

Training Large Language Models (LLMs)

Motivation: Why LLM Training Exists

- Traditional ML paradigm:
 - One model per task
 - Spam detection → separate model
 - Sentiment analysis → separate model
- Key observation:
 - These tasks are **not disjoint**
 - All require **language understanding**

Transfer Learning (Core Idea)

- Train a model once on massive data
 - Reuse learned knowledge across tasks
 - Fine-tune instead of training from scratch
 - **Fundamental paradigm behind LLMs**
-

Two-Stage Training Framework

1. Pre-training (Most Expensive Stage)

- Goal:
 - Learn structure of **language + code**
- Model type:
 - Text-to-text
 - Mostly **decoder-only Transformers** ($\approx 90\%$)
- Objective:
 - **Next-token prediction**
- Trained on **unlabeled web-scale data**

$$\mathcal{L}_{\text{pretrain}} = - \sum_{t=1}^T \log P(x_t | x_1, \dots, x_{t-1})$$

2. Tuning (Adaptation Stage)

- Start from pre-trained weights
- Adapt model to specific tasks:
 - Spam detection
 - Sentiment analysis
 - Classification

- Chat / instruction following
 - Much cheaper than pre-training
-

Pre-training Data

Common Data Sources

- **Common Crawl**
 - ~3 billion web pages per month
 - Wikipedia
 - Reddit conversations
 - Social media text
 - GitHub repositories (code)
 - Stack Overflow
 - Technical forums and blogs
-

Scale of Training Data

- Measured in **number of tokens**
- Typical scale:
 - Hundreds of billions → trillions → tens of trillions

Examples

- **GPT-3** → ~300B tokens
 - **LLaMA-3** → ~15T tokens
-

Compute Notations (Very Important)

1. FLOPs — Floating Point Operations

- Measures **total computation required**
- Used to estimate **training cost**

$$\text{Training FLOPs} \approx 6 \times N_{\text{params}} \times N_{\text{tokens}}$$

Where:

- (N_{params}) = number of model parameters
- (N_{tokens}) = number of training tokens

Order of magnitude

$$\text{LLM Training} \approx 10^{25} \text{ FLOPs}$$

- Exact value depends on architecture:

- Dense models
 - MoE models (lower active FLOPs)
-

2. FLOPS — Floating Point Operations per Second

- Measures **hardware speed**
- Indicates how fast GPUs/TPUs can execute FLOPs

$$\text{Training Time} = \frac{\text{Total FLOPs}}{\text{FLOPS}}$$

 **Important**

- FLOPs = total work
 - FLOPS = speed
 - Papers may mix notation → rely on context
-

Why Pre-training Is So Expensive

- Trillions of training tokens
- Hundreds of billions of parameters
- Long training runs
- Large GPU clusters

Architecture impact

- Dense models → all parameters active
 - MoE models → partial activation → lower compute per step
-

Key Takeaways

- LLM training is built on **transfer learning**
- Pre-training dominates cost and compute
- Data scale is **trillions of tokens**
- Compute understanding requires:
 - **FLOPs** → total work
 - **FLOPS** → hardware speed

Scaling Laws for Language Models

Why Scaling Laws Matter

- Pre-training is extremely expensive:
 - Huge models
 - Massive datasets

- Limited compute budgets
 - Key question:
 - How does performance change as we scale **model size, data size, and compute?**
-

Scaling Laws (Kaplan et al., 2020)

Paper: *Scaling Laws for Neural Language Models*

Experimental Setup

- Trained many Transformer-based LMs
- Varied:
 - Model size (number of parameters)
 - Dataset size (number of tokens)
 - Compute budget
- Objective:
 - **Next-token prediction loss**

Key Findings

- Performance improves predictably with:
 - Larger models
 - More data
 - More compute
- Loss follows a **power-law relationship** with scale

$$\mathcal{L} \propto N^{-\alpha}$$

Where:

- (N) = model size, data size, or compute
 - (α) = scaling exponent
-

Sample Efficiency

- Larger models are **more sample efficient**
 - Meaning:
 - For the same number of training tokens,
 - Larger models achieve **lower loss**
 - Result:
 - Bigger models learn faster **per token**
-

Fixed Compute Problem

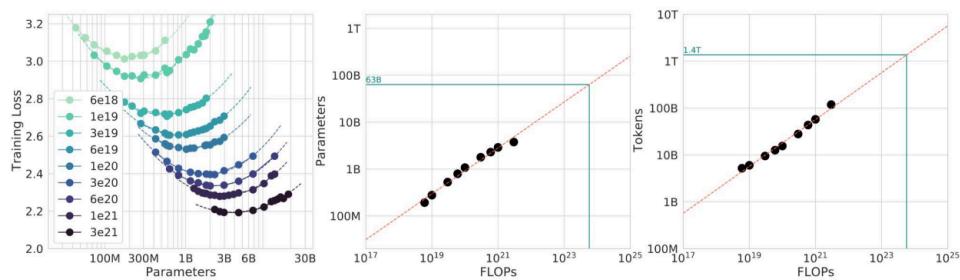
- Compute is **not unlimited**
 - Given a fixed compute budget:
 - How big should the model be?
 - How much data should we train on?
-

Chinchilla Law (DeepMind, 2022)

Core Insight

- Many large models (e.g. GPT-3) were **undertrained**
- Optimal training requires **more data**, not just larger models

Chinchilla law.



Optimal Compute Allocation

$$N_{\text{tokens}} \approx 20 \times N_{\text{parameters}}$$

Where:

- $(N_{\text{parameters}})$ = number of model parameters
 - (N_{tokens}) = number of training tokens
-

Interpretation

- If model is too large and data too small → **undertrained**
- If data is large but model is too small → **capacity-limited**
- Optimal point balances **model size + data size**

Chinchilla law.

| Parameters | FLOPs | FLOPs (in Gopher unit) | Tokens |
|-------------|----------|------------------------|----------------|
| 400 Million | 1.92e+19 | 1/29,968 | 8.0 Billion |
| 1 Billion | 1.21e+20 | 1/4,761 | 20.2 Billion |
| 10 Billion | 1.23e+22 | 1/46 | 205.1 Billion |
| 67 Billion | 5.76e+23 | 1 | 1.5 Trillion |
| 175 Billion | 3.85e+24 | 6.7 | 3.7 Trillion |
| 280 Billion | 9.90e+24 | 17.2 | 5.9 Trillion |
| 520 Billion | 3.43e+25 | 59.5 | 11.0 Trillion |
| 1 Trillion | 1.27e+26 | 221.3 | 21.2 Trillion |
| 10 Trillion | 1.30e+28 | 22515.9 | 216.2 Trillion |

