Wound Healing

Computational Mechanics

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July 6, 2023

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Index

Wound healing using continuum mechanics

Wound configurations

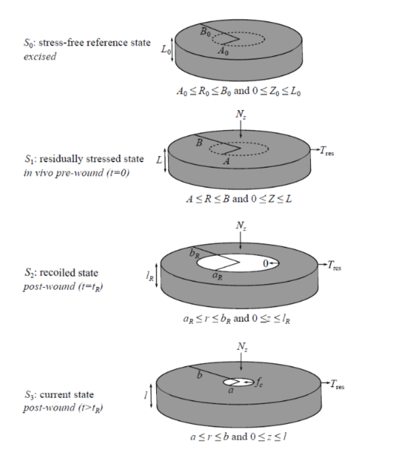


Figure 1: Representation of the mathematical model.

Wound model: main hypothesis

1. Fundamental assumption of morphoelasticity: $F = AG$

   F: geometric deformation tensor

   G: growth tensor

   A: elastic tensor

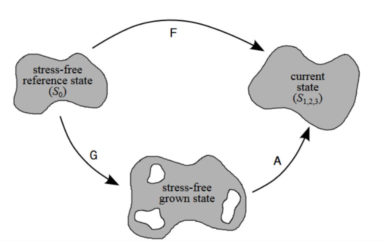


Figure 2: Representation of the deformation of an elastic body.

2. Skin incompressibility: $det(A) = 1$

Wound model: main results

1. Symmetric deformations $\Rightarrow$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\right), \varepsilon_{\theta z} = 0, u_{zr} = \frac{1}{2}\frac{\partial u_z}{\partial r}.$$

2. Balance of momentum:

$$\rho\ddot{\mathbf{x}} = \nabla \cdot \mathbf{T} + \rho \mathbf{f}_b.$$

3. Neo-Hookean material model for dermal tissue $\Rightarrow$

$$\mathbf{T} = \mathbf{A} \cdot \frac{\partial W}{\partial \mathbf{A}} - p\mathbf{I}.$$

4. No body forces + Quasi-static assumption + Plane stress $\Rightarrow$

$$T_{ii} = \alpha_i \frac{\partial W}{\partial \alpha_i} - p = \mu\alpha_i^2 - p, \quad i \in \{r, \theta, z\}$$

Wound model: main results

Then a miracle occurs...

$$T_{rr} = T_{res} = \mu\alpha_r^2 - p = \mu(\alpha_r^2 - \alpha_z^2) = \mu\left(\frac{B^2}{B_0^2} - \frac{B_0^4}{B^4}\right). \tag{1}$$

$$\frac{1}{2\lambda_2}\left(\log\left(\frac{\left(b_R^2 - a_R^2\right)\lambda_2 + A_0^2}{A_0^2}\right) - \log\left(\frac{b_R^2}{a_R^2}\right) + \left(\frac{A_0^2}{\lambda_2} - a_R^2\right)\left(\frac{1}{b_R^2} - \frac{1}{a_R^2}\right)\right) + \left(\frac{B_0^4}{B^4} - \frac{B^2}{B_0 2}\right) = 0 \tag{2}$$

$$N_z = \int_{a_R}^{b_R} T_{zz}(r)dr = 0 \tag{3}$$

With Equations 1, 2 and 3 we may determine A_0, B_0, L_0 and T_{res} from states 1 and 2.

Effect of a wound on an elastic domain: Recoil

Description of the problem

We study the effect of a wound on an elastic domain, in which a constant traction t orthogonal to the slit and at each side of the domain is applied.

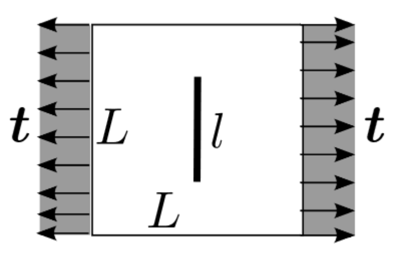


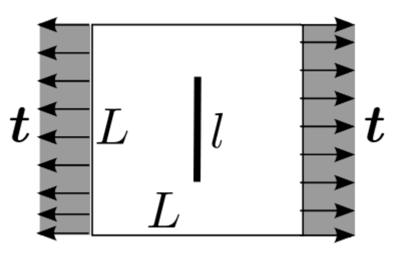
Figure 3: Sketch of the problem

Model differences

Paper model	Our model
Different states	Recoil
Cylindrical geometry	Square geometry
Non-linear elasticity	Linear elasticity

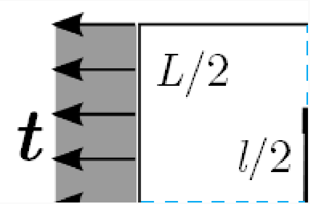
Symmetries of the problem

The sketch of Figure (3) is both horizontally and vertically symmetric, and therefore, the solution is completely determined by that of any of its "quadrants".



Boundary conditions of the symmetric problem

- The top layer and the slit surface have free Neumann boundary conditions.
- The left layer has non-homogeneous Neumann conditions, equivalent to that of a uniform traction load.
- The part of the right layer which is not on the slit has null horizontal displacements.
- The bottom layer has null vertical displacements.



