Mathematics, Grade 8

Expectations by strand

A. Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes

This strand focuses on students' development and application of social-emotional learning skills to support their learning of math concepts and skills, foster their overall well-being and ability to learn, and help them build resilience and thrive as math learners. As they develop SEL skills, students demonstrate a greater ability to understand and apply the mathematical processes, which are critical to supporting learning in mathematics. In all grades of the mathematics program, the learning related to this strand takes place in the context of learning related to all other strands, and it should be assessed and evaluated within these contexts.

Overall expectations

Throughout this grade, in order to promote a positive identity as a math learner, to foster well-being and the ability to learn, build resilience, and thrive, students will:

A1. Social-Emotional Learning (SEL) Skills and the Mathematical Processes

apply, to the best of their ability, a variety of social-emotional learning skills to support their use of the mathematical processes and their learning in connection with the expectations in the other five strands of the mathematics curriculum

<table>
<tr><th>To the best of their ability, students will learn to:</th><th>… as they apply the mathematical processes:</th><th>… so they can:</th></tr>
<tr><td>1. identify and manage emotions</td><td rowspan="5">• problem solving: develop, select, and apply problem-solving strategies
• reasoning and proving: develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counter-examples) to justify thinking, make and investigate conjectures, and construct and defend arguments
• reflecting: demonstrate that as they solve problems, they are pausing, looking back, and monitoring their thinking to help clarify their understanding (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal)
• connecting: make connections among mathematical concepts, procedures, and representations, and relate mathematical ideas to other contexts (e.g., other curriculum areas, daily life, sports)
• communicating: express and understand mathematical thinking, and engage in mathematical arguments using everyday language, language resources as necessary, appropriate mathematical terminology, a variety of representations, and mathematical conventions
• representing: select from and create a variety of representations of mathematical ideas (e.g.,</td><td>1. express and manage their feelings, and show understanding of the feelings of others, as they engage positively in mathematics activities</td></tr>
<tr><td>2. recognize sources of stress and cope with challenges</td><td>2. work through challenging math problems, understanding that their resourcefulness in using various strategies to respond to stress is helping them build personal resilience</td></tr>
<tr><td>3. maintain positive motivation and perseverance</td><td>3. recognize that testing out different approaches to problems and learning from mistakes is an important part of the learning process, and is aided by a sense of optimism and hope</td></tr>
<tr><td>4. build relationships and communicate effectively</td><td>4. work collaboratively on math problems – expressing their thinking, listening to the thinking of others, and practising inclusivity – and in that way fostering healthy relationships</td></tr>
<tr><td>5. develop self-awareness and sense of identity</td><td>5. see themselves as capable math learners, and strengthen their sense of ownership of their learning, as part of their emerging sense of identity and belonging</td></tr>
</table>

6. think critically and creatively	representations involving physical models, pictures, numbers, variables, graphs), and apply them to solve problems • ***selecting tools and strategies:*** select and use a variety of concrete, visual, and electronic learning tools and appropriate strategies to investigate mathematical ideas and to solve problems	6. make connections between math and everyday contexts to help them make informed judgements and decisions

B. Number

Overall expectations

By the end of Grade 8, students will:

B1. Number Sense

demonstrate an understanding of numbers and make connections to the way numbers are used in everyday life

Specific expectations

By the end of Grade 8, students will:

B1.1 Rational and Irrational Numbers

represent and compare very large and very small numbers, including through the use of scientific notation, and describe various ways they are used in everyday life

Teacher supports

Key concepts

- The numerals 0 to 9 are used to form numbers. They are referred to as the digits in a number and each digit corresponds to a place value. A billion is “a thousand millions”, and a trillion is “a thousand billions” or “a million millions”. After the trillions period come quadrillions, quintillions, sextillions, septillions, octillions, and so on. Each period is 1000 times the preceding one.

- Reading numbers involves interpreting them as a quantity when they are expressed in words, in standard notation, in expanded notation, or in scientific notation. Large numbers may be expressed as a decimal number with the unit expressed in words. For example, 36.24 trillion is equivalent to 36 240 000 000 000 = 36.24×10^{12}.
- When a number is expressed in scientific notation, there is only one non-zero digit to the left of the decimal point. Thus, 36.24×10^{12} is not in scientific notation because there are two digits to the left of the decimal point. In scientific notation, 36 240 000 000 000 is written as 3.624×10^{13}.
- In words, 37 020 005 205 is written and said as "thirty-seven billion twenty million five thousand two hundred five". Sometimes an approximation to a large number is used to describe a quantity. For example, the number 37 020 005 205 may be rounded to 37 billion or 37.02 billion, depending on the amount of precision needed.
- Understanding the magnitude of a large number may be done by comparing it to other numbers and quantities. For example:
  - One million seconds is around 11.5 days.
  - One billion seconds is around 32 years.
  - One trillion seconds is around 32 000 years.
- A number greater than 1 that is written in scientific notation can be written in standard notation by multiplying the decimal number by ten the number of times indicated by its exponent. For example, for 3.2×10^5, 3.2 is multiplied by ten, five times. The result is 320 000.
- A number written in standard notation can be written in scientific notation. For a number greater than 1, a decimal point is positioned so that the first non-zero digit is to the left of the decimal point, and then the exponent for the base ten is determined by counting the number of times that decimal number needs to be multiplied by 10 to produce that number in standard notation. For example, 156 000 000 000 = 1.56×10^{11}.
- Very small numbers refer to numbers between 0 and 1. The closer the number is to zero the smaller the number is. These numbers can also be written in scientific notation. A negative exponent is used to indicate that the decimal number needs to be divided by 10 that many times. For example, for 5.2×10^{-8}, 5.2 is divided by 10 eight times to become 0.000000052.
- To write a small number in scientific notation, the decimal point is positioned so that the first non-zero digit is to the left of the decimal point, and then the exponent is determined by counting the number of times that decimal number needs to be divided by 10 to produce that number in standard notation. For example, 0.0034 = 3.4×10^{-3}.
- Numbers expressed in scientific notation can be compared by considering the number of times the decimal number is multiplied or divided by ten. The more times it is multiplied by ten, the greater the number. The more times it is divided by ten, the smaller the number.

Note

- Every strand of mathematics relies on numbers.
- Some numbers have cultural significance.
- Real-life contexts can provide an understanding of the magnitude of large and small numbers.
- The number 1 in scientific notation is 1×10^0.
- The exponent on the base ten, in scientific notation, indicates the number of times the decimal number is multiplied or divided by ten, not how many zeros need to be included for a number to be written in standard notation.
- When inputting numbers electronically, the “^” sign is used for exponents; for example, 10^6 would be entered as 10^6.

B1.2 Rational and Irrational Numbers

describe, compare, and order numbers in the real number system (rational and irrational numbers), separately and in combination, in various contexts

Teacher supports

Key concepts

- Real numbers are a set of numbers that contain all rational and irrational numbers.
- Rational numbers are those that can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers; for example, $-\frac{4}{3}$, 3.12, $\frac{1}{2}$ −7, 0, 205, $6.\dot{4}$, −32.5.
- Fractions (positive and negative) are rational numbers. Any fraction can be expressed as a decimal number that either terminates or repeats.
- Fractions can be written in a horizontal format (e.g., 1/2 or ½) as well as stacked format (e.g., $\frac{1}{2}$).
- Whole numbers are rational numbers since any whole number can be expressed as a fraction (e.g., $5 = \frac{5}{1}$).
- Integers (whole numbers and their opposites) are rational numbers since any integer can be expressed as a fraction (e.g., $-4 = \frac{-4}{1}$, $+8 = \frac{8}{1}$).
- Irrational numbers are numbers that cannot be expressed as a fraction. Examples of irrational numbers include decimal numbers that never repeat or terminate (e.g., 3.12122122212222…), pi (π), and square roots of non-perfect squares (e.g., $\sqrt{2}$).