Example: GPT-3

- Parameters: ~175B
- Tokens: ~300B
- According to Chinchilla:
 - Should have used **~3.5T tokens**
- Conclusion:
 - GPT-3 was **compute-inefficient**

| Model | Pretraining size (# tokens) |
|---------|-----------------------------|
| GPT-3 | 300 billion |
| LLaMA 3 | 15 trillion |

Architecture Assumptions

- Scaling laws assume:
 - Transformer-based
 - Decoder-only
- Architecture choice matters **less** than:
 - Model size
 - Data size
 - Compute budget

Challenges of Pre-training

1. Cost

- Training cost:
 - Millions → tens of millions → hundreds of millions USD
- Requires:
 - Large GPU clusters
 - Long training runs

2. Environmental Impact

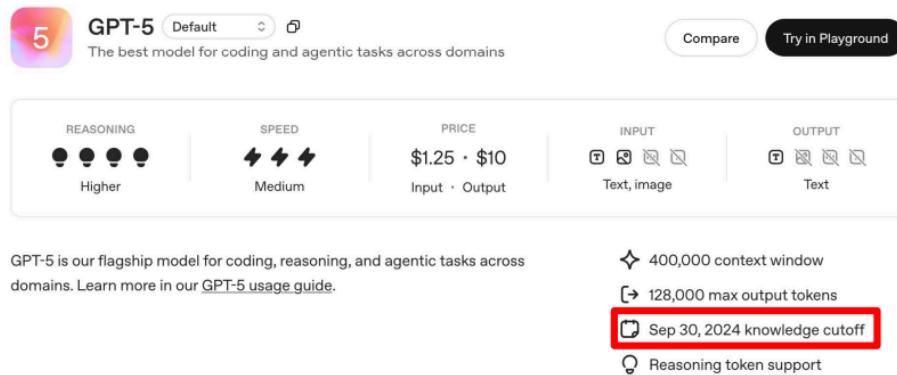
- High energy consumption
- Carbon footprint increasingly reported in papers

3. Knowledge Cutoff

- Model only learns from data **available before training ends**
- Cannot natively know:
 - New events
 - New research
 - New facts

📌 Knowledge Cutoff Date

- Explicitly mentioned in model cards
- Example:
 - GPT-5 → Knowledge cutoff: *September 30*



4. Knowledge Editing Is Hard

- Updating weights post-training can:
 - Break other knowledge
 - Cause regressions
- No clean, safe way to edit knowledge in-place

5. Memorization & Plagiarism

- Risk:
 - Model reproduces training data verbatim
- Especially problematic for:
 - Private data
 - Copyrighted content

Key Takeaways

- Scaling improves performance predictably
- Bigger models are more sample efficient
- Compute is the real bottleneck
- **Chinchilla Law:**

- Data matters as much as parameters
- Pre-training introduces:
 - Cost
 - Environmental concerns
 - Knowledge cutoff limitations

How Are Large Language Models Trained in Practice?

Why Training Is Hard

- LLMs have:
 - Billions → hundreds of billions of parameters
 - Trillions of training tokens
 - Training involves **massive matrix multiplications**
 - Hardware that excels at this:
 - **GPUs** (or TPUs at Google)
-

Training Pipeline of an LLM

1. Model Initialization

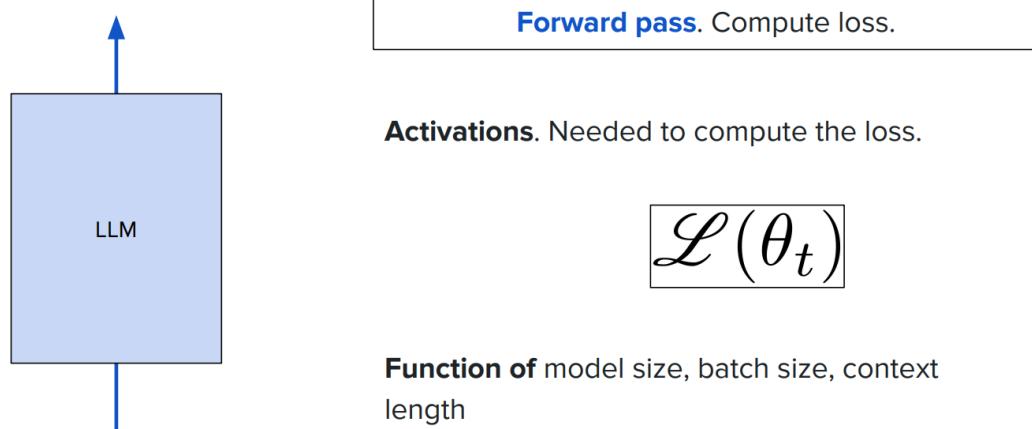
- Decoder-only Transformer
 - Parameters:
 - $(10^9) \rightarrow (10^{11+})$
 - Randomly initialized before training
-

Core Training Steps

1 Forward Pass

- Input:
 - Batch of token sequences
- Operations:
 - Embedding lookup
 - Self-attention
 - Feedforward layers
- Output:
 - Next-token probability distribution
- Compute **training loss**

$$\mathcal{L} = - \sum_{t=1}^T \log P(x_t | x_1, \dots, x_{t-1})$$



Activations (Very Important)

- Activations = intermediate layer outputs
- Must be **stored in memory**
- Needed for backpropagation

Activation memory depends on:

- Model size
- Batch size
- Context length
- Self-attention complexity

$$\text{Attention Complexity} = O(N^2)$$

Where:

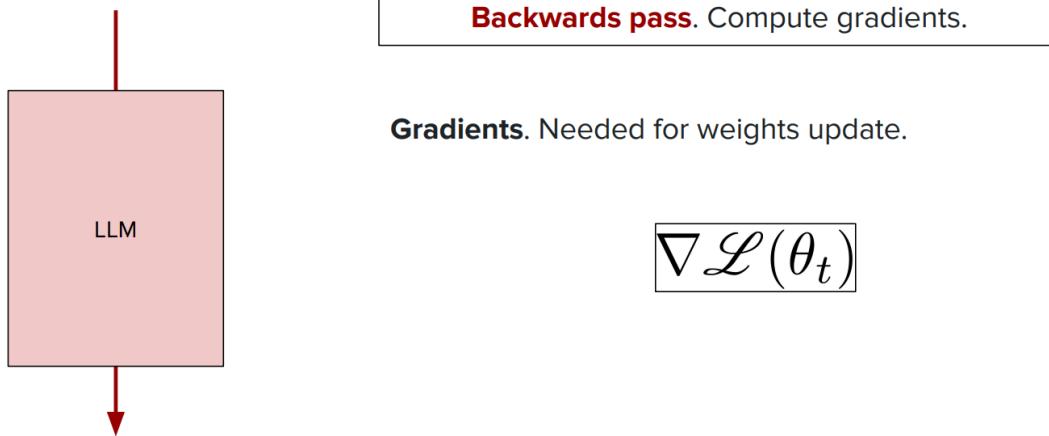
- (N) = sequence length

2 Backward Pass (Backpropagation)

- Compute gradients of loss w.r.t. parameters

$$\nabla_{\theta}\mathcal{L}$$

- Gradients must be stored in memory



Gradients. Needed for weights update.

