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PHYS 414 Final Project Report

NEWTON

(b) A data set containing masses (in solar mass units) and base-10 logarithm of surface gravity (in CGS units) of white dwarf stars was obtained from a .csv file (available in the GitHub repository) and plotted as a scatter plot, as shown in the **Figure 1** below:

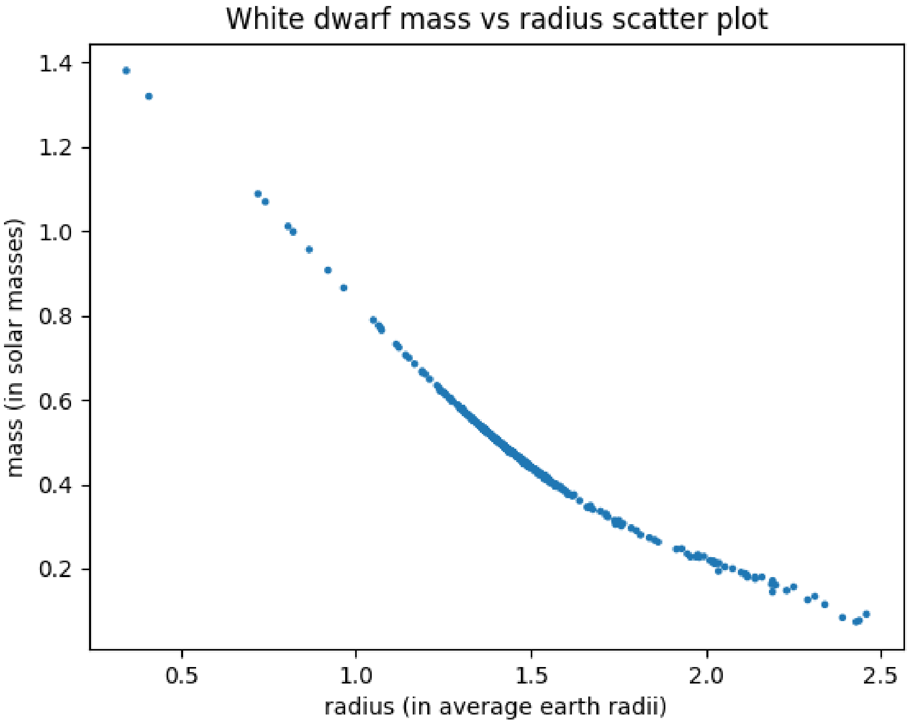

Figure 1. White dwarf mass vs radius scatter plot.

(c) The data in Figure 1 implies an inverse relationship between white dwarf radius and mass. To quantify the relationship, a log – log linear fit was first performed on low-mass white dwarfs in the data set, with a cutoff radius of 1.45 earth radii.

The slope of the fit was found to be -2.76. It was then related to the estimate of the polytropic index n as:

Slope = -2.76 = $\frac{3-n*}{1-n*}$, from which n∗ was found to be 1.53.

The obtained n value was then related to the non-linear fitting parameter q as

$n_* = \frac{q}{5-q}$, from which q was found as 3.02 and, from theory, rounded to the closest integer as q = 3. The value of n_* was then refined according to the previous n_* - q dependence formula as n_* = 1.5

The newly found n_* value was used to fix the slope of the fit as equal to

Slope = $\frac{3-1.5}{1-1.5}$ = -3. The corresponding y – intercept of the fit was found using the y-intercept formula for least-squares linear regression as:

$y_{int} = \overline{\log(y)} - (-3) \times \overline{\log(x)}$ = 0.44, where x – radius data, y – mass data, $\bar{s}$ denotes average of the data points.

