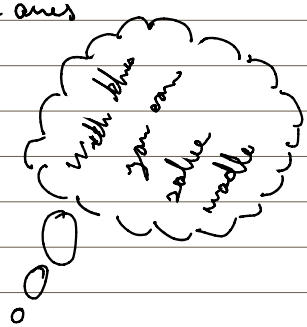
Inference

Derive new ones from old ones

Does? $KB \models \alpha$



Model checking

- To determine if $KB \models \alpha$
  - Enumerate all possible models
  - if in every model where KB is true, α is true, then $KB \models \alpha$
  - Otherwise $KB \not\models \alpha$

Knowledge Engineering

Clue:

- Propositional symbols (suspects, weapons, rooms)

$(mustard \vee scarlet \vee plum) \wedge (room1 \vee room2 \vee room3) \wedge$

$\wedge (weapon_1 \vee weapon_2 \vee weapon_3)$

$\neg weapon_1 \neg mustard \neg room_1$

KNOWLEDGE

Knowledge-based-agents

↳ agents that reason based on an internal representation of knowledge

Sentences, assertion about the world, on a knowledge representation language

Propositional Logic

- Propositon symbols → Ex.: P Q R
  ↳ Sentences or facts.

- Logical connectives

¬ NOT ∧ AND ∨ OR

⇒ IMPLICATION ⇔ BICONDITIONAL

NOT

P	¬P
1	0
0	1

AND

P	Q	P ∧ Q
0	0	0
0	1	0
1	0	0
1	1	1

IMPLICATION

P	Q	P → Q
0	0	1
0	1	1
1	0	0
1	1	1

BICONDITIONAL

P	Q	P ↔ Q
0	0	1
0	1	0
1	0	0
1	1	1

model → assignment of a truth value to every propositional symbol (a "possible world")

Knowledge base → a set of sentence that we know it's true

Entailment

$\alpha \models \beta$ — In every model that α is true, β is true

α entails β

Inference Rules:

Modus Ponens

$$\frac{\alpha \leftrightarrow \beta \quad \alpha}{\beta}$$

And Elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

$$\frac{\alpha \wedge \beta}{\beta}$$

Double negation Elimination

$$\frac{\neg(\neg\alpha)}{\alpha}$$

Implication elimination

$$\frac{\alpha \leftrightarrow \beta}{\neg\alpha \vee \beta}$$

Biconditional Elimination

$$\frac{\alpha \Leftrightarrow \beta}{(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)}$$

De Morgan's Law

$$\frac{\neg(\alpha \wedge \beta)}{\neg\alpha \vee \neg\beta}$$

Distributive Law

Distributive, $\frac{(\alpha \wedge (\beta \vee \gamma))}{(\alpha \wedge \beta) \vee (\alpha \vee \gamma)}$

Use these sentences as states in a search problem

Resolution:

$$\frac{P \vee Q \quad \neg P}{Q}$$

$$\frac{P \vee Q \quad \neg P \vee R}{Q \vee R}$$

clause → disjunction of literals

↳ elements joined with OR

conjunctive normal form → conjunction of clauses (CNF)

Convert to CNF:

Eliminate bicondition

$\alpha \leftrightarrow \beta \Rightarrow (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$

Eliminate implication

$\alpha \rightarrow \beta \Rightarrow \neg\alpha \vee \beta$

Move $\neg$ inwards

$\neg(\alpha \wedge \beta) \Rightarrow \neg\alpha \vee \neg\beta$

Use distributive law

$(P \vee Q) \rightarrow R$

$\neg(P \vee Q) \vee R$

$(\neg P \wedge \neg Q) \vee R$

$(\neg P \vee R) \wedge (\neg Q \vee R)$

Inference by Resolution

$$\frac{P \vee Q \vee S \qquad \neg P \vee R \vee S}{(Q \vee R \vee S)}$$ → no repetition

$$\frac{P \qquad \neg P}{()}$$ → empty clause → false

Determine $KB \models \alpha$

Check if $(KB \wedge \neg \alpha)$ is a contradiction
- If so $KB \models \alpha$
- Otherwise no

Determine $KB \models \alpha$
- Turn $(KB \wedge \neg \alpha)$ to Conjunctive Normal Form
- Keep checking to see if we can use resolution to produce a new clause.

  - If we ever produce an empty clause (a clause a contradiction) so $KB \models \alpha$

Universal Quantification

$\forall x$ BelongsTo(x, Gryffindor) $\rightarrow$
$\neg$ BelongsTo(x, Hufflepuff)

Existential Quantification

$\exists x$ House(x) $\wedge$ BelongsTo(Monarca, x)

$\forall x$ Person(x) $\rightarrow$ ($\exists y$ House(y) $\wedge$ BelongsTo(x, y))

$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

Assume $\neg A$

$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$

$\neg B$

A

( ) Empty ⇒ False

Bt does entail Δ

First-Order Logic

In propositional logic either True or False

Constant Symbol	Predicate Symbol
PPL	Properties that hold true or false for each constant symbol
Houses	