$$\nabla \mathcal{L}(\theta_t)$$

3 Weight Update

- Update parameters using an optimizer

Adam Optimizer (Common)

Maintains:

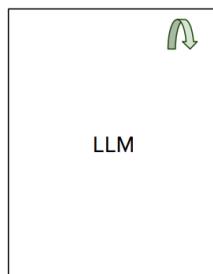
- First moment (mean of gradients)
- Second moment (mean of squared gradients)

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

- These optimizer states also consume **memory**

Weights update. Update model parameters.



$$\theta_{t+1} \leftarrow \theta_t - \alpha \frac{m_t}{\sqrt{v_t + \epsilon}} \quad \text{Adam optimizer}$$

with: $m_{t+1} \leftarrow \beta_1 m_t + (1 - \beta_1) \nabla \mathcal{L}(\theta_t)$
 $v_{t+1} \leftarrow \beta_2 v_t + (1 - \beta_2) (\nabla \mathcal{L}(\theta_t))^2$

What Must Be Stored in GPU Memory?

For **each parameter**:

- Model weights

- Gradients
- Optimizer states (Adam: 2 extra tensors)

For **each layer & token**:

- Activations
- 👉 Memory grows very fast.
-

GPU Memory Is Limited

Example: NVIDIA H100

- GPU Memory \approx **80 GB**
 - Sounds large ✗
 - Insufficient for:
 - Full model
 - Gradients
 - Optimizer states
 - Activations
-

Memory Bottleneck Summary

| Component | Memory Cost |
|------------------|----------------|
| Parameters | High |
| Gradients | High |
| Optimizer states | Very high |
| Activations | Extremely high |

👉 **Memory, not compute, is often the bottleneck**

Technical Specifications

| | H100 SXM | H100 NVL |
|---------------------------------------|--|--|
| FP64 | 34 teraFLOPS | 30 teraFLOPS |
| FP64 Tensor Core | 67 teraFLOPS | 60 teraFLOPS |
| FP32 | 67 teraFLOPS | 60 teraFLOPS |
| TF32 Tensor Core* | 989 teraFLOPS | 835 teraFLOPS |
| BFLOAT16 Tensor Core* | 1,979 teraFLOPS | 1,671 teraFLOPS |
| FP16 Tensor Core* | 1,979 teraFLOPS | 1,671 teraFLOPS |
| FP8 Tensor Core* | 3,958 teraFLOPS | 3,341 teraFLOPS |
| INT8 Tensor Core* | 3,958 TOPS | 3,341 TOPS |
| GPU Memory | 80GB | 94GB |
| GPU Memory Bandwidth | 3.35TB/s | 3.9TB/s |
| Decoders | 7 NVDEC 7 JPEG | 7 NVDEC 7 JPEG |
| Max Thermal Design Power (TDP) | Up to 700W (configurable) | 350-400W (configurable) |
| Multi-Instance GPUs | Up to 7 MIGs @ 10GB each | Up to 7 MIGS @ 12GB each |
| Form Factor | SXM | PCIe dual-slot air-cooled |
| Interconnect | NVIDIA NVLink™: 900GB/s PCIe Gen5: 128GB/s | NVIDIA NVLink: 600GB/s PCIe Gen5: 128GB/s |
| Server Options | NVIDIA HGX H100 Partner and NVIDIA-Certified Systems™ with 4 or 8 GPUs NVIDIA DGX H100 with 8 GPUs | Partner and NVIDIA-Certified Systems with 1-8 GPUs |
| NVIDIA Enterprise | Add-on | Included |

Key Problem

How do we train such large models if **one GPU cannot hold everything?**

Solution

- Use **multiple GPUs**
- **Distribute:**
 - Model
 - Data
 - Computation

→ Leads to **distributed training strategies**

(Next section: Data Parallelism, Model Parallelism, Pipeline Parallelism, ZeRO)

Key Takeaways

- Training involves:
 - Forward pass
 - Backward pass
 - Weight updates
- Memory requirements dominate
- GPU memory is limited
- Single GPU is insufficient
- Distributed training is **mandatory** for LLMs

Distributed Training Strategies for LLMs

Large Language Models (LLMs) **cannot be trained on a single GPU** because:

- Model parameters are extremely large (billions to hundreds of billions)
- Activations scale with:
 - Batch size
 - Sequence length
 - Number of layers
- Optimizer states multiply memory usage

Hence, **distributed training across multiple GPUs is mandatory**.

1 Data Parallelism (DP)

Core Idea

- Each GPU holds a **full replica of the model**
- Training data is **split across GPUs**
- Each GPU performs:
 - Forward pass
 - Loss computation
 - Backward pass

Detailed Workflow

Let there be **N GPUs**.

1. Global batch size (B) is split:

$$B = \sum_{i=1}^N B_i$$

2. Each GPU computes gradients independently:

$$\nabla \mathcal{L}_i = \frac{\partial \mathcal{L}_i}{\partial \theta}$$

3. Gradients are **synchronized (All-Reduce)**:

$$\nabla \mathcal{L} = \frac{1}{N} \sum_{i=1}^N \nabla \mathcal{L}_i$$

4. Each GPU applies the **same weight update**

What Is Replicated on Each GPU

- Model parameters
- Activations
- Gradients
- Optimizer states (Adam moments)

Heavy memory duplication

Advantages

- Simple conceptual model
 - Easy to scale batch size
 - Widely supported in frameworks (PyTorch DDP, Horovod)
-

Limitations

-  Entire model **must fit on one GPU**
 -  Memory grows linearly with model size
 -  Communication overhead from gradient synchronization
 -  Inefficient for very large models
-

ZeRO: Zero Redundancy Optimizer

Motivation

In standard Data Parallelism:

- Every GPU stores **identical copies** of:
 - Parameters
 - Gradients
 - Optimizer states

This leads to **massive memory waste**.

Core Idea

Shard (partition) training states across GPUs

instead of duplicating them.