- Rational and irrational numbers can be represented as points on a number line to show their relative distance from zero.
- The farther a number is to the right of zero on a horizontal number line, the greater the number.
- The farther a number is to the left of zero on a horizontal number line, the lesser the number.
- There are an infinite number of numbers in the real number system.

Note

- Since Grade 1, students have been working with whole numbers. The set of whole numbers (W) is a subset of integers (I), which are a subset of rational numbers (Q).
- Since Grade 1, students have been working with positive fractions, which are rational numbers. Negative fractions are introduced in Grades 7 and 8 as students represent, compare, and order negative fractions. Students will perform operations with negative fractions in secondary school because they are still developing the skills in Grade 8 to perform operations with integers.
- In Grade 7, students were introduced to pi, which is an irrational number. They may have worked with approximations of pi (3.14 or $\frac{22}{7}$), which are rational numbers. In Grade 8, students are introduced to other types of irrational numbers.
- In Grade 8, the focus is supporting students in making connections among the different number systems and the way they have been building knowledge through the years about the real number system.

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B1.3 Rational and Irrational Numbers

estimate and calculate square roots, in various contexts

Teacher supports

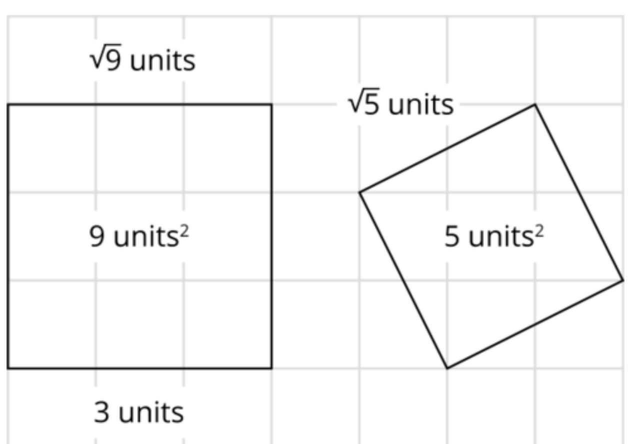
Key concepts

- The inverse of squaring a number is to take its square root.

- Each positive number has two possible square roots. For example, the square roots of 9 are +3 and −3 because (+3)(+3) = 9 and (−3)(−3) = 9.
- The symbol $\sqrt{}$ means the positive square root. The symbol $\pm\sqrt{}$ means both the positive and the negative square root.
- Depending on the context, only the positive square root may be appropriate. For example, given the area of a square, the length of its side is determined by taking the square root of the area. Since the side is a dimension, it makes sense to determine only the positive square root.
- Square roots of non-perfect squares are irrational and are left in radical form (e.g., $\sqrt{3}$) or approximated to a decimal number.
- Estimating the square roots of non-perfect squares involves identifying the two perfect squares that are closest to it. For example, $\sqrt{60}$ is between $\sqrt{49}$ and $\sqrt{64}$, so the first step is to determine the square root of those perfect squares, that is, $\sqrt{49} = 7$ and $\sqrt{64} = 8$. The next step is to estimate a value that is close to the closest square root. Since 60 is closer to 64 than to 49, then $\sqrt{60}$ can be estimated as 7.8.

Note

- Solving for a length using the Pythagorean theorem involves applying squares and square roots of numbers.
- A spatial interpretation of a square number is to think of the area of a square with side length the square root of the area (*side* × *side* or s^2).
- If the area of a square is 9, its side length is $\sqrt{9}$ or 3.
    - 9 is a perfect square.
- If the area of a square is 5, its side length is $\sqrt{5}$.
    - 5 is an imperfect square and its square root is an irrational number, with a decimal that never repeats or terminates.



- Perfect squares can be calculated. Imperfect squares can only be estimated. Calculators give approximations of all square roots of non-perfect square numbers.

B1.4 Fractions, Decimals, and Percents

use fractions, decimal numbers, and percents, including percents of more than 100% or less than 1%, interchangeably and flexibly to solve a variety of problems

Teacher supports

Key concepts

- Converting between fractions, decimals, and percents often makes calculations and comparisons easier to understand and carry out.
- Fractions, decimals, and percents all describe relationships to a whole. While fractions may use any number as a denominator, decimal units are in powers of ten (tenths, hundredths, and so on) and percents express a rate out of 100 ("per cent" means "per hundred").
- Relationships of quantities relative to a whole can be expressed as a fraction, a decimal number, and a percent. The choice of using a fraction, decimal number, or a percent can vary depending on the context of a problem.
- When fractions are considered as a quotient, the numerator is divided by a denominator and the result is a decimal representation that can be converted to a percent.
- To convert a percent to a fraction, it can first be represented out of 100 and then an equivalent proper fraction or mixed number can be made. For example, 104.6% = $\frac{104.6}{100} = \frac{1046}{1000} = 1\frac{46}{1000} = 1\frac{23}{500}$.
- Some decimal numbers when converted to a percent result in whole number percents, and others result in decimal percents (e.g., 0.15 = 15%, 0.642 = 64.2%, 3.425 = 342.5%).
- Percents can be whole number percents (e.g., 32%, 168%) or decimal percents (0.5%, 43.6%, 108.75%). Percents can be understood as decimal hundredths.
- Percents can be composed from other percents. A 15% discount combines a 10% discount and a 5% discount. A 13% tax adds 10% and another 3% (3 × 1%).
- To convert a percent to a decimal number, the percent is divided by 100 (e.g., 35.4% = 0.354, 0.1% = 0.001).
- There are three types of problems that involve percents – determining the percent a quantity represents relative to a whole; finding the percent of a number; and finding a number given the percent.
- Common benchmark fractions, decimals, and percents include:

- $150\% = 1\frac{1}{2} = 1.50$
- $100\% = 1 = 1.00$
- $75\% = \frac{3}{4} = 0.75$
- $50\% = \frac{1}{2} = 0.50$
- $25\% = \frac{1}{4} = 0.25$
- $20\% = \frac{1}{5} = 0.20$
- $10\% = \frac{1}{10} = 0.10$
- $5\% = \frac{1}{20} = 0.05$
- $1\% = \frac{1}{100} = 0.01$
- $0.1\% = \frac{1}{1000} = 0.001$

- Unit fraction conversions can be scaled to determine non-unit conversions. For example:
    - $\frac{1}{4} = 0.25 = 25\%$, so $\frac{1}{8} = 0.125 = 12.5\%$ (half of one fourth).
    - $\frac{1}{8} = 0.125 = 12.5\%$, so $\frac{3}{8} = 0.375 = 37.5$ (three times one eighth).

Note

- Sometimes when working with percents, students may work with complex fractions in which a decimal number is the numerator. An equivalent proper fraction or mixed number can be made by multiplying both the numerator and denominator by the appropriate number of tens.
- More than one strategy can be used to solve problems involving percents. For example, a coat is on sale for 25% off. The cost of the coat can be determined by finding 25% of the original price and then subtracting that discount value from the original price. Another strategy could be to determine 75% of the original price.

