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Probabilistic criteria for time-series predictability estimation

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Abstract

Assessing the time series predictability is necessary for forecasting models validating, for classifying series to optimize the choice of the model and its parameters, and for analyzing the results. The difficulties in assessing predictability occur due to large heteroscedasticity of errors obtained when predicting several series of different nature and characteristics. In this work, the internal predictability of predictive modeling objects is investigated. Using the example of time series forecasting, we explore the possibility of quantifying internal predictability in terms of the probability (frequency) of obtaining a forecast with an error greater than some certain level. We also try to determine the relationship of such a measure with the characteristics of the time series themselves. The idea of the proposed method is to estimate the internal predictability by the probability of an error exceeding a predetermined threshold value. The studies were carried out on data from open sources containing more than seven thousand time series of stock market prices. We compare the probability of errors which exceed the allowable value (miss probabilities) for the same series on different forecasting models. We show that these probabilities differ insignificantly for different forecasting models with the same series, and hence, the probability can be a measure of predictability. We also show the relationship of the miss probability values with entropy, the Hurst exponent, and other characteristics of the series according to which the predictability can be estimated. It has been established that the resulting measure makes it possible to compare the predictability of time series with pronounced heteroscedasticity of forecast errors and when using different models. The measure is related to the characteristics of the time series and is interpretable. The results can be generalized to any objects of predictive modeling and forecasting quality scores. It can be useful to developers of predictive modeling algorithms, machine learning specialists in solving practical problems of forecasting.

Keywords

intrinsic predictability, forecasting error, misprediction

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Вероятностный критерий оценки предсказуемости временных рядов

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Аннотация

Предмет исследования. Задача оценки предсказуемости временных рядов возникает при валидации моделей прогнозирования, при классификации рядов с целью оптимизации выбора модели и ее параметров, при анализе результатов. Большая гетероскедастичность ошибок, получаемых при прогнозировании нескольких различных по природе и характеристикам рядов, часто приводит к затруднениям при оценке предсказуемости. В работе исследована внутренняя предсказуемость объектов предсказательного моделирования. На примере прогнозирования временных рядов определена возможность количественной оценки внутренней предсказуемости по вероятности (частоте) получения прогноза с ошибкой, больше заданного уровня, и связь такой меры с характеристиками самих временных рядов. **Метод.** Суть предлагаемого метода состоит в оценивании внутренней предсказуемости по вероятности возникновения ошибки, большей заранее заданного порогового значения. **Основной результат.** Исследования выполнены на данных из открытых источников, содержащих более 7000 временных рядов биржевых котировок. Проведено сопоставление полученных значений вероятности возникновения ошибок, превосходящих допустимое значение (вероятностей промаха) для одних и тех же рядов на различных моделях прогнозирования. Показано, что при использовании моделей с одним и тем же рядом эти вероятности отличаются незначительно и могут служить мерой предсказуемости. Выявлена связь полученных значений вероятности с энтропией, показателем Хёрста и иными характеристиками рядов, по которым оценивается предсказуемость. Установлено, что полученная мера позволяет сравнивать предсказуемость временных рядов при выраженной гетероскедастичности ошибок прогнозирования и при применении разных моделей. Мера связана с характеристиками временного ряда и интерпретируема. **Практическая значимость.** Полученные результаты могут быть обобщены на любые объекты предсказательного моделирования и меры оценки качества прогноза. Результаты исследования будут полезны разработчикам алгоритмов предсказательного моделирования и специалистам по машинному обучению, при решении практических задач прогнозирования.

Ключевые слова
внутренняя предсказуемость, ошибка прогнозирования, вероятность промаха

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Introduction

In almost every case when we deal with time series forecasting, we need a meaningful and understandable measure of predictability to evaluate the results. In other words, we want to know whether our model achieved the best possible quality or not. Hence, some measure of intrinsic predictability is necessary which could explain how likely a misprediction is, or what the range of errors might occur, or what variance of the errors is expected for the certain series. And of course, it would be very nice if we could get all these aspects before the model is constructed, fitted, and running. So, we're going to find the way to answer these questions by time series analysis. For a rather big dataset we can calculate different features whose connection to predictability was approved by related works, and build some regression that hypothetically can connect these features to misprediction probability or errors variance. Obviously, we can't avoid involving some forecasting model which must produce these probabilities and variances, but for this research we presume that this experience with one model can be generalized to the class or classes of models. At least, for the future work we plan to use several models of different classes in ensemble which will approximate the real metrics of intrinsic predictability more accurately.

