Travelling Salesman Problem

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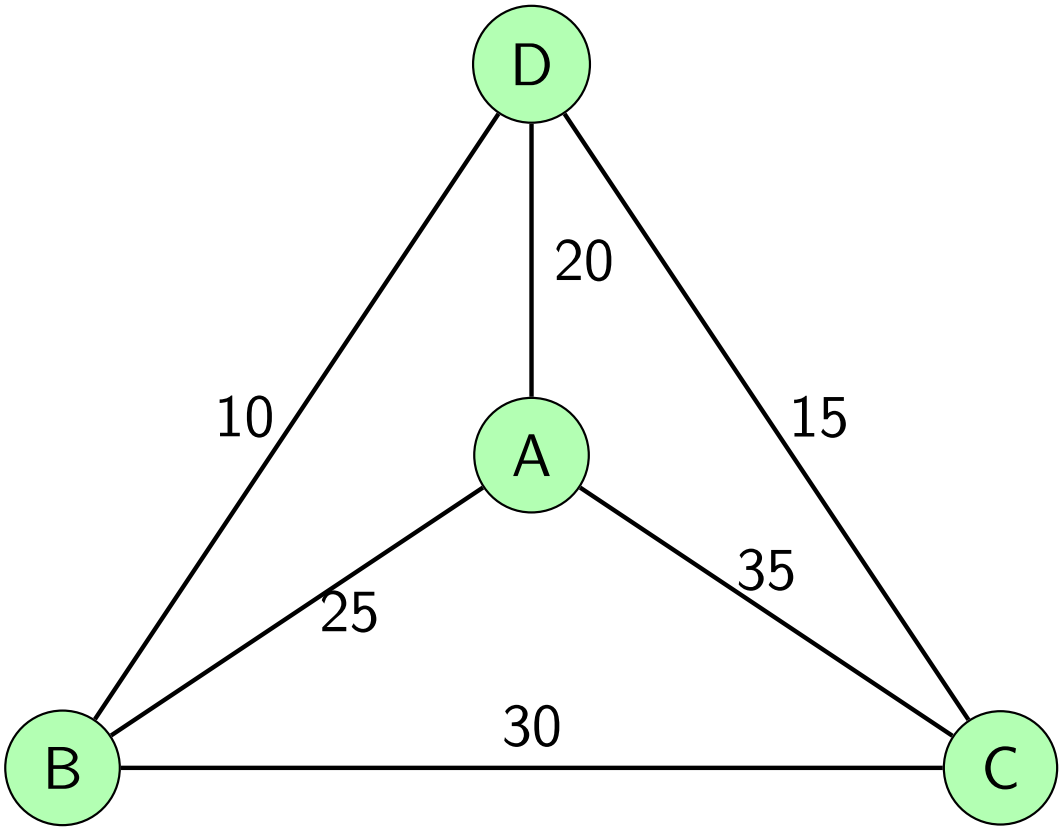
Problem Statement

Travelling Salesman Problem

Given a set of cities and the distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point