B2. Operations

use knowledge of numbers and operations to solve mathematical problems encountered in everyday life

Specific expectations

By the end of Grade 8, students will:

B2.1 Properties and Relationships

use the properties and order of operations, and the relationships between operations, to solve problems involving rational numbers, ratios, rates, and percents, including those requiring multiple steps or multiple operations

Teacher supports

Key concepts

- Properties of operations are helpful for carrying out calculations.
    - The identity property: $a + 0 = a$, $a - 0 = a$, $a \times 1 = a$, $\frac{a}{1} = a$.
    - The commutative property: $a + b = b + a$, $a \times b = b \times a$.
    - The associative property: $(a + b) + c = a + (b + c)$, $(a \times b) \times c = a \times (b \times c)$.
    - The distributive property: $a \times b = (c + d) \times b = c \times b + d \times b$.
- The commutative, associative, and identity properties can be applied for any type of number.
- When an expression includes multiple operations, there is a convention that determines the order in which those operations are performed:
    - Do calculations in the brackets first.
    - Then evaluate the exponents and roots (exponentiation).
    - Then multiply and divide in the order that these operations appear from left to right (multiplication/division).
    - Then add and subtract in the order that these operations appear from left to right (addition/subtraction).
- Multi-step problems may involve working with a combination of whole numbers, decimal numbers, and positive fractions.
- Multi-step problems may involve working with a combination of relationships, including ratios, rates, and percents.
- There may be more than one way to solve a multi-step problem.

Note

- This expectation supports most other expectations in the Number strand and is applied throughout the grade. Whether working with numbers or operations, recognizing and applying properties and relationships builds a strong foundation for doing mathematics.
- Problems that involve rational numbers in this grade include whole numbers, integers, positive decimal numbers, and positive fractions.
- Solving problems with more than one operation involves similar processes to solving problems with a single operation. For both types of problems:
    - Identify the actions and quantities in a problem and what is known and unknown.
    - Represent the actions and quantities with a diagram (physically or mentally).
    - Choose the operation(s) that match the actions to write the equation.
    - Solve by using the diagram (counting) or the equation (calculating).
- In multi-operation problems, sometimes known as two-step problems, there is often an *ultimate* question (asking for the final answer or result being sought), and a *hidden* question (a step or calculation that must be taken to get to the final result). Identifying both questions is an important part of solving these types of problems.
- The actions in a situation inform the choice of operation. The same operation can describe different situations.
    - Does the situation involve changing (joining, separating), combining or comparing? Then it can be represented with addition or subtraction.
    - Does the situation involve equal groups (or rates), ratio comparisons (scaling), or arrays? Then it can be represented with multiplication or division.
    - Representing a situation with an equation is often helpful in solving it.
- The same situation can be represented with different operations. Each operation has an "inverse" operation – an opposite that "undoes" the other. The inverse operation can be used to rewrite an equation to make it easier to calculate, or to check whether a calculation is true.
    - The inverse of addition is subtraction, and the inverse of subtraction is addition. So, for example, $\frac{1}{2} + ? = \frac{3}{4}$ can be rewritten as $\frac{3}{4} - \frac{1}{2} = ?$.
    - The inverse of multiplication is division, and the inverse of division is multiplication. So, for example, $\frac{1}{2} \times ? = \frac{3}{8}$ can be rewritten as $\frac{3}{8} \div \frac{1}{2} = ?$.

B2.2 Math Facts

understand and recall commonly used square numbers and their square roots

Teacher supports

Key concepts

- A perfect square can be represented as a square with its area the value of the perfect square and a side length that is the positive square root of that perfect square number. In general, the area (A) of a square is *side* (s) × *side* (s), $A = s^2$.
- Any integer multiplied by itself produces a square number, or a perfect square, and can be represented as a power with an exponent of 2. For example, 9 is a square number because 3 × 3 = 9 or $3^2 = 9$.
- A square number can be composed of a product of a perfect square and an even number of tens. For example, the square roots for 9, 900, 90 000, 9 000 000 are 3, 30, 300, 3000.

Note

- Negative integers expressed in exponential notation need to have a bracket around them to indicate it is the base of the power. Without the bracket it would not have the same result. For example, $-3^2 = -(3 \times 3) = -9$ versus $(-3)^2 = (-3 \times -3) = 9$.

B2.3 Mental Math

use mental math strategies to multiply and divide whole numbers and decimal numbers up to thousandths by powers of ten, and explain the strategies used

Teacher supports

Key concepts

- Multiplying a number by 0.1 is the same as dividing a number by 10. Therefore, it can be visualized by shifting the digit(s) to the right by one place. For example, 50 × 0.1 = 50, 50 × 0.1 = 5 and 5 × 0.1 = 0.5.
- Multiplying a number by 0.01 is the same as dividing a number by 100. Therefore, it can be visualized by shifting the digit(s) to the right by two places. For example, 500 × 0.01 = 5, 50 × 0.01 = 0.5, and 5 × 0.01 = 0.05.
- Mentally multiplying and dividing whole numbers and decimals by powers of ten builds on the constant 10:1 ratio that exists between place-value columns. For example, 1000 is

ten times greater than 100, or 100 is one tenth of 1000. Similarly, one hundredth (0.01) is ten times greater than one thousandth (0.001), or 0.001 is one tenth of 0.01.

- Multiplying a whole number and a decimal number by a positive power of ten can be visualized as shifting the digits to the left by one for each multiplication by 10.
    - For example, since 54.3×10^4 means $5.43 \times 10 \times 10 \times 10 \times 10$, the digits “543” shift to the left four places to become 543 000. This is true for whole numbers and decimals.
- Dividing a whole number and a decimal number by a power of 10 can be visualized as shifting the digits to the right by one for each division by 10.
    - For example, for $5.43 \div 10 \div 10 \div 10$, the digits “543” shift to the right three spaces to become 0.00543.
    - Dividing by 10 is the same as multiplying by 0.1, thus $5.43 \div 10 \div 10 \div 10$ is equal to $5.43 \times 0.1 \times 0.1 \times 0.1 = 0.00543$.

Note

- Making connections between division by 10 and multiplication by 0.1 can support students in converting a number in scientific notation of the form 5.43×10^{-3} to a number in standard form.
- Mental math refers to doing calculations in one’s head. Sometimes the numbers or the number of steps in a calculation are too complex to completely hold in one’s head, so jotting down partial calculations and diagrams can be used to complete the calculations.
- Estimation is a useful mental strategy when either an exact answer is not needed or there is insufficient time to work out a calculation.

B2.4 Addition and Subtraction

add and subtract integers, using appropriate strategies, in various contexts

Teacher supports

Key concepts

- When given a context, considerations can support the selection of an appropriate model and operation to solve the problem. For example:
    - Are the integers representing a quantity or change?
    - Is the situation involving the addition of integers with like signs or different signs?

  - Is the situation involving the comparison of two integers?
  - When modelling the situation, do zero pairs need to be used to carry out the operation?
  - What will the result of the calculation mean in relation to the problem being solved?

- When adding and subtracting integers, it is important to pay close attention to all of the elements of the statement. For example:

  - (+4) + (−3) may be interpreted as combining positive four and negative three.
  - 4 + (−3) may be interpreted as adding negative three to positive four.
  - (−4) − (+3) may be interpreted as determining the difference between negative four and positive three by comparing them.
  - (−4) − 3 may be interpreted as taking away positive three from negative four.
  - 4 − 3 may be interpreted as taking away positive three from positive four.
  - −4 − 3 may be interpreted as taking away positive three from negative four.

- The order that integers are written in an addition statement does not matter because the commutative property holds true (e.g., −5 + 3 = 3 + (−5)). It is important to note that the sign directly in front of the number belongs to the number.
- The order in which integers are written in a subtraction statement does matter because the commutative property does not hold true. For example, (−5) − (+3) = −8 and (+3) − (−5) = +8; the two expressions do not produce the same result.
- Addition and subtraction are inverse operations; therefore, a subtraction expression can be rewritten as an addition expression by adding its opposite (e.g., (−5) − (+ 3) becomes (−5) + (−3); 2 − (−4) = 2 + (+4)).
- When two positive integers are added together, the result is positive. This can be visualized on a number line as:

  - two vectors moving in a positive direction (right or up);
  - a vector moving in a positive direction from a positive starting position.

