traffic ✗ GPU compute units stay idle waiting for memory

Key Insight Behind Flash Attention

Softmax Is Row-wise

For a row vector (x):

$$\text{softmax}(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

Important property:

- Softmax **can be computed incrementally**
 - You do **not need the full matrix at once**
-

Flash Attention Core Idea

👉 **Tile (block) the computation** so that:

- Small blocks fit into **SRAM**
- Entire attention computation for a block is done **end-to-end**
- Minimize reads/writes to **HBM**

This technique is called **tiling**.

Flash Attention Algorithm (High-Level)

For each **query block** (Q_i):

1. Load a small block of (Q_i) into **SRAM**
2. Loop over key/value blocks ((K_j, V_j)):
 - Load (K_j, V_j) into SRAM
 - Compute partial attention scores
 - Update softmax statistics **incrementally**
3. Accumulate final output
4. Write output once to **HBM**

- ✅ One HBM write per output
 - ✅ No intermediate matrices stored in HBM
-

Incremental Softmax Trick (Key Formula)

Let attention scores be split into blocks:

$$S = [S_1, S_2, \dots, S_n]$$

Instead of computing:

$$\text{softmax}(S) = \frac{e^S}{\sum e^S}$$

We maintain per-row:

- Running maximum (m)
- Running normalization factor (l)

Update rule (conceptual):

$$m_{\text{new}} = \max(m_{\text{old}}, \max(S_i))$$
$$l_{\text{new}} = e^{m_{\text{old}} - m_{\text{new}}} \cdot l_{\text{old}} + \sum e^{S_i - m_{\text{new}}}$$

This allows **exact softmax computation block-by-block**.

✦ No approximation — mathematically identical to full softmax.

Why This Is Faster

| Aspect | Vanilla Attention | Flash Attention |
|------------------|-------------------|-----------------|
| HBM Reads/Writes | Very High | Minimal |
| SRAM Usage | Minimal | Heavy |
| FLOPs | Same | Same |
| Runtime | Slow | Much Faster |

Flash Attention is **IO-aware**, not FLOP-aware.

Backward Pass Optimization (Recomputation)

Standard Training

- Store **activations** during forward pass
 - Use them during backward pass
 - Activation memory dominates GPU usage
-

Flash Attention Trick

- **Do not store activations**
- Recompute them during backward pass
- Recompute is cheap due to:
 - SRAM locality
 - Fast kernels

This is called **activation recomputation (checkpointing)**.

Counterintuitive Result

Even though:

- More FLOPs are executed (recomputation)

We get:

- **Less runtime**
- **Much less memory usage**

Example from paper:

- HBM reads reduced by $\sim 10\times$
 - Runtime reduced
 - Training fits larger batch sizes / longer context
-

Key Properties of Flash Attention

- ☒ Exact attention (no approximation)
 - ☒ Lower memory footprint
 - ☒ Faster forward + backward pass
 - ☒ Enables longer context lengths
 - ☒ Widely used in modern LLMs
-

Practical Notes

- Flash Attention v1, v2, v3:
 - Hardware-specific optimizations
 - Adapted to newer GPUs (A100, H100)
 - Used in:
 - PyTorch
 - Triton
 - xFormers
 - Most modern LLM stacks
-

Final Mental Model

Flash Attention **does not change what attention computes**
It changes **where and how** the computation happens.

Compute in **SRAM**, not **HBM**.

Quantization: Reducing Precision for Efficient LLM Training & Inference

Large Language Models store **all parameters, activations, gradients, and optimizer states** as **floating-point numbers**.

A natural question arises:

Do we really need *that much numerical precision* to achieve good performance?

Quantization addresses this question.

What Is Quantization?

