Mathematics, Grade 6

Expectations by strand

A. Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes

This strand focuses on students' development and application of social-emotional learning skills to support their learning of math concepts and skills, foster their overall well-being and ability to learn, and help them build resilience and thrive as math learners. As they develop SEL skills, students demonstrate a greater ability to understand and apply the mathematical processes, which are critical to supporting learning in mathematics. In all grades of the mathematics program, the learning related to this strand takes place in the context of learning related to all other strands, and it should be assessed and evaluated within these contexts.

Overall expectations

Throughout this grade, in order to promote a positive identity as a math learner, to foster well-being and the ability to learn, build resilience, and thrive, students will:

A1. Social-Emotional Learning (SEL) Skills and the Mathematical Processes

apply, to the best of their ability, a variety of social-emotional learning skills to support their use of the mathematical processes and their learning in connection with the expectations in the other five strands of the mathematics curriculum

<table>
<tr><th>To the best of their ability, students will learn to:</th><th>… as they apply the mathematical processes:</th><th>… so they can:</th></tr>
<tr><td>1. identify and manage emotions</td><td rowspan="5">problem solving: develop, select, and apply problem-solving strategies
reasoning and proving: develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counter-examples) to justify thinking, make and investigate conjectures, and construct and defend arguments
reflecting: demonstrate that as they solve problems, they are pausing, looking back, and monitoring their thinking to help clarify their understanding (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal)
connecting: make connections among mathematical concepts, procedures, and representations, and relate mathematical ideas to other contexts (e.g., other curriculum areas, daily life, sports)
communicating: express and understand mathematical thinking, and engage in mathematical arguments using everyday language, language resources as necessary, appropriate mathematical terminology, a variety of representations, and mathematical conventions
representing: select from and create a variety of representations of mathematical ideas (e.g.,</td><td>1. express and manage their feelings, and show understanding of the feelings of others, as they engage positively in mathematics activities</td></tr>
<tr><td>2. recognize sources of stress and cope with challenges</td><td>2. work through challenging math problems, understanding that their resourcefulness in using various strategies to respond to stress is helping them build personal resilience</td></tr>
<tr><td>3. maintain positive motivation and perseverance</td><td>3. recognize that testing out different approaches to problems and learning from mistakes is an important part of the learning process, and is aided by a sense of optimism and hope</td></tr>
<tr><td>4. build relationships and communicate effectively</td><td>4. work collaboratively on math problems – expressing their thinking, listening to the thinking of others, and practising inclusivity – and in that way fostering healthy relationships</td></tr>
<tr><td>5. develop self-awareness and sense of identity</td><td>5. see themselves as capable math learners, and strengthen their sense of ownership of their learning, as part of their emerging sense of identity and belonging</td></tr>
</table>

6. think critically and creatively	representations involving physical models, pictures, numbers, variables, graphs), and apply them to solve problems • ***selecting tools and strategies:*** select and use a variety of concrete, visual, and electronic learning tools and appropriate strategies to investigate mathematical ideas and to solve problems	6. make connections between math and everyday contexts to help them make informed judgements and decisions

B. Number

Overall expectations

By the end of Grade 6, students will:

B1. Number Sense

demonstrate an understanding of numbers and make connections to the way numbers are used in everyday life

Specific expectations

By the end of Grade 6, students will:

B1.1 Rational Numbers

read and represent whole numbers up to and including one million, using appropriate tools and strategies, and describe various ways they are used in everyday life

Teacher supports

Key concepts

- Reading numbers involves interpreting them as a quantity when they are expressed in words, in standard notation, or in expanded notation.
- The numerals 0 to 9 are used to form numbers. They are referred to as the digits in a number and each digit corresponds to a place value. For example, in the number 945 107, the digit 9 represents 9 hundred thousands, the digit 4 represents 4 ten thousands, the

digit 5 represents 5 thousands, the digit 1 represents 1 hundred, the digit 0 represents 0 tens, and the digit 7 represents 7 ones.

- There are patterns to the way numbers are formed. Each place value column, or period, repeats the 0 to 9 counting sequence.
- Any quantity, no matter how great, can be described in terms of its place value. For example, 1500 may be said as fifteen hundred or one thousand five hundred.
- A number can be represented in expanded form (e.g., 634 187 = 600 000 + 30 000 + 4000 + 100 + 80 + 7, or 6 × 100 000 + 3 × 10 000 + 4 × 1000 + 1 × 100 + 8 × 10 + 7) to show place value relationships.
- Numbers are used throughout the day, in various ways and contexts. Most often numbers describe and compare quantities. They express magnitude, and provide a way to answer questions such as “how much?” and “how much more?”.

Note

- Every strand of mathematics relies on numbers.
- Numbers may have cultural significance.
- Seeing how a quantity relates to *other* quantities supports students in developing an understanding of the magnitude or “how muchness” of a number.
- There are patterns in the place value system that support the reading, writing, saying, and understanding of numbers and that suggest important ways for numbers to be composed and decomposed.
    - The *place* (or position) of a digit determines its *value* (*place value*). The 5 in 511, for example, has a value of 500, not 5.
    - A zero in a column indicates that there are no groups of that size in the number. It serves as a placeholder, holding the other digits in their correct “place”.
    - The value of the columns increases by a constant “times 10” multiplicative pattern. For example, as the digit 5 shifts to the left, from 5000 to 50 000, the digit’s value becomes 10 times as great. As it shifts to the right, from 5000 to 500, its value becomes one tenth as great.
    - To find the value of a digit in a number, the value of the digit is multiplied by the value of its place. For example, in the number 52 036, the 5 represents 50 000 (5 × 10 000) and the 2 represents 2000 (2 × 1000).
    - Expanded notation represents the value of each digit separately, as an expression. Using expanded form, 7287 is written as 7287 = 7000 + 200 + 80 +7, or 7 × 1000 + 2 × 100 + 8 × 10 + 7 × 1.
    - Each period – thousands, millions, billions, trillions – is 1000 times the previous period.

- A “hundreds-tens-ones” pattern repeats within each period (ones, thousands, millions, billions, and so on), and each period is 1000 times the one preceding it. Exposure to these patterns, and the names of these periods, also satisfies a natural curiosity around “big numbers” and could lead to conversations about periods beyond millions (billions, trillions, quadrillions, and so on).

Place Value Patterns

one billions	hundred millions	ten millions	one millions	hundred thousands	ten thousands	one thousands	hundreds	tens	ones

- The number “five hundred eight thousand thirty-seven” is written as “508 037” and not “508 1000 37” (as if being spelled out with numbers). Listening for the period name (508 thousand), and the hundreds-tens-ones pattern that precedes the period, gives structure to the number and signals where a digit belongs. If there are no groups of a particular place value, 0 is used to describe that amount, holding the other digits in their correct place.
- Large numbers are difficult to visualize. Making connections to real-life contexts helps with this, as does comparing large numbers to other numbers using proportional reasoning. For example, a small city might have a population of around 100 000, and 1 000 000 would be 10 of these cities.

B1.2 Rational Numbers

read and represent integers, using a variety of tools and strategies, including horizontal and vertical number lines

Teacher supports

Key concepts

- Integers are whole numbers and their opposites.
- Zero is neither negative nor positive.
- On a horizontal number line, positive integers are displayed to the right of zero and negative integers are displayed to the left of zero.
- On a vertical number line, positive integers are displayed above the zero and negative integers are displayed below the zero.
- Integers can be represented as points on a number line, or as vectors that shows magnitude and direction. The integer −5 can be shown as a point positioned 5 units to the left of zero or 5 units below zero. The integer −5 can also be shown as a vector with

its tail positioned at zero and its head at −5 on the number line, to show that it has a length of 5 units and is moving in the negative direction.
- Each integer has an opposite, and both are an equal distance from zero. For example, −4 and +4 are opposite integers and both are 4 units from zero.
- Zero can be represented with pairs of opposite integers. For example, (+3) and (−3) = 0.
- Integers measure "whole things" relative to a reference point. For example, 1 degree Celsius is used to measure temperature. Zero degrees is freezing (reference point). The temperature +10˚C is ten degrees above freezing. The temperature −10˚ C is ten degrees below freezing.

Note

- Engaging with everyday examples of negative integers (e.g., temperature, elevators going up and down, sea level, underground parking lots, golf scores, plus/minus in hockey, saving and spending money, depositing and withdrawing money from a bank account, walking forward and backwards) helps build familiarity and a context for understanding numbers less than zero.
- Pairs of integers such as (+2) and (−2) are sometimes called "zero pairs".
- The Cartesian plane (see **Spatial Sense, SE E1.3**) uses both horizontal and vertical integer number lines to plot locations, and negative rotations to describe clockwise turns (see **Spatial Sense, SE E1.4**). Both are mathematical contexts for using and understanding positive and negative integers.

B1.3 Rational Numbers

compare and order integers, decimal numbers, and fractions, separately and in combination, in various contexts

Teacher supports

Key concepts

- Numbers with the same units can be compared directly (e.g., 72.5 cm^2 compared to 62.4 cm^2).
- Sometimes numbers without the same unit can be compared, such as 6.2 kilometres and 6.2 metres. Knowing that the unit *kilometre* is greater than the unit *metre* can allow one to infer that 6.2 kilometres is greater than 6.2 metres.
- Sometimes numbers without the same unit may need to be rewritten with the same unit in order to be compared. For example, 1.2 metres and 360 centimetres can be compared

as 120 centimetres and 360 centimetres. Thus, 360 centimetres is greater than 1.2 metres.

- Whole numbers (zero and positive integers) and decimal numbers can be compared and ordered according to their place value.
- Benchmark numbers can be used to compare quantities. For example, $\frac{5}{6}$ is greater than $\frac{1}{2}$ and 0.25 is less than $\frac{1}{2}$, so $\frac{5}{6}$ is greater than 0.25.
- If two fractions have the same denominator, then the numerators can be compared. In this case the numerator with the greater value is the greater fraction because the number of parts considered is greater (e.g., $\frac{2}{3} > \frac{1}{3}$).
- If two fractions have the same numerators, then the denominators can be compared. In this case the denominator with the greater value is the smaller fraction because the size of each partition of the whole is smaller (e.g., $\frac{5}{6} < \frac{5}{3}$).
- Having more digits does not necessarily mean that a number is greater. For example, −7528 has four digits but it is less than +3 because −7528 is less than zero and +3 is greater than zero.
- Any positive number is greater than any negative number.
- When comparing positive numbers, the greater number is the number with the greater magnitude. On a horizontal number line, the greater number is the farthest to the right of zero. On a vertical number line, the greater number is the farthest above zero.
- When comparing negative integers, the least number is the negative integer with the greater magnitude. On a horizontal number line, the lesser number is the farthest to the left of zero. On a vertical number line, the lesser number is the farthest below zero.
- Numbers can be ordered in ascending order – from least to greatest – or can be ordered in descending order – from greatest to least.

Note

- Comparing numbers helps with understanding and describing their order and magnitude.
- Visual models like the number line – particularly if they are proportional – can be used to order numbers; show the relative magnitude of numbers; and highlight equivalences among fractions, decimals, and whole numbers.
- The absolute value of a number is its distance from zero, or its magnitude. Both −5 and +5 have an absolute value of 5, because both are 5 units from zero on the number line.