- When two negative integers are added together, the result is negative. This can be visualized on a number line as:

  - two vectors moving in a negative direction (left or down);
  - a vector moving in a negative direction from a negative starting position.

- When a positive and a negative integer are added together, the result is negative if the absolute value of the negative integer is greater than the absolute value of the positive integer. This can be visualized on a number line as:

- one vector moving in a positive direction and the other vector with a greater magnitude moving in a negative direction, the sign of the resultant vector is negative;
- a vector moving in a negative direction from a positive starting position and the head of the vector is to the left (or below) zero;
- a vector moving in a positive direction from a negative starting position and the head of the vector is to the left (or below) zero.

- When a positive and a negative integer are added together, the result is positive if the absolute value of the positive integer is greater than the absolute value of the negative integer. This can be visualized on a number line as:

  - one vector moving in a negative direction and the other vector with a greater magnitude moving in a positive direction, the sign of the resultant vector is positive;
  - a vector moving in a positive direction from a negative starting position and the head of the vector is to the right (or above) zero;
  - a vector moving in a negative direction from a positive starting position and the head of the vector is to the right (or above) zero.

Note

- If two integers added together have the same sign, then their magnitudes are added together.
- If two integers added together have different signs, then their magnitude is determined by taking the absolute difference between them.
- Depending on the models and the integers that are involved in a subtraction, zero pairs may need to be introduced in order to act out the situation. For example, if the situation involves taking away a negative amount but only positive amounts are shown, then adding zero pairs will allow for the negative amount to be removed.
- If the situation involves comparing two integers, the two integers can be represented as positions on a number line to determine the distance between the two points (magnitude).
- The order the subtraction statement is written is important in determining the sign. The direction of the sign is based on the movement from the point represented by the integer behind the minus sign (subtrahend) to the point represented by the integer in front of the minus sign (minuend). For example:

  - For $10 - (+2) = +8$, the distance between positive 10 and positive 2 is 8; the movement from positive 2 to 10 is in a positive direction.
  - For $(+2) - (+10) = -8$, the distance between positive 2 and positive 10 is 8; the movement from positive 10 to positive 2 is in a negative direction.

    - For $(2) - (-10) = +12$, the distance between positive 2 and negative 10 is 12; the movement from negative 10 to positive 2 is in a positive direction.

- Situations involving addition and subtraction can be modelled using tools such as a number line and integer tiles.
- Change can be represented by a positive or negative integer (e.g., rise of 4 expressed as +4, drop of 4 expressed as −4).
- A quantity relative to zero can be represented by a positive or negative integer (e.g., temperature is 3 degrees, temperature is −5 degrees).
- The integers in a situation may be interpreted as changes or as quantities. For example, if the temperature outside drops 5 degrees and then 3 degrees, this may be expressed as the addition of two drops $[(-5) + (-3)]$ or as a subtraction of 3 degrees $(-5 - 3)$. Both statements result in the same answer (−8), meaning the temperature decreased by 8 degrees.
- Familiar real-world contexts for negative and positive integers (temperature, elevators going up and down, parking garages, sea level, golf scores, plus/minus in hockey, gaining and losing money, walking forward and backwards, debts and surplus) provide an authentic opportunity to understand how integers are used in real life to describe a quantity or change.

B2.5 Addition and Subtraction

add and subtract fractions, using appropriate strategies, in various contexts

Teacher supports

Key concepts

- When adding and subtracting proper and improper fractions with the same denominator, the numerators are added, and the denominator remains the same. When the denominators are the same (e.g., three fourths and nine fourths) they have the same units and so can be added (twelve fourths).
- Strategies to add and subtract fractions with unlike denominators depends on the types of fractions that are given. For example:

    - Mental math can be used to create wholes (ones). For example, for $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$, knowing that three fourths is composed of one half and one fourth, the two halves are combined to make one, and then one fourth is added on.

- Equivalent fractions are created so that both fractions have a common denominator (e.g., $\frac{2}{3} + \frac{1}{2}$ can be scaled so that both have a denominator of 6, which results in the equivalent expression $\frac{4}{6} + \frac{3}{6}$).

- One strategy to add and subtract mixed fractions is to decompose the mixed fraction into its whole and fractional parts. The wholes are added or subtracted, and the fractional parts are added or subtracted. If the result of the fractional part is greater than one, it is rewritten as a mixed number and the whole combined with the other wholes. Another strategy to add and subtract mixed fractions is to first rewrite them as improper fractions.

Note

- Fractions are commonly added and subtracted in everyday life, particularly when using imperial units (inches, feet, pounds, cups, teaspoons). Imperial units are commonly used in construction and cooking.
- Only common units can be added or subtracted, whether adding or subtracting whole numbers, decimals, or fractions. Adding fractions with like denominators is the same as adding anything with like units:
  - 3 apples and 2 apples are 5 apples.
  - 3 fourths and 2 fourths are 5 fourths.
- The numerator in a fraction represents the count of unit fractions. The denominator represents what is being counted (the unit). To add or subtract fractions is to change the total count of units. This is why only the numerator is added or subtracted. There are helpful ways to visualize the addition and subtraction of fractions. Drawings, fraction strips, clock models, and rulers in imperial units can be used to generate equivalent fractions and model how these common units can be combined or separated.
- The three types of addition and subtraction situations (see **SE B2.1**) also apply to fractions.

B2.6 Multiplication and Division

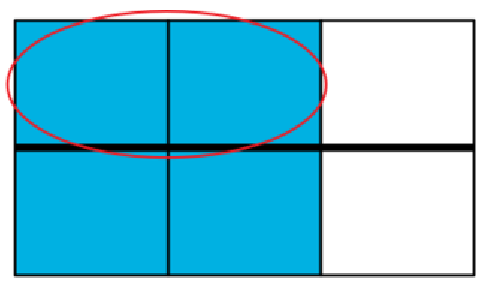
multiply and divide fractions by fractions, as well as by whole numbers and mixed numbers, in various contexts

Teacher supports

Key concepts

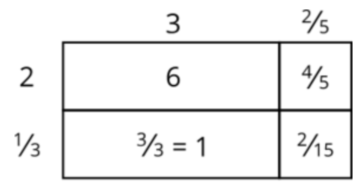
- The multiplication and division of two fractions can be interpreted based on the different ways fractions are used: as a quotient, as parts of a whole, as a comparison (ratio), and as an operator.
- Multiplication as scaling is one way to multiply a fraction by a whole number. For example, $2 \times \frac{2}{3}$ can be interpreted as doubling two one thirds, which is four one thirds or $\frac{4}{3}$.
- The multiplication of two proper fractions as operators can be modelled as follows:
    - For $\frac{1}{2} \times \frac{2}{3}$, the fraction two thirds can be shown as two thirds of a rectangle:

    - $\frac{1}{2}$ as an operator can be shown by taking one half of the two thirds:



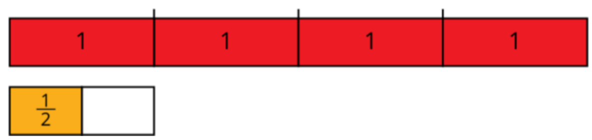
    - In general, the result of a fraction multiplied by a fraction can be obtained by multiplying the numerators and multiplying the denominators. In this example the product of the denominators are the partitions that were created in the rectangle, and the numerator is the resulting count of these partitions.