ZeRO Stage 1 — Optimizer State Partitioning

- Optimizer states (Adam moments) are sharded
- Parameters and gradients are still replicated

Memory per GPU:

$$\text{Params} + \text{Grads} + \frac{1}{N}(\text{Optimizer States})$$

 Memory reduction $\approx 2\times$

ZeRO Stage 2 — Gradient Partitioning

- Shard:
 - Optimizer states
 - Gradients
- Parameters still replicated

Memory per GPU:

$$\text{Params} + \frac{1}{N}(\text{Grads} + \text{Optimizer States})$$

 Memory reduction $\approx 4\times$

ZeRO Stage 3 — Parameter Partitioning

- Shard:
 - Parameters
 - Gradients
 - Optimizer states
- **No redundant copies**

Memory per GPU:

$$\frac{1}{N}(\text{Params} + \text{Grads} + \text{Optimizer States})$$

 Maximum memory savings

 Highest communication overhead

Communication Trade-off

- Higher ZeRO stage → more parameter gathering
 - Frequent all-gather and reduce-scatter operations
 - Slower step time but enables much larger models
-

2 Model Parallelism (MP)

Core Idea

Instead of splitting data, **split the model itself** across GPUs.

Used when:

- Model parameters cannot fit on a single GPU
 - Activation memory is too large
-

a) Tensor Parallelism

- Split **large matrix operations**
- Example: Linear layer

$$Y = XW$$

If ($W \in \mathbb{R}^{d \times d}$), split across GPUs:

- Column-wise or row-wise partition

Each GPU computes a partial result, then aggregates

Benefits:

- Reduces memory per GPU
- Reduces per-GPU compute

Used in:

- Megatron-LM
 - DeepSpeed
 - Most large LLMs
-

b) Pipeline Parallelism

- Split model **by layers**
- Each GPU handles a subset of layers

Example:

- GPU 1 → Layers 1–4

- GPU 2 → Layers 5–8
- GPU 3 → Layers 9–12

Activations are passed between GPUs.

Pipeline Challenges

- Pipeline bubbles (idle GPUs)
 - Requires micro-batching to improve utilization
 - Adds latency
-

c) Expert Parallelism (MoE Models)

- Used in **Mixture-of-Experts (MoE)** LLMs
- Each expert FFN placed on different GPUs
- Router sends tokens to experts dynamically

Only selected experts are active per token.

Used in:

- Switch Transformer
 - Mixtral
 - DeepSeek
 - GShard
-

Key Observation

- Data Parallelism → scales **data**
- ZeRO → removes **memory redundancy**
- Model Parallelism → splits **model computation**

These techniques are **composable** and almost always used together in practice.

Flash Attention (Exact Attention Optimization)

Flash Attention is an **exact attention algorithm** introduced in **2022 (Stanford)**. It **does not approximate attention** — it computes the *same result* as standard self-attention, but **much faster and with far less memory usage**.

The key idea is to **exploit GPU memory hierarchy**.

GPU Memory Hierarchy (Why Flash Attention Exists)

GPs have **two very different memory types**:

1 HBM (High Bandwidth Memory)

- Large (\approx tens of GB, e.g., 80 GB on H100)
- Relatively **slow**
- Bandwidth: \sim 1–3 TB/s
- This is what we usually call *GPU memory*

2 SRAM (On-chip Memory)

- Very small (\approx tens of MB)
- **Extremely fast**
- Bandwidth: \sim 10–20+ TB/s
- Located next to compute units

⚠ Key bottleneck:

Modern GPUs are **memory-bound**, not compute-bound.

Most time is spent **reading/writing HBM**, not computing FLOPs.

Standard Self-Attention (Vanilla Implementation)

Self-attention formula:

$$\text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d}} \right) V$$

Where:

- $(Q \in \mathbb{R}^{N \times d})$
 - $(K \in \mathbb{R}^{N \times d})$
 - $(V \in \mathbb{R}^{N \times d})$
 - (N) = sequence length
-

Naive Attention Memory Pattern

1. Load (Q, K) from **HBM**
2. Compute (QK^T) \rightarrow store to **HBM**
3. Read (QK^T) from **HBM**
4. Apply softmax \rightarrow write to **HBM**
5. Read softmax result + (V) from **HBM**
6. Multiply \rightarrow write output to **HBM**

✗ Multiple HBM reads/writes **✗** Massive memory traffic **✗** GPU compute units stay idle waiting for memory

Key Insight Behind Flash Attention

Softmax Is Row-wise

For a row vector (x):

$$\text{softmax}(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

Important property:

- Softmax **can be computed incrementally**
 - You do **not need the full matrix at once**
-

Flash Attention Core Idea

👉 **Tile (block) the computation** so that:

- Small blocks fit into **SRAM**
- Entire attention computation for a block is done **end-to-end**
- Minimize reads/writes to **HBM**

This technique is called **tiling**.

Flash Attention Algorithm (High-Level)

For each **query block** (Q_i):

1. Load a small block of (Q_i) into **SRAM**
2. Loop over key/value blocks ((K_j, V_j)):
 - Load (K_j, V_j) into SRAM
 - Compute partial attention scores
 - Update softmax statistics **incrementally**
3. Accumulate final output
4. Write output once to **HBM**

- ✓ One HBM write per output
 - ✓ No intermediate matrices stored in HBM
-

Incremental Softmax Trick (Key Formula)

Let attention scores be split into blocks:

$$S = [S_1, S_2, \dots, S_n]$$

Instead of computing:

$$\text{softmax}(S) = \frac{e^S}{\sum e^S}$$

We maintain per-row:

- Running maximum (m)
- Running normalization factor (l)

Update rule (conceptual):

$$m_{\text{new}} = \max(m_{\text{old}}, \max(S_i))$$

$$l_{\text{new}} = e^{m_{\text{old}} - m_{\text{new}}} \cdot l_{\text{old}} + \sum e^{S_i - m_{\text{new}}}$$

This allows **exact softmax computation block-by-block**.

↗ No approximation — mathematically identical to full softmax.

Why This Is Faster

| Aspect | Vanilla Attention | Flash Attention |
|------------------|-------------------|-----------------|
| HBM Reads/Writes | Very High | Minimal |
| SRAM Usage | Minimal | Heavy |
| FLOPs | Same | Same |
| Runtime | Slow | Much Faster |

Flash Attention is **IO-aware**, not FLOP-aware.

Backward Pass Optimization (Recomputation)

Standard Training

- Store **activations** during forward pass
 - Use them during backward pass
 - Activation memory dominates GPU usage
-

Flash Attention Trick

- **Do not store activations**
- Recompute them during backward pass
- Recompute is cheap due to:
 - SRAM locality
 - Fast kernels

This is called **activation recomputation (checkpointing)**.