B1.4 Fractions, Decimals, and Percents

read, represent, compare, and order decimal numbers up to thousandths, in various contexts

Teacher supports

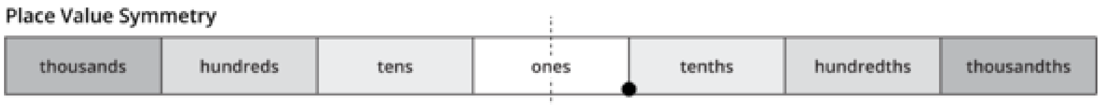
Key concepts

- The place value of the first position to the right of the decimal point is tenths. The second position to the right of the decimal point is hundredths. The third position to the right of the decimal point is thousandths.
- Decimal numbers can be less than one (e.g., 0.654) or greater than one (e.g., 24.723).
- The one whole needs to be shown or explicitly indicated when decimal numbers are represented visually since their representation is relative to the whole.
- Decimal numbers can be represented as a composition or decomposition of numbers according to their place value. For example, decimals can be written in expanded notation 3.628 = 3 + 0.6 + 0.02 + 0.008, or 3 × 1 + 6 × 0.1 + 2 × 0.01 + 8 × 0.001.
- Decimal numbers can be compared by their place value. For example, when comparing 0.8250 and 0.845, the greatest place value where the numbers differ is compared. For this example, 2 hundredths (from 0.825) and 4 hundredths (from 0.845) are compared. Since 4 hundredths is greater than 2 hundredths, 0.845 is greater than 0.825.
- Numbers can be ordered in ascending order – from least to greatest or can be ordered in descending order – from greatest to least.

Note

- Between any two consecutive whole numbers are decimal thousandths. For example, the number 3.628 describes a quantity between 3 and 4 and, more precisely, between 3.6 and 3.7 and, even more precisely, between 3.62 and 3.63.
- Decimals are sometimes called decimal fractions because they represent fractions with denominators of 10, 100, 1000, and so on. Decimal place value columns are added to describe smaller partitions. Decimals, like fractions, have a numerator and a denominator; however, with decimals, only the numerator is visible. The denominator (or unit) is “hidden” within the place value convention.
- Decimals can be composed and decomposed like whole numbers. Expanded notation shows place value subdivisions (e.g., 3.628 = 3 + 0.6 + 0.02 + 0.008, or 3 × 1 + 6 × 0.1 + 2 × 0.01 + 8 × 0.001).
- The decimal point indicates the location of the unit. The unit is always to the left of the decimal point. There is symmetry around the unit column, so tens are matched by tenths, and hundreds are matched by hundredths. Note that the symmetry does not revolve around the *decimal*, so there is no “oneth”:

Place Value Symmetry



- Between any two places in the base ten system, there is a constant 10 : 1 ratio, and this is true for decimals as well. As a digit shifts one space to the right it becomes one tenth as great and if it shifts two spaces to the right it becomes one hundredth as great. So, 0.005 is one tenth as great as 0.05, one hundredth as great as 0.5, and one thousandth as great as 5. This also means that 5 is 1000 times as great as 0.005.
- As with whole numbers, a zero in a decimal indicates that there are no groups of that size in the number:
    - 5.007 means that there are 5 wholes, 0 tenths, 0 hundredths, and 7 thousandths.
    - 5.100 means that there are 5 wholes, 1 tenth, 0 hundredths, and 0 thousandths.
    - 5.1 (five and one tenth), 5.10 (5 and 10 hundredths), and 5.100 (5 and 100 thousandths) are all equivalent (although writing zero in the tenths and hundredths position can indicate the precision of a measurement; for example, the race was won by 5.00 seconds and the winning time was 19.29 seconds). Writing zero in the tenths, hundredths, and thousandths position can indicate the precision of a measurement (e.g., baseball batting averages are given to the nearest thousandths).
- Decimals are read in a variety of ways in everyday life. Decimals like 2.5 are commonly read as two point five; in math, the term pi (π) is commonly approximated as three point one four; the decimal in baseball averages is typically ignored. However, to reinforce the decimal's connection to fractions, and to make visible its place value denominator, it is recommended that decimals be read as their fraction equivalent. So, 2.573 is read as "2 and 573 thousandths."

B1.5 Fractions, Decimals, and Percents

round decimal numbers, both terminating and repeating, to the nearest tenth, hundredth, or whole number, as applicable, in various contexts

Teacher supports

Key concepts

- Rounding makes a number simpler to work with and is often used when estimating computations, measuring, and making quick comparisons.
- A decimal number is rounded to the nearest hundredth, tenth, or whole number based on which hundredth, tenth or whole number it is closest to. If it is the same distance, it is usually rounded up. However, depending on context it may be rounded down.

Note

- Decimal numbers that terminate are like 3.5, 46.27, and 0.625.
- Decimal numbers that repeat are like 3.555555… and can be represented using the symbol with a dot above the repeating digit, (e.g., $3.\dot{5}$). If a string of digits repeats, a bar can be shown above the string, or dots above the first and last digits (e.g., 3.546754675467 is written as $3.\overline{5467}$ or $3.\dot{5}46\dot{7}$.
- Rounding involves making decisions about what level of precision is needed and is often used in measurement. How close a rounded number is to the actual amount depends on the unit it is being rounded to: the larger the unit, the broader the approximation; the smaller the unit, the more precise. Whether a number is rounded up or down depends on the context.

B1.6 Fractions, Decimals, and Percents

describe relationships and show equivalences among fractions and decimal numbers up to thousandths, using appropriate tools and drawings, in various contexts

Teacher supports

Key concepts

- Any fraction can become a decimal number by treating the fraction as a quotient (e.g., $\frac{8}{5} = 8 \div 5 = 1.6$).
- Some fractions as quotients produce a repeating decimal. For example, $\frac{1}{3} = 1 \div 3 = 0.333…$ or $\frac{1}{7} = 1 \div 7 = 0.\overline{142857}$. When decimal numbers are rounded they become approximations of the fraction.
- If a fraction can be expressed in an equivalent form with a denominator of tenths, hundredths, thousandths, and so on, it can also be expressed as an equivalent decimal. For example, because $\frac{1}{4}$ can be expressed as $\frac{25}{100}$, it can also be expressed as 0.25.
- A terminating decimal can be expressed in an equivalent fraction form. For example, $0.625 = \frac{625}{1000}$, which can be expressed as other equivalent fractions, $\frac{125}{200}$ or $\frac{25}{40}$ or $\frac{5}{8}$.
- Any whole number can be expressed as a fraction and as a decimal number. For example, $3 = \frac{3}{1} = 3.0$.

Note

- Decimals are how place value represents fractions and are sometimes called decimal fractions. While fractions may use any number as a denominator, decimals have denominators (units) that are based on a system of tens (tenths, hundredths, and so on).

B2. Operations

use knowledge of numbers and operations to solve mathematical problems encountered in everyday life

Specific expectations

By the end of Grade 6, students will:

B2.1 Properties and Relationships

use the properties of operations, and the relationships between operations, to solve problems involving whole numbers, decimal numbers, fractions, ratios, rates, and whole number percents, including those requiring multiple steps or multiple operations

Teacher supports

Key concepts

- Properties of operations are helpful for carrying out calculations:
    - The identity property: $a + 0 = a$, $a - 0 = a$, $a \times 1 = a$, $\frac{a}{1} = a$.
    - The commutative property: $a + b = b + a$, $a \times b = b \times a$.
    - The associative property: $(a + b) + c = a + (b + c)$, $(a \times b) \times c = a \times (b \times c)$.
    - The distributive property: $a \times (b + c) = (a \times b) + (a \times c)$.
- The commutative, associative, and identity properties can be applied for any type of number.
- The order of operations property needs to be followed when given a numerical expression that involves multiple operations. Any calculations in the brackets are done first. Multiplication and division are done before addition and subtraction. Multiplication and division are done in the order they appear in the expression from left to right. Addition and subtraction are done in the order they appear in the expression from left to right.

- Multi-step problems may involve working with a combination of whole numbers, decimal numbers, and fractions.
- Multi-step problems may involve working with a combination of relationships, including ratios, rates, and percents.
- There may be more than one way to solve a multi-step problem.

Note

- This expectation supports most other expectations in the Number strand and is applied throughout the grade. Whether working with numbers or operations, recognizing and applying properties and relationships builds a strong foundation for doing mathematics.
- Solving problems with more than one operation involves similar processes to solving problems with a single operation. For both types of problems:
    - Identify the actions and quantities in a problem and what is known and unknown.
    - Represent the actions and quantities with a diagram (physically or mentally).
    - Choose the operation(s) that match the actions to write the equation.
    - Solve by using the diagram (counting) or the equation (calculating).
- In multi-step problems, sometimes known as two-step problems, there is an *ultimate* question (asking for the final answer or result being sought), and a *hidden* question (a step or calculation that must be taken to get to the final result). Identifying both questions is a critical part of solving these types of problems.
- The actions in a situation inform the choice of operation. The same operation can describe different situations:
    - Does the situation involve changing (joining, separating), combining, or comparing? Then the situation can be represented with addition and subtraction.
    - Does the situation involve equal groups (or rates), ratio comparisons, or arrays? Then the situation can be represented with multiplication and division.
- Representing a situation with an equation is often helpful for solving a problem. Identifying what is known and unknown in a situation informs how an equation is structured.

B2.2 Math Facts

understand the divisibility rules and use them to determine whether numbers are divisible by 2, 3, 4, 5, 6, 8, 9, and 10

Teacher supports

Key concepts

- There are number patterns that can be used to quickly test whether a number can be evenly divided by another number.
- Divisibility rules can be used to determine factors of numbers.

Note

- Divisibility rules can be applied to all integers; the signs can be ignored.
- Divisibility rules do not apply to decimal numbers that are not whole numbers.

B2.3 Mental Math

use mental math strategies to calculate percents of whole numbers, including 1%, 5%, 10%, 15%, 25%, and 50%, and explain the strategies used

Teacher supports

Key concepts

- Percents represent a rate out of 100 ("per cent" means "per hundred") and are always expressed in relation to a whole. To visually determine the percent of an amount, the whole is subdivided into 100 parts (percent) and described using the percent symbol (%).
- Since 1% is 1 hundredth of an amount, and 10% is 1 tenth, other percents can be calculated by mentally multiplying an amount by tenths and hundredths. For example, $0.01 \times 500 = 5$ or $\frac{1}{100}$ of $500 = 5$.
- Calculating the percent of a whole number can be determined by decomposing the percent as a multiple of 1%. For example, 3% of 500 can be determined by decomposing 3% as $3 \times 1\%$. Since 1% of 500 is 5, then 3% of 500 is $3 \times 1\%$ of $500 = 3 \times 5 = 15$.

Note

- Multiplying a whole number by a percent is the same as multiplying a whole number by a fraction with a denominator of 100 (e.g., $3\% \times 500 = \frac{3}{100} \times 500$ or 0.03×500).
- Dividing an amount by 100 is the same as multiplying it by 0.01. Since 0.01×500 is 5 (i.e., $\frac{1}{100}$ of 500), then 1% of 500 is also 5.

- Percents can be composed from other percents. Since 1% of 500 is 5, then 3% of 500 is 15. This builds on an understanding of fractions and the meaning of the numerator: $\frac{3}{100}$ is the same as 3 one hundredths.
- Relationships between fractions, decimals, and percents also provide building blocks for mentally calculating unknown percents. For example: $\frac{1}{4}$ = 25%; $\frac{1}{2}$ = 50%; $\frac{3}{4}$ = 75%. Five percent is half of 10%, and 15% is 10% plus 5%.
- Calculating a percent is a frequently used skill in everyday life (e.g., when determining sales tax, discounts, or gratuities).
- Mental math is not always quicker than paper and pencil strategies, but speed is not its goal. The value of mental math is in its portability and flexibility, since it does not require a calculator or paper and pencil. Practising mental math strategies also deepens understanding of numbers and operations.

B2.4 Addition and Subtraction

represent and solve problems involving the addition and subtraction of whole numbers and decimal numbers, using estimation and algorithms

Teacher supports

Key concepts

- Situations involving addition and subtraction may involve:
    - adding a quantity onto an existing amount or removing a quantity from an existing amount;
    - combining two or more quantities;
    - comparing quantities.
- If an exact answer is not needed, an estimation can be used. The estimation can be made by rounding the numbers and then adding or subtracting.
- Estimation may also be used prior to a calculation so that when the calculation is performed, one can determine if it seems reasonable or not.
- If an exact answer is needed, a variety of strategies can be used, including algorithms.