Quantization is the process of converting numbers from:

- **Higher precision** → **Lower precision**

Goal:

- Reduce **memory usage**
- Increase **compute throughput**
- Preserve **model quality** as much as possible

Floating Point Numbers: How They Are Represented

A floating-point number is stored as **bits**, split into three components:

| Component | Role |
|------------------------|-------------------------|
| Sign | Positive / Negative |
| Exponent | Scale of the number |
| Mantissa (Significand) | Precision / granularity |

Different floating-point formats allocate bits differently.

| Name | Description | Illustration |
|----------|--|--------------|
| Sign | Controls whether the number is positive or negative. Typically takes up to 1 bit. | |
| Exponent | Controls the magnitude of the number. Also called <i>range</i> . | |
| Mantissa | Controls the granularity of the number, i.e. what is after the decimal point. Also called <i>significand</i> or <i>fraction</i> . | |

| | Sign | Exponent | Mantissa |
|-------------------------------------|------|----------|----------|
| FP16 (Floating-Point 16) | 1 | 5 | 10 |
| FP32 (Floating-Point 32) | 1 | 8 | 23 |
| FP64 (Floating-Point 64) | 1 | 11 | 52 |
| BFLOAT16 (Brain Float 16) | 1 | 8 | 7 |

Common Floating-Point Formats

| Format | Total Bits | Exponent Bits | Mantissa Bits | Notes |
|---------------|------------|---------------|---------------|------------------------------|
| FP64 (double) | 64 | 11 | 52 | Very high precision, slow |
| FP32 (single) | 32 | 8 | 23 | Standard training precision |
| FP16 (half) | 16 | 5 | 10 | Faster, less precise |
| bfloat16 | 16 | 8 | 7 | Better range, less precision |

✦ Key observation

FP16 / bfloat16 use **half the memory** of FP32.

Why Lower Precision Helps

1 Memory Savings

- FP32 → 4 bytes per value
- FP16 / bfloat16 → 2 bytes per value

This affects:

- Model parameters
- Activations
- Gradients
- Optimizer states

Result:

- Larger batch sizes
- Longer context lengths
- Bigger models per GPU

2 Compute Speed Improvements

GPUs are optimized for **lower precision arithmetic**.

Example (NVIDIA H100, approximate):

| Precision | Peak Compute |
|-------------|----------------|
| FP64 | ~34 TFLOPS |
| FP32 | ~60 TFLOPS |
| FP16 / BF16 | ~1,000+ TFLOPS |
| INT8 | Even higher |

✦ Lower precision = **higher throughput**

Precision vs Accuracy Trade-off

Lower precision:

- ✖ Less granularity
- ✖ More numerical noise

But:

- Neural networks are **robust to noise**
- Most LLM training works well in FP16 / BF16

This makes quantization practical.

Mixed Precision Training

Modern LLMs **do not use a single precision everywhere**.

Instead, they use **mixed precision**:

- Weights: FP16 / BF16
 - Activations: FP16 / BF16
 - Gradients: FP16 / BF16
 - Accumulators & loss scaling: FP32
-

Loss Scaling (Why It's Needed)

FP16 has limited precision → small gradients may underflow.

Solution:

- Multiply loss by a scale factor (s)
- Compute gradients
- Divide gradients by (s)

This preserves gradient signal.

Quantization Beyond FP16

Quantization can go even further:

Integer Quantization

- INT8
- INT4
- Even INT2 (research)

General idea:

$$x_q = \text{round} \left(\frac{x}{\Delta} \right)$$

Where:

- (x) = original value
 - (Δ) = quantization scale
-

Types of Quantization

| Type | Description |
|-----------------------------|---------------------------------------|
| Post-Training Quantization | Quantize after training |
| Quantization-Aware Training | Simulate quantization during training |

| Type | Description |
|--------------------------|----------------------------|
| Weight-only Quantization | Only weights are quantized |
| Activation Quantization | Weights + activations |

Where Quantization Is Used

Training

- FP16 / BF16 mixed precision
- Saves memory + speeds training

Inference

- INT8 / INT4 common
- Huge memory savings
- Faster decoding
- Enables deployment on smaller GPUs

Why Quantization Works Well for LLMs

- Transformers are **overparameterized**
- Small numerical errors do not significantly affect predictions
- Attention + FFNs tolerate noise

✦ Performance drop is often **minimal or negligible**

Practical Takeaways

- Quantization reduces **memory + compute**
- FP16 / BF16 is standard for LLM training
- Lower-bit quantization is crucial for inference
- Hardware is explicitly optimized for low precision

Mental Model

Quantization trades **numerical precision** for **efficiency** and LLMs are robust enough to handle it.

Mixed Precision Training

Motivation

LLMs involve:

- Massive models
- Huge datasets
- Extremely high memory and compute costs

Key question:

Can we use **lower numerical precision** without hurting model performance?