Counterintuitive Result

Even though:

- More FLOPs are executed (recomputation)

We get:

- **Less runtime**
- **Much less memory usage**

Example from paper:

- HBM reads reduced by $\sim 10\times$
 - Runtime reduced
 - Training fits larger batch sizes / longer context
-

Key Properties of Flash Attention

- Exact attention (no approximation)
 - Lower memory footprint
 - Faster forward + backward pass
 - Enables longer context lengths
 - Widely used in modern LLMs
-

Practical Notes

- Flash Attention v1, v2, v3:
 - Hardware-specific optimizations
 - Adapted to newer GPUs (A100, H100)
 - Used in:
 - PyTorch
 - Triton
 - xFormers
 - Most modern LLM stacks
-

Final Mental Model

Flash Attention **does not change what attention computes**
It changes **where and how** the computation happens.

Compute in **SRAM**, not **HBM**.

Quantization: Reducing Precision for Efficient LLM Training & Inference

Large Language Models store **all parameters, activations, gradients, and optimizer states** as **floating-point numbers**.

A natural question arises:

Do we really need *that much numerical precision* to achieve good performance?

Quantization addresses this question.

What Is Quantization?

Quantization is the process of converting numbers from:

- **Higher precision → Lower precision**

Goal:

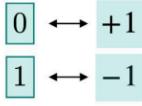
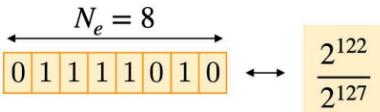
- Reduce **memory usage**
 - Increase **compute throughput**
 - Preserve **model quality** as much as possible
-

Floating Point Numbers: How They Are Represented

A floating-point number is stored as **bits**, split into three components:

| Component | Role |
|------------------------|-------------------------|
| Sign | Positive / Negative |
| Exponent | Scale of the number |
| Mantissa (Significand) | Precision / granularity |

Different floating-point formats allocate bits differently.

| Name | Description | Illustration |
|----------|--|---|
| Sign | Controls whether the number is positive or negative. Typically takes up to 1 bit. |  |
| Exponent | Controls the magnitude of the number. Also called <i>range</i> . |  |
| Mantissa | Controls the granularity of the number, i.e. what is after the decimal point. Also called <i>significand</i> or <i>fraction</i> . |  |

| | Sign | Exponent | Mantissa |
|-------------------------------------|------|----------|----------|
| FP16 (Floating-Point 16) | 1 | 5 | 10 |
| FP32 (Floating-Point 32) | 1 | 8 | 23 |
| FP64 (Floating-Point 64) | 1 | 11 | 52 |
| bfloat16 (Brain Float 16) | 1 | 8 | 7 |

Common Floating-Point Formats

| Format | Total Bits | Exponent Bits | Mantissa Bits | Notes |
|---------------|------------|---------------|---------------|------------------------------|
| FP64 (double) | 64 | 11 | 52 | Very high precision, slow |
| FP32 (single) | 32 | 8 | 23 | Standard training precision |
| FP16 (half) | 16 | 5 | 10 | Faster, less precise |
| bfloat16 | 16 | 8 | 7 | Better range, less precision |

★ Key observation

FP16 / bfloat16 use **half the memory** of FP32.

Why Lower Precision Helps

1 Memory Savings

- FP32 → 4 bytes per value
- FP16 / bfloat16 → 2 bytes per value

This affects:

- Model parameters
- Activations
- Gradients
- Optimizer states

Result:

- Larger batch sizes
- Longer context lengths
- Bigger models per GPU

2 Compute Speed Improvements

GPUs are optimized for **lower precision arithmetic**.

Example (NVIDIA H100, approximate):

| Precision | Peak Compute |
|-------------|----------------|
| FP64 | ~34 TFLOPS |
| FP32 | ~60 TFLOPS |
| FP16 / BF16 | ~1,000+ TFLOPS |
| INT8 | Even higher |

✖ Lower precision = **higher throughput**

Precision vs Accuracy Trade-off

Lower precision:

- ✖ Less granularity
- ✖ More numerical noise

But:

- Neural networks are **robust to noise**
- Most LLM training works well in FP16 / BF16

This makes quantization practical.

Mixed Precision Training

Modern LLMs **do not use a single precision everywhere.**

Instead, they use **mixed precision**:

- Weights: FP16 / BF16
 - Activations: FP16 / BF16
 - Gradients: FP16 / BF16
 - Accumulators & loss scaling: FP32
-

Loss Scaling (Why It's Needed)

FP16 has limited precision → small gradients may underflow.

Solution:

- Multiply loss by a scale factor (s)
- Compute gradients
- Divide gradients by (s)

This preserves gradient signal.

Quantization Beyond FP16

Quantization can go even further:

Integer Quantization

- INT8
- INT4
- Even INT2 (research)

General idea:

$$x_q = \text{round} \left(\frac{x}{\Delta} \right)$$

Where:

- (x) = original value
 - (Δ) = quantization scale
-

Types of Quantization

| Type | Description |
|-----------------------------|---------------------------------------|
| Post-Training Quantization | Quantize after training |
| Quantization-Aware Training | Simulate quantization during training |

| Type | Description |
|--------------------------|----------------------------|
| Weight-only Quantization | Only weights are quantized |
| Activation Quantization | Weights + activations |

Where Quantization Is Used

Training

- FP16 / BF16 mixed precision
- Saves memory + speeds training

Inference

- INT8 / INT4 common
- Huge memory savings
- Faster decoding
- Enables deployment on smaller GPUs

Why Quantization Works Well for LLMs

- Transformers are **overparameterized**
 - Small numerical errors do not significantly affect predictions
 - Attention + FFNs tolerate noise
- ↗ Performance drop is often **minimal or negligible**

Practical Takeaways

- Quantization reduces **memory + compute**
- FP16 / BF16 is standard for LLM training
- Lower-bit quantization is crucial for inference
- Hardware is explicitly optimized for low precision

Mental Model

Quantization trades **numerical precision** for **efficiency** and LLMs are robust enough to handle it.

Mixed Precision Training

Motivation

LLMs involve:

- Massive models
- Huge datasets
- Extremely high memory and compute costs

Key question:

Can we use **lower numerical precision** without hurting model performance?

Mixed precision training is the answer.

Core Idea

Use different floating-point precisions for different parts of training:

- **Weights** → High precision (**FP32**)
- **Forward pass** → Lower precision (**FP16 / BF16**)
- **Backward pass** → Lower precision (**FP16 / BF16**)
- **Weight updates** → High precision (**FP32**)

This allows:

- Faster computation
- Lower memory usage
- Minimal performance degradation

Standard Mixed Precision Workflow

1. **Model weights stored in FP32**
 2. Inputs and activations computed in FP16
 3. Gradients computed in FP16
 4. Gradients are accumulated and applied to **FP32 weights**
 5. Updated FP32 weights are used in next iteration
-

Why Keep Weights in FP32?