Person (Mahmoud) ⇒ Mahmoud → Person

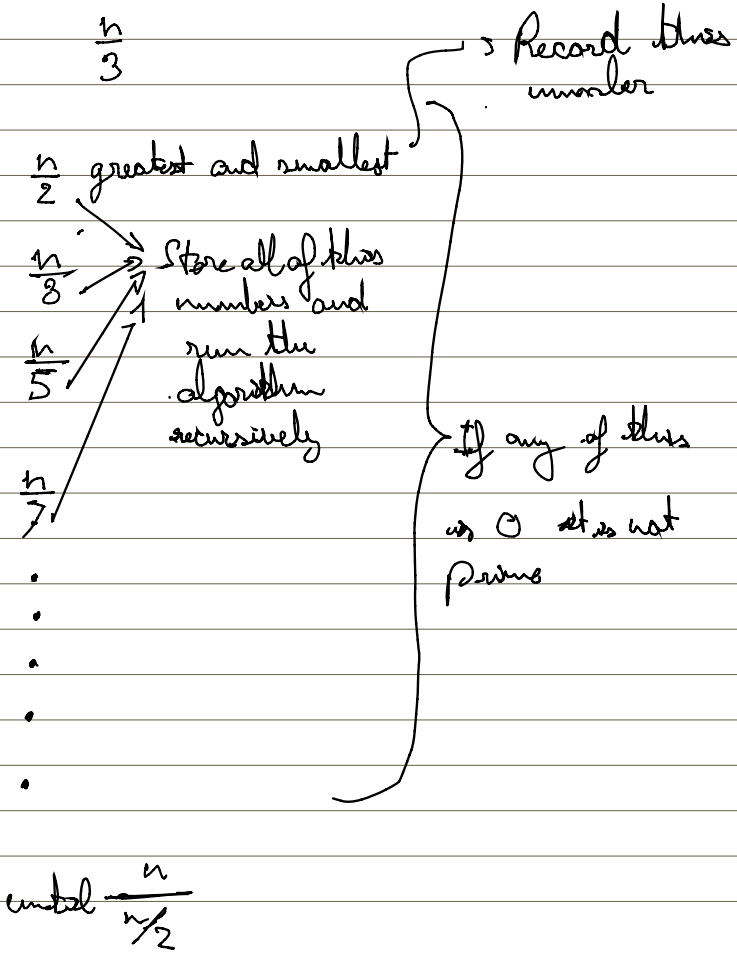
Belongs To (Mahmoud, Gryffyndor) ⇒ Mahmoud belongs to Gryffyndor

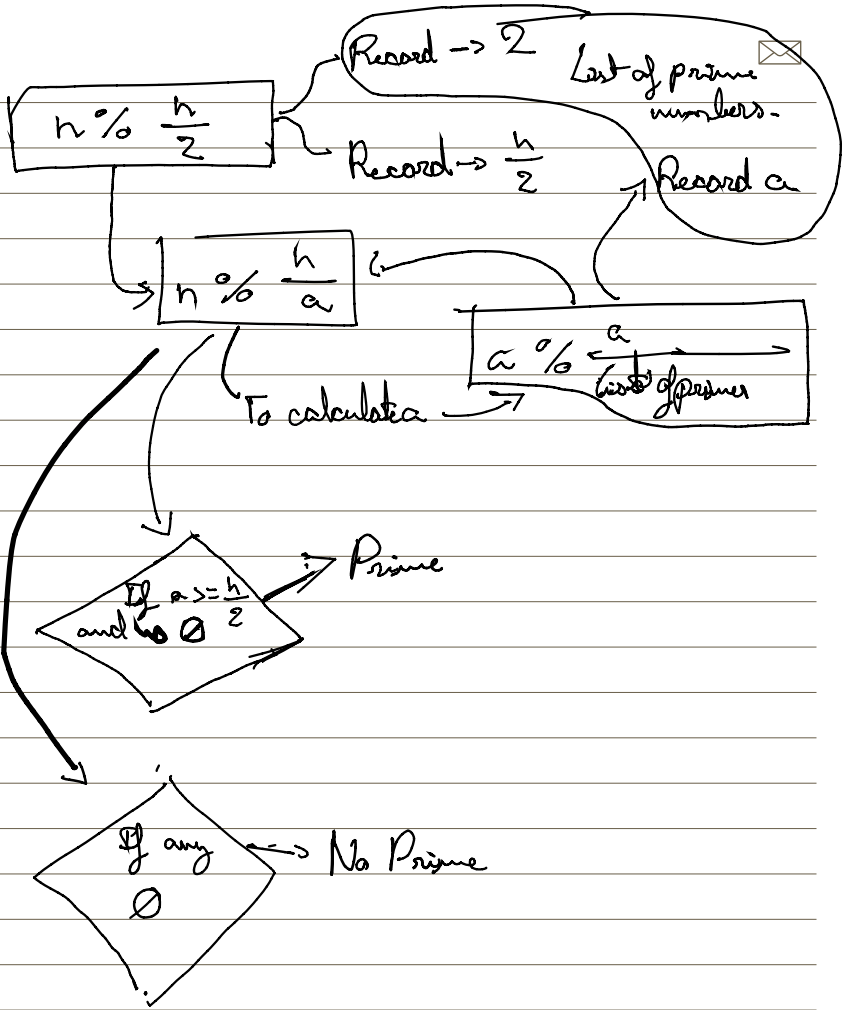
Prime numbers finder

$n \% n = 0$
$n \% 1 = 0$ } only
$n \% (1, n) \neq 0$

Definition of prime numbers

No. factors $< \frac{n}{2}$




n % h/2
Record -> 2
Record -> h/2
Last of prime numbers.
Record a
n % h/a
a % a
To calculate a
If a >= h/2 and
Prime
If any
No Prime