Related works

The earliest mention of the idea to distinguish realized and intrinsic predictability, which we could find, was proclaimed by Edward N. Lorenz in [1] that refers to a 1996 paper. The philosophic discussion on the predictability issues in various senses is going on till nowadays. For instance, Stefan Rummens [2] argues with Victor Gijsbers [3]. Meanwhile, this discussion is as interesting and entertaining as it is far from practical use and everyday needs. At the same time not so many authors attempted to find some quantity measure for time-series intrinsic predictability. The idea to match some time series features or their combination to intrinsic predictability was discussed in [4] where predictability was quantified with permutation entropy, and in [5] where several features including Hurst exponent and Kolmogorov-Sinai entropy were used for series clustering according to their predictability. The similar approach base on transforming a time series to graph is proposed in [6]. All these methods are based on forecasting errors estimation, and they don't consider the fact that the series of bad predictability can perform rather good forecasting quality. Having this fact in mind, the authors of [7] state that intrinsic predictability of chaotic systems might be high, but the realized predictability is expected to be low and difficult to improve substantially. In [8] the rank-based nonlinear predictability score was adapted to time series sampled from time-continuous flows and performed a higher sensitivity for deterministic structure in noisy signals.

One more approach is represented in [9] where intrinsic predictability is estimated by wavelet entropy energy measure after time series wavelet transformation. Besides the forecasting error, they also use Nash–Sutcliffe efficiency for quality estimation.

We failed to find any research where statistical or probabilistic were used for matching of the predictability measure, so off we go.

Real-world data

In our experiments we used the open dataset called Huge Stock Market Dataset from Kaggle which contains historical daily prices and volumes of all U.S. stocks and ETFs. There are more than 7,000 time series mostly of fractal nature [10] with a significant part of the random-walking process. Nevertheless, we expected that this set includes the series of different intrinsic predictability and would be sufficient for the aim of our research.

We dropped the short series with less than 730 observations and shrunk the rest up to last 730 points.

So, our series are neither too short to make the predictive model to do its best, nor too long to satisfy memory and computation time requirements. The number of series after such preprocessing was reduced to 5,121. Every data file in the collection contains the series of opening, high, low, closing prices and trading volume per day. As all the prices are rather close to each other, we take the closing price for the target series.

We use the set of the time series features the same as was used in [5]: Kolmogorov-Sinai entropy, Hurst exponent, embedding dimension, noise measure and random-walk detection. Permutation entropy was explored too, as many researchers usually mention in connection to predictability.

The Kolmogorov-Sinai entropy (h_μ^{KS}) can be calculated based on the entropy rates of finite partitions of the state space of the series [11]. For each of the n finite partitions $\xi = \{C_1, C_2, \dots, C_n\}$ of the state space $M = \cup_{i=1}^{n} C_i$ with dynamics given by a measurable transformation T and defined probability measure $\mu(T^{-1}C_{i_1} \cap \dots \cap T^{-n}C_{i_n})$, the entropy rate is

$$h_\mu(T, \xi) = -\lim_{n\to\infty} \frac{1}{n} \sum_{i_1,\dots,i_n} \mu(T^{-1}C_{i_1} \cap \dots \cap T^{-n}C_{i_n}) \times$$

$$\times \ln\mu(T^{-1}C_{i_1} \cap \dots \cap T^{-n}C_{i_n}).$$

We take the supremum of the entropy rate over all finite partitions:

$$h_\mu^{KS}(T) = \sup_\xi h_\mu(T, \xi).$$

Hurst exponent is used as a measure of long-term memory of time series. According to [12], we estimate it by the re-scaled range:

$$R(\tau) = \max_{1\le t\le\tau} \sum_{j=1}^{\tau} (x_j - \acute{x}_\tau) - \min_{1\le t\le\tau} \sum_{j=1}^{\tau} (x_j - \acute{x}_\tau)$$

and standard deviation:

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau} (x_\tau - \acute{x}_\tau)^2},$$

where $\tau \in [3, N]$ are the time steps for a discrete time series, $x(t)$ is the series value on step t.

