Билет №0

1. $X \sim Exp(2)$

а) $F(x) = \begin{cases} 0, & x<0 \\ 1-e^{-2x}, & x \geq 0 \end{cases}$ $\quad f(x) = \begin{cases} 0, & x<0 \\ +2e^{-2x}, & x \geq 0 \end{cases}$

$$\int_0^x 2e^{-2t}\,dt = -\int_0^x e^{-2t}\,d(-2t) = -e^{-2t}\Big|_0^x =$$

$$= -\left(e^{-2x} - e^{-2\cdot 0}\right) = -\left(e^{-2x} - 1\right) = 1 - e^{-2x}$$

б) $$MX = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^{+\infty} 2x e^{-2x}\,dx = 2\int_0^{+\infty} x e^{-2x}\,dx =$$

$$\cancel{= 2\left(\int_0^{+\infty} e^{-2x}\,d\frac{x^2}{2} = \int_0^{+\infty} e^{-2x}\,dx^2 = x^2 e^{-2x}\Big|_0^{+\infty} - \int_0^{+\infty} x^2\,de^{-2x}\right.}$$

$$= -\int_0^{+\infty} x\,de^{-2x} = -\left(x e^{-2x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-2x}\,dx\right) =$$

$$= -\left(0 + \frac{1}{2} e^{-2x}\Big|_0^{+\infty}\right) = -\frac{1}{2} e^{-2x}\Big|_0^{+\infty} = -\frac{1}{2}(0-1) = \frac{1}{2}$$ ✓

$+\infty\ 0\ -($

$$DX = \int_{-\infty}^{+\infty} (x-m)^2 f(x)\,dx = \int_0^{+\infty} \left(x - \frac{1}{2}\right)^2 2e^{-2x}\,dx =$$

$$= 2\int_0^{+\infty} \left(x - \frac{1}{2}\right)^2 e^{-2x}\,dx = \frac{1}{2}\int_0^{+\infty} (2x-1)^2 e^{-2x}\,dx =$$

$$= \frac{1}{4}\int_0^{+\infty} (2x-1)^2 e^{-2x}\,d(2x) = -\frac{1}{4}\int_0^{+\infty} (2x-1)^2\,de^{-2x} =$$

$4x^2 - 4x + 1$

$$= -\frac{1}{4}\left((2x-1)^2 e^{-2x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-2x}\,d(2x-1)^2\right) =$$

$$= -\frac{1}{4}\left(-1 - \int_0^{+\infty} (8x-4) e^{-2x}\,dx\right) = -\frac{1}{4}\left(-1 - 4\int_0^{+\infty} (2x-1) e^{-2x}\,dx\right) = \frac{1}{4}$$

в) $P\{X > MX\} = P\{X > \frac{1}{2}\} = P\{\frac{1}{2} < X < +\infty\} =$

$= \int_{\frac{1}{2}}^{+\infty} f(x)\,dx = \int_{\frac{1}{2}}^{+\infty} 2e^{-2x}\,dx = -\int_{\frac{1}{2}}^{+\infty} e^{-2x}\,d(-2x) = -e^{-2x}\Big|_{\frac{1}{2}}^{+\infty} =$

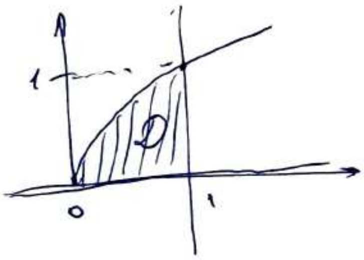
$= -(0 - e^{-1}) = e^{-1} = \frac{1}{e}$ $\qquad p = \frac{1}{e} \quad q = 1 - \frac{1}{e}$

По схеме Бернулли $(n=4;\ k \geq 1)$ $\quad A = \{$хотя успехов 1, 2, 3, 4$\}$

$P(A) = 1 - P(0) = 1 - C_4^0 p^0 q^5 = 1 - \frac{4!}{0!\,4!} \cdot 1 \cdot \left(1 - \frac{1}{e}\right)^5 =$

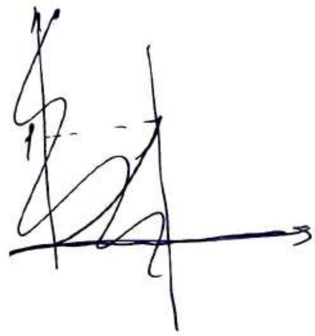
$= \boxed{1 - \left(1 - \frac{1}{e}\right)^5} = 1 - \frac{(e-1)^5}{e^5} = \frac{e^5 - (e-1)^5}{e^5} =$

№2 $\quad f(x,y) = \begin{cases} Ax^2y, & x,y \in D \\ 0, & \text{иначе} \end{cases}$



$y = \sqrt{x}$

$y^2 = x$



а) $1 = \iint_{\mathbb{R}^2} f(x,y)\,dx\,dy = \iint_D Ax^2y\,dx\,dy = A\int_0^1 dx \int_0^{\sqrt{x}} x^2y\,dy =$

$= A\int_0^1 \frac{x^2y^2}{2}\Big|_0^{\sqrt{x}} dx = A\int_0^1 \frac{x^3}{2}\,dx = \frac{A}{2}\int_0^1 x^3\,dx =$

$= \frac{A}{2}\left[\frac{1}{4} - 0\right] = \frac{A}{8} \qquad \frac{A}{8} = 1 \Rightarrow A = 8$

б) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} \int_0^{\sqrt{x}} 8x^2y\,dy & x \in (0,1) \\ 0, & \text{иначе} \end{cases} = \begin{cases} 4x^3, & x \in (0,1) \\ 0, & \text{иначе} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} \int_{y^2}^{1} 8x^2y\,dx & y \in (0,1) \\ 0, & \text{иначе} \end{cases} = \begin{cases} -\frac{8y^7}{3}, & y \in (0,1) \\ 0, & \text{иначе} \end{cases}$

N2

в) $cov(X,Y) = ?$

$$cov(X,Y) = \iint_{\mathbb{R}^2} (x - MX)(y - MY) f(x,y)\, dx\, dy \ominus$$

$$MX = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^1 4x^4\, dx = 4\left[\frac{1}{5} - 0\right] = \frac{4}{5}$$

$$MY = \int_{-\infty}^{+\infty} y f(y)\, dy = \int_0^1 -\frac{8y^8}{3}\, dy = -\frac{8}{3}\left[\frac{1}{9} - 0\right] = -\frac{8}{27}$$

$$\ominus \iint_D \underbrace{\left(x - \frac{4}{5}\right)\left(y + \frac{8}{27}\right) 8x^2 y}_{g(x,y)}\, dx dy =$$

$$= \int_0^1 dx \int_0^{\sqrt{x}} g(x,y)\, dy \approx \frac{16}{1485}$$

Если X, Y - независ $\Rightarrow cov(X,Y) = 0$

Т.к $cov(X,Y) \neq 0 \Rightarrow$ они завис

N3

$$f_\xi(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x+1)^2}{8}}$$

$\xi \sim N(-1, 4) \Rightarrow$ $M\xi = -1$, $D\xi = 4$

ρ ζ ς

$$f_\eta(x) = \begin{cases} 0.1, & x \in (1, 11) \\ 0, & иначе \end{cases}$$

$\eta \sim R[1, 11] \Rightarrow$ $M\eta = 6$, $D\eta = \frac{100}{12}$

$\rho_{\xi\eta} = -0.2$ $\quad \rho_{\xi\eta} = \frac{cov(\xi,\eta)}{\sqrt{D\xi \cdot D\eta}}$

$\zeta = 3\xi - 2\eta + 4$

$$\boxed{M[\zeta]} = 3M\xi - 2M\eta + 4 = 3(-1) - 2\cdot 6 + 4 = -3 - 12 + 4 = -12 + 1 = \boxed{-11}$$

$$\boxed{D\zeta} = D[3\xi - 2\eta + 4] = 3D\xi - 2D\eta + 2(3)\cdot(-2)\cdot cov(\xi,\eta) =$$

$$= 3\cdot\frac{100}{12} - 2\cdot 4 - 12\cdot(-0.2)\sqrt{4\cdot\frac{100}{12}} = \frac{100}{4} - 8 + 2.4\,\frac{2\cdot 10}{\sqrt{12}} =$$

$$= 25 - 8 + \frac{24\cdot \cancel{2}}{\cancel{2}\sqrt{3}} = 17 + \frac{24\sqrt{3}}{3} = \boxed{17 + 8\sqrt{3}}$$

Б21

а) $1 = a\int_{-\infty}^{+\infty} \frac{a}{e^{x}+e^{-x}}\,dx = a\int_{-\infty}^{+\infty}\frac{1}{e^x+e^{-x}}\,dx = \left| \begin{matrix} e^x=u & x=\ln u \\ e^{-x}=\frac1u & dx=\frac1u du \\ x\to -\infty & u=0;\ x=+\infty;\ u=+\infty\end{matrix}\right| =$

$= a\int_{-\infty}^{+\infty}\frac{1}{u+\frac1u}\frac1u\,du = a\int_0^{+\infty}\frac{1}{u^2+1}\,du = a\,\mathrm{arctg}(u)\Big|_0^{+\infty} = a\frac{\pi}{2} = 1$

$\frac{a\pi}{2}=1 \quad a=\frac{2}{\pi}$

б) $F(x)=?$

$F(x)=\int_{-\infty}^x f(t)\,dt = \int_{-\infty}^{x}\frac{a}{e^t+e^{-t}}\,dt = \frac{2}{\pi}\int_{-\infty}^{x}\frac{1}{e^t+e^{-t}}\,dt =$

$= \frac{2}{\pi}\left| \begin{matrix} e^t=u & dt=\frac1u du \\ e^{-t}=\frac1u & t=-\infty \quad u=0 \\ & t=x \quad u=e^x\end{matrix}\right| = \frac{2}{\pi}\int_0^{e^x}\frac{1}{u^2+1}\,du =$

$= \frac2\pi\,\mathrm{arctg}(u)\Big|_0^{e^x} = \frac2\pi\,\mathrm{arctg}(e^x)$

в) $P\{\xi<1\} = P\{-\infty<\xi<1\} = F(1)-F(-\infty) =$

$= \frac2\pi\,\mathrm{arctg}\, e$

По схеме Бернулли: $p=\frac2\pi\,\mathrm{arctg}\, e$

$q=1-p$

$n=2 \quad k=2$

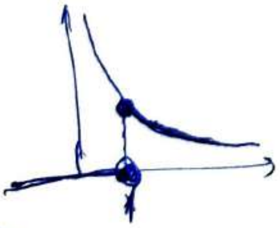
$P\{k=2\} = C_2^2 p^2 q^0 = \frac{2!}{2!\,0!} \quad p^2=\frac{4}{\pi^2}\mathrm{arctg}^2 e$

$y^{+\frac32}y^{-2} = y^{-\frac12}$

№2 $\eta=\frac1\xi$

$\varphi(\xi)=\frac1\xi$ — ~~строго~~ монотонна

$\Downarrow$



$y\in(-\infty,0)\Rightarrow$
$\to \psi(y)<0 \Rightarrow$
$\Rightarrow f_x(\varphi(y))=0$

$f_\eta(y)=f_\xi(\psi(y))\,|\psi'(y)|$ $\quad \frac1y<0$

$x<0$

ψ — обратна к φ $\quad 0<x<1$

$\psi=\frac1\eta \quad \psi'=\frac1\eta = -\eta^{-2} = -\frac{1}{\eta^2}$ $\quad \frac1y<1$

$f_\eta(y) = \begin{cases}0, & y<0,\ y>1\\ \frac{1}{2}\left(\frac{1}{[illegible]}\right)^{-\frac32}\frac{1}{y^2}, & y\in(0,1)\end{cases} = \begin{cases}0, & y<0\\ \frac{1}{2\sqrt y}, & y\in(0,1\end{cases}$

а) $P\{0{,}01 < \eta < 0{,}04) = \int\limits_{0{,}01}^{0{,}04} f_\eta(y)\,dy \int\limits_{0{,}01}^{0{,}04} \frac{1}{2\sqrt{y}}\,dy =$

$= \frac{1}{2} \int\limits_{0{,}01}^{0{,}04} y^{-\frac{1}{2}}\,dy$

в) $M\eta = \int\limits_{-\infty}^{+\infty} y\, f_\eta(y)\,dy = \int\limits_{0}^{1} y \frac{1}{2\sqrt{y}}\,dy = \frac{1}{2}\int\limits_{0}^{1} \sqrt{y}\,dy$

3. $MX = 10 \quad \sigma_x = 0{,}1$

$MY = 5 \quad \sigma_y = 0{,}05$ нормальн

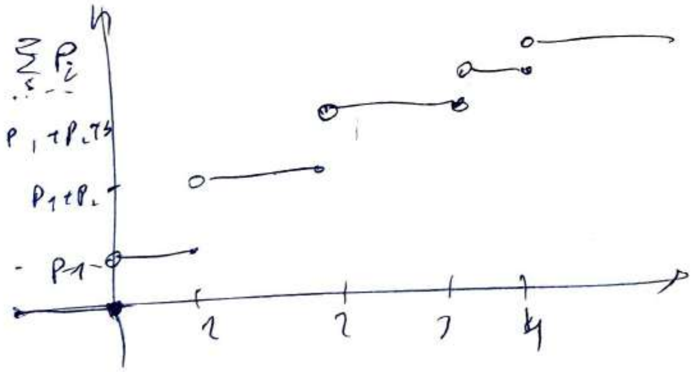
$\rho_{xy} = 0{,}6$

5_2 $\quad p = 0.25$ $\qquad k = 0, 1, 2, 3, 4$

$q = 0.25$

$C_4^k\, p^k\, p^{n-k}$

	0	1	2	3	4
	P_1	P_2	P_3	P_4	P_5



$F(x) = P\{X < x\}$

$MX = \sum\limits_i x_i p_i = 0 p_1 + 1 \cdot P_2 + \ldots + 4 P_5$

$DX = \sum\limits_i (x_i - m) p_i = \ldots = \quad DX = M[X^2] - (MX)^2$

$P\{X \geq 3\} = P\{X = 3\} + P\{X = 4\} = F(+\infty) - F(3) = 1 -$

2.

$$f(x) = \begin{cases} \frac{1}{b-a}, & x \in (a, b) \\ 0, & \text{иначе} \end{cases}$$

~~1 2 3~~ 4 5 6 7 ~~8~~ 9 10

$$f(x) = \begin{cases} \frac{2}{\pi}, & x \in (0, \frac{\pi}{2}) \\ 0, & \text{иначе} \end{cases}$$

а) $Y = X^2$ — кусочно-монотонн.

$\Psi = \sqrt{y}$

$$f_Y(y) = \sum_{j=1}^{2} f_X(\Psi_j(y)) |\Psi'(y)|,$$

где $x_j = \Psi_j(y)$, $j = \overline{1, n}$ — все решения ур-я $\varphi(x) = y$

~~Разбор случаев~~

Разбор случаев:

1) $y \in (-\infty; 0) \Rightarrow$ ур-е $x^2 = y$ не имеет решен. $\Rightarrow f_Y(y) = 0$

2) $y \in (0, \frac{\pi^2}{4}) \Rightarrow$ ур-е $x^2 = y \Rightarrow x = \pm\sqrt{y}$

$\Psi_1(y) = -\sqrt{y}$; $\Psi_2(y) = \sqrt{y}$ $\quad -\sqrt{y}$ $\quad (y^{\frac{1}{2}})' = \frac{1}{2} y^{-\frac{1}{2}}$

Тогда

$$f_Y(y) = \cancel{f_X(-\sqrt{y}) |(-\sqrt{y})'|}^{\,0} + f_X(\sqrt{y}) |(\sqrt{y})'| =$$

$$= \frac{2}{\pi} \frac{1}{2\sqrt{y}}$$

$$\boxed{MY = \int_{-\infty}^{+\infty} y f(y)\, dy = \int_0^{\frac{\pi^2}{4}} y \frac{1}{\pi\sqrt{y}}\, dy = \frac{\pi^2}{12}}$$

3) $y \in (\frac{\pi^2}{4}; +\infty)$ аналогично

$$f_Y(y) = \cancel{f_X(-\sqrt{y})}^{\,0} |(-\sqrt{y})'| + \cancel{f_X(\sqrt{y})}^{\,0} = 0$$

$$f_Y(y) = \begin{cases} \frac{2}{\pi} \frac{1}{2\sqrt{y}}; & y \in (0, \frac{\pi^2}{4}) \\ 0, & \text{иначе} \end{cases}$$

Б3

№1 $\quad \ell \sim N(20,\ 0{,}04) \qquad f(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-m)^2}{2\sigma^2}}$

1) $P\{19{,}7 < \ell < 20.3\} = \displaystyle\int_{19{,}7}^{20.3} f(x)\,dx = \Phi_0\left(\frac{b-m}{\sigma}\right) - \Phi_0\left(\frac{a-m}{\sigma}\right) =$

$$\cancel{= \int_{19{,}7}^{20{,}3} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-m)^2}{2\sigma^2}}\,dx = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{19{,}7}^{20.3}}$$

$$= \Phi\left(\frac{0.3}{0.2}\right) - \Phi_0\left(\frac{-0.3}{0.2}\right) = \Phi_0(1.5) + \Phi_0(1.5) = 2\,\Phi_0(1.5) \approx 2 \cdot 0.35314 \approx$$

$\approx 0{,}706$

2) $P\{|X-20| < \varepsilon\} = 0.95 \qquad \varepsilon = ?$

$$P\{|X-20| < \varepsilon\} = P\{-\varepsilon < X-20 < \varepsilon\} =$$

$$= P\{\overset{m}{20} - \varepsilon < X < \overset{m}{20} + \varepsilon\} = \Phi_0\left(\frac{\cancel{m}+\varepsilon-\cancel{m}}{\sigma}\right) - \Phi_0\left(\frac{\cancel{m}-\varepsilon-\cancel{m}}{\sigma}\right) =$$

$$= \Phi_0\left(\frac{\varepsilon}{\sigma}\right) + \Phi_0\left(\frac{\varepsilon}{\sigma}\right) = 2\Phi_0\left(\frac{\varepsilon}{\sigma}\right) =$$

$$2\Phi_0\left(\frac{\varepsilon}{\sigma}\right) = 0.95$$

$$\Phi_0\left(\frac{\varepsilon}{\sigma}\right) = \frac{0.95}{2}$$

$$\Phi_0\left(\frac{\varepsilon}{\sigma}\right) = 0.475$$

$$\Downarrow$$

$$\frac{\varepsilon}{\sigma} \approx 1.96$$

$$\frac{\varepsilon}{0.04} \approx 1.96$$

$$\cancel{\varepsilon \approx 7{,}98}$$

$$\varepsilon \approx 3{,}8416$$

N2

$X \sim Exp(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & иначе \end{cases}$$

a) $\cancel{F(x) = \int_{-\infty}^{+\infty} f(x)\,dx}$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \left\{ \begin{matrix} \int_0^x \lambda e^{-\lambda t}\,dt, & x > 0 \\ 0, & иначе \end{matrix} \right| = $$

$$\circledast \int_0^x e^{-\lambda t}\,d(\lambda t) = -e^{-\lambda t}\Big|_0^x = -\left(e^{-\lambda x} - 1\right) = 1 - e^{\lambda x}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & иначе \end{cases}$$

б) $MY = M[e^{-X}] = \int_{-\infty}^{+\infty} e^{-x} f(x)\,dx = \int_0^{+\infty} e^{-x} \lambda e^{-\lambda x}\,dx =$

$$= \lambda \int_0^{+\infty} e^{-x(1+\lambda)}\,dx =$$

$$= \lambda \int_0^{+\infty} e^{-x(\lambda+1)}\,dx = \frac{\lambda}{\lambda+1} \int_0^{+\infty} e^{-x(\lambda+1)}\,d(x(\lambda+1)) =$$

$\lambda > 0$

$$= -\frac{\lambda}{\lambda+1} e^{-x(\lambda+1)}\Big|_0^{+\infty} = -\frac{\lambda}{\lambda+1}(0 - 1) = \frac{\lambda}{\lambda+1}$$

? $+\infty - 1 = +\infty$

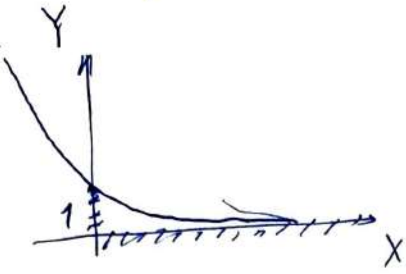
$\lambda > 1$

$0 < \lambda < 1$

0

Б3

б) Найдем плотность распр-я сл. вел $Y = e^{-X}$



Функция монотонно убывает $\Rightarrow$

$\Rightarrow$ восп. формулой

$$f_Y(y) = f_X(\psi(y))\,|\psi'(y)|,$$

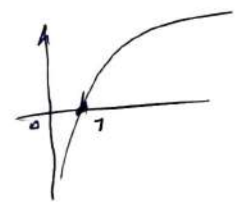
ψ – обратная к e^{-X}

$$Y = e^{-X} \Leftrightarrow \ln Y = -X \Rightarrow X = -\ln Y$$

$\ln y > 0$

$y > 1$

$$\psi(Y) = -\ln Y \qquad \psi'(Y) = -\frac{1}{Y}$$



Т.к. $Y > 0$:

$$f_Y(y) = \begin{cases} 0, & y < 0 \\ f_X(\underbrace{\psi(y)}) \, |\psi'(y)|, & y > 0 \end{cases} =$$

$$= \begin{cases} 0, & y < 0 \\ 0, & -\ln y < 0 \\ \lambda e^{+\lambda \ln y} \left|-\frac{1}{y}\right|, & -\ln y > 0 \end{cases} = \begin{cases} 0, & y < 0 \text{ или } y > 1 \\ \lambda e^{\lambda \ln y} \left|-\frac{1}{y}\right|, & y \in (0,1) \end{cases} =$$

$$= \begin{cases} 0, & \text{—"—} \\ \lambda e^{\lambda \ln y} \frac{1}{y}, & y \in (0,1) \end{cases} = \begin{cases} 0, & \text{—"— иначе} \\ \frac{\lambda y^{\lambda}}{y}, & y \in (0,1) \end{cases} = \begin{cases} \lambda y^{\lambda - 1}, & y \in (0,1) \\ 0, & \text{иначе} \end{cases}$$

$\cancel{MX = \int_{-\infty}^{+\infty} x f(x)}$ $\quad MY = \int_{-\infty}^{+\infty} y f(y)\, dy =$

$$= \int_0^1 y \lambda y^{\lambda - 1}\, dy = \lambda \int_0^1 y^{\lambda}\, dy = \lambda \left. \frac{y^{\lambda+1}}{\lambda+1} \right|_0^1 =$$

$$= \lambda \left[\frac{1}{\lambda+1} + 0 \right] = \frac{\lambda}{\lambda+1}$$

$$DY = \int_{-\infty}^{+\infty} (y-m)^2 f(y)\,dy = \int_0^1 \left(y - \overset{m}{\cancel{\tfrac{\lambda}{\lambda+1}}}\right)^2 \lambda y^{\lambda-1}\,dy =$$

$$= \int_0^1 \left(\frac{y(\lambda+1)-\lambda}{\lambda+1}\right)^2 \lambda y^{\lambda-1}\,dy =$$

$$= \frac{\lambda}{\lambda+1}\int_0^1 \left(y^2(\lambda+1)^2 - 2y\lambda(\lambda+1) + \lambda^2\right) y^{\lambda-1}\,dy =$$

$$= \frac{\lambda}{\lambda+1}\left(\int_0^1 y^2(\lambda+1)^2 y^{\lambda-1}\,dy + \int_0^1 2y\lambda(\lambda+1)\,y^{\lambda-1}\,dy + \right.$$

$$\left. + \int_0^1 \lambda^2 y^{\lambda-1}\,dy\right) =$$

$$= \frac{\lambda}{\lambda+1}\left((\lambda+1)^2\int_0^1 y^{\lambda+1}\,dy + 2\lambda(\lambda+1)\int_0^1 y^{\lambda}\,dy + \right.$$

$$\left. + \lambda^2\int_0^1 y^{\lambda-1}\,dy\right) =$$

[illegible]

$$\cancel{= \frac{\lambda}{\lambda+1}\left((\lambda+1)^2(\lambda+2) + 2\lambda(\lambda+1) + \lambda^2\right) =}$$

$$\cancel{= \frac{\lambda}{\lambda+1}(\lambda+1+\lambda)^2 = \frac{\lambda(2\lambda+1)^2}{\lambda+1}}$$

$$DY = M[Y^2] - (MY)^2 = \frac{\lambda}{\lambda+2} - \frac{\lambda^2}{(\lambda+1)^2} \cancel{\frac{(\lambda+1)^2 - \lambda}{(\lambda+2)(\lambda+1)}} =$$

$$M[Y^2] = \int_{-\infty}^{+\infty} y^2 f(y)\,dy = \int_0^1 y^2 \lambda y^{\lambda-1}\,dy =$$

$$= \lambda\int_0^1 y^{\lambda+1}\,dy = \lambda\left.\frac{y^{\lambda+2}}{\lambda+2}\right|_0^1 = \frac{\lambda}{\lambda+2}$$

$$(MY)^2 = \left(\frac{\lambda}{\lambda+1}\right)^2$$

$$\cancel{= \lambda\left(\frac{\lambda^2+2\lambda+1-\lambda}{\lambda^2+3\lambda+2}\right) = \lambda\left(\frac{\lambda^2+3\lambda+1}{\lambda^2+3\lambda+2}\right)}$$

№4

$MX = -10 \quad MY = 10$

$\sigma_x = 2 \quad \sigma_y = 3 \quad \rho_{xy} = 0{,}7$

$Z = 3X - Y$

$MZ = M[3X-Y] = 3MX - MY = 3\cdot(-10) - 10 = -30 - 10 = -40$

$DZ = D[3X-Y] = 9\underbrace{DX}_{\sigma_x^2} + \underbrace{DY}_{\sigma_y^2} + 2\cdot 3\cdot(-1)\,cov(X,Y) =$

$\rho_{xy} = \dfrac{cov(X,Y)}{\sqrt{DX\cdot DY}} = 0{,}7 = \dfrac{cov(X,Y)}{3\cdot 2} \Rightarrow cov(X,Y) = 4{,}2$

$= 9\cdot 4 + 9 - 6\cdot 4{,}2 = 19{,}8$

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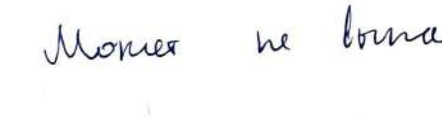
1. 

$\xi = X$

а) Монета не выпала ни одного, выпало 1, 2, или 3

Пусть "успех" – выпадение орла, "неуспех" – вып. решки

X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$q = p = \frac{1}{2}$

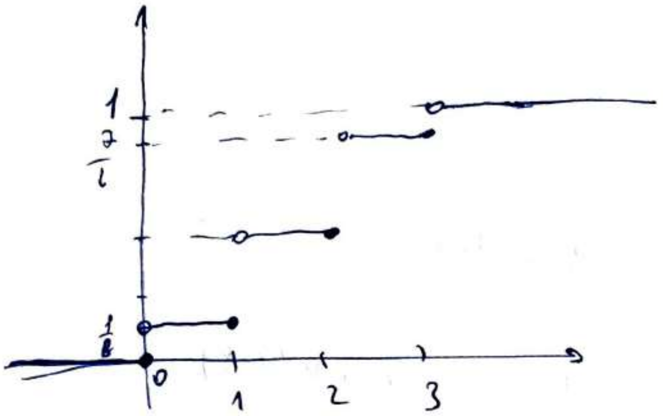
$P\{X=0\} = P\{k=0\} = C_3^0 p^0 q^3 = \frac{1}{8}$

$P\{X=1\} = P\{k=1\} = C_3^1 p^1 q^2 = \frac{3!}{1!\,2!}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$

$P\{X=2\} = P\{k=2\} = C_3^2 p^2 q = \frac{3!}{2!\,1!}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$

$P\{X=3\} = P\{k=3\} = C_3^3 p^3 = \frac{3!}{3!\,0!}\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

б) $F(x) = P\{X < x\}$



$F(0) = P\{X < 0\} = 0$

$F(1) = P\{X < 1\} = P\{X = 0\} = \frac{1}{8}$

nezaluc.

$F(2) = P\{X < 2\} = P\{X = 0\} + P\{X = 1\} = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$F(3) = P\{X < 3\} = P\{X < 2\} + P\{X = 2\} = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$

$$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{8}, & 0 < x \le 1 \\ \frac{1}{2}, & 1 < x \le 2 \\ \frac{7}{8}, & 2 < x \le 3 \\ 1, & x > 3 \end{cases}$$

в) $M\xi = \sum_{i=1}^{k} x_i p_i = \sum_{i=1}^{4} x_i p_i = 0 \cdot \cancel{\frac{1}{8}} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} =$

$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2} = 1.5$

$D\xi = \sum_{i=1}^{4} (x_i - m)^2 p_i = (0 - 1.5)^2 \cdot \frac{1}{8} + (1 - 1.5)^2 \cdot \frac{3}{8} +$

$+ (2 - 1.5)^2 \frac{3}{8} + (3 - 1.5)^2 \frac{1}{8} = 0.75$

г) $P\{\xi \le 1\} = P\{0 \le \xi \le 1\} = F(1) - F(0) = \frac{1}{8}$

Б8

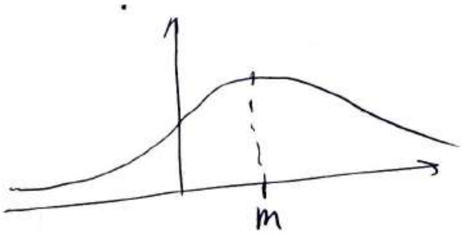
№2 $\xi \sim N(m, \sigma^2)$ $\quad \eta = A\xi + B$

Док-ть, что $\eta \sim N(m_\eta, \sigma_\eta^2)$

$\xi \sim N(m, \sigma^2)$

$$\Downarrow$$

$$f_\xi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$



$\eta = A\xi + B$ – монотонная

$$\Downarrow$$

$\eta = \varphi(\xi) = A\xi + B$

$$f_\eta(y) = f_\xi(\Psi(y)) \, |\Psi'(y)|$$, где $\Psi(y)$ – обратная к φ

$\varphi(\xi) = A\xi + B \Rightarrow \Psi(\eta) = \frac{\eta - B}{A}$ $\quad \left(\frac{1}{A}\eta - \frac{B}{A}\right)' = \frac{1}{A}$

η

$\Psi'(\eta) = \frac{1}{A}$

$\eta - B = A\xi$

$\xi = \frac{\eta - B}{A}$

$M\eta = M[A\xi + B] = A \cdot M\xi + B = Am + B$

$D\eta = D[A\xi + B] = A^2 D\xi = A^2\sigma^2$

$$f_\eta(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\left(\frac{y-B}{A} - m\right)^2}{2\sigma^2}} \left|\frac{1}{A}\right| =$$

$$= \frac{1}{\sqrt{2\pi}\sigma|A|} e^{-\frac{(y - B - mA)^2}{2A^2\sigma^2}} = \left| \begin{matrix} \text{замена} \\ B + mA = m_\eta \\ |A|\sigma = \sigma_\eta \end{matrix} \right| =$$

$$= \frac{1}{\sqrt{2\pi}\sigma_\eta} e^{-\frac{(y - m_\eta)^2}{2\sigma_\eta^2}} \rightarrow \eta \sim N(m_\eta, \sigma_\eta^2)$$

ч т д

№3. $$f(x,y) = \frac{a}{3+x^2+3y^2+x^2y^2}$$

а) Из усл-я нормировки

$$1 = \iint_{\mathbb{R}^2} \frac{a}{\ldots}\,dx\,dy = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} \frac{a}{y^2(3+x^2)+(3+x^2)}\,dy =$$

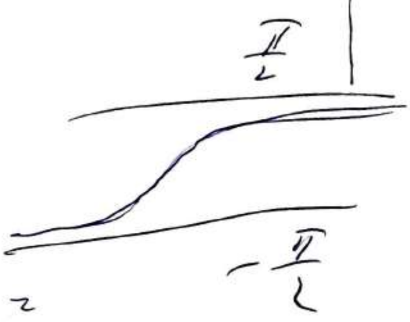
$$= a\int_{-\infty}^{+\infty} \frac{1}{(3+x^2)}\,dx \int_{-\infty}^{+\infty} \frac{1}{y^2+1}\,dy =$$

$$= a\int_{-\infty}^{+\infty} \frac{1}{(3+x^2)}\,dx \; \operatorname{arctg}(y)\Big|_{-\infty}^{+\infty} =$$

$$= a\int_{-\infty}^{+\infty} \frac{1}{3+x^2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)dx =$$

$$= a\pi\int_{-\infty}^{+\infty} \frac{1}{x^2+3}\,dx = a\pi\,\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}\Big|_{-\infty}^{+\infty} =$$

$$= \frac{a\pi}{\sqrt{3}}\,\pi = \frac{a\pi^2}{\sqrt{3}} = 1 \qquad a\pi^2 = \sqrt{3} \qquad a = \frac{\sqrt{3}}{\pi^2}$$



б) $$f_\xi(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_{-\infty}^{+\infty} \frac{a}{(3+x^2)(y^2+1)}\,dy =$$

$$= \frac{a}{(3+x^2)}\int_{-\infty}^{+\infty}\frac{1}{y^2+1}\,dy = \frac{a}{(3+x^2)}\operatorname{arctg}(y)\Big|_{-\infty}^{+\infty} =$$

$$= \frac{a}{(3+x^2)}\,\pi = \frac{a\pi}{(3+x^2)} = \frac{\sqrt{3}}{\pi(3+x^2)}$$

$$f_\eta(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \frac{a}{(y^2+1)\sqrt{3}}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right)\Big|_{-\infty}^{+\infty} =$$

$$= \frac{a}{y^2+1}\,\frac{1}{\sqrt{3}}\,\pi = \frac{1}{\pi(y^2+1)}$$

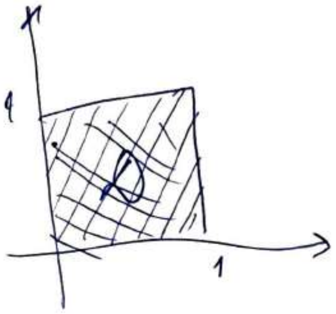
в) ξ, η – независ $\Longleftrightarrow f(x,y) \equiv f_\xi(x) \cdot f_\eta(y)$

$$f_\xi(x) \cdot f_\eta(y) = \frac{\sqrt{3}}{\pi^2(3+x^2)(y^2+1)} = \frac{\sqrt{3}}{- \,//\, -} = f(x,y)$$

$\Downarrow$

ξ и η – независимы

г)



$$P\{(x,y) \in D\} = \iint\limits_D f(x,y)\,dx\,dy =$$

$$= \int_0^1 dx \int_0^1 \frac{\sqrt{3}}{\pi^2(3+x^2)(y^2+1)}\,dy = \frac{\sqrt{3}}{\pi^2} \int_0^1 dx \frac{1}{(3+x^2)} \, dx \operatorname{arctg}(y)\Big|_0^1 dx =$$

$$= \frac{\sqrt{3}}{4\pi} \int_0^1 \frac{1}{(3+x^2)}\,dx = \frac{\sqrt{3}}{4\pi} \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right)\Big|_0^1 =$$

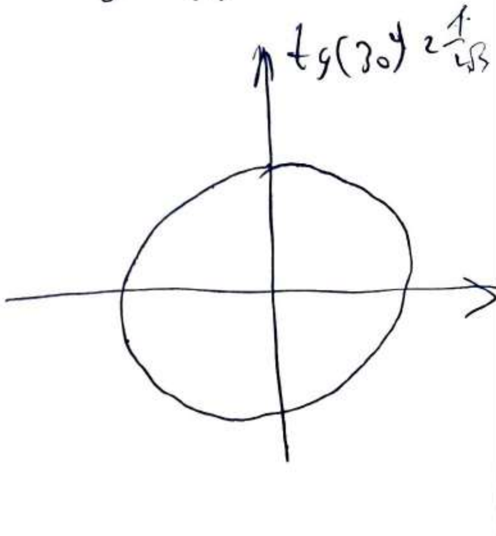
$$= \frac{1}{4\pi} \operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{24}$$

$\operatorname{arctg}(1) = \frac{\pi}{4}$

$\sin(30°) = \frac{1}{2}$

$\sin(\frac{\pi}{3}) = \cos(30°) = \frac{\sqrt{3}}{2}$

$\operatorname{tg}(30°) = \frac{1}{\sqrt{3}}$



1. $T \sim N(\mu, \sigma^2) \Rightarrow f_T(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in \mathbb{R}$

2. $C = \frac{a}{T}$, т. е $y = \varphi(x) = \frac{a}{x} \Longleftrightarrow x = \frac{a}{y}$, т.е $\psi(y) = \frac{a}{y}$

$\Downarrow$

$\psi'(y) = \frac{-a}{y^2}$

3. $f_C(y) = f_T(\psi(y)) |\psi'(y)| = \left\{ \begin{matrix} T \text{ принимает} \\ \text{значение } 0 \text{ с} \\ \text{вер-тью } 0 \end{matrix} \right\} =$

$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\frac{a}{y}-\mu)^2}{2\sigma^2}} \left|-\frac{a}{y^2}\right| = \left\{ \begin{matrix} a > 0 \\ y^2 > 0 \end{matrix} \right\} = \frac{a}{\sqrt{2\pi}\sigma y^2} e^{-\frac{(a-\mu y)^2}{2\sigma^2 y^2}}, y \in \mathbb{R}$

$R[a,b]$ $f(x) = \begin{cases} \frac{1}{b-a}, & x \in (a,b) \\ 0 & \text{иначе} \end{cases}$ $MX = \frac{a+b}{2}$ $DX = \frac{(b-a)^2}{12}$

$Exp(\lambda)$ $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ \text{иначе} \end{cases}$

$\underbrace{① ② ③ ④ ⑤ ⑥}_{\text{бра}} \underbrace{7 \ldots 25}_{\text{не брак}}$

$\{x_1, x_2, x_3\}$, где x_i - номер уе

$\Uparrow$ размещения без повт из 25 по 3 и легче упор.

~~число $\frac{3}{25}$~~ ~~повт~~ C_{25}^3

$\cancel{C_{19}^3}$

~~φ~~ $= 3$

$P\{$

C_{25}^3

1) вытащили 0 из брак и 3 из небрак $C_6^0 \cdot C_{19}^3$

1) $f(x) = \frac{2}{\pi} \cos^2 x$, $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

a) $\eta = 2\xi + 3$ - монотонн

2) $\eta = 2\xi + 3 \Rightarrow y = \varphi(x) = 2x+3 \Rightarrow \psi(y) = \frac{y-3}{2} = \frac{y}{2} - \frac{3}{2}$

$\Downarrow$

$\psi'(y) = \frac{1}{2}$

3) $x \in (-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow y \in (-\pi+3; \pi+3) \Rightarrow$

$\Rightarrow f_\eta(y) = f_\xi(\psi(y))|\psi'(y)| =$

$= \frac{\cancel{2}}{\pi} \cos^2\left(\frac{y-3}{2}\right) \cdot \frac{1}{\cancel{2}} = \frac{\cos^2\left(\frac{y-3}{2}\right)}{\pi} = \frac{1+\cos(y-3)}{2\pi}$

4) $x \notin (\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow y \in (-\pi+3; \pi+3) \Rightarrow f_\eta(y) = 0$

если y - такой, то x удв. второго части <0, тогда $f_Y(y) = 0$

$MX = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{-\pi+3}^{\pi+3} x \left(\frac{1+\cos(x-3)}{2\pi}\right) dx =$

$\pi + 3 + \pi - 3 -$

$= \frac{1}{2\pi}\left(\int_{-\pi+3}^{\pi+3} x\,dx + \frac{1}{2\pi}\int_{-\pi+3}^{\pi+3} x\cos(x-3)\,dx\right) =$

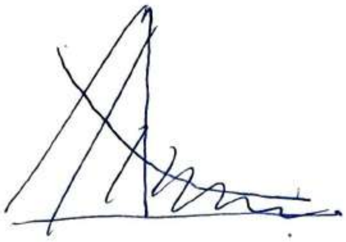
$= \frac{1}{2\pi}\left(2\pi\frac{x^2}{2} + \frac{1}{2\pi}\int_{-\pi+3}^{\pi+3} x\,d\sin(x-3) = \frac{x^2}{2}\Big|_a^b + \int x\sin(x-3)\Big|_a^b -\right.$

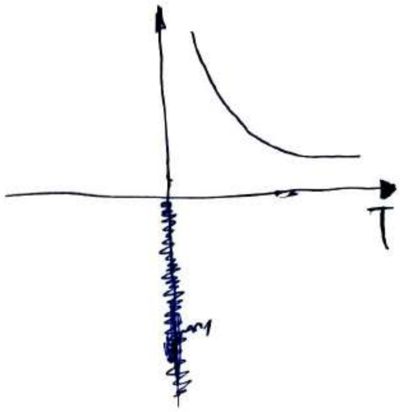
$- \int_a^b \sin(x-3)\,dx = \ldots = 3$

$P\{\eta < 3\} = \int P\{-\infty < \eta < 3\} = \int_{-\infty}^{3} f(x)\,dx = \frac{1}{2}$

1 $T \sim N(\mu, \sigma^2)$

$$f_T(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$





$C = \frac{a}{T}$, $a > 0$ $\qquad$ $a>0$, $T>0$

монотонна

$\Downarrow$

$$f_C(y) = f_T(\Psi(y))\,|\Psi'(y)|,$$

где $\Psi(y)$ — обратная к $\varphi(x) \overset{=y=}{=} \frac{a}{x} \Rightarrow \Psi = \frac{a}{y}$

$$\Psi' = (a y^{-1})' = -a y^{-2} = \frac{-a}{y^2}$$

$$f_C(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(\frac{a}{y}-\mu)^2}{2\sigma^2}} \cdot \left|\frac{a}{y^2}\right| = \left\{\begin{matrix} a>0 \\ y^2>0 \end{matrix}\right\} =$$

$$= \frac{a}{\sqrt{2\pi}\,\sigma y^2} e^{-\frac{(a-\mu y)^2}{2\sigma^2 y^2}}$$

9, 1 – стр.

Функция Лапласа

$$\Phi_0(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}}\,dx \qquad \Phi(x) = \int_{-\infty}^{x} e^{-\frac{x^2}{2}}\,dx'$$

1° $\Phi(x) = \frac{1}{2} + \Phi_0(x)$

2° $\Phi_0(x) = \Phi_0(x)$

3° $\Phi_0(-\infty) = -\frac{1}{2}$

4° $\Phi_0(+\infty) = \frac{1}{2}$

1) $y \in (-\infty; 0) \Rightarrow \Psi(y) = \frac{a}{y} \Rightarrow$ решение $x = \frac{a}{y}$

$f_C(y) = f_T(\Psi(y))\,|\Psi'(y)|$

2) $y = 0$ $\quad P\{C = 0\} = 0$

3) $y \in (0, +\infty)$