PRACTICAL EXERCISES
WED/THURS 20/21-3-2024

CLASS 6

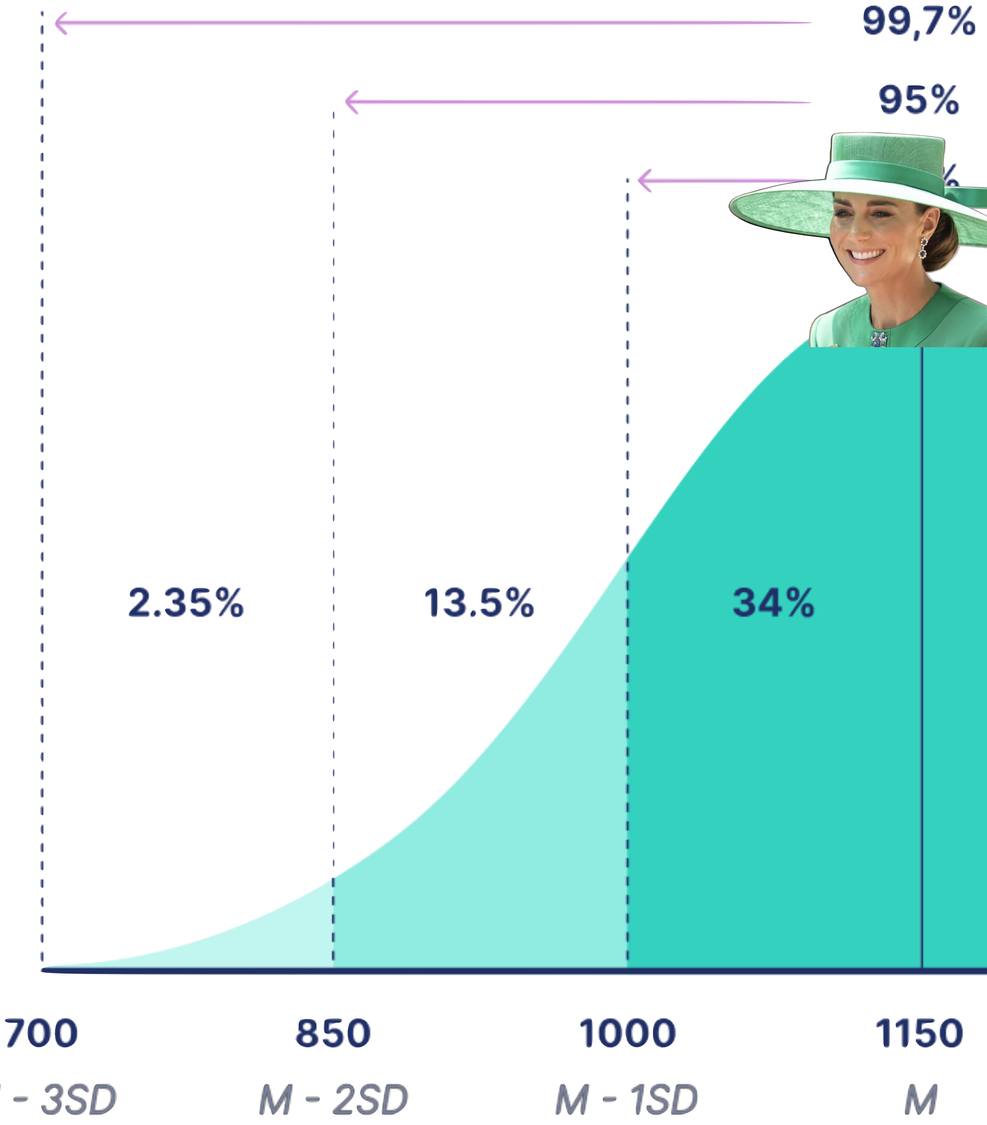
MATRIX AND VECTOR OPERATIONS + LEAST SQUARES APPROXIMATION

"GENERATE A PICTURE OF LINEAR ALGEBRA AND KATE MIDDLETON DISAPPEARING"

STUDENT INSTRUCTOR | PERNILLE HØJLUND BRAMS (AU650502@CAS.AU.DK)
COURSE | METHODS 2: THE GENERAL LINEAR MODEL

AGENDA

- Recap, runthrough + group exercises
- Midterm
- Break
- Quick talk about midterm
- Exercises 📝



https://www.scribbr.com/statistics/normal-distribution/

https://people.com/kevin-pietersen-says-seen-kate-middleton-last-couple-days-amid-conspiracy-theories-8611234

MATRIX MULTIPLICATION (PT. 1/3)

WHEN CAN I MULTIPLY TWO MATRICES?

Two matrices can be multiplied, if

The number of columns in the **1st** matrix equals number of rows in the **2nd**

CAN I MULTIPLY THESE?

COMPATIBILITY STATUS

1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

2 $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$

3 $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$$

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$$

👍👍👍👍💞👍👍👍👍

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$$

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$$

🙅‍♀️🤺😑😡&$!#%NO🙅‍♀️🤺😑😡&$!#%

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$$F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$$

😍🥰 MATRIX SPEED DATING 🥰😍

Two matrices can be multiplied, if

The number of columns in the **1st** matrix
equals number of rows in the **2nd**

ARE THEY COMPATIBLE?

$$E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$$F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$$

👍👍👍👍💞👍👍👍👍

MATRIX MULTIPLICATION (PT. 1/3)

WHEN CAN I MULTIPLY TWO MATRICES?

Two matrices can be multiplied, if

The number of columns in the **1st** matrix equals number of rows in the **2nd**

CAN I MULTIPLY THESE?

COMPATIBILITY STATUS

1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

2 $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$

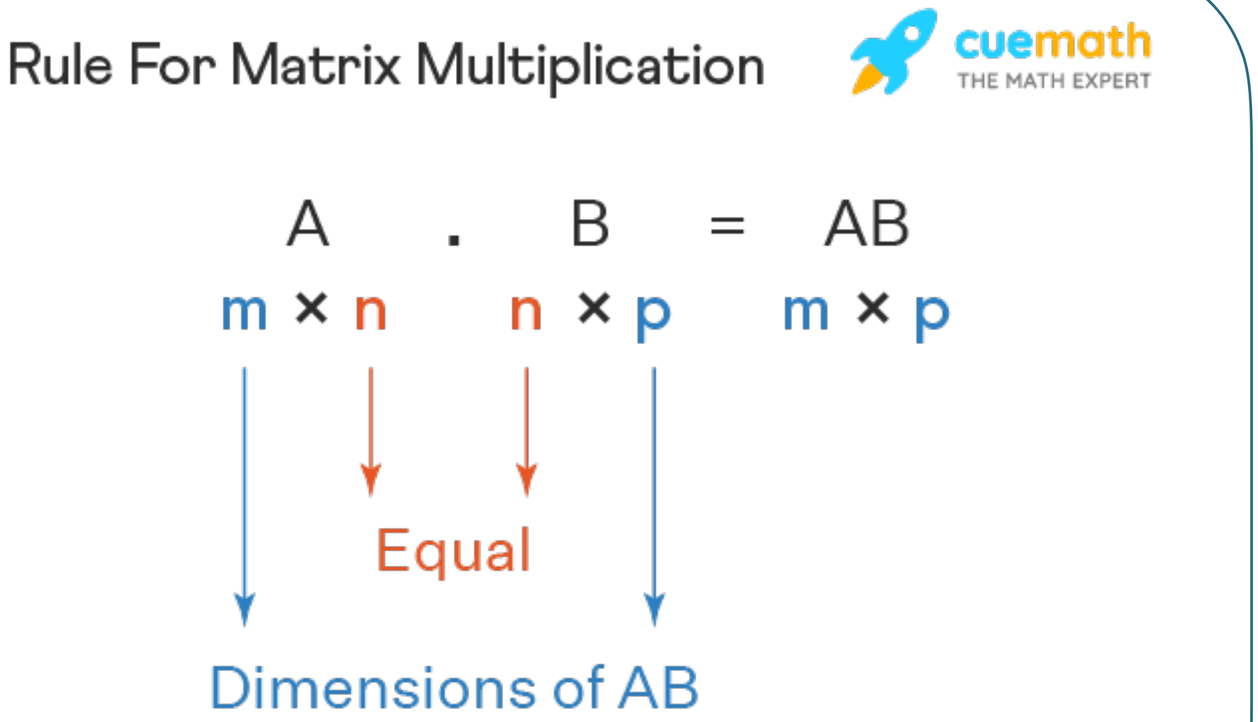
C (2x2) and D (3x2)

3 $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

MATRIX MULTIPLICATION (PT. 2/3)

WHAT DIMENSIONS SHOULD THE PRODUCT MATRIX HAVE?



1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

? WHAT DIMENSIONS WILL A*B HAVE?

3 $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

? WHAT DIMENSIONS WILL E*F HAVE?

MATRIX MULTIPLICATION (PT. 2/3)

IN GROUPS
CALCULATE THE PRODUCT MATRICES AB AND EF (AND SECRET MATRIX GH):

1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

GH $G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}, H = \begin{pmatrix} 9 & 10 \\ 11 & 12 \end{pmatrix}$

3 $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

MATRIX MULTIPLICATION (PT. 2/3)

IN GROUPS
CALCULATE THE PRODUCT MATRICES AB AND EF (AND SECRET MATRIX GH):

1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

$AB = \begin{pmatrix} 58 & 64 \\ 139 & 154 \end{pmatrix}$

3 $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

$EF = \begin{pmatrix} 29 & 32 & 35 & 38 \\ 65 & 72 & 79 & 86 \\ 101 & 112 & 123 & 134 \end{pmatrix}$

GH $G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}, H = \begin{pmatrix} 9 & 10 \\ 11 & 12 \end{pmatrix}$

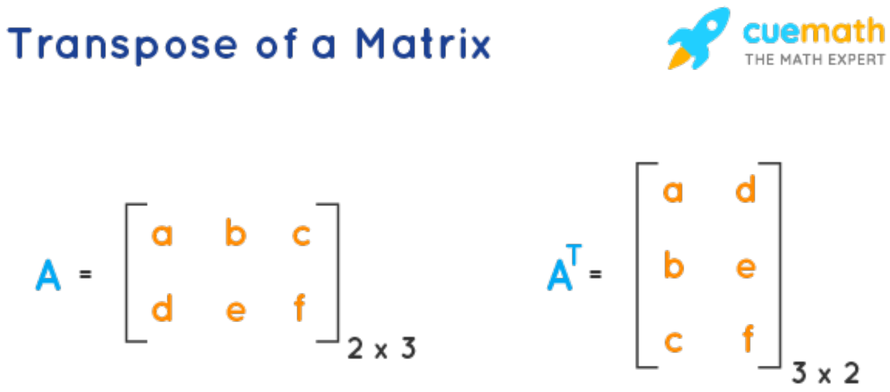
Hehe we can't take the product of these, but why not??

G (2x4) and H (2x2)

MATRIX MULTIPLICATION (PT. 3/3)

TRANSPOSING

You transpose a matrix by flipping a matrix on its diagonal:



https://www.cuemath.com/algebra/transpose-of-a-matrix/

IN GROUPS
TRANSPOSE THE FOLLOWING MATRICES:

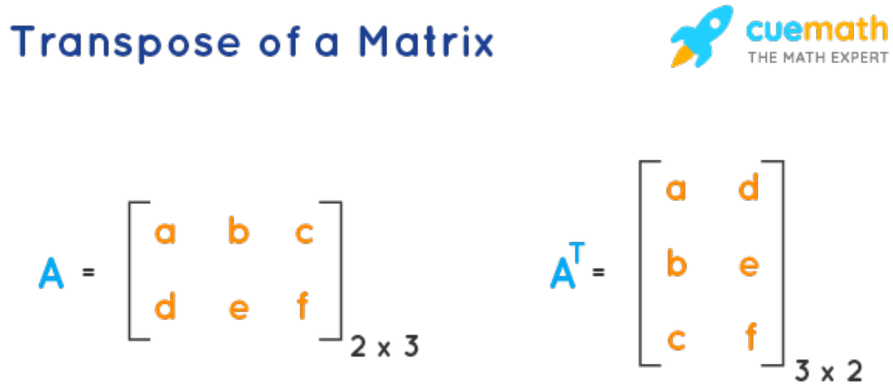
1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

2 $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

MATRIX MULTIPLICATION (PT. 3/3)

TRANSPOSING

You transpose a matrix by flipping a matrix on its diagonal:



https://www.cuemath.com/algebra/transpose-of-a-matrix/

IN GROUPS
TRANSPOSE THE FOLLOWING MATRICES:

1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

2 $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

SOLUTIONS

$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

$B^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$

EXTRA INFO

Curious about how linear equations and matrices connect?

https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html

Notice how we can do this,

because the two matrices are compatible:

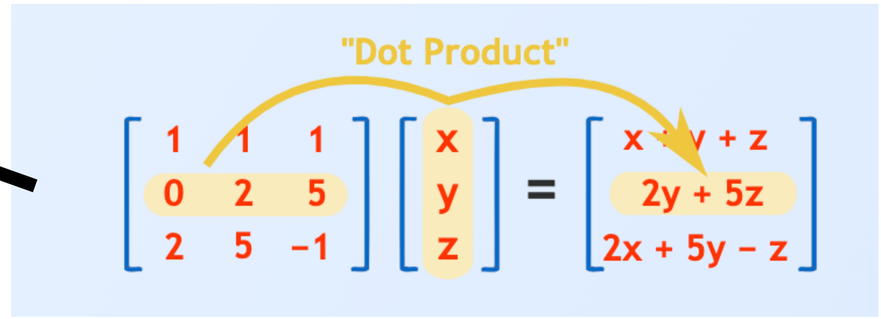
First matrix has 3 cols, second has 3 rows ✅

Well, think about the equations:

$$\begin{aligned} x + y + z &= 6 \\ 2y + 5z &= -4 \\ 2x + 5y - z &= 27 \end{aligned}$$

<=>

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

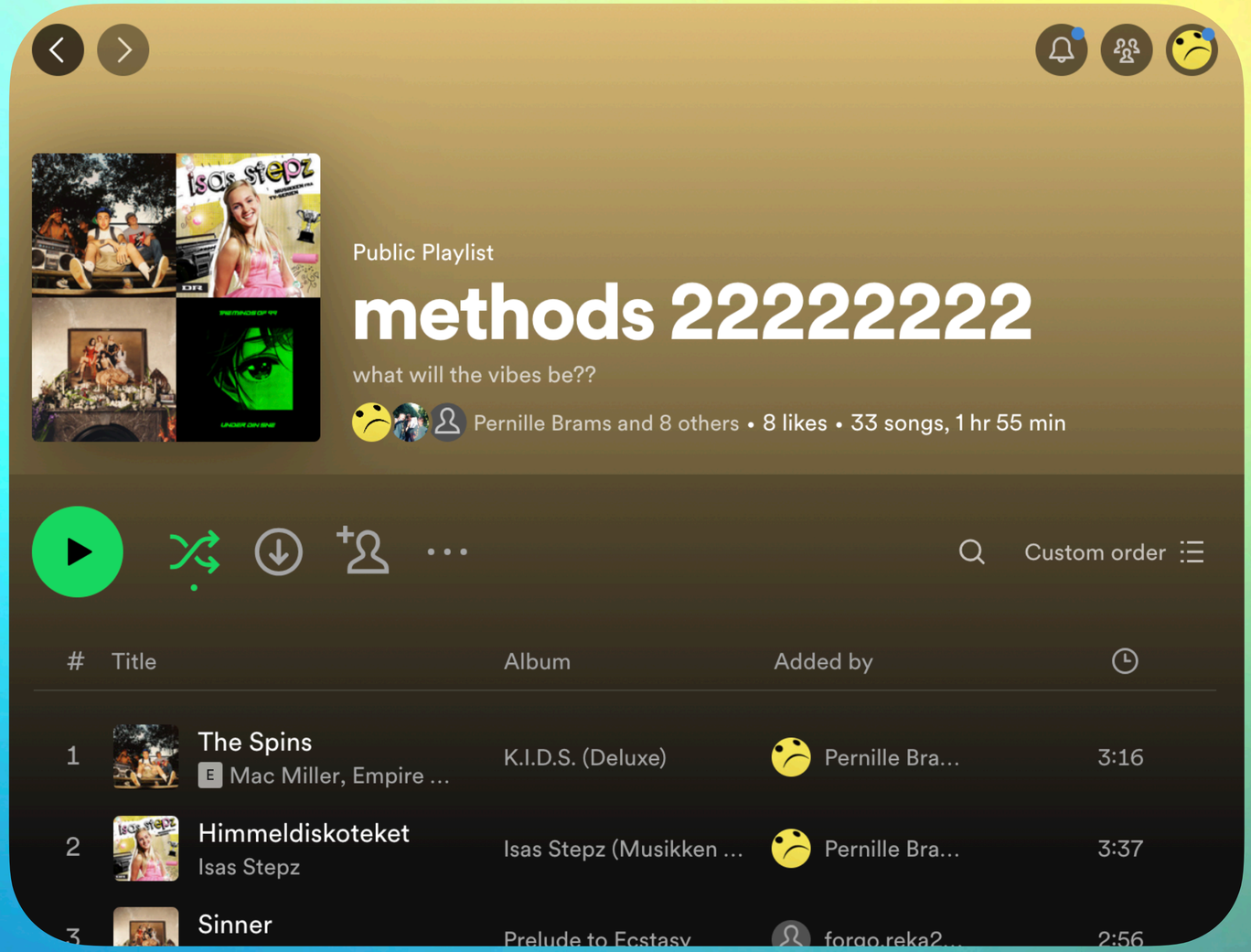


MIDTERM ☀️

https://www.menti.com/alanoe1s4vco

MENTI CODE: 2782 1638

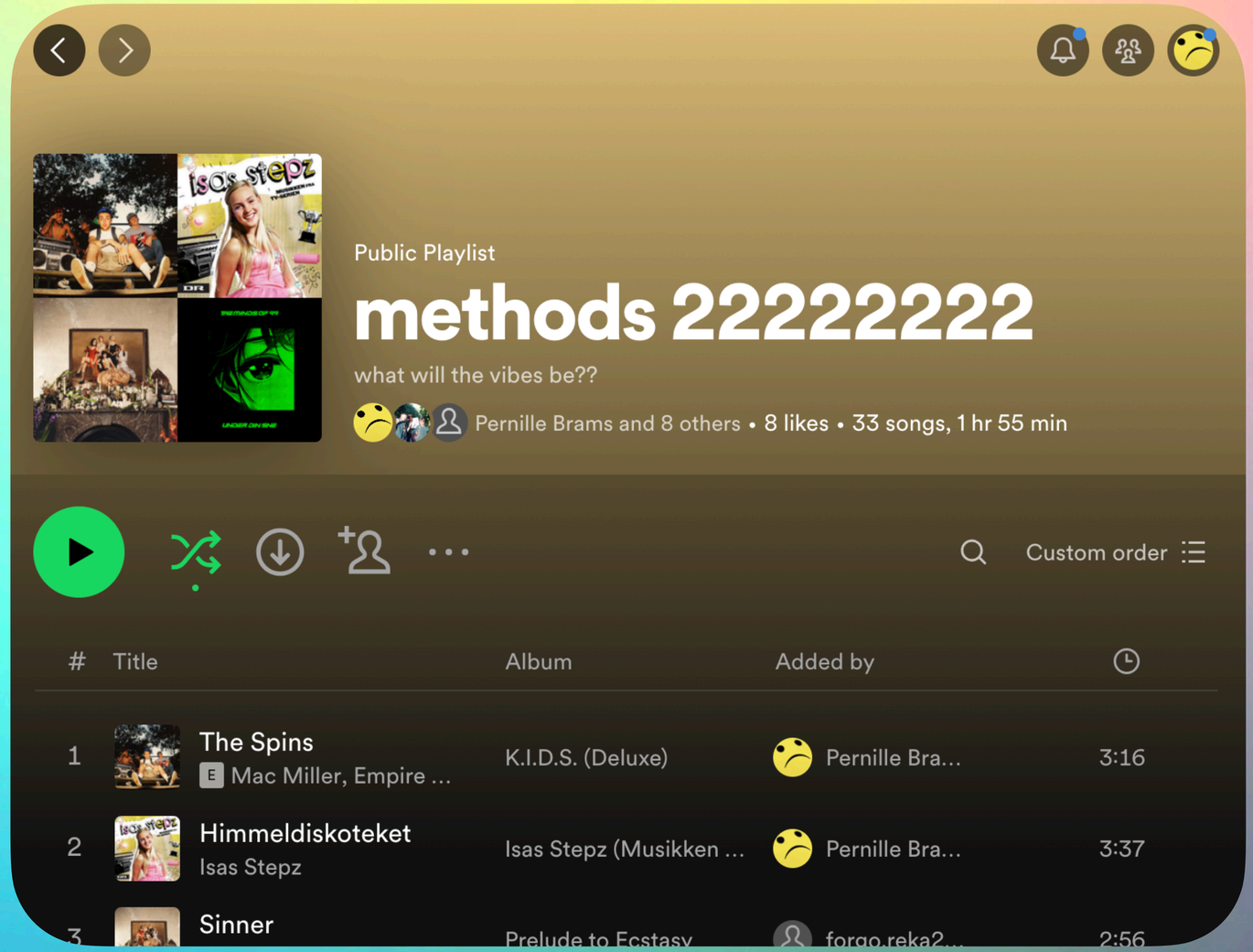
BREAK



Collab: https://open.spotify.com/playlist/5UUiKD15vyFwymQ4qLur9V?si=cf80f6c72721427c&pt=a27065eeba53e23fe2dd160612aa5598

EXERCISES

In today's markdown under **class 6**. Exercises A-C probably easiest to do (first time) in hand.



Collab: https://open.spotify.com/playlist/5UUiKD15vyFwymQ4qLur9V?si=cf80f6c72721427c&pt=a27065eeba53e23fe2dd160612aa5598