Note

- There are three types of situations that involve addition and subtraction. A problem may combine several situations with more than one operation to form a multi-step or multioperation problem (see **SE B2.1**). Recognizing the type and structure of a situation provides a helpful starting point for solving problems.

  - *Change* situations, where one quantity is changed, either by having an amount *joined* to it or *separated* from it. Sometimes the result is unknown; sometimes the starting point is unknown; sometimes the change is unknown.
  - *Combine* situations, where two quantities are *combined*. Sometimes one part is unknown; sometimes the other part is unknown; sometimes the result is unknown.
  - *Compare* situations, where two quantities are being *compared*. Sometimes the greater amount is unknown; sometimes the lesser amount is unknown; sometimes the difference between the two amounts is unknown.

- The most common standard algorithms for addition and subtraction in North America use a compact organizer to decompose and compose numbers based on place value. They begin with the smallest unit – whether it is the unit column, decimal tenths, or decimal hundredths – and use regrouping or trading strategies to carry out the computation. (See **Grade 4, SE B2.4** for a notated subtraction example with decimals, and **Grade 3, SE B2.4** for a notated addition example with whole numbers; the same process applies to decimal hundredths.)
- When carrying out an addition or subtraction algorithm, only common units can be combined or separated. This is particularly noteworthy when using the North American standard algorithms with decimals because, unlike with whole numbers, the smallest unit in a number is not always common (e.g., 90 − 24.7). The expression “line up the decimal” is about making sure that common units are aligned. Using zero as a placeholder is one strategy to align values.

B2.5 Addition and Subtraction

add and subtract fractions with like and unlike denominators, using appropriate tools, in various contexts

Teacher supports

Key concepts

- The type of models (e.g., linear model, area model) and tools (e.g., concrete materials) that are used to represent the addition or subtraction of fractions can vary depending on the context.
- Addition and subtraction of fractions with the same denominator may be modelled using fraction strips partitioned into the units defined by the denominators with the counts of

the units (numerators) being combined or compared. The result is based on the counts of the same unit.

- For example, if adding, 3 one fourths (three fourths) and 2 one fourths (two fourths) are 5 one fourths (five fourths), or $\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$.
- For example, if subtracting, taking 2 one fourths (two fourths) from 7 one fourths (seven fourths) leaves 5 one fourths (five fourths), or $\frac{7}{4}-\frac{2}{4}=\frac{5}{4}$. Or, when thinking about the difference, 5 one fourths (five fourths) is 2 one fourths less than 7 one fourths (seven fourths).

- Addition and subtraction of fractions with unlike denominators may be modelled using fraction strips of the same whole that are partitioned differently. When these fractions are combined or compared, the result is based on the counts of one of the denominators or of a unit that both denominators have in common.
- Hops on a number line may represent adding a fraction on to an existing amount or subtracting a fraction from an existing amount.

Note

- The three types of addition and subtraction situations (see **B2.4**) also apply to fractions.
- As with whole numbers and decimals (see **SE B2.4**), only common units can be added or subtracted. This is also true for fractions. Adding fractions with like denominators is the same as adding anything with like units:
    - 3 apples and 2 apples are 5 apples.
    - 3 fourths and 2 fourths are 5 fourths.
- When adding and subtracting fractions as parts of a whole, the fractions must be based on the same whole. Thus, avoid using a set model because the tendency is to change the size of the whole.
- The numerator in a fraction represents the count of unit fractions. The denominator represents what is being counted (the unit). To add or subtract fractions is to change the total count of units, so only the numerator is added or subtracted.
- If students are adding and subtracting fractions with unlike denominators, they may need to estimate the sum and difference, depending on the tools they are using. This kind of estimation will support fraction sense.
- Without a context, the addition and subtraction of fractions are assumed to be treating the fractions as parts of a whole. Fractions as parts of a whole are commonly added and subtracted in everyday life (e.g., construction, cooking), particularly when combining or comparing units that are commonly used, such as imperial units (inches, feet, pounds, cups, teaspoons).

- Adding and subtracting fractions as comparisons may also have everyday applications. For example, when adding up test scores – a student got 3 of the 4 possible marks ($\frac{3}{4}$) for question 1 and got 4 of the 5 possible marks ($\frac{4}{5}$)for question 2. For the two questions together, the student got 7 of 9 possible marks ($\frac{7}{9}$). In this example, the fractions are comparing what a student got compared to what was possible.

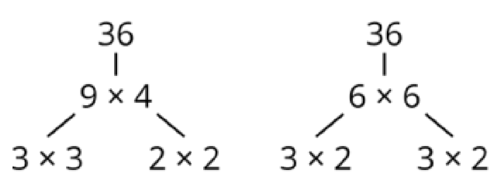
B2.6 Multiplication and Division

represent composite numbers as a product of their prime factors, including through the use of factor trees

Teacher supports

Key concepts

- A number can be decomposed as a product of its factors.
- A prime number can only be expressed as a product of two unique factors, the number itself and 1, for example, 1 = 11 × 1.
- A composite number can be expressed as a product of two or more factors. For example, 8 can be written as a product of the factors 1 × 8, 2 × 4, and 2 × 2 × 2.
- The number 1 is neither prime, nor composite, since it has only one unique factor: itself. It is called a unit.
- Any whole number can be written as a product of its prime factors. Factor trees can be used to show how a number can be repeatedly decomposed until all of its factors are prime.



$$36 = 2 \times 2 \times 3 \times 3$$

Note

- Prime and composite numbers can be visualized using rectangles. Rectangles with areas that are prime numbers have only one possible set of whole number dimensions; rectangles with areas that are composite numbers have more than one. For example, there is only one rectangle with whole number dimensions that has an area of 11 cm^2

(1 cm × 11 cm), but there are two rectangles that have an area of 4 cm^2 (1 cm × 4 cm and 2 cm × 2 cm).

- A factor may also be decomposed into other factors.
- The factors of a number can assist with mental calculations. For example, 36 × 4 might be challenging to do mentally, but thinking of this as the product 4 × 3 × 3 × 4 means that the known fact 12 × 12 can be used to determine the product.

B2.7 Multiplication and Division

represent and solve problems involving the multiplication of three-digit whole numbers by decimal tenths, using algorithms

Teacher supports

Key concepts

- An area model can be used to visualize multiplication with decimals.
  - The two numbers being multiplied can be the dimensions of a rectangle.
  - The dimensions can be decomposed by their place value.
  - The number of smaller rectangles formed will depend on how the dimensions have been decomposed.
  - Known facts can be used to determine each of the smaller areas.
  - The smaller areas are added together resulting in the product.

235 × 0.3

	200	30	5
0.3	60.0	9.0	1.5

$$\text{Total} = \begin{array}{r} 60.0 \\ 9.0 \\ +\ 1.5 \\ \hline 70.5 \end{array}$$

- There are many different algorithms that can be used for multiplication. Students may use one of these algorithms, or their own, and are not required to know all or more than one method. Standard multiplication algorithms for whole numbers can also be applied to decimal numbers. As with whole numbers, these algorithms add partial products to create a total. For example, with 235 × 0.3, the partial products are formed by multiplying each whole number by three tenths. Note the connection between this and multiplying a whole number by 30% (see **SE B2.3**) and by a fraction (see **SE B2.9**).

How It May Be Written

What Is Happening

$$\begin{array}{r l}
23.5 & \\
\times\ 0.3 & \\
\hline
0.15 & \Rightarrow 0.3 \times 0.5 = 0.15 \\
0.90 & \Rightarrow 0.3 \times 3.0 = 0.9 \\
+\ 6.00 & \Rightarrow 0.3 \times 20.0 = 6.0 \\
\hline
7.05 &
\end{array}$$

- Another algorithm approach uses factoring and properties of operations. It enables multiplication by tenths to be treated as a whole number calculation, which is then multiplied by a tenth (0.1). For example:
  - 235 × 0.3 can be thought of as 235 × 3 × 0.1.
  - A standard algorithm determines that 235 × 3 equals 705.
  - 705 is then multiplied by one tenth (0.1).
  - One tenth of 705 is 70.5.

Multiply by Whole Numbers	Then Multiply by 0.1
$\begin{array}{r} 235 \\ \times\ 3 \\ \hline 705 \end{array}$	$\Rightarrow 705 \times 0.1 = 70.5$

- The context of multiplication problems may involve:
  - repeated equal groups, including rates.
  - scale factors – ratio comparisons, rates, and scaling.
  - area and certain other measurement attributes.
  - the number of possible combinations of attributes given two or more sets (see **Data, SE D2.2**).

- Connections can be made between the multiplication of a whole number by a decimal number and multiplying a whole number by a percent. For example, 235 × 0.3 is connected to multiplying a whole number by 30% (see **SE B2.3**) and by a fraction (see **SE B2.9**).
- Multiplication of a whole number by a decimal number between 0 and 1 will result in a product much less than the original number.
- Estimating a product prior to a calculation helps with judging if the calculation is reasonable.

B2.8 Multiplication and Division

represent and solve problems involving the division of three-digit whole numbers by decimal tenths, using appropriate tools, strategies, and algorithms, and expressing remainders as appropriate

Teacher supports

Key concepts

- A strategy to divide whole numbers by decimal numbers is to create an equivalent division statement using whole numbers. For example, 345 ÷ 0.5 will have the same result as 3450 ÷ 5.
- Often division does not result in whole number amounts. In the absence of a context, remainders can be treated as a leftover quantity, or they can be distributed equally as fractional parts across the groups.
  - When using the standard “long-division” algorithm, the whole number dividend can be expressed as a decimal number by adding zeroes to the right of the decimal point until a terminating decimal number can be determined, or until a decimal number is rounded to an appropriate number of places. For example, 27 ÷ 8 can be expressed as 27.000 ÷ 8 to accommodate an answer of 3.375.
  - A remainder can be expressed as a fraction (e.g., $27 \div 8 = 3\frac{3}{8}$).

Note

- Multiplication and division are related (see **SE B2.1**).
- When dividing by tenths, contexts often use quotative division and ask “How many tenths are in this amount?” It is more difficult to think of division with decimals as partitive, where an amount is shared evenly among a tenth, although it is possible. For example, thinking of 22 ÷ 0.5 partitively means thinking that if 22 is only 5 tenths of the whole, what is the whole?
- The context of a division problem may involve:
  - repeated equal groups, including rates;
  - scale factor – ratio comparisons, rates, and scaling;
  - the area of rectangles;
  - the number of possible combinations of attributes given two or more sets (see **Data, SE D2.2**).
- In real-world situations, the context determines how a remainder should be dealt with:

- Sometimes the remainder is ignored, leaving a smaller amount (e.g., how many boxes of 5 can be made from 17 items?).
- Sometimes the remainder is rounded up, producing a greater amount (e.g., how many boxes are needed if 17 items are packed in boxes of 5?).
- Sometimes the remainder is rounded to the nearest whole number, producing an approximation (e.g., if 5 people share 17 items, approximately how many will each receive?).

- Division of a whole number by a decimal number between 0 and 1 will result in a quotient greater than the original whole number.
- Estimating a quotient prior to a calculation helps with judging if the calculation is reasonable.

B2.9 Multiplication and Division

multiply whole numbers by proper fractions, using appropriate tools and strategies

Teacher supports

Key concepts

- A proper fraction can be decomposed as a product of the count and its unit fraction (e.g., $\frac{3}{4} = 3 \times \frac{1}{4}$ or $\frac{1}{4} \times 3$).
- The strategies used to multiply a whole number by a proper fraction may depend on the context of the problem.