Problem Statement



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Application

Genetics and Biology

- to optimize the order in which different genes are sequenced

Image : https://bit.ly/3KyXsqz

Application

Logistics and Transportation

-to optimize the delivery routes of goods, services, or people

Image : https://bit.ly/3IMfl3V

Application

Computer Wiring

-connecting together computer components using minimum wire length

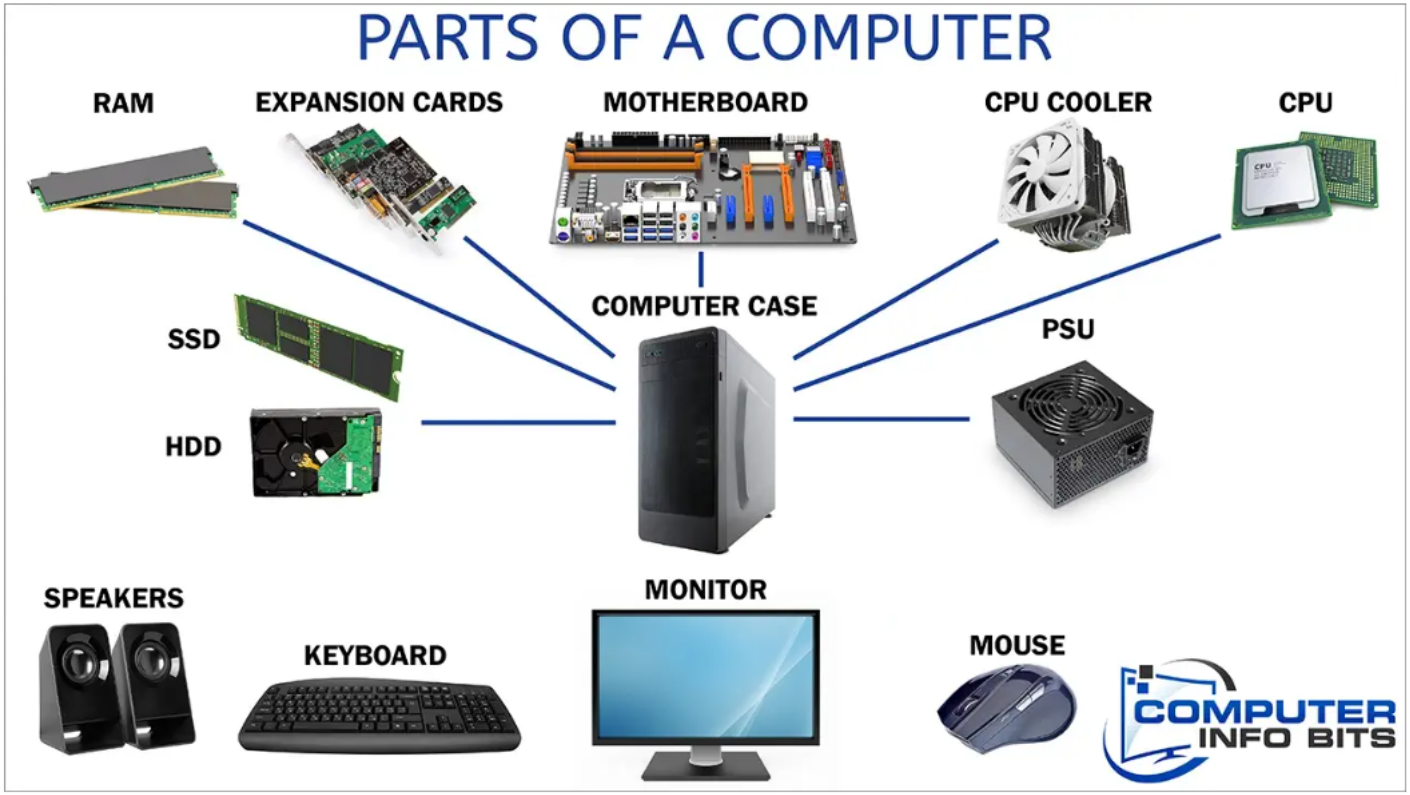

Image : https://bit.ly/3YYr44W

Application

- *Network Design*
- *Circuit Board Design*
- *Job Sequencing*
- *Airplane Scheduling*

And many more ...