The final fit – M = $-3 \times R + 0.44$ – is shown in the **Figure 2** below:

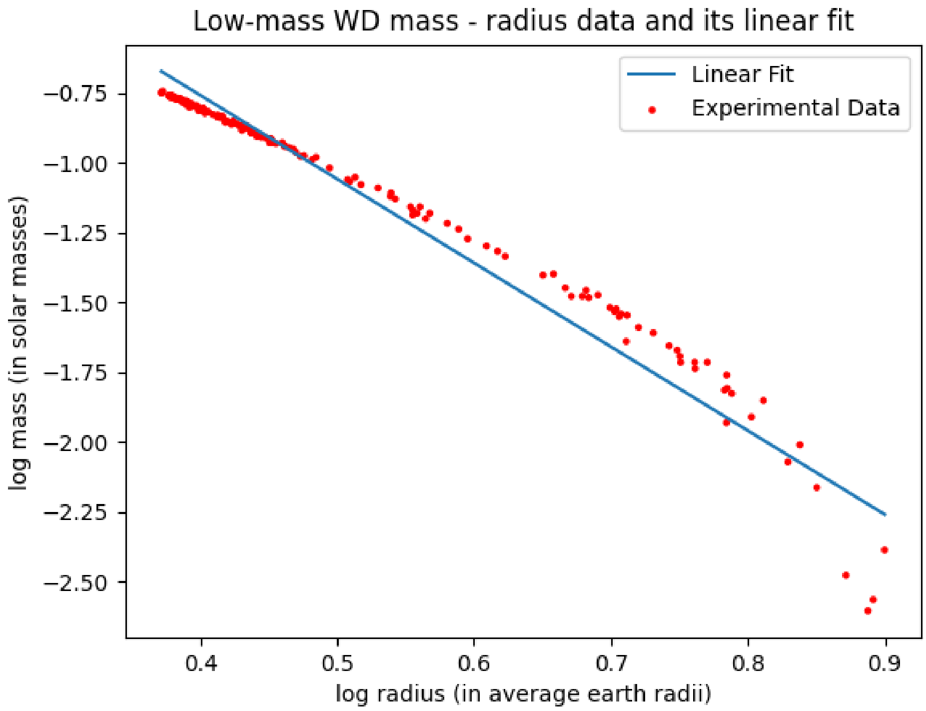

Figure 2. Low – mass WD log mass – log radius data and its linear fit.

The y – intercept of the linear fit was related to the non-linear fitting parameter K_* as:

$y_{int} = \frac{n_*}{n_*-1} \times \ln(\frac{K_*}{G\ N_n})$, where $N_n = \frac{(4\pi)^{1/n}}{n+1} \times (\left[-\xi^2 \frac{d\theta}{d\xi}\right] \big|_{\xi\ =\ \xi_n})^{\frac{1-n}{n}} \times \xi_n^{\frac{n-3}{n}}$,

as given by the Lane – Emden equation. Thus, Lane – Emden equation was evolved with n = 1.5, and ξ_n (Lane – Emden radius of the star) and $\frac{d\theta}{d\xi}$ at the surface were obtained to evaluate the above expression. As a result, K_* was found to be 0.11, as shown in the **Figure 3** below:

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Radius from Lane - Emden integration:  3.6568827081200674
K* =  0.10975146657664457
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Figure 3. Console output for the obtained Lane – Emden radius ξ_n and non-linear fitting parameter K_*

Finally, solving the Lane – Emden also allowed the central densities of the stars, ρ_c, in the data set to be calculated according to: $M = 4\pi\rho_c R^3 \left(\frac{-1}{\xi_n}\frac{d\theta}{d\xi}\Big|_{\xi = \xi_n}\right)$

In the equation above, M and R are known from the data set, and ξ_n, $\frac{d\theta}{d\xi}\Big|_{\xi = \xi_n}$ are obtained from Lane – Emden equation solution. The central densities are plotted as a function of star masses in the **Figure 4** below:

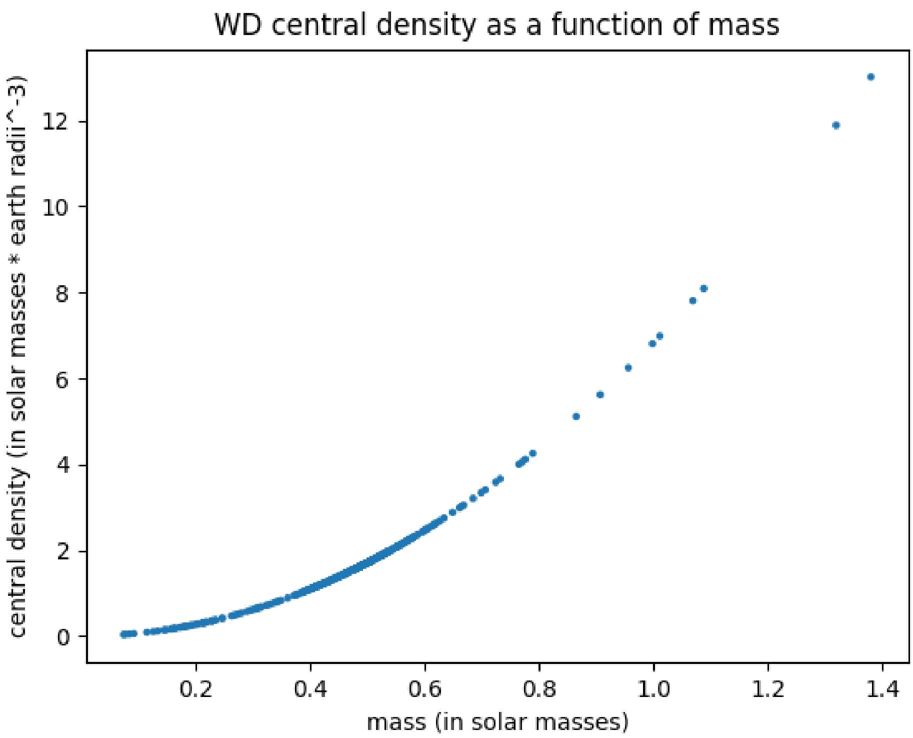

Figure 4. WD central density as a function of mass.

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(a) 50 central density values were generated on the order of 10^{-3} (physically meaningful scale for neutron stars in geometric units for density). The central densities were related to pressure through polytropic equation of state with the polytropic index n = 1 and the coefficient K = 100 (in geometric units). Tolman – Oppenheimer – Volkoff (TOM) equations were evolved, and the corresponding masses and radii of the NSs were obtained, as shown in the **Figure 5** below:

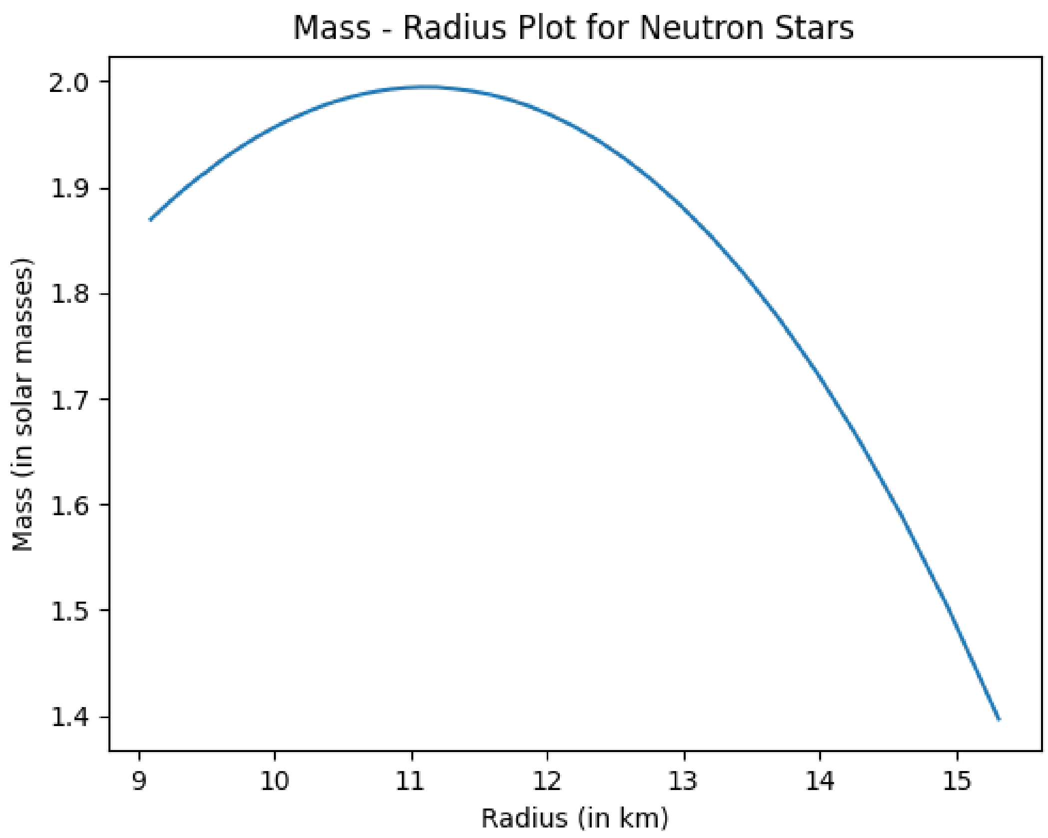

Figure 5. Mass – radius plot for neutron stars.

(b) Baryonic mass, M_P, ODE was evolved together with the TOM equations, and the fractional binding energy was calculated as: $\Delta = \frac{M_P - M}{M}$, as shown in the **Figure 6** below:

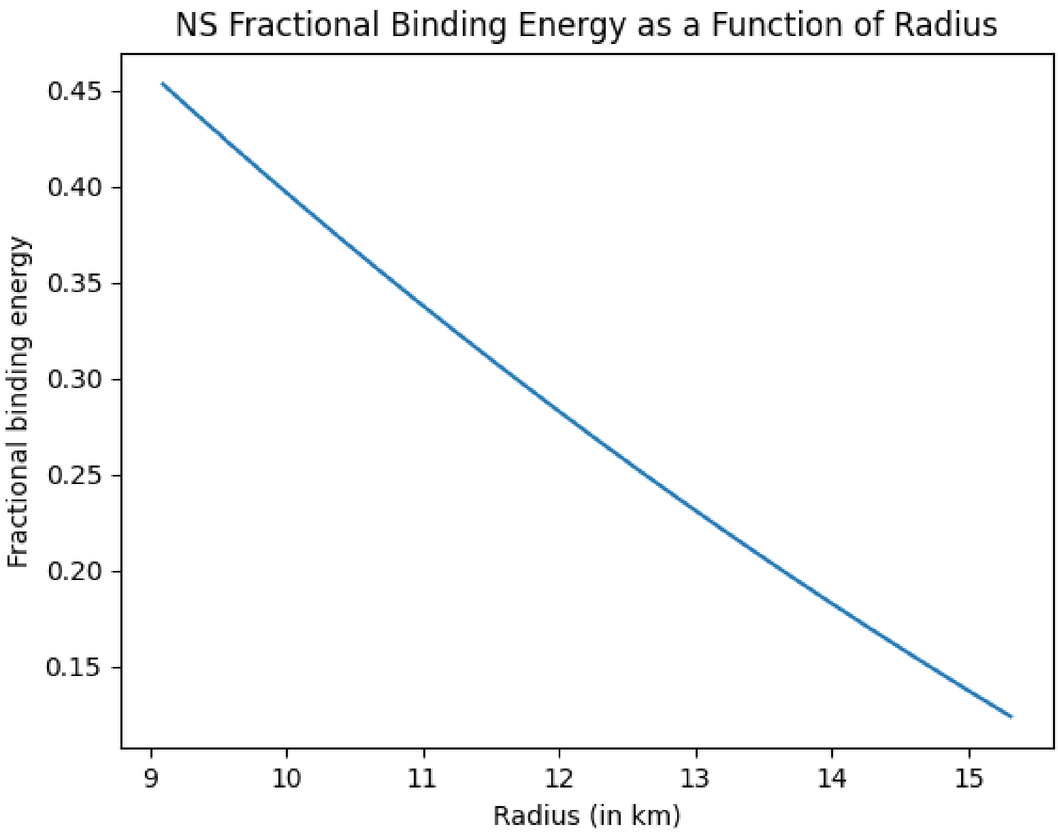

Figure 6. Neutron star fractional binding energy as a function of its radius.

According to the calculations in Figure 6, the binding energy increases as the radius of the neutron star decreases, which is expected, as smaller distance implies stronger interaction between the particles.

(c) The masses obtained in part (a) were plotted as a function of corresponding central densities, as shown in the **Figure 7** below:

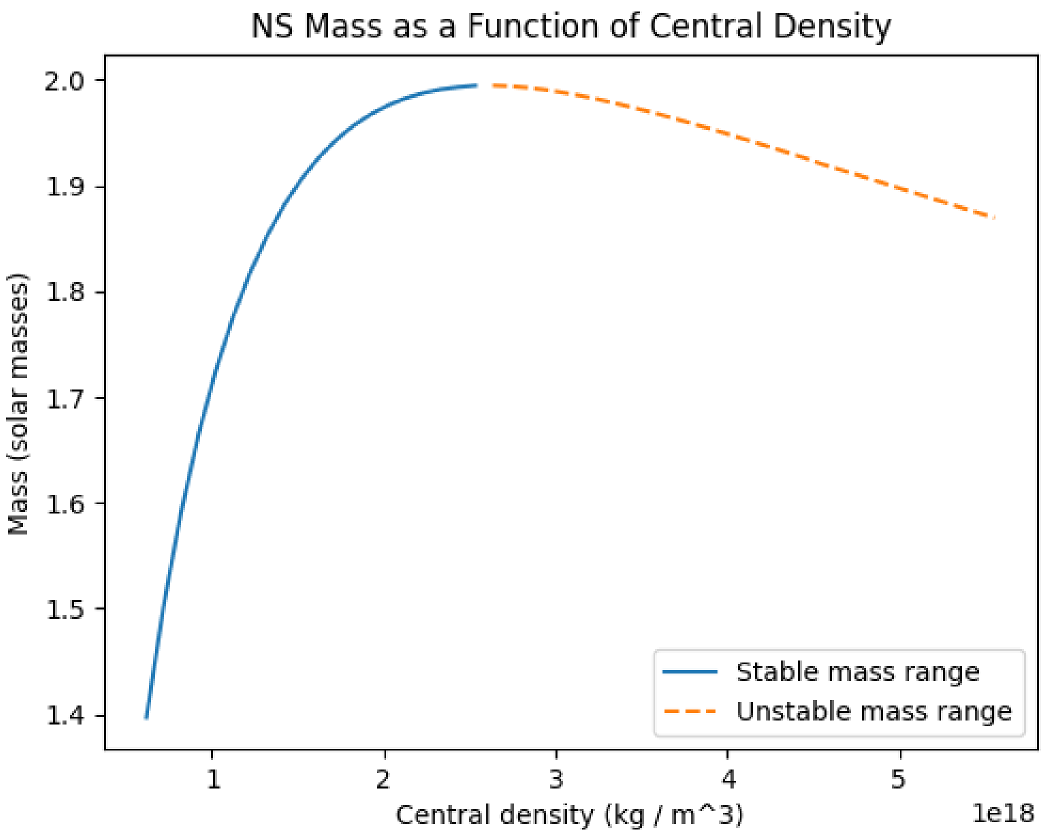

Figure 7. Neutron star mass as a function of central density.

The neutron stars in the negative mass – central density slope region are unstable, and the maximum allowed neutron star mass according to the polytropic equation of state with n = 1 and K = 100 corresponds to **2 solar masses.**

(d) The coefficient K of the polytropic equation of state with n = 1 was varied in the range [80, 100], in geometric units. The maximum mass of a stable neutron star was obtained for each K, as in part (c). The results are shown in the **Figure 8** below:

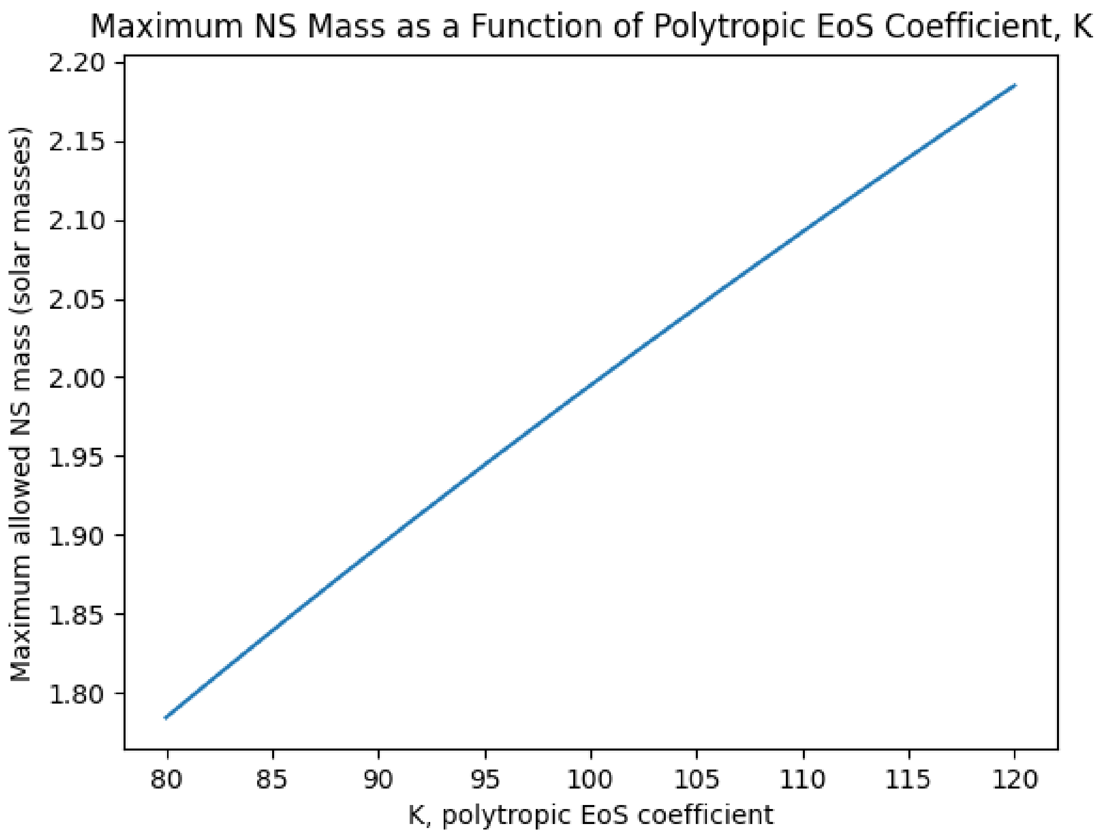

Figure 8. Maximum neutron star mass as a function of polytropic equation of state coefficient K.

If we assume that the currently most massive experimentally observed neutron star – corresponding to 2.14 solar masses – dictates the acceptable values of K, then there is an upper bound at K = 114, corresponding to M = 2.14 solar masses.