- Multiplying a mixed fraction by a mixed fraction can be modelled as the product of the area of a rectangle with its dimensions decomposed into wholes and fractions. For example, the rectangle to model $2\frac{1}{3} \times 3\frac{2}{5}$ has a width of two and one third and a length of three and two fifths. The areas of the four smaller rectangles formed by the decomposition are $2 \times 3 = 6$, $2 \times \frac{2}{5} = \frac{4}{5}$, $\frac{1}{3} \times 3 = 1$, and $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$. The sum of all the areas is $6\,\frac{4}{5} + 1\,\frac{2}{15} = 7 + \frac{12}{15} + \frac{2}{15} = 7\frac{14}{15}$.

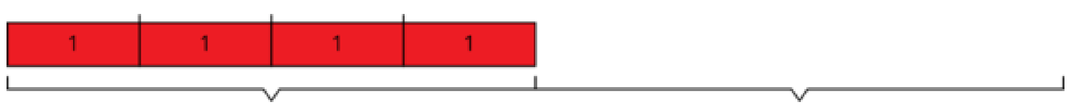


$$\begin{aligned} &6 + 1 + \tfrac{4}{5} + \tfrac{2}{15} \\ &= 7 + \tfrac{12}{15} + \tfrac{2}{15} \\ &= 7\,\tfrac{14}{15} \end{aligned}$$

- Division of fractions can be interpreted in two ways:
    - $4 \div \frac{1}{2} = ?$ can be interpreted as “How many one halves are in four?”

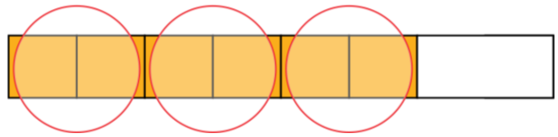


        - Two one halves make 1, so eight one halves make 4. Therefore, $4 \div \frac{1}{2} = 8$.
    - $4 \div \frac{1}{2} = ?$ can also be interpreted as “If 4 is one half of a number, what is the number?”

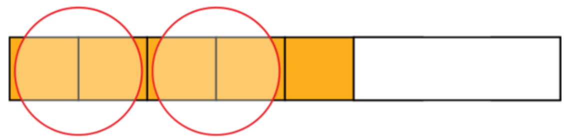


        - Since 4 is one half of a number, the other one half is also 4. Therefore, $4 \div \frac{1}{2} = 8$.

- Division of a fraction by its unit fraction (e.g., $\frac{5}{8} \div \frac{1}{8}$) can be interpreted as how many counts of the unit are in the fraction (i.e., how many one eighths are in five eighths)?" The result is the number of counts (e.g., there are 5 counts of one eighth).
- Dividing a fraction by a fraction with the same denominator (e.g., $\frac{6}{8} \div \frac{2}{8}$) can be interpreted as “How many divisors are in the dividend?” In the fraction strips below, notice there are three counts of two eighths that are in six eighths. Similar to the division of a fraction by its unit fraction, the result is the count.



- Sometimes the division of a fraction by a fraction with the same denominator has a fractional result. For example, $\frac{5}{8} \div \frac{2}{8}$.



  - Notice there are 2 two eighths in five eighths, and then $\frac{1}{2}$ of another two eighths.
  - Therefore, $\frac{5}{8} \div \frac{2}{8} = 2\frac{1}{2}$.

- In general, when dividing a fraction by a fraction with the same denominator, the result can be obtained by dividing the numerators and dividing the denominators.
- To multiply fractions by fractions that have unlike denominators, a strategy is to create equivalent fractions so that the two fractions have a common denominator and then divide numerators and divide denominators. For example:

$\frac{3}{4} \div \frac{5}{6}$

$= \frac{9}{12} \div \frac{10}{12}$

$= \frac{9 \div 10}{12 \div 12}$

$= \frac{\frac{9}{10}}{1} = \frac{9}{10}$

- A fraction divided by a whole number can use the same strategy. For example:

$\frac{3}{4} \div 5$

$= \frac{3}{4} \div \frac{20}{4}$

$= \frac{3}{20}$

- When division involves mixed numbers, a strategy is to convert them to improper fractions and then multiply accordingly.

Note

- When multiplying a fraction by a fraction using the area of a rectangle, first the rectangle is partitioned horizontally or vertically into the same number of sections as one of the denominators. Next, the region represented by that fraction is shaded to show that fraction of a rectangle. Next, the shaded section of the rectangle is partitioned in the other direction into the same number of sections as the denominator of the second fraction. Now it is possible to identify the portion of the shaded area that is represented by that fraction.
- Any whole number can be written as a fraction with one as its denominator. A whole number divided by a fraction can be used to support students in understanding the two ways division can be interpreted. If context is given, usually only one or the other way is needed. Dividing a whole number by a fraction also supports making connections with division of a fraction as the multiplication of its reciprocal.
- In general, dividing fractions with the same denominator can be determined by dividing the numerators and dividing the denominators.
- Since division is the inverse operation of multiplication, the division statement can be rewritten in terms of multiplication. For example, if $\frac{5}{4} \div \frac{2}{3} = n$, then $\frac{2}{3} \times n = \frac{5}{4}$.
    - To solve for *n*, each side of the equal sign is multiplied by $\frac{3}{2}$:
    - $\frac{3}{2} \times \frac{2}{3} \times n = \frac{5}{4} \times \frac{3}{2}$
    - therefore $n = \frac{5}{4} \times \frac{3}{2}$
    - Now the numerators can be multiplied, and the denominators can be multiplied.
    - $n = \frac{15}{8}$
    - The fraction $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$ because the product of these two fractions is 1.
    - Therefore, another strategy to divide two fractions is to multiply the dividend by the reciprocal of the divisor.

- Multiplying fractions follows a developmental progression that may be helpful in structuring tasks for this grade:
    - A proper or improper fraction by a whole number.
    - A whole number by a proper or improper fraction.
    - A unit fraction by a unit fraction.
    - A unit fraction by a proper or improper fraction.
    - A proper or improper fraction by a proper or improper fraction.
    - A mixed fraction by a proper or improper fraction.
    - A mixed fraction by a mixed fraction.

- Dividing fractions follows a developmental progression that may be helpful in structuring tasks for this grade:

    - A whole number divided by a whole number.
    - A proper or improper fraction divided by a whole number.
    - A whole number divided by a unit fraction.
    - A whole number divided by a proper or improper fraction.
    - A proper or improper fraction divided by a unit fraction.
    - A proper or improper fraction divided by a proper or improper fraction with the same denominators and a result that is a whole number.
    - A proper or improper fraction divided by a proper or improper fraction with the same denominators and a result that is a fractional amount (e.g., $\frac{7}{5} \div \frac{2}{5}$).
    - A proper or improper fraction divided by a proper or improper fraction with unlike denominators.
    - A proper or improper fraction divided by a whole number or mixed fraction.
    - A mixed fraction divided by a whole number.
    - A mixed fraction divided by a mixed fraction.

B2.7 Multiplication and Division

multiply and divide integers, using appropriate strategies, in various contexts

Teacher supports

Key concepts

- Multiplication and division facts for whole numbers can be used for multiplying and dividing integers. The difference is consideration of the sign, which is determined by the numbers being worked with.
- A positive integer multiplied or divided by a positive integer has a result that is positive.
- A positive integer multiplied by a negative integer has a result that is negative. Since multiplication can be understood as repeated equal groups, then the positive integer can represent the number of groups and the negative integer can represent the quantity in each group. For example, $3 \times (-4)$ can be modelled as $(-4) + (-4) + (-4) = -12$.
- The commutative property holds true for the multiplication of integers, so $(+3) \times (-4) = (-4) \times (+3)$. Therefore $(-4) \times (+3) = -12$ or (-12).
- Since division is the inverse operation, the rules for the signs with multiplication are the same for division:

- A positive number divided by a positive number has a result that is positive.
- A positive number divided by a negative number has a result that is negative.
- A negative number divided by a positive number has a result that is negative.
- A negative number divided by a negative number has a result that is positive.