Mixed precision training is the answer.

Core Idea

Use different floating-point precisions for different parts of training:

- **Weights** → High precision (**FP32**)
- **Forward pass** → Lower precision (**FP16 / BF16**)
- **Backward pass** → Lower precision (**FP16 / BF16**)
- **Weight updates** → High precision (**FP32**)

This allows:

- Faster computation
 - Lower memory usage
 - Minimal performance degradation
-

Standard Mixed Precision Workflow

1. **Model weights stored in FP32**
 2. Inputs and activations computed in FP16
 3. Gradients computed in FP16
 4. Gradients are accumulated and applied to **FP32 weights**
 5. Updated FP32 weights are used in next iteration
-

Why Keep Weights in FP32?

Intuition:

- **Forward / backward computations**
 - Operate on data batches
 - Data itself is noisy
 - Extreme numerical precision is not critical
- **Weights**

- Accumulate updates over millions of steps
- Quantization errors can **accumulate**
- FP32 prevents long-term numerical drift

Think of gradients as *directional hints*

Think of weights as *long-term memory*

Benefits of Mixed Precision Training

1 Memory Savings

- FP16 uses **half the memory** of FP32
 - Enables:
 - Larger batch sizes
 - Longer context lengths
 - Larger models per GPU
-

2 Faster Training

- GPUs are optimized for FP16 / BF16
 - Much higher throughput than FP32
 - Especially effective on modern GPUs (A100, H100)
-

3 Minimal Performance Loss

- Empirically shown:
 - Little to no degradation in final model quality
 - Widely adopted in production LLM training
-

Loss Scaling (Important Detail)

Problem:

- FP16 has limited dynamic range
- Small gradients may **underflow to zero**

Solution: **Loss Scaling**

1. Multiply loss by a scaling factor (s)
2. Compute gradients
3. Divide gradients by (s) before update

This preserves gradient signal.

Do We Apply Mixed Precision Everywhere?

Short answer: **Not always**

- Some layers are more sensitive to precision:
 - LayerNorm
 - Softmax
 - Attention score computation

Different setups may:

- Keep some operations in FP32
- Use BF16 instead of FP16 for stability

There is **no single universal recipe**.

Relationship to Scaling Laws

Mixed precision is **orthogonal** to:

- Model size vs token size tradeoffs
- Chinchilla-style scaling laws

In practice:

- Teams run **small-scale experiments**
 - Determine:
 - Optimal precision
 - Optimal model size
 - Optimal dataset size
 - Then extrapolate to large-scale training
-

Practical Notes

- FP16 and BF16 are the most common choices
 - BF16 has:
 - Wider exponent range
 - Better numerical stability
 - Many modern LLMs prefer **BF16**
-

Key Takeaway

Mixed precision training:

- Reduces memory
- Speeds up training

- Preserves model quality

It is a **core technique** in modern LLM training pipelines.

Supervised Fine-Tuning (SFT)

Why Pre-training Is Not Enough

- Pre-training teaches the model:
 - Grammar
 - Syntax
 - Broad world patterns
- But **not helpful behavior**

Example:

“Can I put my teddy bear in the washer?”

A pre-trained LLM:

- Predicts likely next tokens
- Does **not reason as an assistant**
- Mimics patterns from internet text

✗ Not helpful

✗ Not aligned with user intent

Motivation for Fine-Tuning

Goal:

- Turn a **next-token predictor**
- Into a **helpful assistant**

This is achieved via **Supervised Fine-Tuning (SFT)**.

What Is SFT?

SFT = Supervised Fine-Tuning

- Start from **pre-trained weights**
 - Train on **(input, output)** pairs
 - Objective:
 - Produce *useful, human-like responses*
 - Supervised = labeled data
-

Nature of SFT Data

Each training example contains:

- **Input (prompt)**
- **Target output (ideal response)**

Example:

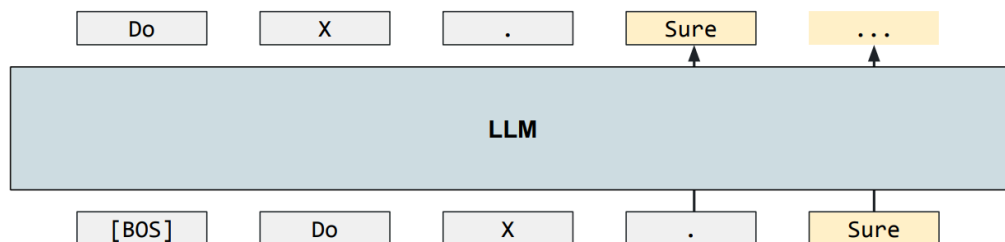
Input: "Can I put my teddy bear in the washer?"

