ECO 372: Introduction to Econometrics

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Hypothesis Tests and Confidence Intervals

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Outline

Our objectives will be to explore:

- Hypothesis Testing: Recap
- Single Coefficient Hypothesis Test
- Hypothesis Test with Regressions: Example

Hypothesis Testing: Recap

- We can use the information in the sample to make inferences about the population.
- We will always have two hypotheses that go together, the null hypothesis (denoted H_0) and the alternative hypothesis (denoted H_1).
- The null hypothesis is the statement or the statistical hypothesis that is actually being tested. The alternative hypothesis represents the remaining outcomes of interest.
- For example, suppose given the regression results, we are interested in the hypothesis that the true value of β is in fact 0.5. We would use the notation.

  $$H_0 : \beta = 0.5$$
  $$H_1 : \beta \neq 0.5$$

  This would be known as a two sided test.

Single Coefficient Hypothesis Test

➢ Multiple regression model with K explanatory variables:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i$$

Steps for Testing Hypothesis for Single Coefficient:

1. Compute the standard error of $\hat{\beta}_j$ $[i.e., SE(\hat{\beta}_j)]$
2. Compute the t-statistic: $t = \frac{\widehat{\beta}_j - \widehat{\beta}_{j,0}}{SE\ (\widehat{\beta}_j)}$
3. Compute the p-value: $2\phi(-|t^{act}|)$
4. Make decision

➢ Where, t^{act} is the t-statistic actually computed. Reject the hypothesis at the 5%-10% significance level if the p-value is less than 0.05-0.10 or, equivalently $t^{crit} = 1.96 - 1.85$ (can be obtained from t-table).

➢ It is worth noting that 10% significance level is considered as weak significance. Less than or equal 5% significance level is considered as high significance

Note: The standard error the t-statistic and p-value are typically computed by the software.

Hypothesis Test with Regressions: Example

- Let us take a model to start with:

$$Wage_i = \alpha_0 + \beta_1 EXP_i + \beta_2 EDU_i + u_i$$

Source	SS	df	MS
Model	1951.0667	2	975.53335
Residual	4570.6764	28,529	.160211588
Total	6521.7431	28,531	.228584455

Number of obs	=	28,532
F(2, 28529)	=	6089.03
Prob > F	=	0.0000
R-squared	=	0.2992
Adj R-squared	=	0.2991
Root MSE	=	.40026

ln_wage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ttl_exp	.0344367	.0005242	65.69	0.000	.0334092	.0354642
grade	.0741641	.0010493	70.68	0.000	.0721075	.0762208
_cons	.5314258	.0129982	40.88	0.000	.5059487	.5569029

Single Coefficient Hypothesis Test: Example

- Our objective is to test null that the coefficient of variable experience (ttl_exp) is actually zero.
- So, out set is as follows:

$$H_0 : \beta = 0$$

$$H_1 : \beta \neq 0$$

Now, we follow the steps as discussed previously:

1. Standard error of β_1=0.0005242
2. t statistics= $\frac{0.0344367-0}{0.0005242}$ which is 65.70
3. P-value =0.000
4. We can reject null at 5% since p-value is less than 5% and t-value in this case is well above 3 (from student's t-table)

Question: can we reject the null at 1%?

Task: Can you do the same hypothesis testing for the education variable?

Single Coefficient Hypothesis Test: Example

Table A.2

t* Distribution: Critical Values of *t

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646

Multiple Coefficient or Joint Hypothesis Test

- From our wage model, can we say there is no difference between $\hat{\beta}_1$ and $\hat{\beta}_2$?
- That is,

$$H_0: \hat{\beta}_1 = \hat{\beta}_2 = 0$$

$$H_1: \hat{\beta}_1 \neq \hat{\beta}_2 \neq 0$$

- This is joint hypothesis. We use F-statistics for the joint hypothesis test.
- Why F-statistics? Because it helps to understand the model's nested character.
- Suppose if $\hat{\beta}_1$ and $\hat{\beta}_2$ are zero or either one of them are zero, with the help of F-test we, we can determine whether there is any need of any variable in the regression. General formula is:

$$F = \frac{(M_{restricted} - M_{unrestricted})/q}{M_{restricted}/(N - k_{unrestricted} - 1)}$$

N= sample size, k=number of predictors, $M_{restricted}$=restricted entity, $M_{unrestricted}$=unrestricted entity, q=number of restrictions(just the numbers of parameters tested).

Multiple Coefficient or Joint Hypothesis Test

Source	SS	df	MS			
				Number of obs	=	28,532
				F(2, 28529)	=	6089.03
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Residual	4570.6764	28,529	.160211588	R-squared	=	0.2992
				Adj R-squared	=	0.2991
Total	6521.7431	28,531	.228584455	Root MSE	=	.40026

ln_wage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ttl_exp	.0344367	.0005242	65.69	0.000	.0334092	.0354642
grade	.0741641	.0010493	70.68	0.000	.0721075	.0762208
_cons	.5314258	.0129982	40.88	0.000	.5059487	.5569029

```
. test ttl_exp = grade=0

 ( 1)  ttl_exp - grade = 0
 ( 2)  ttl_exp = 0

       F(  2, 28529) = 6089.03
            Prob > F =    0.0000
```

We can reject the null at 5% since F static P-value is 0.000
Also, Notice STATA output reports the same result!

<u>STATA Command</u>

reg ln_wage ttl_exp grade
test ttl_exp=grade=0

Multiple Coefficient or Joint Hypothesis Test

TABLE A.3

***F* Distribution: Critical Values of *F* (5% significance level)**

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84

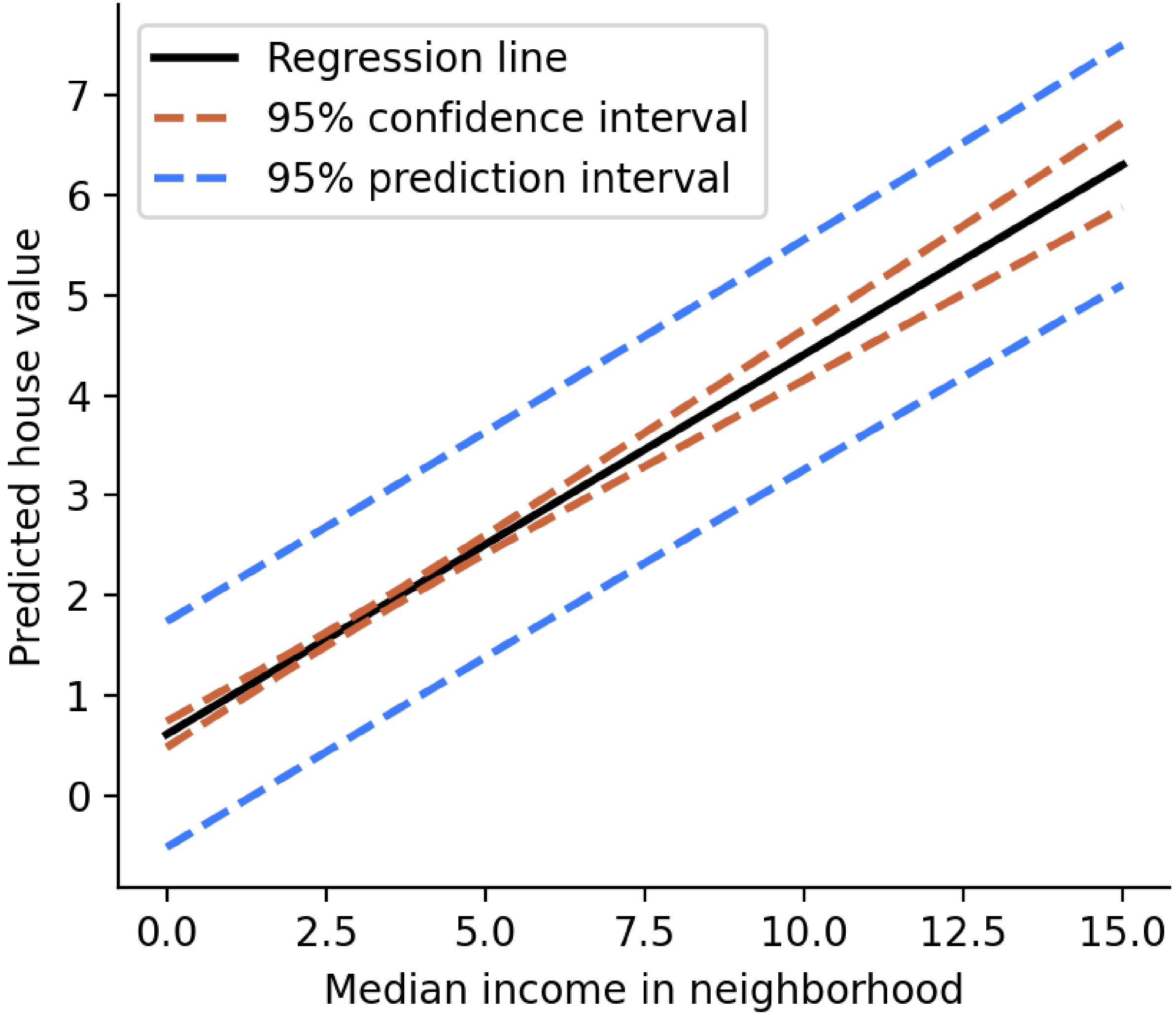
Confidence Interval (CI)

- A Confidence Interval (CI) is an interval which is expected to typically contain the parameter being estimated.
- 95% CI is computed by finding the set of values of the coefficients that are not rejected using a t-statistic at the 5% significance level.
- This approach can be extended to the case of multiple coefficients.
- CI = point estimate ± margin of error margin of error, which can widely known as following:

$\hat{\beta} \pm z\frac{\sigma}{\sqrt{N}}$, where z=z statistics, σ=standard error, N= sample size

- CI actually creates a band within which all the significant values fall.

Confidence Interval (CI)



Confidence Interval (CI)

- Suppose we want to show the residual within a 95% CI for the wage model we have seen earlier.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i$$

- How to find residual? Recall our last class.
- Residual : $\hat{u}_i = Y_i - \hat{Y}_i$
- Now all we have to do is to tell STATA to compute and show residual with 95% CI
  - Run the regression with "reg" command
  - Find residual with "predict" command
  - Prepare a table of residual summary with "ci residual" command
  - "ci means" gives CI for all variables in the dataset

STATA Command

reg ln_wage ttl_exp grade
predict residual, resid
ci residual
ci means

Variable	Obs	Mean	Std. Err.	[95% Conf.	Interval]
residual	28,532	7.78e-12	.0023695	-.0046444	.0046444