Hurst exponent can be calculated as:

$$H(\tau) = \lim_{\tau\to\infty} \frac{\log \frac{R(\tau)}{S(\tau)}}{\log(\alpha\tau)},$$

where $\alpha \approx 0.5$ is a Hirst's empirically found constant.

These re-scaled range $R(\tau)$ and standard deviation $S(\tau)$ are used to represent the R/S-trajectory $RS(\tau) = \frac{R(\tau)}{S(\tau)}$ that helps to estimate the *memory depth* of the time series.

Embedding dimension is a measure of the dimensionality of the space occupied by a set of random points, often referred to as a type of fractal dimension. It's less noisy when only a small number of points is available and is often in agreement with other calculations of dimension. It can be calculated by means of the correlative integral for time series of finite length:

$$C(r) = \sum_{i=1}^{m} \sum_{j=i+1}^{m} \frac{\theta(r - \rho(i, j))}{m(m-1)},$$

where $\rho_k(i, j) = \sqrt{\sum_{l=1}^{k} (x_{i-k+l} - x_{j-k+l})^2}$; $\theta(x)$ — Heaviside function; r — characteristic phase space cell size.

The value of embedding dimension is the slope of the logarithmic graph of the correlation integral [11] and it can be evaluated as the following limit:

$$d_k = \lim_{r\to0} \lim_{m\to\infty} \frac{\ln C(r)}{\ln r}.$$

Noise measure feature is based on the idea of Robert M. May, firstly published in 1976 [13], to compare the standard deviation of a time-series with the standard deviation of its first-order differences.

$$F_N = 1 - \sqrt{\frac{N\sum_{i=1}^{N-1}(x_i' - \acute{x}')^2}{(N-1)\sum_{i=1}^{N-1}(x_i - \acute{x})^2}},$$

where $x_i' = x_{i+1} - x_i$ is the first-order differences; $\acute{x}$, $\acute{x}'$ are mean values of the initial time series and corresponding differenced series. This measure can be used to find a random walk process for which the first-order differences should be noise.

The idea is to detect high-frequency noise that increases the comparative deviation of value change on each time step; so, the higher value of this measure means the lower noise influence in time series.

Forecasting and realized predictability

In order to collect information about forecasting quality statistics, we launched the Extreme Gradient Boosting (XGB) forecasting model for each time series of the data set. Hyperparameters were chosen empirically for the best performance on 30-days horizon for most of the series. Thus, we got 20 estimators and maximal depth equal to 8. Every single time series is forecasted 107 times. We take 365-days observations as the training period and 30-days forecast as a test, then we repeat the same with a 3-day time shift. The stock market time series are chaotic enough to eliminate the effect of dependencies in the sequential forecasting experiments. The procedure is like a rolling window. So, for each of the series, we get 107 values of Mean Absolute Percentage Error (MAPE) and calculate the mean value and standard deviation for these error series, and for the part of errors which is greater than 10 %, we consider that an error of less than 10 % is sufficient for the good forecast quality. This part helps to estimate the misprediction probability.

Of course, this method of predictability estimation is inseparably connected with the forecasting model. This connection could be broken if we show that the same metrics for some other models of different kinds are either close or at least correlated with those for our basic model.

Local approximation (LA) and Maximal Similarity (MS) were applied in the same conditions as the alternate models. The first one was designed by prof. Alexander Loskutov [11], and it is based on neighborhood of points of system trajectory in the state space; the other one was invented by Irina Chuchueva [14], and it uses the rescaled patterns of a certain length in the series to make a forecast. Both models are described in detail in [5] and [6]. In Fig. 1, *a* we can see that squared misprediction probability for MS correlates with that for XGB with coefficient of 0.94. As the values distribution is obviously far from normal, and it will be shown further, we use the Spearman's rank coefficient to estimate the correlations. In the case of LA there is no need to deal with squared values. The correlation is 0.92, the scatter plot is presented in Fig. 1, *b*.

So, we can conclude that at least for three types of forecasting models the scores of misprediction probability are correlated and this measure could be considered model independent.

Regression model for intrinsic predictability measuring

As we have got a set of time series features and the values which we consider to be the measure of predictability, we can try to find some dependence between them. The scatter matrix for the features shown in Fig. 2 looks not very promising. Pairwise correlation coefficients between the features and scores are collected in Table. They are not great as well, but nevertheless we try to build a regression model. This picture also confirms that, as we noted earlier, the value distributions are not normal. Besides, it illustrates the large heteroscedasticity of errors. For instance, the more noise, the wider is standard deviation range.

Presumably, models of linear regression would fail with such sort of predictors. We use an Extra-Trees regressor which fits a number of randomized decision trees on various sub-samples of the dataset and uses averaging to improve the predictive accuracy and control over-fitting. Its

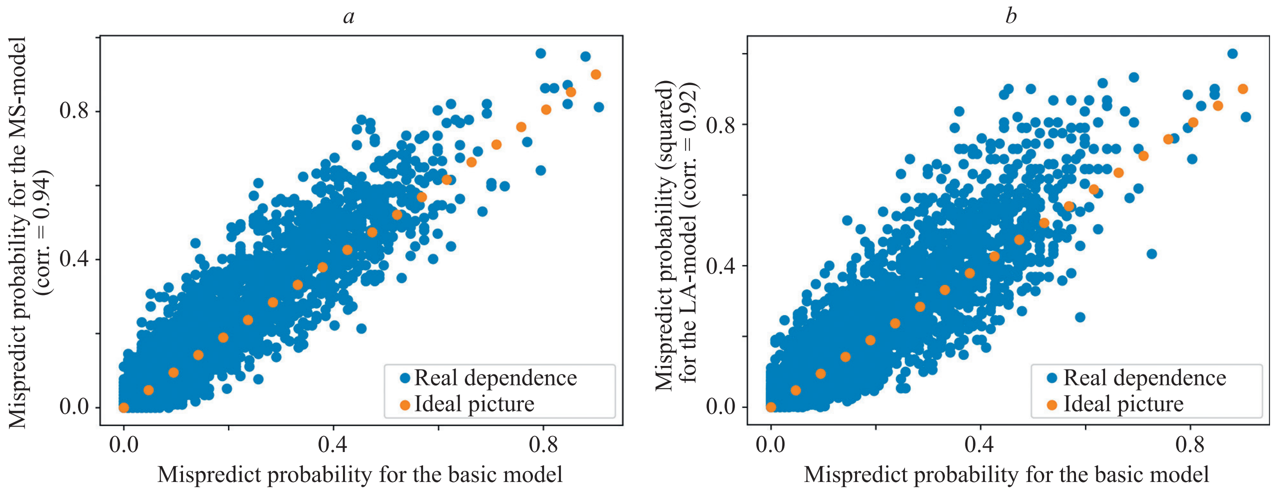

Fig. 1. Misprediction probability for MS (*a*) and LA (*b*) models regarding to XGB model

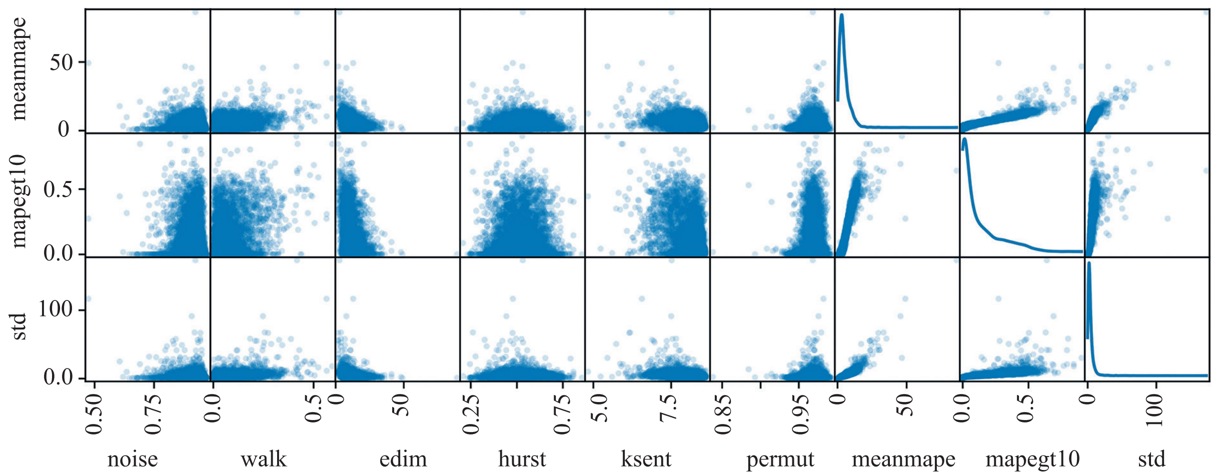

Fig. 2. Scatter matrix for features and scores.
Features: noise — noise measure; walk — random walk measure; edim — embedding dimension; hurst — Hurst exponent; ksent — Kolmogorov-Sinai entropy; permut — permutation entropy. Scores: meanmape — mean MAPE value for all samples; mapegt10 — the part of samples with MAPE>10 %, std — standard deviation

Table. Spearman correlation coefficients for features and scores

	Noise measure	Random walk	Embedding dimension	Hurst exponent	Kolmogorov-Sinai entropy	Permutation entropy
Mean MAPE value	–0.12	0.320	–0.17	0.02	–0.370	0.13
Part of samples with MAPE >10 %	–0.11	0.290	–0.18	0.04	–0.383	0.11
Standard deviation	–0.12	0.339	–0.16	–0.01	–0.330	0.09

hyper-parameters were chosen empirically: 128 estimators with minimal samples split of 16 and min samples leaf of 2. We use coefficient of determination (R^2) for regression scoring. This regressor performance score was $R^2 = 0.60$ on the train set, $R^2 = 0.32$ and $R^2 = 0.24$ on 50-times splitting cross-validation. The Mean Absolute Error is 0.12 for 1706 test samples. The error distribution analysis shows that most of errors are in the range of ±0.2 and there is a bias about +0.07.

We also tried Random Forest Regression model, Gradient Boosting, and based not on random trees model of K-Neighbors Regression. None of them performs better regression quality.

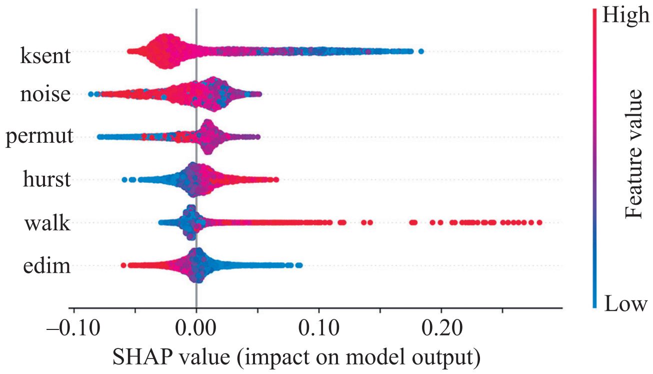

Fig. 3. Features impact analysis by Shapley values (Designations are the same as those in Fig. 2)

Experiment results and discussion

As we can see in the previous section, the regressor performance is rather poor, so, our experiments are aimed to find out whether it's possible to get some use in spite of that. Table shows that correlations between the features and responses are weak which explains the low score of the regression.

The further feature analysis by means of Shapley values shows that the impact of the features on the result could be opposite for different series which is reflected as the mess of the red and blue dots on the bee-swarm plot in Fig. 3. All the features are actually important for Extra-Trees regression. Kolmogorov-Sinai entropy is most important with relative importance score of 0.33, the least one is Hurst exponent with score of 0.08. We also can notice that Kolmogorov-Sinai entropy is much more correlated with misprediction probability, and its importance is more than those for usually used permutation entropy. The mess of Shapley values at least for our dataset is not that big too.

In the last item of our research agenda, we have got the misprediction probability values for the real model and estimated it by the regressor. The distribution of these values is illustrated in Fig. 4, *a* as a histogram which shows that the regressor tends to overestimate the probability due to its bias, and the range of real values is wider.

To get a more detailed picture of the regressor performance, we can see the scatter plot in Fig. 4, *b* with

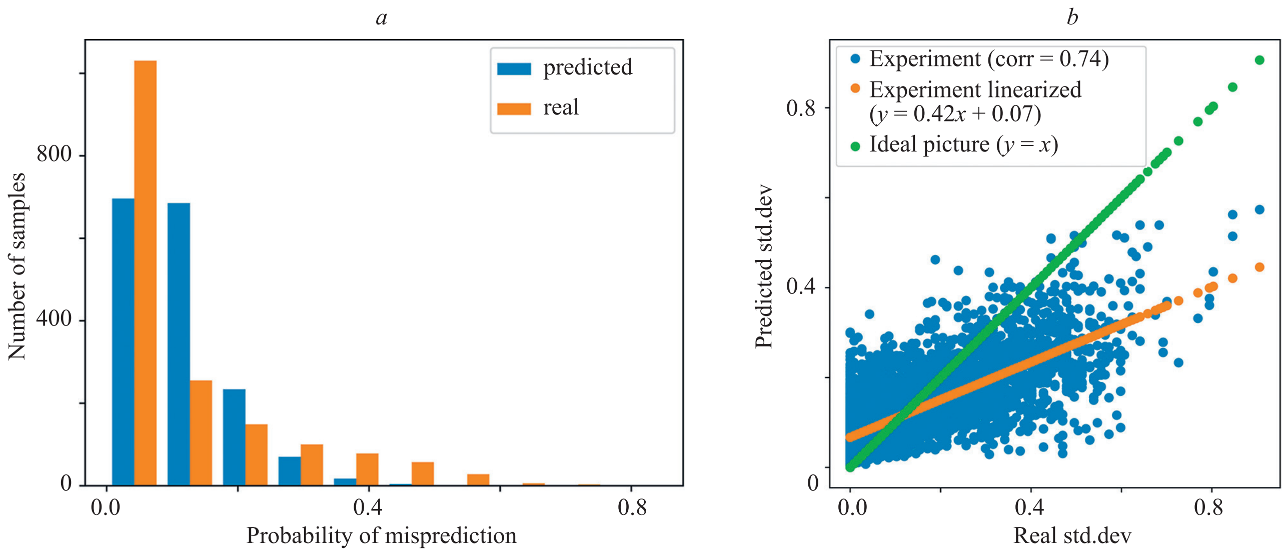

Fig. 4. Distribution of real and estimated misprediction probability as a histogram (*a*) and as a scatter plot with the least squares approximation (*b*)

the least squares approximation and ideal line shown. The correlation and distribution of the real and predicted values are quite suitable for time series predictability estimation. We suppose that for some not very precise real-world tasks that estimation would be enough.

Future work

For our future work we plan to increase the number and variety of our experiments on predictability. Of course, it's necessary to track the dependence of misprediction probability on the forecasting horizon. The time series of different shapes and nature should be tested as well. It would be better to get more different models for experiments and maybe to take the best quality from among all models for the regressor training.

Besides, as we said above, the other score could be useful for time-series predictability measuring. It is forecasting errors standard deviation. This score also worth to be explored. To continue the predictability exploring, it could be useful to take into account not only errors themselves but the shape of their distribution.

Conclusion

Our probabilistic method of intrinsic predictability evaluation is an attempt to pay attention to the numerously noticed fact that the bad forecasting quality not obligatory can occur for the series which features indicate bad predictability. Our regressor helps to estimate the probability of poor-quality forecast despite the rather big errors of the regressor.

The proposed method of predictability evaluation looks very much like the simple confidence interval calculation. But there is some difference. First, making up the confidence interval we first define the confidence level and then calculate the thresholds for predicted values. Here we approach from the opposite side. We settle the thresholds and count how often the shooter hits it. Then, for the statistical calculation of the confidence interval the normally distributed value is strongly desired. In our case we were never tied by such a requirement. The confidence interval is used mostly for continuous values and doesn't work with classification and ranking models. Our approach can be applied to any quality score for almost every type of model. We may set a threshold on F-score or accuracy and count the amount of hits.

Besides, the time series predictability is usually estimated by one of the series features like entropy or Hurst exponent or something else. We try to get use of a set of several features which turns out to be rather helpful in such estimation.

The benefit of proposed approach to time series predictability scoring is in direct evaluation of misprediction probability in quite understandable terms which suits for all sorts of time series and depends only on the forecasting horizon.

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