Output: "It depends on the material. Check the care label..."

Goal. "Graduate" the model to being a helpful assistant.

| | |
|---|---|
| <p>Story writing</p> <p>Write a short story about a teddy bear who likes to read poetry.</p> <p>Once upon a time, a bear, Teddy, stumbled upon verses from Attar. The bear became so captivated with the rhythm of the poem's sentences that he got filled with happiness.</p> | <p>Lists generation</p> <p>List three fun activities a teddy bear might do on a rainy day.</p> <p>Sure! 1. Read poetry with friends. 2. Be cute. 3. Hug its owner tightly.</p> |
| <p>Poem creation</p> <p>Create a poem about my cute teddy bear.</p> <p>Soft and cuddly, full of charm, Always keeps me safe from harm, With button eyes and fur so neat, My teddy bear is oh so sweet.</p> | <p>Explanation</p> <p>Explain why a teddy bear is a great friend.</p> <p>A teddy bear is a great friend because it provides comfort and companionship. Its soft fur and cuddly nature is perfect for hugs. Teddy bears are always there in both happy and sad moments.</p> |

Objective function. Predict next token *given the input*.



Key Difference: Pre-training vs SFT Objective

Pre-training

- Model predicts **every token**
- Includes the input itself

$$\mathcal{L}_{\text{pretrain}} = - \sum_{t=1}^T \log P(x_t | x_1, \dots, x_{t-1})$$

- Loss applies to **all tokens**
 - Teacher forcing everywhere
-

Supervised Fine-Tuning (SFT)

- Input is **conditioning context**
- No loss on input tokens
- Loss applies **only to output tokens**

Let:

- (x) = input (prompt)
- $(y = (y_1, \dots, y_N))$ = target output

$$\mathcal{L}_{\text{SFT}} = - \sum_{t=1}^N \log P(y_t \mid x, y_1, \dots, y_{t-1})$$

- ✓ Input is **given**
 - ✓ Model is trained to **respond**, not repeat
-

Teacher Forcing in SFT

- Teacher forcing is used **only on output**
 - The model:
 - Sees the full prompt
 - Predicts the response token-by-token
 - Ground truth output tokens are provided during training
-

Intuition Behind SFT

- Pre-training:
 - "What token usually comes next?"
- SFT:
 - "Given this instruction, what is the best response?"

This shifts the model from:

- Language modeling
to
 - Instruction following
-

Why SFT Works Well

- Uses **human-written answers**

- Teaches:
 - Helpfulness
 - Politeness
 - Structure
 - Safety-aware behavior
 - Much cheaper than pre-training
 - Strong performance gains
-

Role of SFT in LLM Training Pipeline

1. **Pre-training**
 - Learn language & world patterns
 2. **Supervised Fine-Tuning**
 - Learn how to respond
 3. (Next) Alignment & preference optimization
-

Key Takeaways

- SFT is **mandatory** for useful LLMs
- Input tokens are **not predicted**
- Loss is applied **only to outputs**
- Same next-token loss, different conditioning
- Turns LLM into an assistant

Instruction Tuning (A Special Case of SFT)

From Language Modeling → Helpful Assistant

- Pre-training:
 - Learns **how language looks**
 - Predicts likely next tokens
- Supervised Fine-Tuning (SFT):
 - Teaches **how to respond**
- **Instruction Tuning:**
 - Teaches the model to **follow instructions**

Instruction tuning is a **subcategory of SFT** focused specifically on:

- Answering user instructions
 - Acting as an assistant
-

Instruction Tuning Objective

Given:

- An **instruction** (input)
- A **desired response** (output)

The model is trained to:

- Condition on the instruction
- Predict only the response tokens

Loss Application



-  No loss on instruction tokens
-  Loss applied only to response tokens

Illustration:

[Instruction / Prompt] → [Response Tokens] ↑ Loss computed here

Instruction-Tuning Data Composition

Unlike pre-training (raw internet text), instruction tuning uses **curated data** that demonstrates *helpful behavior*.