    - If the situation involves scaling, $5 \times \frac{3}{4}$ may be interpreted as "the total number of unit fractions is five times greater". Thus, $5 \times \frac{3}{4} = 5 \times 3 \times \frac{1}{4} = 15 \times \frac{1}{4} = \frac{15}{4}$ (15 fourths).
    - If the situation involves equal groups, $5 \times \frac{3}{4}$ may be interpreted as "five groups of three fourths". Thus, $5 \times \frac{3}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} =$ or $3\frac{3}{4}$.
    - If the situation involves area, $5 \times \frac{3}{4}$ may be interpreted as "the area of a rectangle with a length of five units is multiplied by its width of three fourths of a unit". The area could be determined by finding the area of a rectangle with dimension 5 by 1 and then subtracting the extra area, which is 5 one fourths. Therefore:
    - $5 \times \frac{3}{4}$
      $= (5 \times 1) - 5 \times (\frac{1}{4})$
      $= 5 - \frac{5}{4}$

$= 5 - 1\frac{1}{4}$

$= 3\frac{3}{4}$

Note

- How tools are used to multiply a whole number by a proper fraction can be influenced by the contexts of a problem. For example:
  - A double number line may be used to show multiplication as scaling.
  - Hops on a number line may be used to show multiplication as repeat addition.
  - A grid may be used to show multiplication as area of a rectangle.
- The strategies that are used to multiply a whole number by a proper fraction may depend on the type of numbers given. For example, in the case of $8 \times \frac{3}{4} = 8 \times 3 \times \frac{1}{4}$. Using the associative property, the product of $8 \times \frac{1}{4}$ may be multiplied first and then multiplied by 3. This results in $2 \times 3 = 6$. Another approach is to multiply 8×3 first, which results in 24, which is then multiplied by $\frac{1}{4}$ and resulting in 6.

B2.10 Multiplication and Division

divide whole numbers by proper fractions, using appropriate tools and strategies

Teacher supports

Key concepts

- Multiplication and division are related. The same situation or problem can be represented with a division or a multiplication sentence. For example, the division question $6 \div \frac{3}{4} = ?$ can also be thought of as a multiplication question, $\frac{3}{4} \times ? = 6$.
- The strategies used to divide a whole number by a proper fraction may depend on the context of the problem.
  - If the situation involves scaling, $24 \div \frac{3}{4}$ may be interpreted as "some scale factor of three fourths gave a result of 24".
    - Therefore, $\frac{3}{4} \times ? = 24$
    - $3 \times \frac{1}{4} \times ? = 24$ or $\frac{1}{4} \times ? = 8$
    - Therefore, the quotient is 32 because 32 one fourths is 8.

- If the situation involves equal groups, $24 \div \frac{3}{4}$ may be interpreted as “How many three fourths are in 24?” Either three fourths is repeatedly added until it has a sum of 24 or it is repeatedly subtracted until the result is zero.
- If the situation involves area, $24 \div \frac{3}{4}$ may be interpreted as “What is the length of a rectangle that has an area of 24 square units, if its width is three fourths of a unit?” Therefore, $\frac{3}{4} \times ? = 24$ may be determined by physically manipulating 24 square units so that a rectangle is formed such that one dimension is three fourths of one whole.

Note

- In choosing division situations that divide a whole number by a fraction, consider whether the problem results in a full group or a partial group (remainder). In Grade 6, students should solve problems that result in full groups.

B2.11 Multiplication and Division

represent and solve problems involving the division of decimal numbers up to thousandths by whole numbers up to 10, using appropriate tools and strategies

Teacher supports

Key concepts

- Multiplication and division are related. The same situation or problem can be represented with a division or a multiplication sentence.
- The strategies used to divide a decimal number by a single digit whole number may depend on the context of the problem and the numbers used.
  - If the situation involves scaling, $2.4 \div 8$ may be interpreted as “some scale factor of 8 gave a result of 2.4” or “What is the scale factor of 8 to give a result of 2.4?” Therefore, $8 \times ? = 2.4$. The result of 0.3 could be determined using the multiplication facts for 8 and multiplying it by one tenth.
  - If the situation involves equal groups, $3.24 \div 8$ may be interpreted as “How much needs to be in each of the 8 groups to have a total of 3.24?” The result of 0.405 could be determined using the standard algorithm.
  - If the situation involves area, $48.16 \div 8$ may be interpreted as “What is the width of a rectangle that has an area of 48.16 square units, if its length is 8 units?”

Therefore, 8 × ? = 48.16. The result of 6.02 could be determined using short division.

Note

- Using the inverse operation of multiplication is helpful for estimating and for checking that a calculation is accurate. For example, 1.935 ÷ 9 = ? can be written as 9 × ? = 1.935, which verifies that the missing factor must be less than 1.

B2.12 Multiplication and Division

solve problems involving ratios, including percents and rates, using appropriate tools and strategies

Teacher supports

Key concepts

- A ratio describes the multiplicative relationship between two or more quantities.
- Ratios can compare one part to another part of the same whole, or a part to the whole. For example, if there are 25 beads in a bag, of which 10 are red and 15 are blue:
  - The ratio of blue beads to red beads is 15 : 10 or $\frac{15}{10}$, and this can be interpreted as there are one half times more blue beads than red beads.
  - The ratio of red beads to the total number of beads is 10 : 25 or $\frac{10}{25}$, and this can be interpreted as 40% of the beads are red.
- Any ratio can be expressed as a percent.
- A rate describes the multiplicative relationship between two quantities expressed with different units. For example, walking 10 km per 2 hours or 5 km per hour.
- Problems involving ratios and rates may require determining an equivalent ratio or rate. An equivalent ratio or rate can be determined by scaling up or down. For example:
  - The ratio of blue marbles to red marbles (10 : 15) can be scaled down to 2 : 3 or scaled up to 20 : 30. In all cases, there are $\frac{2}{3}$ or approximately 66% as many blue marbles as red marbles.
  - The walking rate 10 km per 2 hours (10 km/2h) can be scaled down to 5 km/h (unit rate) or scaled up to 50 km/10 h.

Note

- Ratios compare two (or more) different quantities to each other using multiplication or division. This means the comparison is *relative* rather than *absolute*. For example, if there are 10 blue marbles and 15 red marbles:
  - An absolute comparison uses addition and subtraction to determine that there are 5 more red marbles than blue.
  - A relative comparison uses proportional thinking to determine that:
    - for every 2 blue marbles there are 3 red marbles;
    - there are $\frac{2}{3}$ as many blue marbles as red marbles;
    - there are 1.5 times as many red marbles as blue marbles;
    - 40% of the marbles are blue and 60% of the marbles are red.
- A three-term ratio shows the relationship between three quantities. The multiplicative relationship can differ among the three terms. For example, there are 6 yellow beads, 9 red beads, and 2 white beads in a bag. This situation can be expressed as a ratio of yellow : red : white beads = 6 : 9 : 2. The multiplicative relationship between yellow to white is 6 : 2 or 3 : 1, meaning there are three times more yellow beads than white beads. The multiplicative relationship between yellow and red beads is 6 : 9 or 2 : 3, meaning there are two thirds as many yellow beads as there are red beads.
- Ratio tables can be used for noticing patterns when a ratio or rate is scaled up or down. Ratio tables connect scaling to repeated addition, multiplication and division, and proportional reasoning.

C. Algebra

Overall expectations

By the end of Grade 6, students will:

C1. Patterns and Relationships

identify, describe, extend, create, and make predictions about a variety of patterns, including those found in real-life contexts

Specific expectations

By the end of Grade 6, students will:

C1.1 Patterns

identify and describe repeating, growing, and shrinking patterns, including patterns found in real-life contexts, and specify which growing patterns are linear

Teacher supports

Key concepts

- Repeating patterns have a pattern core that is repeated over and over.
- In growing patterns, there is an increase in the number of elements or the size of the elements from one term to the next.
- Some linear growing patterns have a direct relationship between the term number and the term value; for example, a pattern where each term value is four times its term number. Growing patterns that are linear can be plotted as a straight line on a graph.
- Each iteration of a pattern can be referred to as the term number, and the number of elements in each iteration can be referred to as the term value. The term value is dependent on the term number. The relationship between the term number and the term value can be generalized.
- In shrinking patterns, there is a decrease in the number of elements or the size of the elements from one term to the next.

Note

- Growing and shrinking patterns are not limited to linear patterns.
- Many real-life objects and events can be viewed as having more than one type of pattern.

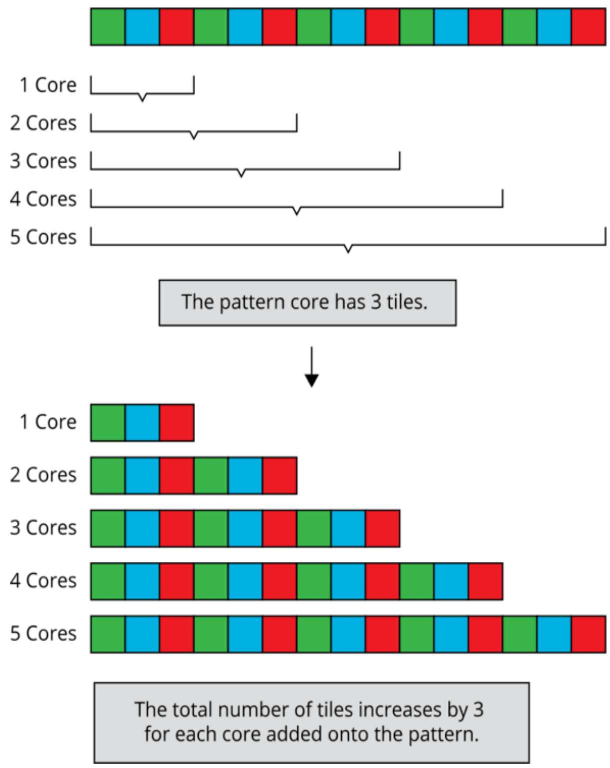
C1.2 Patterns

create and translate repeating, growing, and shrinking patterns using various representations, including tables of values, graphs, and, for linear growing patterns, algebraic expressions and equations

Teacher supports

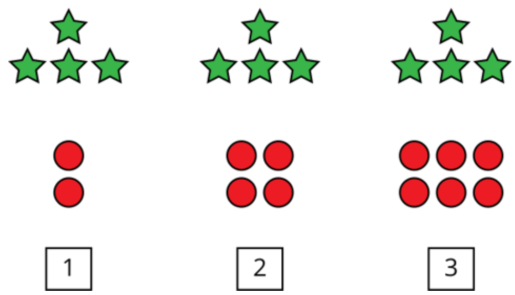
Key concepts

- Growing patterns are created by increasing the number of elements or the size of the elements in each iteration (term).
- A growing pattern can be created by repeating a pattern's core. Each iteration shows how the total number of elements grows with each addition of the pattern core.



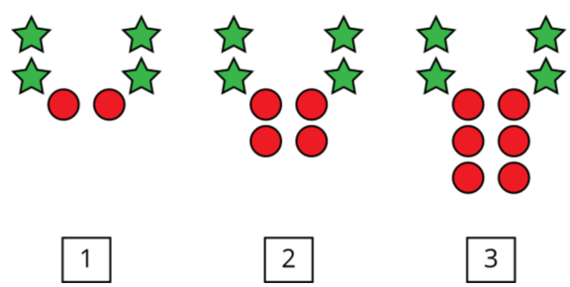
- Shrinking patterns are created by decreasing the number of elements or the size of the elements in each iteration.
- In translating a pattern from a concrete representation to a graph, the term number (x) is represented on the horizontal axis of the Cartesian plane, and the term value (y) is represented on the vertical axis. Each point (x, y) on the Cartesian plane is plotted to represent the pattern. In a table of values, the term number is shown in the left-hand column and the term value is shown in the right-hand column.
- A linear growing pattern can be represented using an algebraic expression or equation to show the relationship between the term number and the term value.
- Examining possible physical structures of a linear growing pattern can provide insight into the different algebraic equations that show the relationship between the term number and term value. For example, in Diagram 1, each term value can be viewed as four more than double the term number, which can be expressed as
  term value = 2*(term number) + 4 or $y = 2x + 4$.