Intuition:

- **Forward / backward computations**
 - Operate on data batches
 - Data itself is noisy
 - Extreme numerical precision is not critical
- **Weights**

- Accumulate updates over millions of steps
 - Quantization errors can **accumulate**
 - FP32 prevents long-term numerical drift
- Think of gradients as *directional hints*
Think of weights as *long-term memory*
-

Benefits of Mixed Precision Training

1 Memory Savings

- FP16 uses **half the memory** of FP32
 - Enables:
 - Larger batch sizes
 - Longer context lengths
 - Larger models per GPU
-

2 Faster Training

- GPUs are optimized for FP16 / BF16
 - Much higher throughput than FP32
 - Especially effective on modern GPUs (A100, H100)
-

3 Minimal Performance Loss

- Empirically shown:
 - Little to no degradation in final model quality
 - Widely adopted in production LLM training
-

Loss Scaling (Important Detail)

Problem:

- FP16 has limited dynamic range
- Small gradients may **underflow to zero**

Solution: **Loss Scaling**

1. Multiply loss by a scaling factor (s)
2. Compute gradients
3. Divide gradients by (s) before update

This preserves gradient signal.

Do We Apply Mixed Precision Everywhere?

Short answer: **Not always**

- Some layers are more sensitive to precision:
 - LayerNorm
 - Softmax
 - Attention score computation

Different setups may:

- Keep some operations in FP32
- Use BF16 instead of FP16 for stability

There is **no single universal recipe.**

Relationship to Scaling Laws

Mixed precision is **orthogonal** to:

- Model size vs token size tradeoffs
- Chinchilla-style scaling laws

In practice:

- Teams run **small-scale experiments**
 - Determine:
 - Optimal precision
 - Optimal model size
 - Optimal dataset size
 - Then extrapolate to large-scale training
-

Practical Notes

- FP16 and BF16 are the most common choices
 - BF16 has:
 - Wider exponent range
 - Better numerical stability
 - Many modern LLMs prefer **BF16**
-

Key Takeaway

Mixed precision training:

- Reduces memory
- Speeds up training

- Preserves model quality

It is a **core technique** in modern LLM training pipelines.

Supervised Fine-Tuning (SFT)

Why Pre-training Is Not Enough

- Pre-training teaches the model:
 - Grammar
 - Syntax
 - Broad world patterns
- But **not helpful behavior**

Example:

"Can I put my teddy bear in the washer?"

A pre-trained LLM:

- Predicts likely next tokens
- Does **not reason as an assistant**
- Mimics patterns from internet text

✗ Not helpful

✗ Not aligned with user intent

Motivation for Fine-Tuning

Goal:

- Turn a **next-token predictor**
- Into a **helpful assistant**

This is achieved via **Supervised Fine-Tuning (SFT)**.

What Is SFT?

SFT = Supervised Fine-Tuning

- Start from **pre-trained weights**
- Train on **(input, output)** pairs
- Objective:
 - Produce *useful, human-like responses*
- Supervised = labeled data

Nature of SFT Data

Each training example contains:

- **Input (prompt)**
- **Target output (ideal response)**

Example:

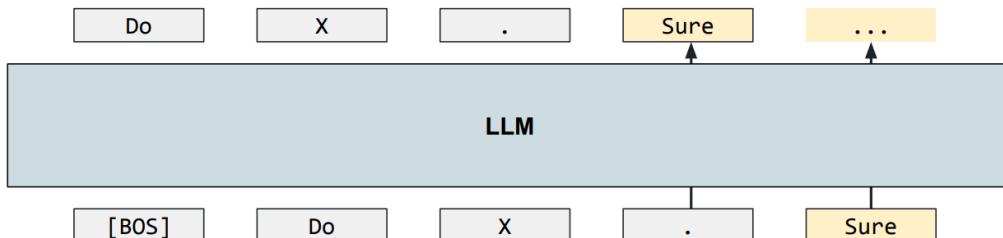
Input: "Can I put my teddy bear in the washer?"

Output: "It depends on the material. Check the care label..."

Goal. "Graduate" the model to being a helpful assistant.

| | |
|--|--|
| Story writing Write a short story about a teddy bear who likes to read poetry. Once upon a time, a bear, Teddy, stumbled upon verses from Attar. The bear became so captivated with the rhythm of the poem's sentences that he got filled with happiness. | Lists generation List three fun activities a teddy bear might do on a rainy day. Sure! 1. Read poetry with friends. 2. Be cute. 3. Hug its owner tightly. |
| Poem creation Create a poem about my cute teddy bear. Soft and cuddly, full of charm, Always keeps me safe from harm, With button eyes and fur so neat, My teddy bear is oh so sweet. | Explanation Explain why a teddy bear is a great friend. A teddy bear is a great friend because it provides comfort and companionship. Its soft fur and cuddly nature is perfect for hugs. Teddy bears are always there in both happy and sad moments. |

Objective function. Predict next token given the input.



Key Difference: Pre-training vs SFT Objective

Pre-training

- Model predicts **every token**
- Includes the input itself

$$\mathcal{L}_{\text{pretrain}} = - \sum_{t=1}^T \log P(x_t | x_1, \dots, x_{t-1})$$

- Loss applies to **all tokens**
 - Teacher forcing everywhere
-

Supervised Fine-Tuning (SFT)

- Input is **conditioning context**
- No loss on input tokens
- Loss applies **only to output tokens**

Let:

- (x) = input (prompt)
- $(y = (y_1, \dots, y_N))$ = target output

$$\mathcal{L}_{\text{SFT}} = - \sum_{t=1}^N \log P(y_t | x, y_1, \dots, y_{t-1})$$

- Input is **given**
 - Model is trained to **respond**, not repeat
-

Teacher Forcing in SFT

- Teacher forcing is used **only on output**
 - The model:
 - Sees the full prompt
 - Predicts the response token-by-token
 - Ground truth output tokens are provided during training
-

Intuition Behind SFT

- Pre-training:
 - "What token usually comes next?"
- SFT:
 - "Given this instruction, what is the best response?"