Common Instruction Categories

- Story writing
- Poem generation
- List creation
- Explanations
- Summarization
- Question answering
- Code generation
- Reasoning tasks
- Assistant-style dialogues

Each example is a **(instruction, ideal response)** pair.

Assistant Dialogues

- Many SFT datasets are framed as **assistant conversations**
- User asks a question
- Assistant provides a helpful answer

This trains the model to:

- Be conversational

- Follow intent
 - Produce coherent responses
-

Human vs Synthetic Instruction Data

Early Instruction Tuning

- Instructions written by humans
- Responses written by expert annotators
- High quality but **slow and expensive**

Modern Instruction Tuning

- Use existing strong LLMs to:
 - Generate candidate responses
 - Augment datasets
- Humans or other models:
 - Review
 - Filter
 - Rank quality

This **scales instruction data creation** significantly.

Safety-Oriented Instruction Data

Instruction tuning data also includes **safety-focused examples**.

Purpose

- Ensure model is:
 - Helpful
 - Harmless
 - Responsible

Examples

- Refusing harmful requests
- Avoiding dangerous instructions
- Hedging uncertain claims
- Rejecting disallowed content

Example behavior:

"I'm sorry, I can't help with that request."

⚠ This behavior is **learned**, not hard-coded via rules.

Why Not Use Rules or Regex?

- Rule-based systems:
 - Not scalable
 - Fragile
 - Instead:
 - Safety behavior is **embedded into model weights**
 - Learned through supervised examples
-

Generalization From Instruction Tuning

Example:

- Dataset includes:
 - "Write a story"
- Prompt at inference:
 - "Write a sci-fi poem about space-time"

Why does this work?

- Pre-training:
 - Teaches what sci-fi, poems, space-time are
- Instruction tuning:
 - Teaches **how to follow instructions**

➡ Model learns the *pattern*, not the exact content.

Instruction Tuning Scale (Orders of Magnitude)

Instruction tuning uses **far less data** than pre-training.

Reported Numbers

- GPT-3:
 - ~13,000 instruction examples
- LLaMA-3:
 - ~10 million instruction examples

Rough Token Estimate

Assume:

- ~1,000 tokens per example

Then:

- SFT data \ll Pre-training data
- Several **orders of magnitude smaller**

Mental Model: Pre-training vs Instruction Tuning

| Stage | Data Size | Data Quality | Purpose |
|--------------------------|----------------------|--------------|------------------------|
| Pre-training | Trillions of tokens | Noisy | Learn language |
| SFT / Instruction Tuning | Millions of examples | High-quality | Learn helpful behavior |

Effect of Instruction Tuning (Teddy Bear Example)

Before (Pre-trained model):

- Continues text probabilistically
- Does not answer the question

After Instruction Tuning:

- Responds directly
- Gives helpful advice
- Aligns with user intent

Example:

"You should handwash the teddy bear instead of using a washer."

Key Observation

- Instruction tuning **aligns model behavior**
- Makes LLMs usable by humans
- Still uses next-token prediction
- But with **task-aware conditioning**

Challenges of Supervised Fine-Tuning (SFT) and Motivation for Optimization

1. High-Quality Data Requirement

- SFT requires **high-quality, curated data**
- "High-quality" usually implies:
 - Human involvement
 - Expert-written responses
 - Compliance with safety and policy rules

- Early SFT datasets were:
 - Almost entirely **human-generated**
 - Expensive in time, cost, and coordination

Modern pipelines:

- Mix **human-written** and **model-generated** data
- Still require:
 - Human review
 - Filtering
 - Quality control

✅ Advantage:

- Once created, datasets can be **reused and extended over time**

❌ Disadvantage:

- Still expensive in human and computational resources
-

2. Prompt Distribution Mismatch

- SFT data comes from a **specific prompt distribution**
- Real-world inference prompts may differ significantly

Example:

- Training prompt:
 - "Write a story"
- Inference prompt:
 - "Write a story inspired by a specific movie plot"

This creates an **out-of-distribution (OOD)** issue.