Diagram 1



- Diagram 2 shows that for the same pattern, each term value can also be viewed as twice the term number plus two, which can be expressed as term value = term number + two + term number + two or $y = x + 2 + x + 2$. This expression for Diagram 2 can be simplified to $y = 2x + 4$, which is the same expression derived for Diagram 1.

Diagram 2



Note

- The creation of growing and shrinking patterns in this grade is not limited to linear patterns.
- The general equation for a linear growing pattern is $y = mx + b$, where x represents the term number, m represents the value of the multiplier, b represents a constant value, and y represents the term value.

C1.3 Patterns

determine pattern rules and use them to extend patterns, make and justify predictions, and identify missing elements in repeating, growing, and shrinking patterns, and use algebraic representations of the pattern rules to solve for unknown values in linear growing patterns

Teacher supports

Key concepts

- Patterns can be extended because they are repetitive by nature.
- Pattern rules are generalizations about a pattern, and they can be described in words.
- Patterns can be extended in multiple directions, showing what comes next and what came before.
- To make a near prediction about a pattern is to state or show what a pattern will look like just beyond the given representation of that pattern. The prediction can be verified by extending that pattern.
- To make a far prediction about a pattern is to state or show what a pattern will look like well beyond the given representation of that pattern. Often calculations are needed to make an informed prediction that can be justified.
- Identifying the missing elements in a pattern represented using a table of values may require determining the term number (x) or the term value (y).
- Identifying the missing elements in a pattern represented on a graph may require determining the point (x, y) within the given representation or beyond it, in which case the pattern will need to be extended.
- The algebraic expression that represents a linear growing pattern is also referred to as the general term or the nth term. It can be used to solve for the term value or the term number.

Note

- Determining a point within the graphical representation of a pattern is called interpolating.
- Determining a point beyond the graphical representation of a pattern is called extrapolating.

C1.4 Patterns

create and describe patterns to illustrate relationships among whole numbers and decimal numbers

Teacher supports

Key concepts

- Patterns can be used to demonstrate relationships among numbers.
- There are many patterns within the decimal number system.

Note

- Many number strings are based on patterns and on the use of patterns to develop a mathematical concept.
- The use of the word “strings” in coding is different from its use in “number strings”.

C2. Equations and Inequalities

demonstrate an understanding of variables, expressions, equalities, and inequalities, and apply this understanding in various contexts

Specific expectations

By the end of Grade 6, students will:

C2.1 Variables and Expressions

add monomials with a degree of 1 that involve whole numbers, using tools

Teacher supports

Key concepts

- A monomial with a degree of 1 has a variable with an exponent of one. For example, the exponent of *m* for the monomial 2*m* is 1. When the exponent is not shown, it is understood to be one.
- Monomials with a degree of 1 with the same variables can be added together; for example, 2*m* and 3*m* can be combined as 5*m*.

Note

- Examples of monomials with a degree of 2 are x^2 and *xy*. The reason that *xy* has a degree of 2 is because both *x* and *y* have an exponent of 1. The degree of the monomial is determined by the sum of all the exponents of its variables.
- Adding monomials using tools supports students in understanding which monomials can be combined. Only monomials with the same variables (like terms) can be combined.

C2.2 Variables and Expressions

evaluate algebraic expressions that involve whole numbers and decimal tenths

Teacher supports

Key concepts

- To evaluate an algebraic expression, the variables are replaced with numerical values, and calculations are performed based on the order of operations.

Note

- When students are working with formulas, they are evaluating expressions.
- Replacing the variables with numerical values often requires the use of brackets. For example, the expression 4.5*m* becomes 4.5(*m*) and then 4.5(7) when *m* = 7. The operation between 4.5 and (7) is understood to be multiplication.
- Many coding applications involve algebraic expressions being evaluated. This may be carried out in several steps. For example, the instruction: “input 'the side of a square’, sideA” is instructing the computer to define the variable “sideA” and store whatever the user inputs into the temporary location called “sideA”. The instruction: “calculate sideA*sideA, areaA” instructs the computer to take the value that is stored in “sideA” and multiply it by itself, and then store that result in the temporary location, which is another variable called “areaA”.

C2.3 Equalities and Inequalities

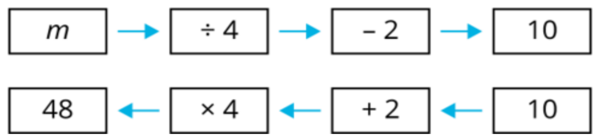
solve equations that involve multiple terms and whole numbers in various contexts, and verify solutions

Teacher supports

Key concepts

- Equations are mathematical statements such that the expressions on both sides of the equal sign are equivalent.
- In equations, variables are used to represent unknown quantities.
- There are many strategies to solve equations including guess-and-check, the balance model, and the reverse flow chart.

- The strategy of using a reverse flow chart can be used to solve equations like $\frac{m}{4} - 2 = 10$. The first diagram shows the flow of operations performed on the variable *m* to produce the result 10. The second diagram shows the reverse flow chart, or flow of the reverse operations, in order to identify the value of the variable *m*.



- Formulas are equations in which any of the variables can be solved for. When solving for a variable in a formula, values for the variables are substituted in, and then further calculations may be needed depending on which variable is being solved for. For example, for $A = lw$, if $l = 10$ and $w = 3$, then $A = (10)(3) = 30$. If $A = 50$ and $l = 10$, then $50 = 10w$, and solving this will require either using known multiplication facts or dividing both sides by 10 to solve for *w*.

Note

- Some equations may require monomials to be added together before they can be solved using the reverse flow chart method.
- The flow chart used in coding is different from the reverse flow chart that can be used to solve equations.

C2.4 Equalities and Inequalities

solve inequalities that involve two operations and whole numbers up to 100, and verify and graph the solutions

Teacher supports

Key concepts

- Inequalities can be solved like an equation, and then values need to be tested to identify those that hold true for the inequality.
- A number line shows the range of values that hold true for an inequality by placing a dot at the greatest or least possible value. An open dot is used if the inequality involves “less than” or “greater than”; if the inequality includes the equal sign (=), then a closed dot is used.

Note

- The solution for an inequality that has one variable, such as $2x + 3 < 9$, can be graphed on a number line.
- The solution for an inequality that has two variables, such as $x + y < 4$, can be graphed on a Cartesian plane, showing the set of points that hold true.

C3. Coding

solve problems and create computational representations of mathematical situations using coding concepts and skills

Specific expectations

By the end of Grade 6, students will:

C3.1 Coding Skills

solve problems and create computational representations of mathematical situations by writing and executing efficient code, including code that involves conditional statements and other control structures

Teacher supports

Key concepts

- A flow chart can be used to plan and organize thinking. The symbols used in flow charts have specific meanings, including those that represent a process, a decision, and program input/output.
- Efficient code may involve using the instructions to solve a problem, using the smallest amount of space to store program data, and/or executing as fast as possible.
- Using loops whenever possible is one way to make code more efficient.
- Conditional statements are a representation of binary logic (yes or no, true or false, 1 or 0).
- A conditional statement evaluates a Boolean condition, something that can either be true or false.
- Conditional statements are usually implemented as “if…then” statements, or “if…then…else” statements. If a conditional statement is true, then there is an

interruption in the current flow of the program being executed and a new direction is taken or the program will end.
- Conditional statements, like loops, can be nested to allow for a range of possible outcomes or to implement decision trees.

Note

- Coding can support students in developing a deeper understanding of mathematical concepts.
- The reverse flow chart that is used to solve equations is not the same as the flow chart used in coding.
- More efficient code can reduce execution time and reduce computer storage space.
- Coding can be used to learn how to automate simple processes and enhance mathematical thinking. For example, students can code expressions to recall previously stored information (defined variables), then input values (e.g., from a sensor, count, or user input) and redefine the value of the variable. For examples of these, refer to the notes in **SEs C2.2** and **C2.3**.
- The construction of the code should become increasingly complex and align with other developmentally and grade-appropriate learning.

C3.2 Coding Skills

read and alter existing code, including code that involves conditional statements and other control structures, and describe how changes to the code affect the outcomes and the efficiency of the code

Teacher supports

Key concepts

- Reading code is done to make predictions as to what the expected outcome will be. Based on that prediction, one can determine if the code needs to be altered prior to its execution.
- Reading code helps with troubleshooting why a program is not able to execute.
- Code is altered so that an expected outcome can be achieved.
- Code can be altered to be used for a new situation.
- Altering code to make it more efficient often involves refining algorithms so that there are no unnecessary steps and using control structures effectively.
- Loops can be used to create efficient code.

Note

- When students are reading code, they are exercising problem-solving skills related to predicting and estimating.
- When code is altered with the aim of reaching an expected outcome, students get instant feedback when it is executed. Students exercise problem-solving strategies to further alter the program if they did not get the expected outcome. If the outcome is as expected, but it gives the wrong answer mathematically, students will need to alter their thinking.
- Efficient code can be altered more easily than inefficient code to adapt to new mathematical situations. For example, in a probability simulation, the number of trials can be increased by changing the number of repeats rather than writing additional lines of code for each of the new trials.

C4. Mathematical Modelling

apply the process of mathematical modelling to represent, analyse, make predictions, and provide insight into real-life situations

This overall expectation has no specific expectations. Mathematical modelling is an iterative and interconnected process that is applied to various contexts, allowing students to bring in learning from other strands. Students' demonstration of the process of mathematical modelling, as they apply concepts and skills learned in other strands, is assessed and evaluated.

Read more about the mathematical modelling process.

Teacher supports

Key concepts

- The process of mathematical modelling requires: understanding the problem; analysing the situation; creating a mathematical model; and analysing and assessing the model.

Note

- A mathematical modelling task is different from a real-life application due to the cyclic nature of modelling, which involves examining a problem from outside mathematics, modelling it, and then checking the model back against the real-life situation and adjusting as necessary.

- The process of mathematical modelling should not be confused with using a "model" to represent or solve a problem that does not require the whole process.
- Mathematical modelling tasks can be utilized in many ways and can support students with making connections among many mathematical concepts across the math strands and across other curricula.

D. Data

Overall expectations

By the end of Grade 6, students will:

D1. Data Literacy

manage, analyse, and use data to make convincing arguments and informed decisions, in various contexts drawn from real life

Specific expectations

By the end of Grade 6, students will:

D1.1 Data Collection and Organization

describe the difference between discrete and continuous data, and provide examples of each

Teacher supports

Key concepts

- Quantitative data is either discrete or continuous.
- Discrete data includes variables that can be counted using whole numbers, such as the number of students in a class, the number of pencils in a pencil case, or the number of words in a sentence.
- Continuous data can have an infinite number of possible values for a given range of a variable (e.g., height, length, distance, mass, time, perimeter, and area). Continuous data can take on any numerical value, including decimals and fractions.

Note

- A variable is any attribute, number, or quantity that can be measured or counted.

D1.2 Data Collection and Organization

collect qualitative data and discrete and continuous quantitative data to answer questions of interest about a population, and organize the sets of data as appropriate, including using intervals

Teacher supports

Key concepts

- The type and amount of data to be collected will be based on the question of interest.
- Some questions of interest may require answering multiple questions that involve any combination of qualitative data and quantitative data.
- Depending on the question of interest, the data may need to be collected from a primary or a secondary source.
- Depending on the question of interest, a random sample of the population may need to be taken. Types of sampling methods include simple random sampling, stratified random sampling, and systematic random sampling.
- When continuous data is collected, it can be recorded and organized using intervals in frequency tables.