Plot

Plot obtained using material parameters:

$$\nu = 0.3,\ E = 1,\ |t| = 0.02,\ L = 1.$$

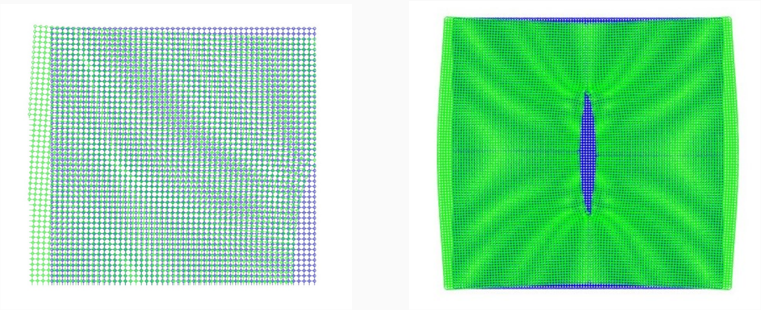


Figure 4: Original (blue) and deformed (green) FEM nodal configurations of the specimen, for both the upper-left quadrant (left) and the complete domain (right).

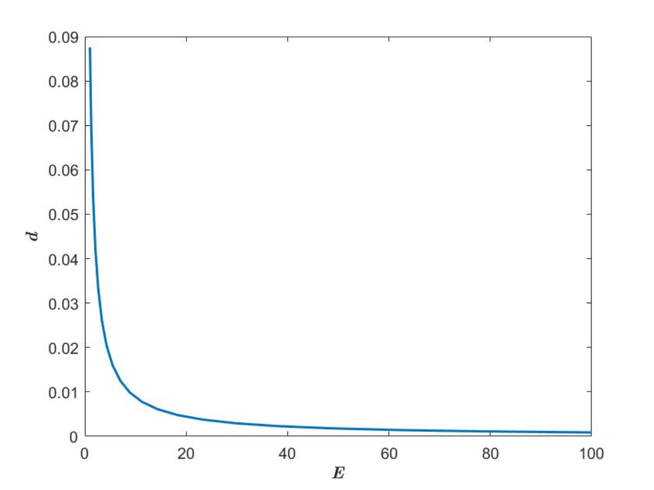


Figure 5: Maximum opening d versus the Young modulus E.

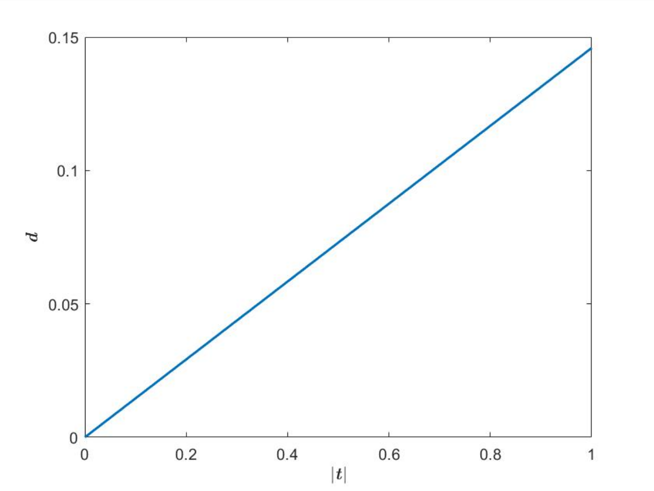


Figure 6: Maximum opening d versus the traction magnitude $|t|$.

Results

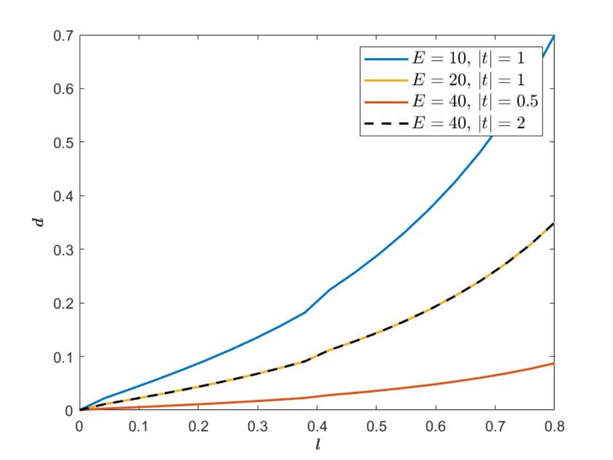


Figure 7: Maximum opening *d* versus slit length *l* for different material parameters and boundary traction.

Conclusions

Conclusions

1. The presence of symmetries facilitates simplification and reduces computational costs.
2. Linear elasticity leads to linear results.
   Linear system: $\mathbf{K}\mathbf{u} = \mathbf{f}$, with $\mathbf{K} \propto E$ and $\mathbf{f} \propto |\mathbf{t}|$.
   Let us define the quantities:
   $\mathbf{K}_\mathbf{E} := \mathbf{K}/E$, $\mathbf{f}_t := \mathbf{f}/|\mathbf{t}|$, and $\alpha = |\mathbf{t}|/E$ to write

$$\mathbf{K}\mathbf{u} = \mathbf{f} \Rightarrow (E\mathbf{K}_\mathbf{E})u = (|\mathbf{t}|\mathbf{f}_t) \Rightarrow \mathbf{K}_\mathbf{E} u = \alpha \mathbf{f}_t.$$

References

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[2] Larry Taber. *Nonlinear Theory of Elasticity: Applications in Biomechanics*. Jan. 2004. DOI: 10.1142/9789812794222.