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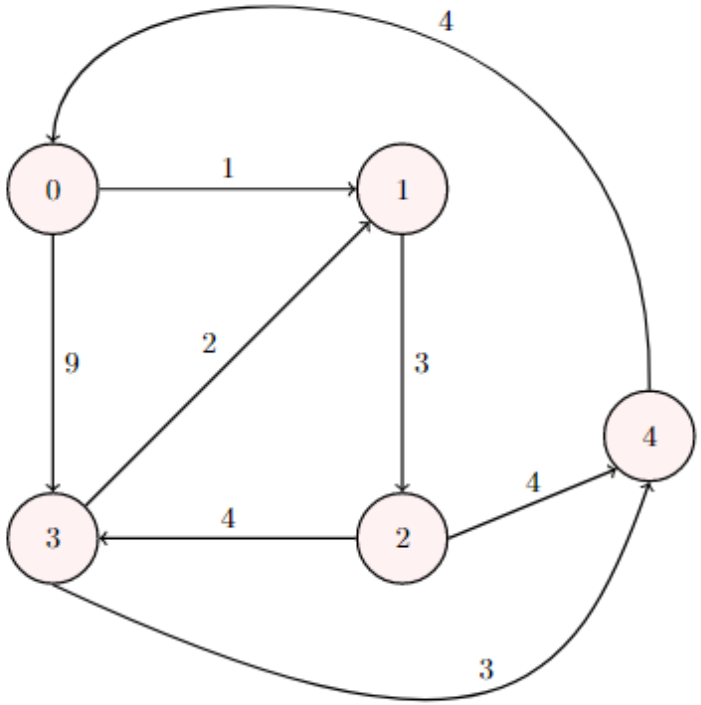
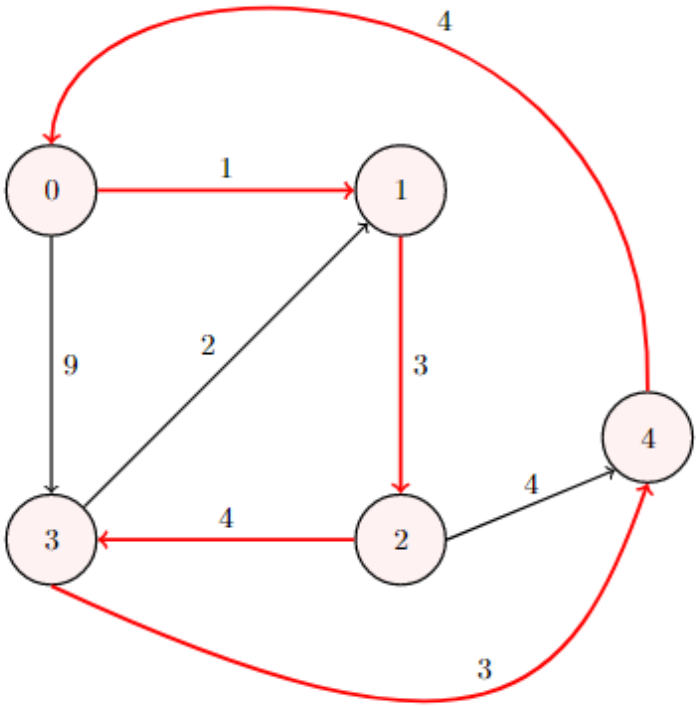
How to implement?



What will be the minimum cost?

How to implement?

Path 1:

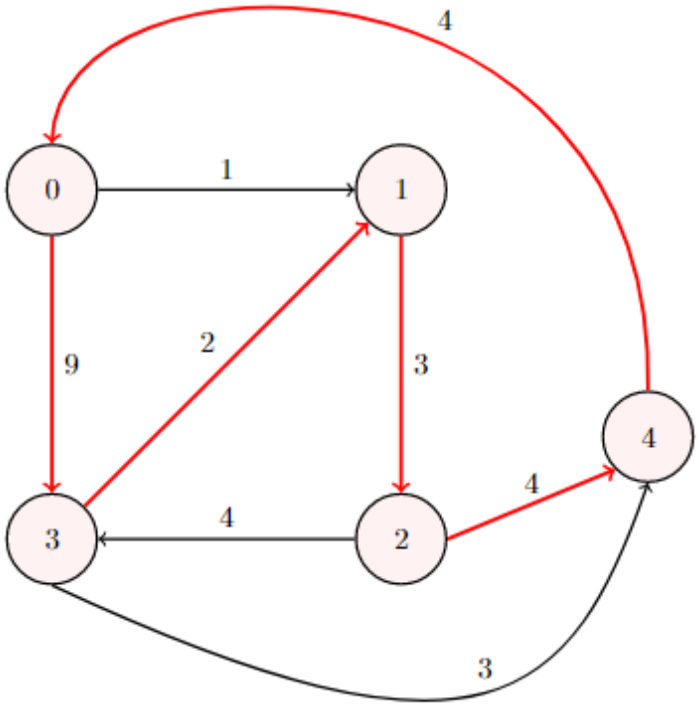


- $0 - 1 - 2 - 3 - 4 - 0$

Cost: $1(0\text{-}1) + 3(1\text{-}2) + 4(2\text{-}3) + 3(3\text{-}4) + 4(4\text{-}0) = 15$

How to implement?

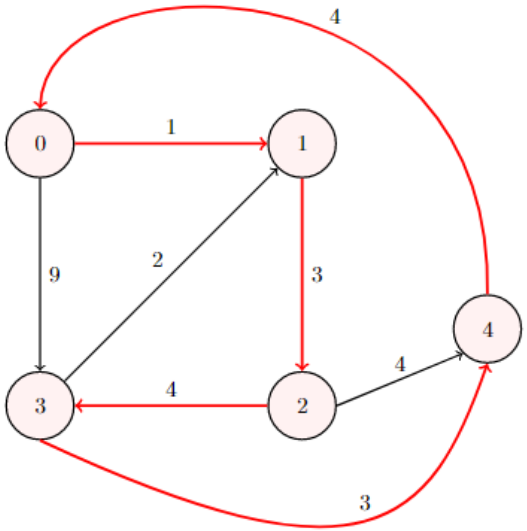
Path 2:

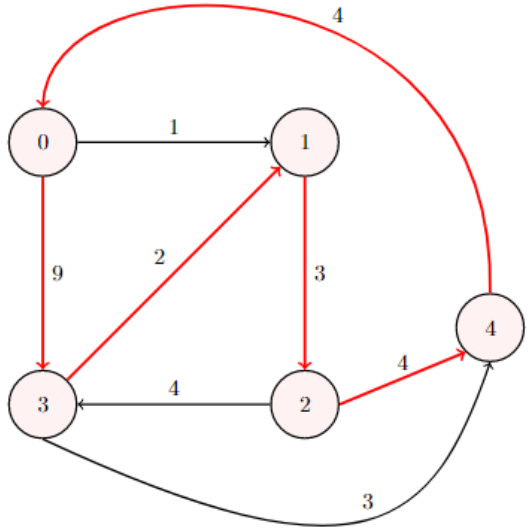


- $0 - 3 - 1 - 2 - 4 - 0$

Cost: $9(0\text{-}3) + 2(3\text{-}1) + 3(1\text{-}2) + 4(2\text{-}4) + 4(4\text{-}0) = 22$

How to implement?





Here the minimal cost is 15

- A complete graph with n vertex gives n! different path combinations
- Which requires $O(n^2 n!)$ runtime
- No Polynomial time solution
- NP-hard problem

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  3. Bitmask Dynamic Programming

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  2. Approximation using MST
  3. Bitmask Dynamic Programming
- Here we only see Bitmask DP algorithm

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- *But what is the problems?*
  - Can't travel one city more than once.
  - Must remember which city we already visited.

Why Bit Mask?

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..Cont'd Subproblem

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..Cont'd Subproblem

- *Add additional parameter mask.*
  - Our updated function, $f(i, mask)$.
  - i indicates present city.
  - mask contains information about the tour.
- *How do we implement bit masking?*

Bit Masking

If started from city 0.

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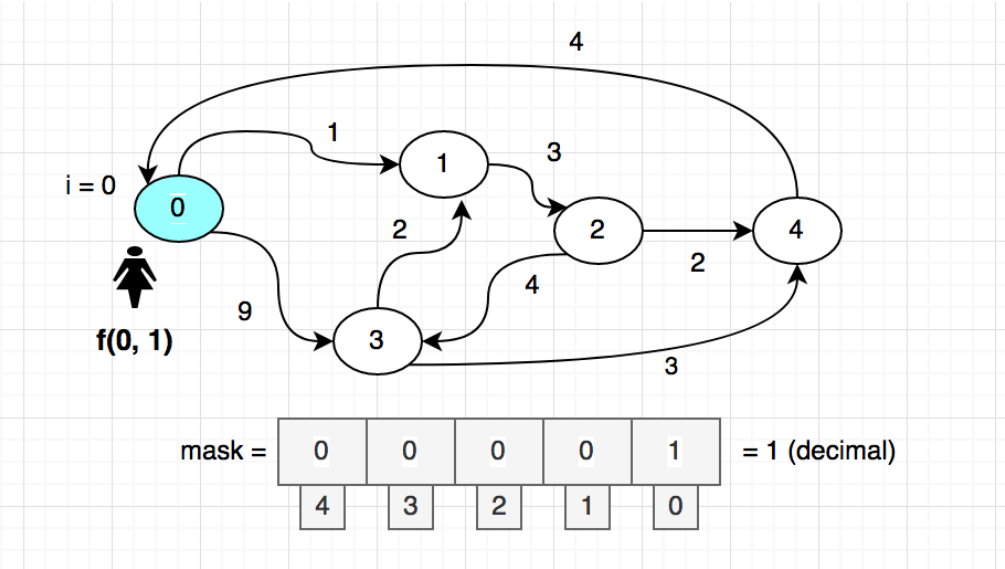
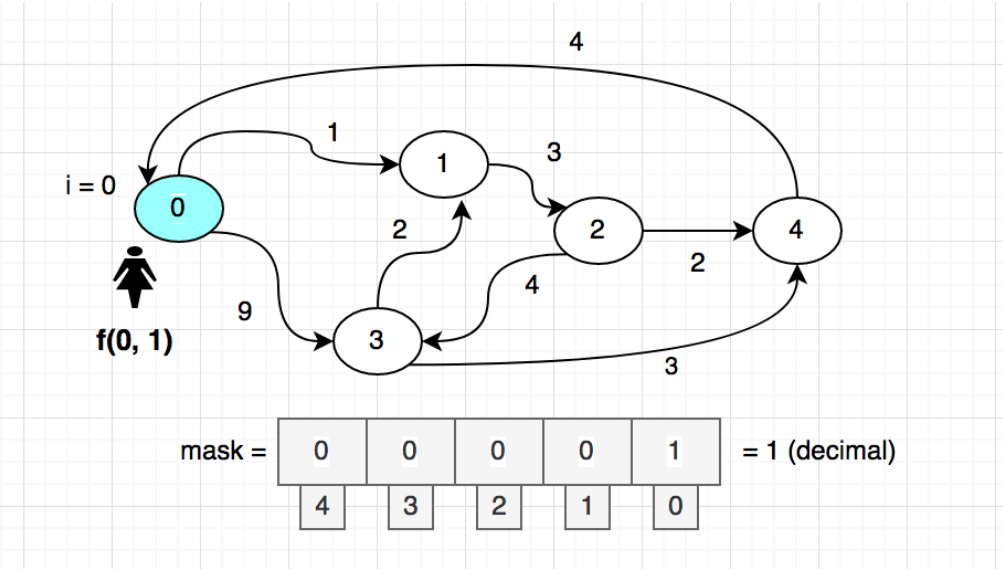


Bit Masking

If started from city 0.

Then he goes to city 3.



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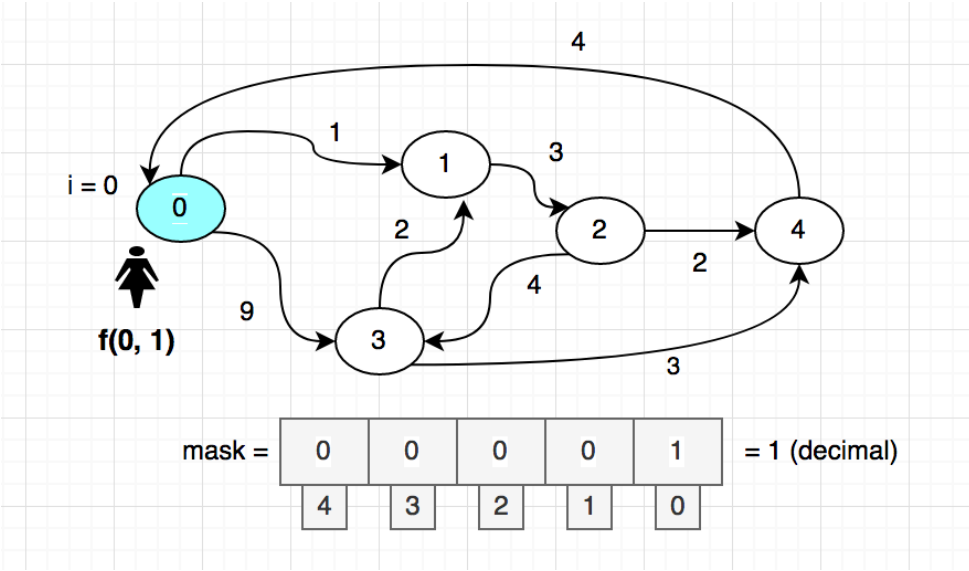
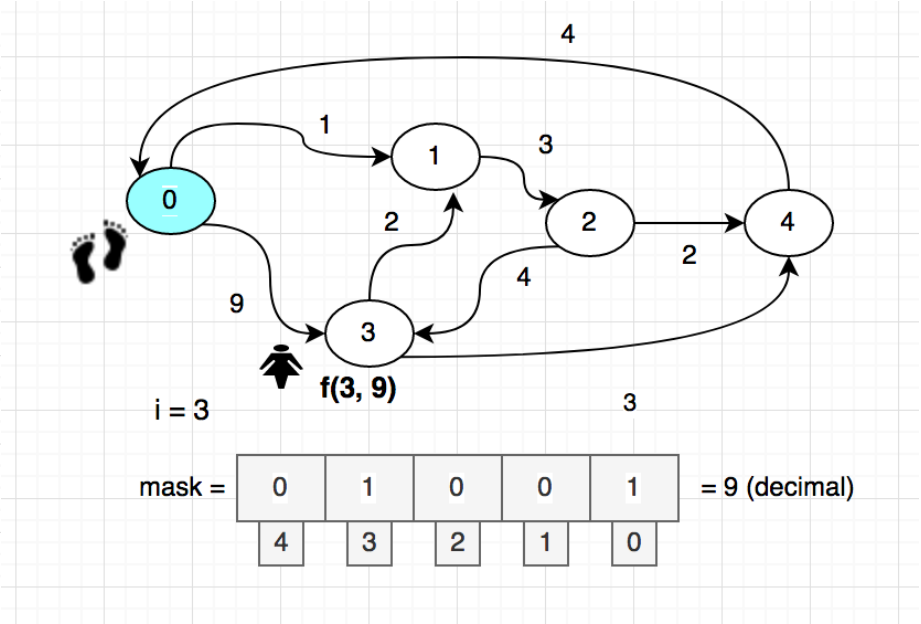


Then he goes to city 3.



Bit Masking

```
int turnOn(int x,int pos)
{
    return x | (1<<pos);
}
```

Code snippet for turning on a bit

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Code snippet for turning on a bit

```
bool isOn(int N,int pos)
{
    return (bool)(N & (1<<pos));
}
```

Code snippet for checking a bit

Recursive Formula

- We have to solve $f(0, 1)$

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- Base case $f(i, 2^{n-1})$

Recursive Formula

$f(i, 2^{n-1}) = dis[i][0]$
$f(i, mask) = min(f(j, turnOn(mask, j)) + w[i][j]) where i, j \in E$

Algorithm

```
#define EMPTY_VALUE -1
#define MAX_N 10
#define INF 1061109567

int w[MAX_N][MAX_N];
int mem[MAX_N][1<<MAX_N];
int n;

int f(int i, int mask)
{
    if (mask == (1<<n) - 1)
        return w[i][0];


    if (mem[i][mask] != -1)
        return mem[i][mask];


    int ans = INF;
    for (int j = 0;j < n;j++)
    {
        if (w[i][j] == INF) continue;

        if (isOn(mask,j) == 0)
        {
            int result = f(j, turnOn(mask, j)) + w[i][j];
            ans = min(ans, result);
        }
    }

    return mem[i][mask] = ans;
}
```

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- For n city mask value can be 0 to 2^n.

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- Total Run time: $O(n^2 * 2^n)$

Thank you

Any Question?