Key idea:

- Model generalization depends on:
 - Coverage of prompt space
 - Diversity of training examples
-

3. Memorization vs Generalization

- Question:
 - If we give the model a prompt identical to one seen during SFT, will it reproduce the same response?

Answer:

- Usually **no**, because:

- Sampling is stochastic
- Temperature > 0
- Output may:
 - Have the same *flavor*
 - But not the same wording

Influencing factors:

- Temperature
 - Sampling strategy
 - Pre-training diversity
-

4. Role of Temperature

- Higher temperature:
 - More diversity
 - More creative outputs
 - Higher chance of unlikely tokens
- Lower temperature:
 - More deterministic
 - Less variation

Temperature directly controls:

- How much the model explores the probability space
-

5. Improving Generalization

- Primary lever: **data**
- More diverse and sparse coverage of prompt space:
 - Better generalization
 - Less overfitting to specific examples
- Repeating similar examples \neq better learning

Key idea:

- Teach the **concept**, not the exact instance
-

Evaluation Challenges for Instruction-Tuned Models

Why Evaluation Is Hard

- Helpfulness is **subjective**
 - No single metric captures user satisfaction
-

Benchmark-Based Evaluation

Common benchmark categories:

- General language understanding
- Reasoning
- Math
- Code generation

Examples:

- **MLU** (Massive Multitask Language Understanding)
 - ~50 tasks
 - Single aggregate score
 - **GSM8K**
 - Grade-school math reasoning
 - ~8K problems
-

The “Training on the Test Task” Problem

Observed phenomenon:

- Sudden benchmark performance jumps
- Often unexplained by architecture changes

Cause:

- Model trained on **data resembling the benchmark task**

Important distinction:

- Training on the **test task**
- NOT necessarily training on the **test set**

Implication:

- Benchmark scores depend heavily on training mixture

⚠ Fair comparison requires:

- Parity in training exposure
 - Transparency about auxiliary data
-

Benchmark Saturation

- Models optimize for known benchmarks
- New benchmarks appear to fill gaps
- Leads to:
 - Score inflation

- Weak correlation with real-world usefulness
-

Human Preference-Based Evaluation

Chatbot Arena

- Users compare two model responses
- Choose preferred one
- Rankings computed via pairwise comparisons

Captures:

- “Vibes”
 - User preference
 - Conversational quality
-

Limitations of Preference-Based Evaluation

1. Early Noise Sensitivity

- Initial comparisons heavily influence rankings

2. Leaderboard Manipulation Risk

- Models can detect:
 - Who they are being compared against
- Adversarial behavior possible

3. Factuality vs Helpfulness

- Users may prefer:
 - Detailed but incorrect answers
- Users may lack domain knowledge to verify correctness

4. Subjective Preferences

- Style preferences differ:
 - Emojis vs no emojis
 - Conciseness vs verbosity
- Expert preferences \neq general population preferences

5. Safety Bias

- Users dislike refusals
 - Preference systems may favor unsafe compliance
 - Conflicts with intended safety policies
-

No Single “Best” Metric

- Benchmarks \rightarrow objective but narrow

- Human preference → subjective but broad
- Safety → policy-driven
- Real usefulness depends on **use case**

Evaluation must be:

- Multi-dimensional
 - Context-aware
 - Goal-driven
-

Alignment: Where SFT Fits

Alignment Definition

Alignment = making the model:

- Helpful
- Harmless
- Honest

Alignment Stages

1. **Supervised Fine-Tuning (SFT)**
2. **Preference Tuning** (next lecture)

Together:

- Form the **alignment pipeline**
-

Emerging Concept: Mid-Training

- Occurs **after pre-training**
- Uses the same next-token objective
- But on **task-relevant data**
- Bridges gap between:
 - General pre-training
 - Task-specific fine-tuning

Mid-training is:

- New
 - Actively researched
 - Increasingly adopted
-

Big Picture (So Far)

1. Pre-training:
 - Learn language
2. SFT / Instruction Tuning:
 - Learn to help
3. Preference Tuning:
 - Learn what users prefer
4. Evaluation:
 - Hard, multi-faceted, imperfect

Parameter-Efficient Fine-Tuning (PEFT): LoRA and QLoRA

Motivation: Why PEFT?