Note

- A census is an attempt to collect data from an entire population.
- Every subject in the sample must be collected in the same manner in order for the data to be representative of the population.

D1.3 Data Visualization

select from among a variety of graphs, including histograms and broken-line graphs, the type of graph best suited to represent various sets of data; display the data in the graphs with proper sources, titles, and labels, and appropriate scales; and justify their choice of graphs

Teacher supports

Key concepts

- Understanding the features and purposes of different kinds of graphs is important when selecting appropriate displays for a set of data.

- Pictographs, line plots, bar graphs, multiple-bar graphs, and stacked-bar graphs are used to display qualitative data and discrete quantitative data.
- Histograms display continuous quantitative data using intervals. The bars on a histogram do not have gaps between them due to the continuous nature of the data. This contrasts with bar graphs, which do have gaps between the bars to show the discrete categories.
- Broken-line graphs are used to show change over time and are helpful for identifying trends. To create a broken-line graph, students apply their understanding of scales and estimation.
- The source, titles, labels, and scales provide important information about data in a graph or table:
    - The source indicates where the data was collected.
    - The title introduces the data contained in the graph.
    - Labels provide additional information, such as the intervals that have been used in a histogram.
    - Scales identify the possible values of a variable along an axis of a graph.

Note

- It is important for students to understand the difference between a bar graph and a histogram and to recognize that they are not the same.
- At least one of the variables of a broken-line graph is not continuous.

D1.4 Data Visualization

create an infographic about a data set, representing the data in appropriate ways, including in tables, histograms, and broken-line graphs, and incorporating any other relevant information that helps to tell a story about the data

Teacher supports

Key concepts

- Infographics are used in real life to share data and information on a topic, in a concise and appealing way.
- Infographics contain different representations, such as tables, plots, and graphs with limited text.
- Information to be included in an infographic needs to be carefully considered so that it is clear, concise, and connected.

- Infographics tell a story about the data with a specific audience in mind. When creating infographics, students need to create a narrative about the data for that audience.

Note

- Creating infographics has applications in other subject areas, such as communicating key findings and messages in STEM projects.

D1.5 Data Analysis

determine the range as a measure of spread and the measures of central tendency for various data sets, and use this information to compare two or more data sets

Teacher supports

Key concepts

- The mean, median, and mode are the three measures of central tendency. The mean, median, and mode can be determined for quantitative data. Only the mode can be determined for qualitative data.
- A variable can have one mode, multiple modes, or no modes.
- The use of the mean, median, or mode to make an informed decision is relative to the context.
- The range is one type of measure to describe the spread of a data set, and it is the difference between the greatest and least data values.
- Data sets are compared by the mean, median, or mode of the same variable.
- If the data sets that are both representative of a similar population, then it is possible to compare the mean, median, and mode of data sets that have a different number of data values.
- If the data sets are representing different populations, then it is important for the comparison of the mean, median, and mode be based on the same number of data values.

Note

- The range and the measures of central tendency provide information about the shape of the data and how this can be visualized graphically (e.g., when the three measures of central tendency are the same, then a histogram is symmetrical).

D1.6 Data Analysis

analyse different sets of data presented in various ways, including in histograms and broken-line graphs and in misleading graphs, by asking and answering questions about the data, challenging preconceived notions, and drawing conclusions, then make convincing arguments and informed decisions

Teacher supports

Key concepts

- A histogram provides a picture of the distribution or shape of the data.
  - A normal distribution results in a symmetrical histogram that looks like a bell. In this case, the mode, mean, and median are the same.
  - If data are skewed to the left (goes up from left to right), then the mean is likely to be less than the median. If the data are skewed to the right (goes down from left to right), then the mean is likely to be greater than the median.
- Broken-line graphs show changes in data over time.
- Sometimes graphs misrepresent data or show it inappropriately and this can influence conclusions about the data. Therefore, it is important to always interpret presented data with a critical eye.
- Data presented in tables, plots, and graphs can be used to ask and answer questions, draw conclusions, and make convincing arguments and informed decisions.
- Sometimes presented data challenges current thinking and leads to new and different conclusions and decisions.
- Questions of interest are intended to be answered through the analysis of the representations. Sometimes the analysis raises more questions that require further collection, representation, and analysis of data.

Note

- Broken-line graphs are not used to make predictions, only to show what has happened to the data over time. Only data values that show a strong relationship between two variables can be used to make predictions.
- There are three levels of graph comprehension that students should learn about and practise:
    - Level 1: information is read directly from the graph and no interpretation is required.
    - Level 2: information is read and used to compare (e.g., greatest, least) or perform operations (e.g., addition, subtraction).
    - Level 3: information is read and used to make inferences about the data using background knowledge of the topic.
- Working with misleading graphs helps students analyse their own graphs for accuracy.

D2. Probability

describe the likelihood that events will happen, and use that information to make predictions

Specific expectations

By the end of Grade 6, students will:

D2.1 Probability

use fractions, decimals, and percents to express the probability of events happening, represent this probability on a probability line, and use it to make predictions and informed decisions

Teacher supports

Key concepts

- The probability of events has numeric values ranging from 0 to 1, and percent values ranging from 0% to 100%.
- Fractions and decimals can be used to express the probability of events across the 0 to 1 continuum.

Note

- Have students make connections between words to describe the likelihood of events (i.e., “impossible”, “unlikely”, “equally likely”, “likely”, and “certain”) and possible fractions, decimals, and percents that can be used to represent those benchmarks on the probability line.

D2.2 Probability

determine and compare the theoretical and experimental probabilities of two independent events happening

Teacher supports

Key concepts

- Two events are independent if the probability of one does not affect the probability of the other. For example, the probability for rolling a die the first time does not affect the probability for rolling a die the second time.
- The more trials completed in an experiment, the closer the experimental probability will be to the theoretical probability.
- The sum of the probabilities of all possible outcomes is 1 or 100%
- The probability of an event can be used to predict the likelihood of that event happening again in the future.
- Tree diagrams are helpful to determine all the possible outcomes for two independent events.

Note

- “Odds in favour” is a comparison of the probability that an event will occur with the probability that the event will not occur (complementary events). For example, the probability that the sum of two dice is 2 is $\frac{1}{36}$ and the probability that the sum of two dice is not 2 is $\frac{35}{36}$. The odds in favour of rolling a sum of 2 is $\frac{1}{36} : \frac{35}{36}$ or 1 : 35, since the fractions are both relative to the same whole.

E. Spatial Sense

Overall expectations

By the end of Grade 6, students will:

E1. Geometric and Spatial Reasoning

describe and represent shape, location, and movement by applying geometric properties and spatial relationships in order to navigate the world around them

Specific expectations

By the end of Grade 6, students will:

E1.1 Geometric Reasoning

create lists of the geometric properties of various types of quadrilaterals, including the properties of the diagonals, rotational symmetry, and line symmetry

Teacher supports

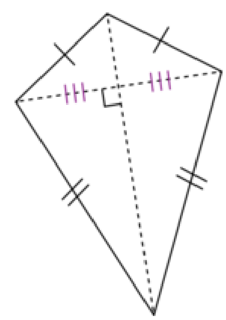
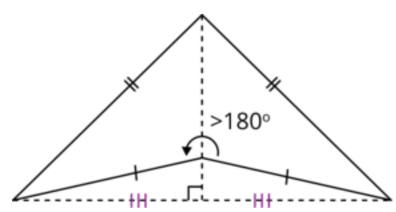
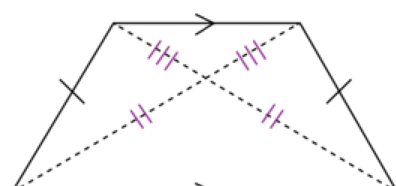
Key concepts

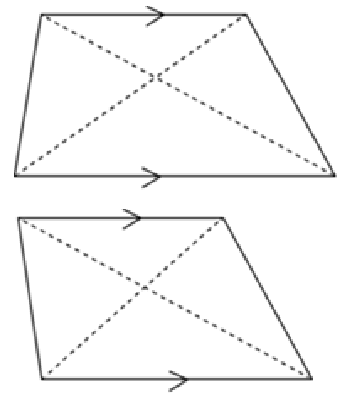
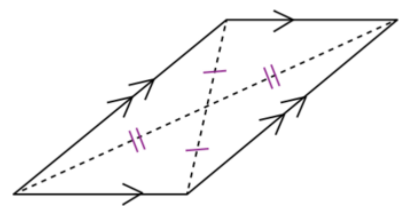
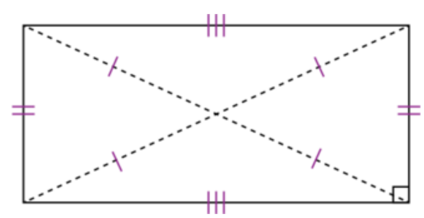
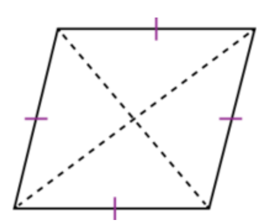
- A geometric property is an attribute that helps define an entire class of shapes.
- Quadrilaterals are polygons with four sides and four interior angles that add up to 360°. These are defining geometric properties of quadrilaterals. If a polygon has one of these attributes, it will automatically have the other and will be a quadrilateral.
- There are many different sub-categories, or classes, of quadrilaterals, and they are defined by their geometric properties. Certain attributes are particularly relevant for defining the geometric properties of shapes:
    - angles:
        - the number of right angles;
        - the number of reflex angles;
    - sides:
        - the number of equal sides;
        - whether the equal sides are adjacent or opposite;
        - the number of parallel sides;

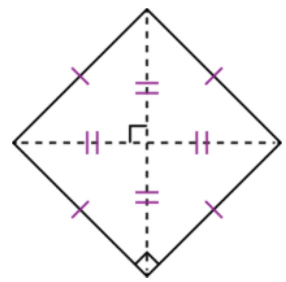
- symmetries:
  - the number of lines of symmetry;
  - the order of rotational symmetry;
- diagonals:
  - whether they are of equal length;
  - whether they intersect at right angles;
  - whether they intersect at their midpoint.

Note

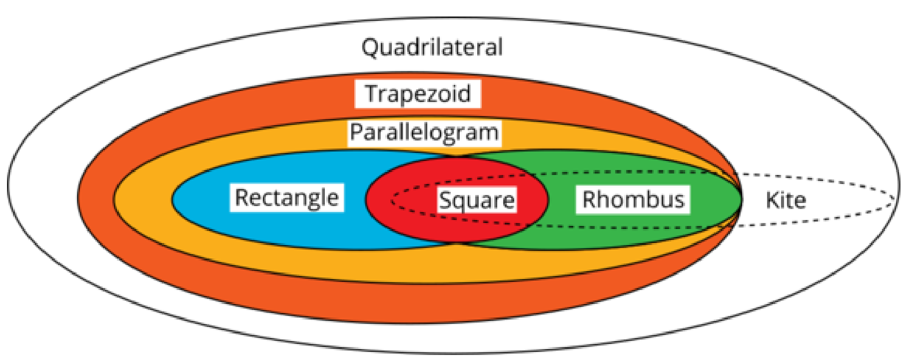
- Quadrilaterals can be sorted and defined by their geometric properties. Analysing geometric properties is an important part of geometric reasoning. The goal is not to memorize these property lists but to generate and use property lists to create spatial arguments.

Quadrilateral	**Sample Properties**	**Example**
Kite	• two pairs of congruent sides that are adjacent (next to each other) • (at least) one line of reflection • (at least) one bisected diagonal (i.e., one diagonal is split in half) • diagonals that intersect at right angles	
Dart	A kite with: • one reflex angle (i.e., greater than 180°) • one diagonal that is not inside the dart	
Trapezoid	• at least one pair of parallel sides *Note* Some definitions of trapezoid specify only one pair of parallel sides. The Ontario mathematics curriculum uses an inclusive definition: any quadrilateral with	 Isosceles Trapezoid

	at least one pair of parallel sides is a trapezoid.	 Trapezoid
Parallelogram	a trapezoid with: • two pairs of parallel sides • bisected diagonals (i.e., diagonals that split each other in half) • congruent opposite sides	
Rectangle	a parallelogram with: • four right angles • congruent diagonals • (at least) two lines of symmetry • rotational symmetry of (at least) order 2	
Rhombus	a parallelogram with: • all congruent sides • diagonals that intersect at right angles • (at least) two lines of symmetry • rotational symmetry of (at least) order 2 a kite with: • all congruent sides • two pairs of parallel sides	

	• (at least) two lines of symmetry • rotational symmetry of (at least) order 2	
Square	a rectangle with: • four congruent sides • congruent diagonals • four lines of symmetry • rotational symmetry of order 4 a rhombus with: • four right angles • diagonals that intersect at right angles • four lines of symmetry • rotational symmetry of order 4 a kite with: • four congruent sides • four right angles • congruent diagonals that intersect at right angles • four lines of symmetry • rotational symmetry of order 4	

- Relationships exist among the properties of quadrilaterals. For example, a square is a special type of rectangle, which is a special type of parallelogram.



- Minimum property lists identify the fewest properties guaranteed to identify the class (e.g., if a quadrilateral has four lines of symmetry it must be a square).

E1.2 Geometric Reasoning

construct three-dimensional objects when given their top, front, and side views

Teacher supports

Key concepts

- Three-dimensional objects can be represented in two dimensions.
- Given accurate top, front, and side views of an object, with sufficient information included, an object can be reproduced in three dimensions. Conventions exist (e.g., shading squares to show different heights) to clarify any ambiguities.
- Architects and builders use plan (top view) and elevation (side view) to guide their construction. Visualizing objects from different perspectives is an important skill used in many occupations, including all forms of engineering. STEM (science, technology, engineering, and mathematics) professionals use three-dimensional modelling apps to model a project before building a prototype. Three-dimensional objects can be represented in two dimensions.
- Given accurate top, front, and side views of an object, with sufficient information included, an object can be reproduced in three dimensions. Conventions exist (e.g., shading squares to show different heights) to clarify any ambiguities.
- Architects and builders use plan (top view) and elevation (side view) to guide their construction. Visualizing objects from different perspectives is an important skill used in many occupations, including all forms of engineering. STEM (science, technology, engineering, and mathematics) professionals use three-dimensional modelling apps to model a project before building a prototype.

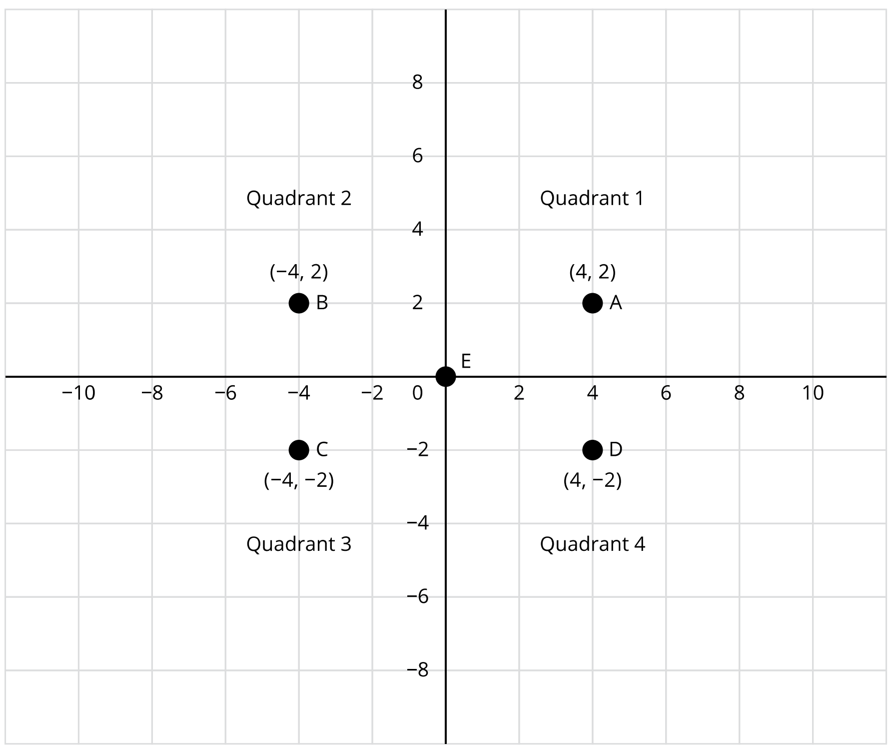
E1.3 Location and Movement

plot and read coordinates in all four quadrants of a Cartesian plane, and describe the translations that move a point from one coordinate to another

Teacher supports

Key concepts

- The X-Y Cartesian plane uses two perpendicular number lines to describe locations on a grid. The *x*-axis is a horizontal number line, the *y*-axis is a vertical number line, and these two number lines intersect perpendicularly at the origin, (0, 0), forming four quadrants.
- Pairs of numbers (coordinates) describe the precise location of any point on the plane. The coordinates are enclosed by parentheses as an ordered pair (*x*, *y*). The first number in the pair describes the horizontal distance and the direction from the origin. The second number describes the vertical distance and the direction from the origin. For example, the point (4, 2) is located four units to the right of the origin and two units up; the point (4, -2) is located four units to the right of the origin and two units down; the point (-4, 2) is located four units to the left of the origin and two units up; the point (-4, -2) is located four units to the left of the origin and two units down.



E1.4 Location and Movement

describe and perform combinations of translations, reflections, and rotations up to 360° on a grid, and predict the results of these transformations

Teacher supports

Key concepts

- Transformations on a shape result in changes to its position or its size. As a shape transforms, its vertices (points on a grid) move. The transformation describes the results of the movement. This explains how transformations involve location and movement.
- Transformations can be combined or composed. Sometimes a single transformation can be created by combining multiple transformations.
- A translation involves distance and direction. Every point on the original shape "slides" the same distance and direction to create a translated image. This is called the translation vector. For example, on a grid, a vector could describe that each point moves "5 units right and 2 units up". It is a mathematical convention that the horizontal distance (*x*) be given first, followed by the vertical distance (*y*).
- A reflection involves a line of reflection that acts like a mirror. Every point on the original shape is "flipped" across the line of reflection to create a reflected image. Every point on the original image is the same distance from the line of reflection as the corresponding point on the reflected image. Reflections are symmetrical.
- A rotation involves a *centre* of rotation and an *angle* of rotation. Every point on the original shape turns around the centre of rotation by the same specified angle. Any point on the original is the same distance to the centre of rotation as the corresponding point on the reflected image.
- Because a rotation is a turn, and 360° produces a full turn, a counterclockwise rotation of 270° produces the same result as a clockwise rotation of 90°. Convention has it that a positive angle describes a counterclockwise turn and a negative angle describes a clockwise turn, based on the numbering system of the Cartesian plane (see **SE E1.3**).

Note

- At this grade level, students can express the translation vector using arrows; for example, (5 →, 2 ↑).
- Dynamic geometry applications are recommended to support students to understand how transformations behave, either as a single transformation, or combined with others.

E2. Measurement

compare, estimate, and determine measurements in various contexts

Specific expectations

By the end of Grade 6, students will:

E2.1 The Metric System

measure length, area, mass, and capacity using the appropriate metric units, and solve problems that require converting smaller units to larger ones and vice versa

Teacher supports

Key concepts

- The choice of an appropriate unit depends on which attribute is being measured and the reason for measuring it.
  - The attribute to be measured determines whether to choose a unit of length, area, mass, or capacity.
  - The reason or context for measuring determines how accurate a measurement needs to be. Large units are used for broad, approximate measurements; small units are used for precise measurements and detailed work.

- When choosing the appropriate size of unit, it is helpful to know that the same set of metric prefixes applies to all attributes (except time) and describes the relationship between the units. Although not all metric prefixes are used commonly in English Canada, understanding the system reinforces the connection to place value:

Metric Prefix	kilo-unit	hecto-unit	deca-unit	**unit**	deci-unit	centi-unit	milli-unit
Unit Value	1000 units	100 units	10 units	**1 unit**	$\frac{1}{10}$ unit	$\frac{1}{100}$ unit	$\frac{1}{1000}$ unit
Place Value	thousand	hundred	ten	**one**	one tenth	one hundredth	one thousandth

- For any metric unit, the next largest unit (e.g., the unit to its left) is 10 times as great, and the next smallest unit (e.g., the unit immediately to its right) is one tenth as great. Both place value and the metric system use the same system of tens, so converting between units parallels multiplying or dividing by powers of 10 (e.g., by 10, 100, 1000). For example, since 1 m is $\frac{1}{1000}$ of 1 km, 28 500 m is 28.5 km (28 500 ÷ 1000), and since 1 cm is $\frac{1}{100}$ of 1m, 58 centimetre cm is 0.58 metres m (58 ÷ 100).
- There is an inverse relationship between the size of a unit and the count of units: larger units produce a smaller measure, and smaller units produce a larger measure. This

principle is important for estimating whether a conversion will result in a having larger or smaller count of units.

- Because both place value and the metric system are based on a system of tens, metric conversions can be visualized as a shifting of digits to the left or right of the decimal point a certain number of places. The amount of shift depends on the relative size of the units being converted. For example, since 1 km is 1000 times as long as 1 m, 28.5 km becomes 28 500 m when the digits shift three places to the left.
- Conversions are ratios, so the same tools that are useful for scaling and finding equivalent ratios are useful for unit conversions (e.g., double number lines, ratio tables, ratio boxes).

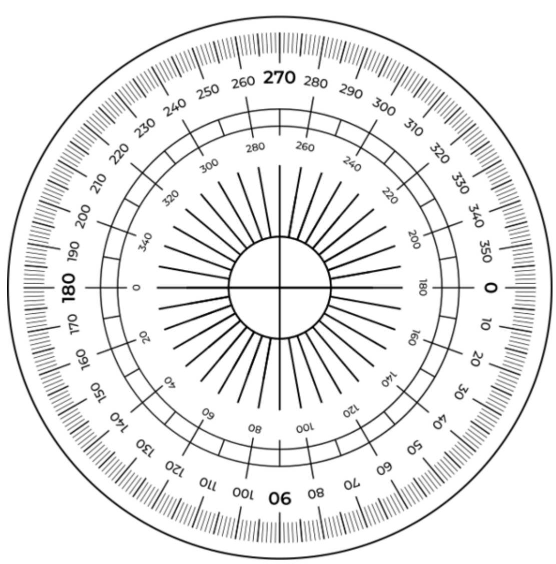
E2.2 Angles

use a protractor to measure and construct angles up to 360°, and state the relationship between angles that are measured clockwise and those that are measured counterclockwise

Teacher supports

Key concepts

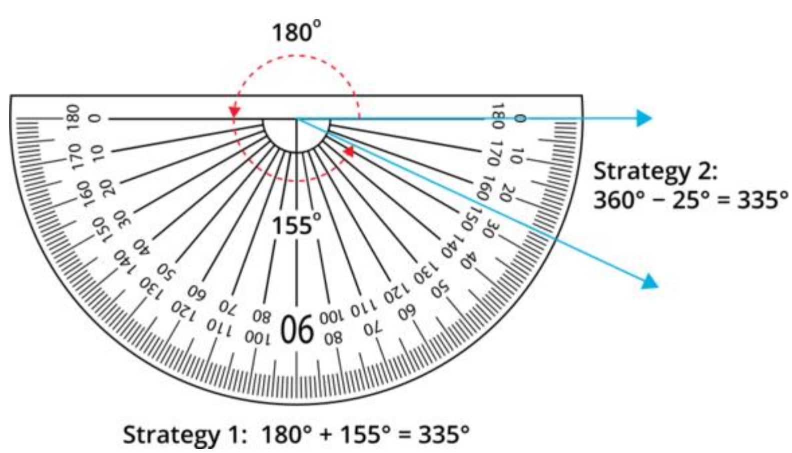
- The lines (rays) that form an angle (i.e., the "arms" of an angle) meet at a vertex. The size of the angle is not affected by the length of its arms.
- Protractors, like rulers or any other measuring tool, replace the need to lay out and count individual physical units. The protractor repeats a unit so there are no gaps or overlaps and includes a scale to keep track of the unit count.
- A degree is a very small angle and is a standard unit for measuring angles. When 360 degrees are placed together, they form a circle.



- Since a degree is such a small unit, standard protractors often use a scale (typically in increments of 10) with markings to show the individual degrees. If every degree were labelled, the protractor would need to be much larger.
- Protractors usually include a double scale to make it easier to count the degrees in angles that open clockwise and those that open counterclockwise. On a 180° protractor, the outer scale goes from 0° to 180° and reads from left to right whereas the inner scale goes from 0° to 180° and reads from right to left.
- To make an accurate measurement (i.e., a count of degrees) using a protractor:

    - align the vertex of the line (ray) with the vertex of the protractor (i.e., the midpoint of the protractor where all the degree angles meet);
    - align one arm of the line (ray) with the zero line of the protractor, similar to measuring from zero with a ruler;
    - choose the scale that begins the count at zero, use the scale to count the degrees in the angle, and read the measurement where the arm of the line (ray) crosses the number scale – that is, if the rays open to the right, use the inner scale, and if the rays open to the left, use the outer scale.

- Many common protractors are semi-circular, meaning the scale only counts 180°. There are two strategies to measure or construct a reflex angle: measure the angle beyond the straight angle and add 180° to that amount or subtract the remaining angle from 360°.



Note

- Smaller angles may be added together to determine a larger angle. This is the additivity principle of measurement.

E2.3 Angles

use the properties of supplementary angles, complementary angles, opposite angles, and interior and exterior angles to solve for unknown angle measures

Teacher supports

Key concepts

- Angles can be measured indirectly (calculated) by applying angle properties. Measuring angles indirectly is often quicker than measuring them directly and is the only choice if the location of an angle is impossible or impractical to measure.
- Smaller angles may be added together to determine a larger angle. This is the additivity principle of measurement.
- Angle properties can be used to determine unknown angles.
    - A straight angle measures 180°: this property is used to determine the measurement of a supplementary angle and is applied when determining the exterior angles of a polygon.
    - A right-angle measures 90°: this property is used to determine the measurement of a complementary angle.
    - Interior angles of quadrilaterals sum to 360°; this property is used to find an unknown angle in a quadrilateral.
    - Interior angles of triangles sum to 180°; this property is used to find an unknown angle in a triangle.
- Angle properties can also be used to determine other unknown measures (e.g., the exterior angle measures of a polygon) or to explain why opposite angles are equal.

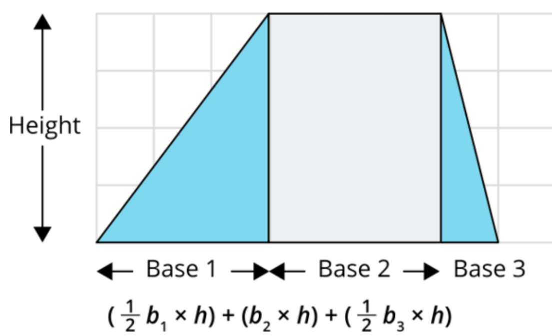
E2.4 Area and Surface Area

determine the areas of trapezoids, rhombuses, kites, and composite polygons by decomposing them into shapes with known areas

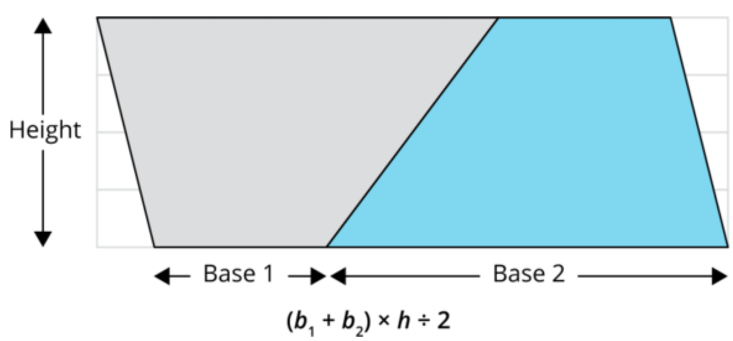
Teacher supports

Key concepts

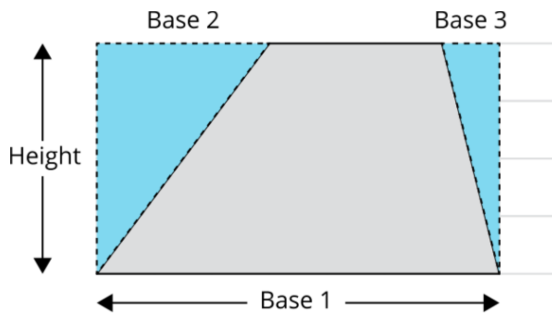
- Partial areas can be added together to find a whole area. If an area is decomposed and rearranged into a different shape (recomposed), the area remains constant. These are applications of the additivity and conservation principles.
- The area of a polygon can be determined by decomposing it into triangles, rectangles, and parallelograms – polygons with known area formulas:
    - Area of a parallelogram or rectangle = $b \times h$, where b represents the base and h represents the height
    - Area of a triangle = $b \times h \div 2$ or $\frac{1}{2} b \times h$
- Spatial relationships among quadrilaterals inform measurement relationships. For example, since all rhombuses, squares, and rectangles are specific types of parallelograms (see **SE E1.1**), the same area (A) formula applies to all: $A = b \times h$.
- Trapezoids can be decomposed into rectangles, parallelograms, or triangles in various ways. The illustrations below show how the four different decomposition strategies result in the same formula on simplification.
- A trapezoid can be decomposed into two triangles and a rectangle and the areas combined:



- A trapezoid can be doubled to create a parallelogram whose area is then halved:

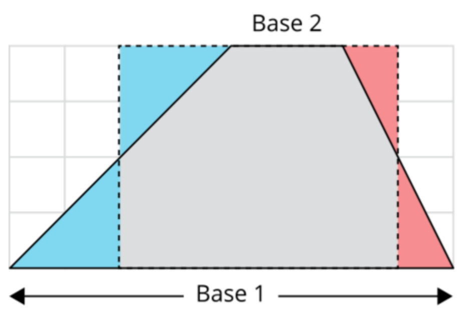


- Two triangles can be added to a trapezoid to create a larger rectangle, and then their areas can be subtracted:



$(b_1 \times h) - (\frac{1}{2} b_2 \times h) - (\frac{1}{2} b_3 \times h)$

- A trapezoid can be decomposed and then recomposed into a rectangle with the same area, where the base of the rectangle is the average of the two bases of the trapezoid (i.e., by creating two triangles from the midpoints of the sides):



average$(b_1 + b_2) \times h$

Note

- The same composition and decomposition strategies used to find the area of a trapezoid can be used to determine the area of kites (by decomposing into two triangles) and composite polygons.

E2.5 Area and Surface Area

create and use nets to demonstrate the relationship between the faces of prisms and pyramids and their surface areas

Teacher supports

Key concepts

- Area is additive: partial areas can be added together to find a whole area. Finding the surface area of a prism or pyramid is an application of the property of additivity.
- Nets help in visualizing the two-dimensional shapes that make up the faces of prisms and pyramids.
- Prisms have two parallel, congruent faces, which are the prism's bases. For an object to be a prism, the bases must be joined by rectangles or parallelograms. Rectangles produce "right" prisms and parallelograms produce "oblique" prisms. The shape of a base gives the prism its name (e.g., a prism with two triangles for bases is a triangle-based prism, or triangular prism).
- Pyramids have a single polygon for a base. For an object to be a pyramid, triangles must be joined to each side of the base and meet at the pyramid's apex. The shape of the base gives the pyramid its name (e.g., a pyramid with a square for a base is a square-based pyramid).

Note

- Visualizing the nets for prisms and pyramids – imagining them in the "mind's eye" – involves identifying the number and type of polygons that form their faces. It also involves recognizing how the dimensions of the prism or pyramid relate to the dimensions of the different faces. Being able to visualize a net is helpful for determining surface area.

E2.6 Area and Surface Area

determine the surface areas of prisms and pyramids by calculating the areas of their two-dimensional faces and adding them together

Teacher supports

Key concepts

- Area is additive: partial areas can be added together to find a whole area. Finding the surface area of a prism or pyramid is an application of the property of additivity.
- The faces joining the bases of a prism are rectangles or parallelograms. The faces joining the base of a pyramid are triangles. The areas of these faces can be determined by using

the formula for the area of a rectangle or parallelogram ($b \times h$) and the formula for the area of a triangle ($\frac{1}{2} b \times h$)

- The base of a prism or a pyramid can be any polygon.
    - If the base is a triangle, parallelogram, or trapezoid, then a formula can be used to measure the area of the base indirectly.
    - If the base is not one of these shapes, then its area may still be measured indirectly, by decomposing the shape and recomposing it into areas with known formulas (see **SE E2.4**), or it may be measured directly by overlaying a grid and counting the square units.

F. Financial Literacy

Overall expectations

By the end of Grade 6, students will:

F1. Money and Finances

demonstrate the knowledge and skills needed to make informed financial decisions

Specific expectations

By the end of Grade 6, students will:

F1.1 Money Concepts

describe the advantages and disadvantages of various methods of payment that can be used to purchase goods and services

Teacher supports

Key concepts

- Various methods of payment can be used when purchasing goods and services.
- Considering the advantages and disadvantages of various payment options helps consumers make informed purchasing decisions.

F1.2 Financial Management

identify different types of financial goals, including earning and saving goals, and outline some key steps in achieving them

Teacher supports

Key concepts

- Setting financial goals, including earning and saving goals, is an important life skill.
- Key steps and considerations are involved in achieving set financial goals.

Note

- An understanding of trade-offs may be helpful when setting achievable financial goals.
- Identifying the process of setting financial goals, including considering various influencing factors, and the steps involved in achieving those goals provides a context for developing social-emotional learning skills that build the confidence and competence students need to manage their finances.

F1.3 Financial Management

identify and describe various factors that may help or interfere with reaching financial goals

Teacher supports

Key concepts

- Anticipating potential barriers and considering factors that may help or interfere with reaching financial goals are part of the financial planning process.
- Achievable financial goals are based on context, research, knowledge, and an understanding of each individual situation.

F1.4 Consumer and Civic Awareness

explain the concept of interest rates, and identify types of interest rates and fees associated with different accounts and loans offered by various banks and other financial institutions

Teacher supports

Key concepts

- There are interest rates and fees associated with financial products such as bank accounts and loans.
- Critically examining and comparing the interest rates and fees offered by different financial institutions allows consumers to make informed choices.

F1.5 Consumer and Civic Awareness

describe trading, lending, borrowing, and donating as different ways to distribute financial and other resources among individuals and organizations

Teacher supports

Key concepts

- Financial and other resources can be distributed through different means depending on the context (e.g., cultural, socio-economic, historical, technological).
- Being aware of the various ways in which financial and other resources can be distributed may provide greater flexibility in choosing an appropriate method in a given situation or context.