This shifts the model from:

- Language modeling
to
 - Instruction following
-

Why SFT Works Well

- Uses **human-written answers**

- Teaches:
 - Helpfulness
 - Politeness
 - Structure
 - Safety-aware behavior
 - Much cheaper than pre-training
 - Strong performance gains
-

Role of SFT in LLM Training Pipeline

1. **Pre-training**
 - Learn language & world patterns
 2. **Supervised Fine-Tuning**
 - Learn how to respond
 3. *(Next)* Alignment & preference optimization
-

Key Takeaways

- SFT is **mandatory** for useful LLMs
- Input tokens are **not predicted**
- Loss is applied **only to outputs**
- Same next-token loss, different conditioning
- Turns LLM into an assistant

Instruction Tuning (A Special Case of SFT)

From Language Modeling → Helpful Assistant

- Pre-training:
 - Learns **how language looks**
 - Predicts likely next tokens
- Supervised Fine-Tuning (SFT):
 - Teaches **how to respond**
- **Instruction Tuning:**
 - Teaches the model to **follow instructions**

Instruction tuning is a **subcategory of SFT** focused specifically on:

- Answering user instructions
 - Acting as an assistant
-

Instruction Tuning Objective

Given:

- An **instruction** (input)
- A **desired response** (output)

The model is trained to:

- Condition on the instruction
- Predict only the response tokens

Loss Application

- No loss on instruction tokens
- Loss applied only to response tokens

Illustration:

[Instruction / Prompt] → [Response Tokens] ↑ Loss computed here

Instruction-Tuning Data Composition

Unlike pre-training (raw internet text), instruction tuning uses **curated data** that demonstrates *helpful behavior*.

Common Instruction Categories

- Story writing
- Poem generation
- List creation
- Explanations
- Summarization
- Question answering
- Code generation
- Reasoning tasks
- Assistant-style dialogues

Each example is a (**instruction**, **ideal response**) pair.

Assistant Dialogues

- Many SFT datasets are framed as **assistant conversations**
- User asks a question
- Assistant provides a helpful answer

This trains the model to:

- Be conversational

- Follow intent
 - Produce coherent responses
-

Human vs Synthetic Instruction Data

Early Instruction Tuning

- Instructions written by humans
- Responses written by expert annotators
- High quality but **slow and expensive**

Modern Instruction Tuning

- Use existing strong LLMs to:
 - Generate candidate responses
 - Augment datasets
- Humans or other models:
 - Review
 - Filter
 - Rank quality

This **scales instruction data creation** significantly.

Safety-Oriented Instruction Data

Instruction tuning data also includes **safety-focused examples**.

Purpose

- Ensure model is:
 - Helpful
 - Harmless
 - Responsible

Examples

- Refusing harmful requests
- Avoiding dangerous instructions
- Hedging uncertain claims
- Rejecting disallowed content

Example behavior:

"I'm sorry, I can't help with that request."

 This behavior is **learned**, not hard-coded via rules.

Why Not Use Rules or Regex?

- Rule-based systems:
 - Not scalable
 - Fragile
 - Instead:
 - Safety behavior is **embedded into model weights**
 - Learned through supervised examples
-

Generalization From Instruction Tuning

Example:

- Dataset includes:
 - "Write a story"
- Prompt at inference:
 - "Write a sci-fi poem about space-time"

Why does this work?

- Pre-training:
 - Teaches what sci-fi, poems, space-time are
 - Instruction tuning:
 - Teaches **how to follow instructions**
- Model learns the *pattern*, not the exact content.
-

Instruction Tuning Scale (Orders of Magnitude)

Instruction tuning uses **far less data** than pre-training.

Reported Numbers

- GPT-3:
 - ~13,000 instruction examples
- LLaMA-3:
 - ~10 million instruction examples

Rough Token Estimate

Assume:

- ~1,000 tokens per example

Then:

- SFT data << Pre-training data
 - Several **orders of magnitude smaller**
-

Mental Model: Pre-training vs Instruction Tuning

| Stage | Data Size | Data Quality | Purpose |
|--------------------------|----------------------|--------------|------------------------|
| Pre-training | Trillions of tokens | Noisy | Learn language |
| SFT / Instruction Tuning | Millions of examples | High-quality | Learn helpful behavior |

Effect of Instruction Tuning (Teddy Bear Example)

Before (Pre-trained model):

- Continues text probabilistically
- Does not answer the question

After Instruction Tuning:

- Responds directly
- Gives helpful advice
- Aligns with user intent

Example:

"You should handwash the teddy bear instead of using a washer."

Key Observation

- Instruction tuning **aligns model behavior**
- Makes LLMs usable by humans
- Still uses next-token prediction
- But with **task-aware conditioning**

Challenges of Supervised Fine-Tuning (SFT) and Motivation for Optimization

1. High-Quality Data Requirement

- SFT requires **high-quality, curated data**
- "High-quality" usually implies:
 - Human involvement
 - Expert-written responses
 - Compliance with safety and policy rules

- Early SFT datasets were:
 - Almost entirely **human-generated**
 - Expensive in time, cost, and coordination

Modern pipelines:

- Mix **human-written** and **model-generated** data
- Still require:
 - Human review
 - Filtering
 - Quality control

Advantage:

- Once created, datasets can be **reused and extended over time**

Disadvantage:

- Still expensive in human and computational resources
-

2. Prompt Distribution Mismatch

- SFT data comes from a **specific prompt distribution**
- Real-world inference prompts may differ significantly

Example:

- Training prompt:
 - "Write a story"
- Inference prompt:
 - "Write a story inspired by a specific movie plot"

This creates an **out-of-distribution (OOD)** issue.

Key idea:

- Model generalization depends on:
 - Coverage of prompt space
 - Diversity of training examples
-

3. Memorization vs Generalization

- Question:
 - If we give the model a prompt identical to one seen during SFT, will it reproduce the same response?

Answer:

- Usually **no**, because:

- Sampling is stochastic
- Temperature > 0
- Output may:
 - Have the same *flavor*
 - But not the same wording

Influencing factors:

- Temperature
 - Sampling strategy
 - Pre-training diversity
-

4. Role of Temperature

- Higher temperature:
 - More diversity
 - More creative outputs
 - Higher chance of unlikely tokens
- Lower temperature:
 - More deterministic
 - Less variation

Temperature directly controls:

- How much the model explores the probability space
-

5. Improving Generalization

- Primary lever: **data**
- More diverse and sparse coverage of prompt space:
 - Better generalization
 - Less overfitting to specific examples
- Repeating similar examples \neq better learning

Key idea:

- Teach the **concept**, not the exact instance
-

Evaluation Challenges for Instruction-Tuned Models

Why Evaluation Is Hard

- Helpfulness is **subjective**
 - No single metric captures user satisfaction
-

Benchmark-Based Evaluation

Common benchmark categories:

- General language understanding
- Reasoning
- Math
- Code generation

Examples:

- **MMLU** (Massive Multitask Language Understanding)
 - ~50 tasks
 - Single aggregate score
 - **GSM8K**
 - Grade-school math reasoning
 - ~8K problems
-

The “Training on the Test Task” Problem

Observed phenomenon:

- Sudden benchmark performance jumps
- Often unexplained by architecture changes

Cause:

- Model trained on **data resembling the benchmark task**

Important distinction:

- Training on the **test task**
- NOT necessarily training on the **test set**

Implication:

- Benchmark scores depend heavily on training mixture

⚠ Fair comparison requires:

- Parity in training exposure
 - Transparency about auxiliary data
-

Benchmark Saturation

- Models optimize for known benchmarks
- New benchmarks appear to fill gaps
- Leads to:
 - Score inflation

- Weak correlation with real-world usefulness
-

Human Preference-Based Evaluation

Chatbot Arena

- Users compare two model responses
- Choose preferred one
- Rankings computed via pairwise comparisons

Captures:

- "Vibes"
 - User preference
 - Conversational quality
-

Limitations of Preference-Based Evaluation

1. Early Noise Sensitivity

- Initial comparisons heavily influence rankings

2. Leaderboard Manipulation Risk

- Models can detect:
 - Who they are being compared against
- Adversarial behavior possible

3. Factuality vs Helpfulness

- Users may prefer:
 - Detailed but incorrect answers
- Users may lack domain knowledge to verify correctness

4. Subjective Preferences

- Style preferences differ:
 - Emojis vs no emojis
 - Conciseness vs verbosity
- Expert preferences ≠ general population preferences

5. Safety Bias

- Users dislike refusals
 - Preference systems may favor unsafe compliance
 - Conflicts with intended safety policies
-

No Single "Best" Metric

- Benchmarks → objective but narrow

- Human preference → subjective but broad
- Safety → policy-driven
- Real usefulness depends on **use case**

Evaluation must be:

- Multi-dimensional
 - Context-aware
 - Goal-driven
-

Alignment: Where SFT Fits

Alignment Definition

Alignment = making the model:

- Helpful
- Harmless
- Honest

Alignment Stages

1. **Supervised Fine-Tuning (SFT)**
2. **Preference Tuning** (next lecture)

Together:

- Form the **alignment pipeline**
-

Emerging Concept: Mid-Training

- Occurs **after pre-training**
- Uses the same next-token objective
- But on **task-relevant data**
- Bridges gap between:
 - General pre-training
 - Task-specific fine-tuning

Mid-training is:

- New
 - Actively researched
 - Increasingly adopted
-

Big Picture (So Far)

1. Pre-training:
 - Learn language
2. SFT / Instruction Tuning:
 - Learn to help
3. Preference Tuning:
 - Learn what users prefer
4. Evaluation:
 - Hard, multi-faceted, imperfect

Parameter-Efficient Fine-Tuning (PEFT): LoRA and QLoRA

Motivation: Why PEFT?

- Full fine-tuning updates **all model parameters**
 - Computationally expensive:
 - High memory usage
 - Large optimizer states
 - Long training time
 - Goal:
 - Adapt large pre-trained models
 - While training **as few parameters as possible**
-

LoRA: Low-Rank Adaptation

Core Idea

- **Freeze** pre-trained weights
 - Learn a **low-rank update** instead of full weight updates
-

Standard Linear Layer

Given a linear transformation:

$$y = W_0 x$$

- $(W_0 \in \mathbb{R}^{d_{out} \times d_{in}})$
 - (W_0) is **pre-trained**
-

LoRA Formulation

Instead of updating (W_0) , we write:

$$W = W_0 + \Delta W$$

Where:

$$\Delta W = BA$$

- $(B \in \mathbb{R}^{d_{out} \times r})$
 - $(A \in \mathbb{R}^{r \times d_{in}})$
 - $(r \ll \min(d_{out}, d_{in}))$
-

Forward Pass with LoRA

$$y = W_0x + B(Ax)$$

- (W_0) : **frozen**
 - (A, B) : **trainable**
-

Parameter Reduction

- Full fine-tuning parameters:

$$d_{out} \times d_{in}$$

- LoRA parameters:

$$r(d_{out} + d_{in})$$

Example:

- $(d_{out}, d_{in} \sim 1000)$
- $(r \sim 4)$ or (8)

➡ Orders of magnitude fewer trainable parameters

Task-Specific Adaptation

- (W_0) : general language knowledge
- (A, B) : task-specific adaptation

Example:

- Spam detection → one set of (A, B)
- Sentiment analysis → another set of (A, B)

Same base model, different LoRA adapters

Where Is LoRA Applied?

Original LoRA Paper

- Applied mainly to **attention projections**:

- (W_Q, W_K, W_V, W_O)
-

Later Findings

- Best performance gains from:
 - **Feed-Forward Networks (FFN / MLP blocks)**
 - Modern practice:
 - Apply LoRA to:
 - Attention layers
 - Feed-forward layers
 - Majority of gains from FFN
-

Practical Training Observations (Empirical)

1. Higher Learning Rate

- LoRA typically uses:

$$\text{LR}_{\text{LoRA}} \approx 10 \times \text{LR}_{\text{full FT}}$$

Hypothesis:

- Low-rank matrices explore a smaller subspace
 - Need larger steps to adapt effectively
-

2. Large Batch Size Hurts

- LoRA performance degrades with very large batch sizes

Possible explanation:

- Training dynamics of **matrix products** differ from full matrices
- Gradient noise may help LoRA adaptation

 Empirical observation, not fully theoretically explained

Rank Selection (Hyperparameter (r))

- Typical values:
 - $(r = 4, 8, 16)$
 - Grid search possible but often unnecessary
 - Popular defaults work well
 - Parameter reduction already massive → diminishing returns beyond this
-

QLoRA: Quantized LoRA

Motivation

- Even frozen weights (W_0) consume VRAM
 - Goal:
 - Quantize **frozen base model**
 - Train LoRA adapters in high precision
-

QLoRA Setup

- (W_0) : **Quantized**
 - (A, B) : **BF16 / FP16**
 - Gradients only flow through LoRA weights
-

NF4 Quantization

- NF4 = NormalFloat 4-bit
- Assumes weights follow a **normal distribution**
- Uses **quantiles**, not uniform buckets

Benefits:

- Better use of limited bits
 - Lower quantization error
-

Double Quantization

1. Quantize weights
2. Quantize quantization constants

Purpose:

- Further reduce memory overhead
-

Memory Savings

- QLoRA achieves:
 - **~16x VRAM reduction**
 - Double quantization:
 - Small additional gains
-

Summary of LoRA vs QLoRA

| Method | Base Weights | Trainable Weights | Precision | Memory |
|---------|--------------|-------------------|-----------|--------------|
| Full FT | Trainable | All | FP16/32 | ✗ Huge |
| LoRA | Frozen | A, B | FP16 | ✓ Low |
| QLoRA | Quantized | A, B | BF16 | ✓ ✓ Very Low |

Key Takeaways

- LoRA enables efficient fine-tuning of LLMs
- Low-rank updates capture task-specific knowledge
- QLoRA makes fine-tuning possible on limited hardware
- Widely used in practice for LLM adaptation

In []: