

# CLASS 6

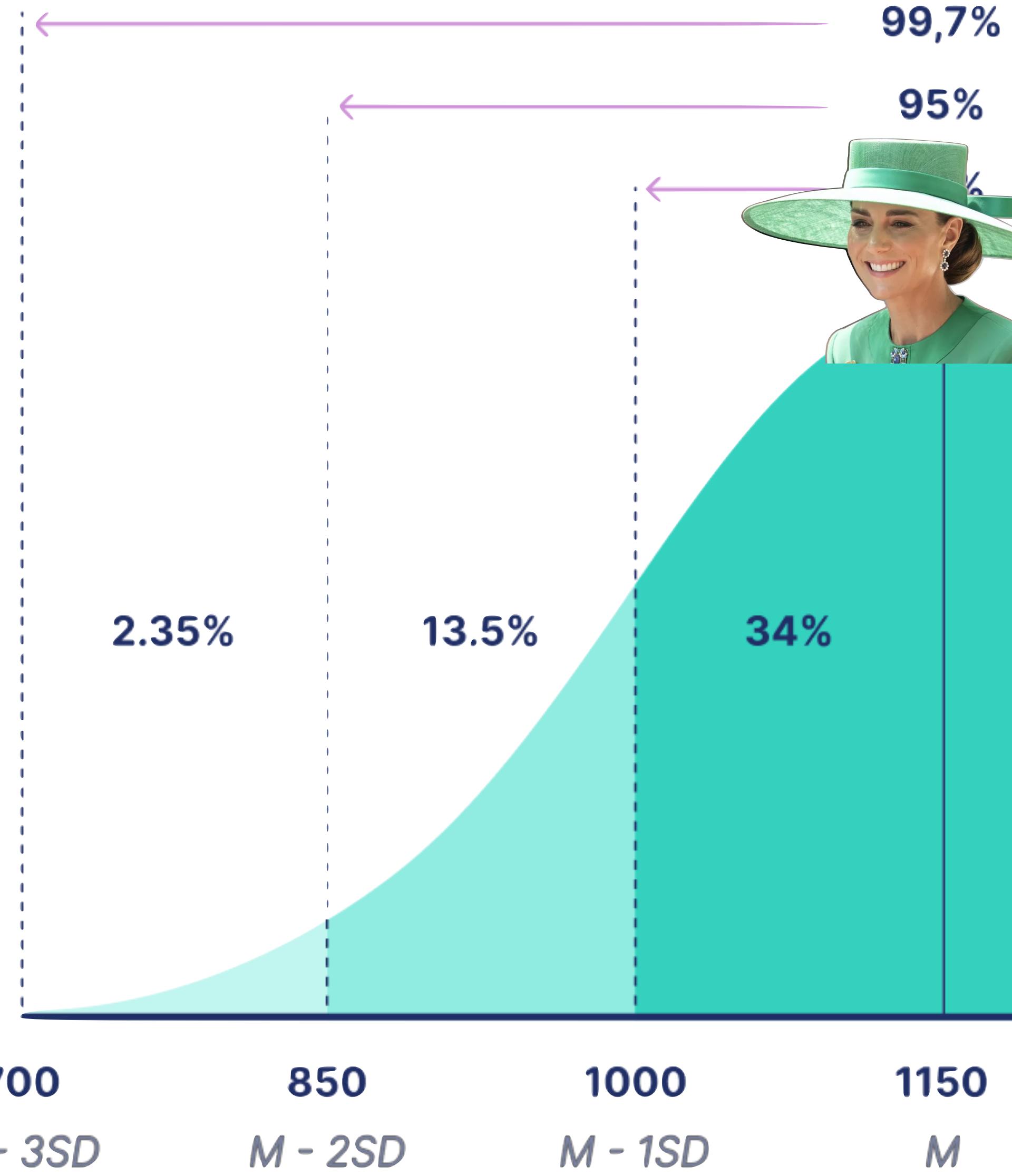
MATRIX AND VECTOR OPERATIONS + LEAST SQUARES  
APPROXIMATION



"GENERATE A PICTURE OF LINEAR ALGEBRA AND KATE MIDDLETON DISAPPEARING"

# AGENDA

- Recap, runthrough + group exercises
- Midterm
- Break
- Quick talk about midterm
- Exercises 



# MATRIX MULTIPLICATION (PT. 1/3)

## WHEN CAN I MULTIPLY TWO MATRICES?

Two matrices can be multiplied, if  
The number of columns in the **1st** matrix  
equals number of rows in the **2nd**

## CAN I MULTIPLY THESE?

1     $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

2     $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$

3     $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

## COMPATIBILITY STATUS

# Matrix Speed Dating

Two matrices can be multiplied, if

The number of columns in the **1st** matrix  
equals number of rows in the **2nd**

**ARE THEY  
COMPATIBLE?**

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

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# MATRIX MULTIPLICATION (PT. 1/3)

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**Two matrices can be multiplied, if**  
The number of columns in the **1st** matrix  
equals number of rows in the **2nd**

## CAN I MULTIPLY THESE?

1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$$

A (2x3) and B (3x2)

2

$$C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, D = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}$$

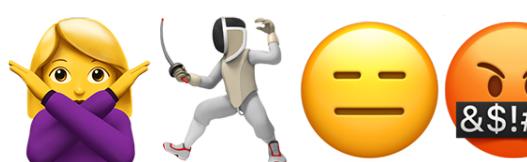
C (2x2) and D (3x3)

3

$$E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$$

E (3x2) and F (2x4)

COMPATIBILITY STATUS



# MATRIX MULTIPLICATION (PT. 2/3)

WHAT DIMENSIONS SHOULD THE PRODUCT MATRIX HAVE?

Rule For Matrix Multiplication



$$A \quad . \quad B = AB$$

$m \times n$        $n \times p$        $m \times p$

↓                  ↓                  ↓

Equal

Dimensions of  $AB$

<https://www.cuemath.com/algebra/multiplication-of-matrices/>

1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$$

A (2x3) and B (3x2)

? WHAT DIMENSIONS WILL  $A^*B$  HAVE?

3

$$E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$$

E (3x2) and F (2x4)

? WHAT DIMENSIONS WILL  $E^*F$  HAVE?

# MATRIX MULTIPLICATION (PT. 2/3)

IN GROUPS

CALCULATE THE PRODUCT MATRICES AB AND EF (AND SECRET MATRIX GH):

1     $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

3     $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

GH     $G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}, H = \begin{pmatrix} 9 & 10 \\ 11 & 12 \end{pmatrix}$

# MATRIX MULTIPLICATION (PT. 2/3)

IN GROUPS

CALCULATE THE PRODUCT MATRICES AB AND EF (AND SECRET MATRIX GH):

1  $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$

A (2x3) and B (3x2)

$$AB = \begin{pmatrix} 58 & 64 \\ 139 & 154 \end{pmatrix}$$

3  $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, F = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$

E (3x2) and F (2x4)

$$EF = \begin{pmatrix} 29 & 32 & 35 & 38 \\ 65 & 72 & 79 & 86 \\ 101 & 112 & 123 & 134 \end{pmatrix}$$

GH

$$G = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}, H = \begin{pmatrix} 9 & 10 \\ 11 & 12 \end{pmatrix}$$

Hehe we can't take the product of these, but why not??

G (2x4) and H (2x2)

# MATRIX MULTIPLICATION (PT. 3/3)

## TRANSPOSING

You transpose a matrix by flipping a matrix on its diagonal:

### Transpose of a Matrix



$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3}$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

<https://www.cuemath.com/algebra/transpose-of-a-matrix/>

**IN GROUPS**  
**TRANSPOSE THE FOLLOWING MATRICES:**

1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

2

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

# MATRIX MULTIPLICATION (PT. 3/3)

## TRANSPOSING

You transpose a matrix by flipping a matrix on its diagonal:

### Transpose of a Matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3}$$

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<https://www.cuemath.com/algebra/transpose-of-a-matrix/>

IN GROUPS  
TRANSPOSE THE FOLLOWING MATRICES:

1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

2

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

## SOLUTIONS

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

# EXTRA INFO

**Curious about how linear equations and matrices connect?**

<https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html>

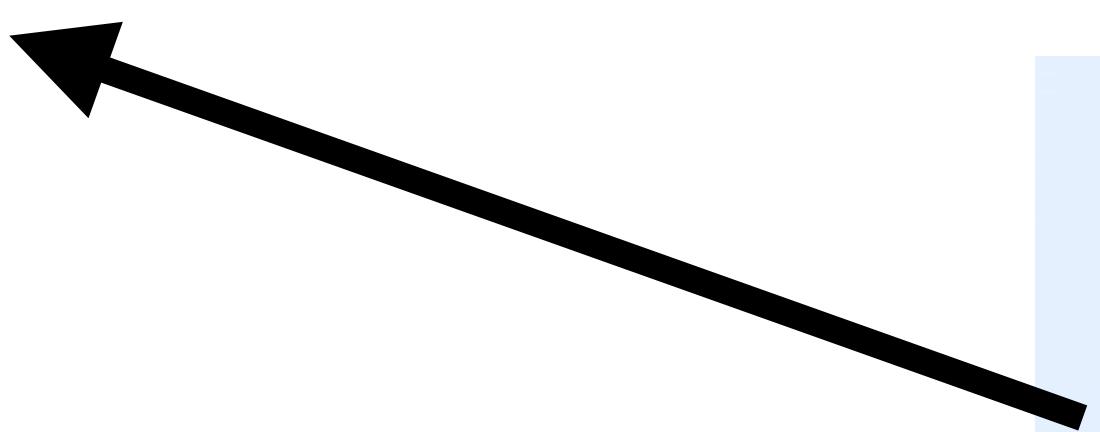
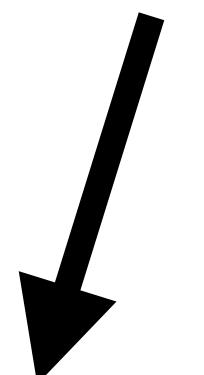
Well, think about the equations:

$$\begin{aligned} x + y + z &= 6 \\ 2y + 5z &= -4 \\ 2x + 5y - z &= 27 \end{aligned}$$

$\Leftrightarrow$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

Notice how we can do this,  
because the two matrices are compatible:  
First matrix has 3 cols, second has 3 rows ✓



"Dot Product"

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y + z \\ 2y + 5z \\ 2x + 5y - z \end{bmatrix}$$

# MIDTERM



<https://www.menti.com/alanoe1s4vco>

**MENTI CODE:**  
**2782 1638**



# BREAK

The image shows a Spotify interface displaying a public playlist. The title of the playlist is "methods 2222222". Below the title, it says "what will the vibes be?". The playlist was created by Pernille Brams and has 8 likes. It contains 33 songs, with a total duration of 1 hr 55 min. The interface includes a play button, shuffle, repeat, and download icons. A search bar and a "Custom order" button are also visible. The table below lists the first three songs:

#	Title	Album	Added by	Length
1	The Spins	K.I.D.S. (Deluxe)	Pernille Bra...	3:16
2	Himmeldiskoteket	Isas Stepz (Musikken ...)	Pernille Bra...	3:37
3	Sinner	Prelude to Ecstasy	forao.reka2...	2:56

Collab: <https://open.spotify.com/playlist/5UUiKD15vyFwymQ4qLur9V?si=cf80f6c72721427c&pt=a27065eeba53e23fe2dd160612aa5598>

# EXERCISES

In today's markdown under **class 6**.  
Exercises A-C probably easiest to do (first time) in hand.

Public Playlist  
**methods 2222222**  
what will the vibes be??  
Pernille Brams and 8 others • 8 likes • 33 songs, 1 hr 55 min

#	Title	Album	Added by	Length
1	The Spins E Mac Miller, Empire ...	K.I.D.S. (Deluxe)	Pernille Bra...	3:16
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