- Full fine-tuning updates **all model parameters**
 - Computationally expensive:
 - High memory usage
 - Large optimizer states
 - Long training time
 - Goal:
 - Adapt large pre-trained models
 - While training **as few parameters as possible**
-

LoRA: Low-Rank Adaptation

Core Idea

- **Freeze** pre-trained weights
 - Learn a **low-rank update** instead of full weight updates
-

Standard Linear Layer

Given a linear transformation:

$$y = W_0 x$$

- $(W_0 \in \mathbb{R}^{d_{out} \times d_{in}})$
 - (W_0) is **pre-trained**
-

LoRA Formulation

Instead of updating (W_0) , we write:

$$W = W_0 + \Delta W$$

Where:

$$\Delta W = BA$$

- $(B \in \mathbb{R}^{d_{out} \times r})$
 - $(A \in \mathbb{R}^{r \times d_{in}})$
 - $(r \ll \min(d_{out}, d_{in}))$
-

Forward Pass with LoRA

$$y = W_0x + B(Ax)$$

- (W_0) : **frozen**
 - (A, B) : **trainable**
-

Parameter Reduction

- Full fine-tuning parameters:

$$d_{out} \times d_{in}$$

- LoRA parameters:

$$r(d_{out} + d_{in})$$

Example:

- $(d_{out}, d_{in} \sim 1000)$
- $(r \sim 4)$ or (8)

 **Orders of magnitude fewer trainable parameters**

Task-Specific Adaptation

- (W_0) : general language knowledge
- (A, B) : task-specific adaptation

Example:

- Spam detection \rightarrow one set of (A, B)
- Sentiment analysis \rightarrow another set of (A, B)

Same base model, different LoRA adapters

Where Is LoRA Applied?

Original LoRA Paper

- Applied mainly to **attention projections**:

- (W_Q, W_K, W_V, W_O)
-

Later Findings

- Best performance gains from:
 - **Feed-Forward Networks (FFN / MLP blocks)**
 - Modern practice:
 - Apply LoRA to:
 - Attention layers
 - Feed-forward layers
 - Majority of gains from FFN
-

Practical Training Observations (Empirical)

1. Higher Learning Rate

- LoRA typically uses:

$$\text{LR}_{\text{LoRA}} \approx 10 \times \text{LR}_{\text{full FT}}$$

Hypothesis:

- Low-rank matrices explore a smaller subspace
 - Need larger steps to adapt effectively
-

2. Large Batch Size Hurts

- LoRA performance degrades with very large batch sizes

Possible explanation:

- Training dynamics of **matrix products** differ from full matrices
- Gradient noise may help LoRA adaptation

⚠ Empirical observation, not fully theoretically explained

Rank Selection (Hyperparameter (r))

- Typical values:
 - $(r = 4, 8, 16)$
 - Grid search possible but often unnecessary
 - Popular defaults work well
 - Parameter reduction already massive → diminishing returns beyond this
-

QLoRA: Quantized LoRA

Motivation

- Even frozen weights (W_0) consume VRAM
 - Goal:
 - Quantize **frozen base model**
 - Train LoRA adapters in high precision
-

QLoRA Setup

- (W_0): **Quantized**
 - (A, B): **BF16 / FP16**
 - Gradients only flow through LoRA weights
-

NF4 Quantization

- NF4 = NormalFloat 4-bit
- Assumes weights follow a **normal distribution**
- Uses **quantiles**, not uniform buckets

Benefits:

- Better use of limited bits
 - Lower quantization error
-

Double Quantization

1. Quantize weights
2. Quantize quantization constants

Purpose:

- Further reduce memory overhead
-

Memory Savings

- QLoRA achieves:
 - **~16× VRAM reduction**
 - Double quantization:
 - Small additional gains
-

Summary of LoRA vs QLoRA

| Method | Base Weights | Trainable Weights | Precision | Memory |
|---------|--------------|-------------------|-----------|--------------|
| Full FT | Trainable | All | FP16/32 | ✗ Huge |
| LoRA | Frozen | A, B | FP16 | ✓ Low |
| QLoRA | Quantized | A, B | BF16 | ✓ ✓ Very Low |

Key Takeaways

- LoRA enables efficient fine-tuning of LLMs
- Low-rank updates capture task-specific knowledge
- QLoRA makes fine-tuning possible on limited hardware
- Widely used in practice for LLM adaptation

In []: