Report 5: No More Gradients

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Discontinuous Drag

The optimal point for the discontinuous drag function was found at an aspect ratio of 17.26556792952655428053 and a reference area of 12.474372829082144065 18 m^2, with drag equal to 191.598055498603685009 58 N. This was found using my particle swarm optimizer using 40 particles. Below, fig. 1 shows the paths that 5 of the 40 particles took to reach the minimum as well as the path the BFGS algorithm took.

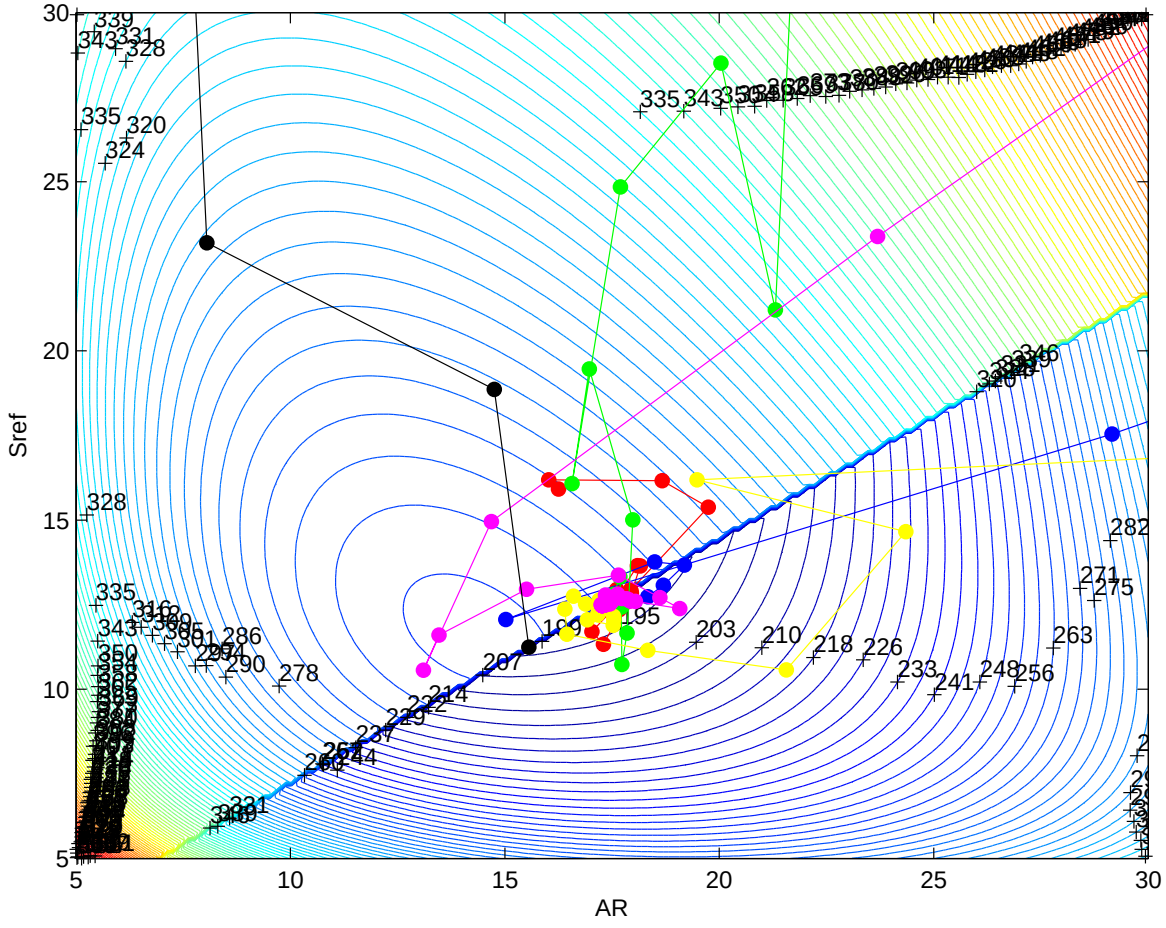


Figure 1: 5 particles converging on the minimum. The black path is for BFGS.

To make the BFGS algorithm work with this type of discontinuous problem, only a minor modification needed to be made. Normally the stop conditions for BFGS look at both the convergence of the function value and the normalized value of the gradient. This becomes a problem with discontinuous problems as the gradient at the minimum point is not necessarily zero. With the default stopping conditions for BFGS, if it was started on one side of the discontinuity it would converge to the drag value of the fully turbulent problem. If started on the other side, it would get confused an stall out. I changed the stopping conditions so it only took into account the convergence rate, and would stop when it was below a certain tolerance.

This didn't fully fix the problem, but it was a significant improvement. Now depending on where BFGS was started it would still converge to slightly different values. For example, with a starting point of $AR = 35$ and $S_{ref} = 10$, BFGS converged to a drag value of 191.90162587272661198767 N. With a starting point of $AR = 15$ and $S_{ref} = 40$, BFGS converged to 194.05158969490773301914 N. I did not have time to further explore this phenomenon.

The particle swarm optimizer would consistently converge within 0.005% of the same value every time. However, to achieve this consistency, a lot of tuning was done to the coefficients. For this problem, the coefficients I found to work the best were, $w = 0.5$, $c_1 = 0.6$, $c_2 = 0.9$, and $\Delta t = 0.9$. However, when optimizing other problems, like the Rosenbrock function (especially at higher dimensions), I found that these were not the best coefficients.

Performance & Convergence

Finding the proper stopping conditions for the PSO was a source of difficulty for me. The problem with using the standard convergence rate of

$$\gamma = \frac{||x_{k+1} - x_k||}{1 + ||x_k||} + \frac{|f(x_{k+1}) - f(x_k)|}{1 + |f(x_k)|} \tag{2.01}$$

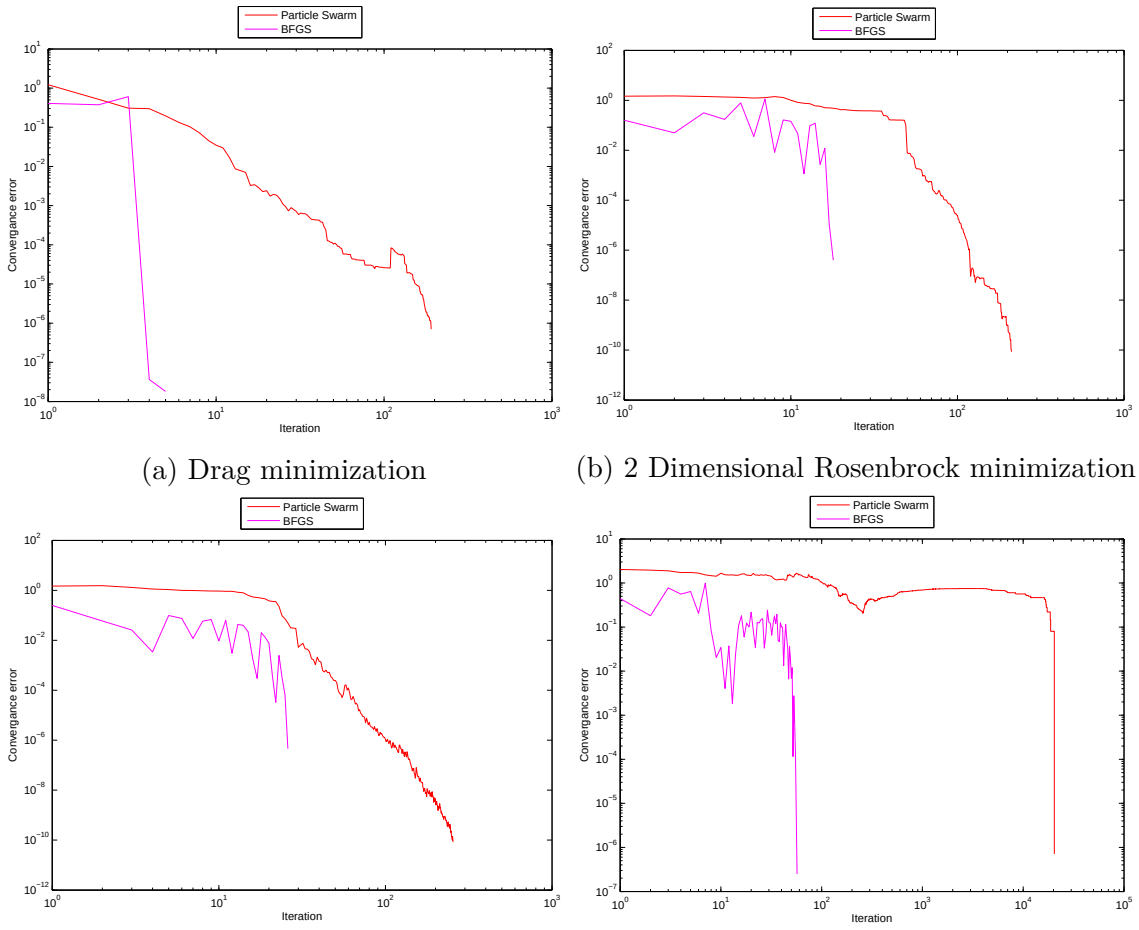
is that successive steps may still have the same global optimum if no new one was found. I tried a few different methods of measuring the convergence, but the one I found to work best was this:

$$\gamma = \frac{||x^g - \bar{x}^l||}{1 + ||\bar{x}^l||} + \frac{|f(x^g) - f(\bar{x}^l)|}{1 + |f(\bar{x}^l)|} \tag{2.02}$$

where x^g is the global best optimum so far, and $\bar{x}^l$ is the average of all particles local optimum. I based this on the idea that as the algorithm progresses, the swarm should move closer to the global optimum. As it approaches the optimal point, all particles should start converging on the same value. This isn't perfect however; if a global optimal point is found that is not the actual optimal there is a chance that that the particles converge on the wrong point. I found this to be especially problematic at higher dimensions. This problem can be combated by using more particles, but this, of course, uses much more processing power.

Relative to BFGS, the particle swarm algorithm does not perform nearly as well, neither in convergence rate, or computational time. This should be intuitively obvious though, as by definition, a particle swarm will require many more function evaluations than any gradient based algorithm. Further, in the very early stages of the algorithm, there is little guidance of the particles, meaning little progress may be made. A gradient based algorithm will have at least an idea of which direction to travel for the very first iteration.

The BFGS algorithm always converged faster. It's shown in fig. 2 that BFGS always, without a doubt converges sooner than PSO. Couple this with the fact that PSO requires many more function evaluations per iteration, and PSO starts looking like a bad idea. However, if accurate derivatives are not viable for whatever reason, then PSO is probably the best of the worst.



(a) Drag minimization

(b) 2 Dimensional Rosenbrock minimization

(c) 4 Dimensional Rosenbrock minimization

(d) 8 Dimensional Rosenbrock minimization

Figure 2: Comparison of convergence rates between BFGS and PSO.

Rosenbrock

Increasing the dimensionality of the problem proved somewhat problematic for PSO. The coefficients I found that worked best for the two dimensional, did not scale up very well. Using those same constants for the eight dimensional case, PSO would consistently converge to the wrong solution. After some further tuning, it would converge to the correct solution, but the computation cost increased dramatically. This could be further improved with more tuning of the constants, but this underlines a major drawback of the algorithm. Each problem set may need different constants for PSO to be effective. This isn't always easy to do, and with an expensive computational model it can become impractical to tune the constants. Frankly, the constants themselves present an optimization problem that could be solved, but I digress.

I decided to look at the solution error as a function of dimensions using the same constants. As shown in fig. 3, an exponential trend emerges as the number of dimensions increase.

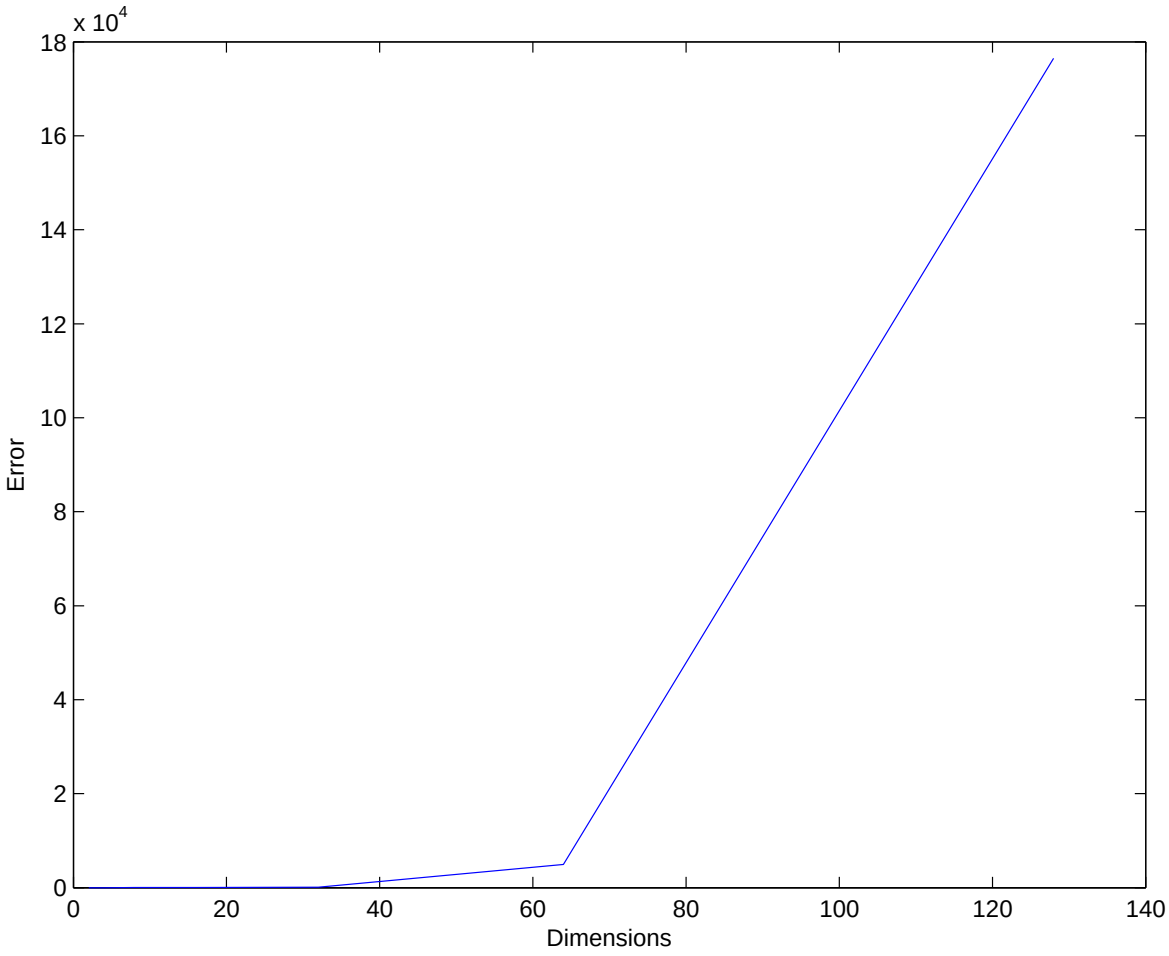


Figure 3: Error as a function of dimensions.

This may very be a result of how I determine convergence, but unfortunately I don't have

enough to fully explore this (something about having to build an airplane).

Measuring computation time can be troublesome as it can vary somewhat significantly between to independent runs of the same problem. However, in theory, increased dimensionality would not have an effect on computation time, if the same number of particles are used over a fixed number of iterations. The objective function will still only be evaluated once for each particle. Of course this does not take into account stopping criteria or the coefficients used for the problem, but again, I did not have time to explore this to the extent I wanted.

Conclusions

The particle swarm optimizer is an effective choice if accurate gradients aren't available, but it has it's problem areas. It can be difficult to tune the constants to a specific problem, something that might be necessary to get a decent convergence rate, it is not deterministic, it will not produce the same results each time it is run, and the choice of good stopping conditions proved difficult. That being said, it handles discontinuities much better than any gradient based solution and implementation wise, it is much simpler to get running. Performance wise, it was very slow compared to BFGS. Even if it converged in the same number of iterations it still takes more computation time if there are an appreciable number of particles. At the end of the day, a gradient based solution is the way to go when possible.