Note

- When two brackets are side by side, it is understood to be multiplication. For example: (−3)(−4).
- Division of two numbers can be indicated using the division symbol or using the division bar. For example, $12 \div (-3) = \frac{12}{-3}$.
- Multiplication can be understood as repeated equal groups, where the first factor is the number of groups and the second factor is the size of the groups. When the first integer is positive, regardless of the sign of the second integer, this is helpful for visualizing the situation since it links multiplication with the repeated addition of a group.

  - A 3° rise in temperature 4 days in a row can be represented as (+4) × (+3) = (+12).
  - A 3° drop in temperature 4 days in a row can be represented as (+4) × (−3) = (−12).

- It is difficult, although not impossible, to conceive of a negative number of groups. To overcome this, properties and reasoning can help:

  - The commutative property says that (−4) × (+3) is the same as (+3) × (−4), so both must equal (−12).
  - Patterning can be used to determine that (−3) × (−4) must be (+12).

- Understanding division of integers requires a strong understanding of the operation and its relationship to multiplication. Grouping division asks, “How many groups of ___ are in ___? ” Sharing division asks, “How many does each receive if ___ are shared among ___? ” Both are helpful for understanding division with integers and their relationship to multiplication, repeated addition, and repeated subtraction:

  - (+20) ÷ (+5) draws on work in earlier grades for an answer of (+4).
  - (−20) ÷ (−5) can mean how many groups of (−5) are in (−20). Since there are 4 groups of (−5) in (−20), (−20) ÷ (−5) = (+4).
  - (−20) ÷ (+5) can mean that (−20) can be shared between 5 groups. Since each group would receive (−4), (−20) ÷ (+5) = (−4).
  - (+20) ÷ (−5) can draw on patterns and the inverse relationship between multiplication and division to rewrite this statement as (−5) × ___ = (−20) to see that (+20) ÷ (−5) = (−4).

- There are conventions for expressing multiplication and division in ways that make algebraic expressions clearer:
    - Multiplication may be shown with the multiplication sign: (−3) × (−4).
    - Multiplication may be shown with no multiplication sign: (−3)(−4).
    - Multiplication may be shown with the dot operator: (−3)·(−4).
    - Division may be shown with the division sign (÷).
    - Division may be shown with the fraction bar (—).

B2.8 Multiplication and Division

compare proportional situations and determine unknown values in proportional situations, and apply proportional reasoning to solve problems in various contexts

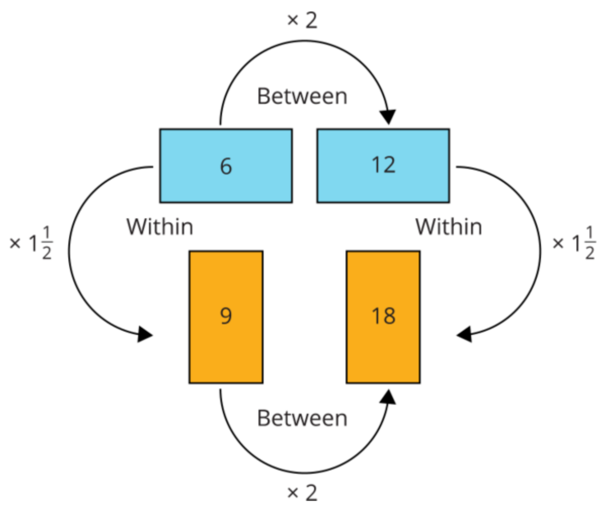
Teacher supports

Key concepts

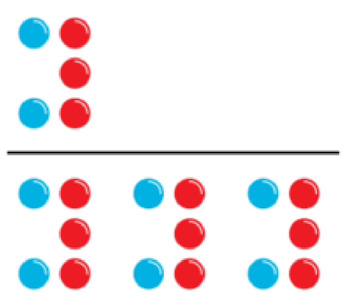
- If two quantities change at the same rate, the quantities are proportional. Proportional growth, when plotted on a graph, forms a straight line (i.e., linear growth) because each point changes at a constant rate.
- Proportions involve multiplicative comparisons (ratios) and are written in the form $a : b = c{:}d$ or expressed using fractional notation as $\frac{a}{b} = \frac{c}{d}$. When ratios are represented using fractional notation, they are usually read as 3 out of 4, or 3 to 4, rather than as a fraction, three fourths. Writing ratios using fractional notation is helpful for making comparisons and calculating proportions.
- There are four ways a proportion can be written for it to hold true. For example, 3.7 km for every 5 hours and 7.4 km for every 10 hours can be expressed as:
    - $\frac{3.7}{5} = \frac{7.4}{10}$ or
    - $\frac{3.7}{7.4} = \frac{5}{10}$ or
    - $\frac{5}{3.7} = \frac{10}{7.4}$ or
    - $\frac{7.4}{3.7} = \frac{10}{5}$.
- Problems involving proportional relationships can be solved in a variety of ways, including using a table of values, a graph, a ratio table, a proportion, and scale factors.

Note

- One strategy when using a proportion to solve for an unknown value is to position that unknown in the upper part of the equation (e.g., $\frac{m}{9} = \frac{3.4}{6.8}$).
- In solving for proportional situations, comparisons can be made *within* a situation (i.e., the unit rate is constant) and *between* situations (i.e.., the scaling factor is constant). So, 6 items costing \$9 is proportional to 12 items costing \$18:

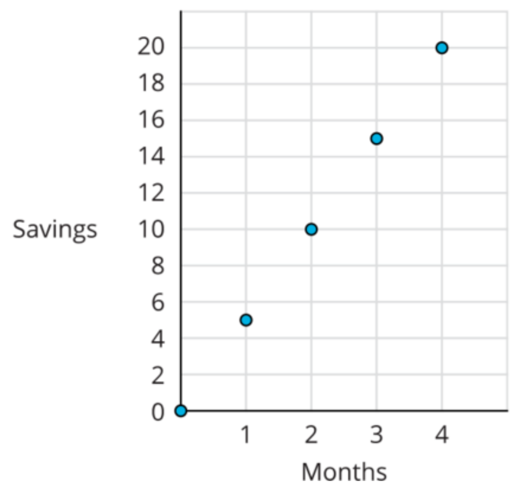


- Scaling a ratio creates other proportional situations. For example, the relationship of 2 blue marbles to 3 red marbles (2 : 3) is in proportion to 6 blue marbles and 9 red marbles (6 : 9). The fractions $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent, so the situations are proportional.



- Ratio tables, double number lines, and between-within diagrams are helpful tools to identify and compare proportional relationships, and to solve for unknown values.
- Tables and graphs are helpful for seeing proportional (or non-proportional) relationships. Any of the points marked on the graph is proportional to each of the other points.

Month	Deposit	Total Saved	Rate of Change	Rate per Month
1	$5	$5	+5	$5/month
2	$5	$10	+5	$5/month
3	$5	$15	+5	$5/month
4	$5	$20	+5	$5/month



C. Algebra

Overall expectations

By the end of Grade 8, students will:

C1. Patterns and Relationships

identify, describe, extend, create, and make predictions about a variety of patterns, including those found in real-life contexts

Specific expectations

By the end of Grade 8, students will:

C1.1 Patterns

identify and compare a variety of repeating, growing, and shrinking patterns, including patterns found in real-life contexts, and compare linear growing and shrinking patterns on the basis of their constant rates and initial values

Teacher supports

Key concepts

- Repeating patterns have a pattern core that is repeated over and over.
- In growing patterns, there is an increase in the number of elements or size from one term to the next.
- In shrinking patterns, there is a decrease in the number of elements or size from one term to the next.
- If the ratio of the change in one variable to the change in another variable is equivalent between any two sets of data points, then there is a constant rate. An example of a real-life application of a constant rate is an hourly wage of $15.00 per hour.
- The initial value (constant) of a linear pattern is the value of the term when the term number is zero. An example of a real-life application of an initial value is a membership fee.
- The relationship between the term number and the term value can be generalized. A linear pattern of the form $y = mx + b$ has a constant rate, m, and an initial value, b. The graph of a linear growing pattern that has an initial value of zero passes through the origin at (0, 0).
- The graphical representation of a linear *growing* pattern is a line that rises to the right; for a linear *shrinking* pattern, the line descends to the right.

Note

- Growing and shrinking patterns are not limited to linear patterns.

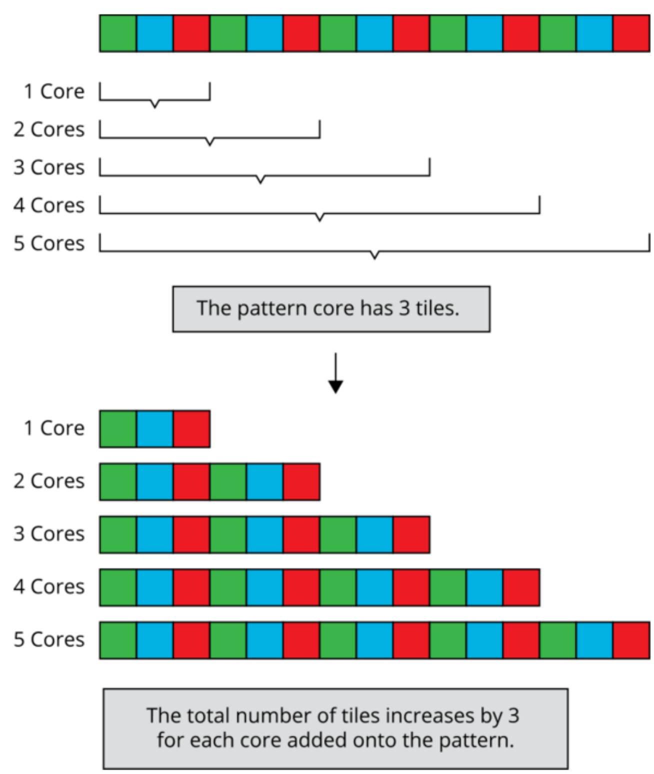
C1.2 Patterns

create and translate repeating, growing, and shrinking patterns involving rational numbers using various representations, including algebraic expressions and equations for linear growing and shrinking patterns

Teacher supports

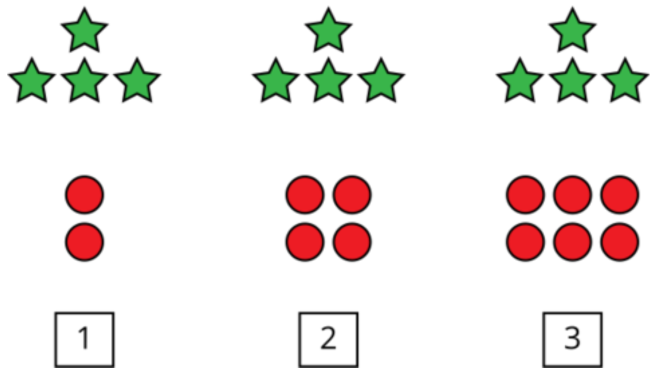
Key concepts

- Growing patterns are created by increasing the number of elements or the size of the elements in each iteration.
- Shrinking patterns are created by decreasing the number of elements or the size of the elements in each iteration.
- Graphical representations of linear growing and shrinking patterns appear as straight lines.
- Graphical representations of non-linear growing and shrinking patterns appear as curves.
- Some patterns are based on continuous variables, such as height, distance, or time. Graphical representations of continuous values are solid lines or curves, illustrating their continuous nature.
- A linear growing pattern can be created by repeatedly representing a pattern to show the total number of elements in each iteration of the pattern core.



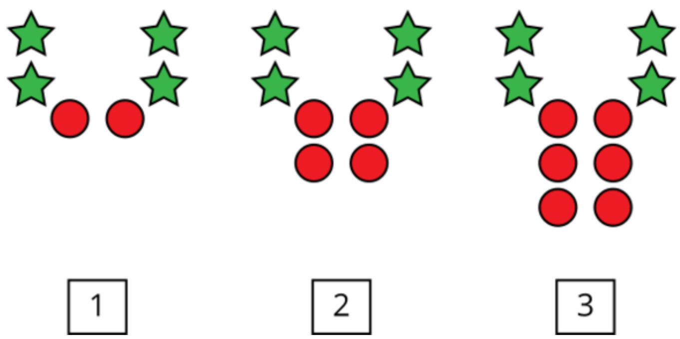
- Examining the physical structure of a linear growing pattern can provide insight into the different algebraic equations that show the relationship between the term number and the term value. For example, in Diagram 1, each term value can be viewed as four more than double the term number, which can be expressed as term value = 2*(term number) + 4 or $y = 2x + 4$.

Diagram 1



- Diagram 2 shows that for the same pattern, each term value can also be viewed as twice the term number plus two, which can be expressed as term value = term number + two + term number + two or $y = x + 2 + x + 2$. This expression for Diagram 2 can be simplified to $y = 2x + 4$, which is the same expression derived for Diagram 1.

Diagram 2



Note

- The creation of growing and shrinking patterns in this grade is not limited to linear patterns.

C1.3 Patterns

determine pattern rules and use them to extend patterns, make and justify predictions, and identify missing elements in growing and shrinking patterns involving rational numbers, and use algebraic representations of the pattern rules to solve for unknown values in linear growing and shrinking patterns

Teacher supports

Key concepts

- Patterns can be extended because they are repetitive by nature.
- Pattern rules are generalizations about a pattern, and they can be described in words.
- Patterns can be extended in multiple directions, showing what comes next and what came before.
- To make a near prediction about a pattern is to state or show what a pattern will look like just beyond the given representation of that pattern. The prediction can be verified by extending that pattern.
- To make a far prediction about a pattern is to state or show what a pattern will look like well beyond the given representation of that pattern. Often calculations are needed to make an informed prediction that can be justified.
- Identifying the missing elements in a pattern represented using a table of values may require determining the term number (x) or the term value (y).
- Identifying the missing elements in a pattern represented on a graph may require determining the point (x, y) within the given representation or beyond it, in which case the pattern will need to be extended.
- The algebraic expression that represents a linear growing and shrinking pattern is also referred to as the general term or the nth term. It can be used to solve for the term value or the term number.

Note

- Determining a point on a graph within a given set of points that fit a pattern is called interpolation. Determining a point on a graph beyond a given set of points that fit a pattern is called extrapolation. This skill set is used in a variety of contexts, including in the science curriculum when students are working on the concept of the mechanical advantage (levers).

C1.4 Patterns

create and describe patterns to illustrate relationships among rational numbers

Teacher supports

Key concepts

- Patterns can be used to demonstrate an understanding of number properties, including the use of exponents to express numbers in scientific notation.

Note

- Using patterns is a useful strategy in developing understanding of mathematical concepts, such as knowing what sign to use when two integers are added or subtracted.

C2. Equations and Inequalities

demonstrate an understanding of variables, expressions, equations, and inequalities, and apply this understanding in various contexts

Specific expectations

By the end of Grade 8, students will:

C2.1 Variables and Expressions

add and subtract monomials with a degree of 1, and add binomials with a degree of 1 that involve integers, using tools

Teacher supports

Key concepts

- A monomial with a degree of 1 has a variable with an exponent of one. For example, the exponent of m for the monomial $2m$ is 1. When the exponent is not shown, it is understood to be one.
- A binomial with a degree of 1 consists of two terms (two binomials) in which at least one of the terms has a variable with an exponent of one (e.g., $2m + 5$ or $2m + 5n$).
- Only like terms can be combined when monomials and binomials are added together. For example:
    - $5m + (-3m + 4n)$
    - $= 5m + (-3m) + 4n$
    - $= 2m + 4n$
- Monomials with a degree of 1 with the same variables can be subtracted (e.g., $-10y - 8y = -18y$).
- Monomials can be subtracted in different ways. One way is to compare them and determine the missing addend (e.g., $3m + ? = 7m$). Another way is to remove them from the expression representation.

- Strategies for performing operations with integers can also be used to add and subtract monomials and binomials.

Note

- Examples of monomials with a degree of 2 are x^2 and xy. The reason that xy has a degree of 2 is because both x and y have an exponent of 1. The degree of the monomial is determined by the sum of all the exponents of its variables.
- Visual representations can support students' understanding of combining like terms.
- When adding binomials, brackets are used around the expressions to show that they are binomials.

C2.2 Variables and Expressions

evaluate algebraic expressions that involve rational numbers

Teacher supports

Key concepts

- To evaluate an algebraic expression, the variables are replaced with numerical values and calculations are performed based on the order of operations.

Note

- When students are working with formulas, they are evaluating expressions.
- Replacing the variables with numerical values often requires the use of brackets. For example, the expression $\frac{3}{4}m$ becomes $\frac{3}{4}(m)$ and then $\frac{3}{4}\left(\frac{2}{5}\right)$ when $m = \frac{2}{5}$. The operation between $\frac{3}{4}$ and $\left(\frac{2}{5}\right)$ is understood to be multiplication.
- Many coding applications involve algebraic expressions being evaluated. This may be carried out in several steps. For example, the instruction: "input 'the radius of a circle', radiusA" is instructing the computer to define the variable "radiusA" and store whatever the user inputs into the temporary location called radiusA. The instruction: "input 'the height of the cylinder', heightA" is instructing the computer to define the variable "heightA" and store whatever the user inputs into the temporary location called heightA. The instruction: "calculate 3.14*radiusA^2*heightA, volumeA" instructs the computer to take the value that is stored in radiusA and multiply it by itself, then multiply it by the value stored in heightA, and then store that result in the temporary location, which is

another variable called “volumeA”. (The caret symbol (^) is used with many forms of technology as the exponent symbol.)

C2.3 Equalities and Inequalities

solve equations that involve multiple terms, integers, and decimal numbers in various contexts, and verify solutions

Teacher supports

Key concepts

- Equations are mathematical statements such that the expressions on both sides of the equal sign are equivalent.
- In equations, variables are used to represent unknown quantities.
- There are many strategies to solve equations, including guess-and-check, the balance model, and using the reverse flow chart.
- Equations need to be simplified in order to use the strategy of using a reverse flow chart to solve equations like $\frac{m}{4} - 2 = -10$. The first diagram shows the flow of operations performed on the variable m to produce the result −10. The second diagram shows the reverse flow chart, or flow of the reverse operations, in order to identify the value of the variable *m*.

$$\boxed{m} \rightarrow \boxed{\div 4} \rightarrow \boxed{-2} \rightarrow \boxed{-10}$$

$$\boxed{-32} \leftarrow \boxed{\times 4} \leftarrow \boxed{+2} \leftarrow \boxed{-10}$$

- Formulas are equations in which any of the variables can be solved for. When solving for a variable in a formula, values for the variables are substituted in and then further calculations maybe needed depending on which variable is being solved for. For example, for $A = lw$, if $l = 10.5$, and $w = 3.5$, then $A = (10.5)(3.5) = 36.75$. If $A = 36.75$ and $l = 10.5$, then $36.75 = 10.5w$, and this will require dividing both sides by 10.5 to solve for w.

Note

- The flow chart used in coding is different from the reverse flow chart that can be used to solve equations.
- Many coding applications involve formulas and solving equations.

C2.4 Equalities and Inequalities

solve inequalities that involve integers, and verify and graph the solutions

Teacher supports

Key concepts

- An inequality can be solved like an equation, and then values need to be tested to identify those that hold true for the inequality.
- When multiplying or dividing by a negative integer, the inequality sign needs to be reversed in order for the solution to hold true.
- A number line shows the range of values that hold true for an inequality by placing a dot at the greatest or least possible value. An open dot is used when an inequality involves "less than" or "greater than"; if the inequality includes the equal sign (=), then a closed dot is used.

Note

- Inequalities that involve multiple terms may need to be simplified before they can be solved.
- The solution for an inequality that has one variable, such as $-2x + 3x < 10$, can be graphed on a number line.
- The solution for an inequality that has two variables, such as $x + y < 4$, can be graphed on a Cartesian plane, showing the set of points that hold true.

C3. Coding

solve problems and create computational representations of mathematical situations using coding concepts and skills

Specific expectations

By the end of Grade 8, students will:

C3.1 Coding Skills

solve problems and create computational representations of mathematical situations by writing and executing code, including code that involves the analysis of data in order to inform and communicate decisions

Teacher supports

Key concepts

- Data can be stored in lists, or input into a program, in order to find solutions to problems and make decisions.
- A flow chart can be used to plan and organize thinking. The symbols used in flow charts have specific meanings, including those that represent a process, a decision, and program input/output.
- Efficient code can include using the fewest number of instructions to solve a problem, using the smallest amount of space to store program data, or executing as fast as possible.
- Loops can be used to create efficient code.
- Conditional statements, like loops, can be nested to allow for a range of possible outcomes or to implement decision trees.
- Sub-programs are used to assemble a complex program by writing portions of the code that can be modularized. This helps to create efficient code.

Note

- By combining mathematical and coding concepts and skills, students can write programs to draw conclusions from data.
- Students can use data, coding, and math concepts and skills to generate a range of possibilities and likelihoods and decide upon a specific course of action (e.g., optimizing packaging, price points, sports performance).
- Coding can be used to automate simple processes and enhance mathematical thinking. For example, students can code expressions to recall previously stored information (defined variables), then input values (e.g., from a sensor, count, or user input) and redefine the value of the variable. (See **SEs C2.2** and **C2.3**.)

C3.2 Coding Skills

read and alter existing code involving the analysis of data in order to inform and communicate decisions, and describe how changes to the code affect the outcomes and the efficiency of the code

Teacher supports

Key concepts

- Reading code is done to make predictions as to what the expected outcome will be. Based on that prediction, one can determine if the code needs to be altered prior to its execution.
- Reading code helps with troubleshooting why a program is not able to execute.
- Code is altered so that an expected outcome can be achieved.
- Code can be altered to be used for a new situation.
- Altering code to make it more efficient often involves refining algorithms so that there are no unnecessary steps and using control structures effectively.
- Using sub-programs makes it easier to debug programs since each sub-program can be tested individually.

Note:

- Altering existing code and describing how changes affect outcomes allows students to investigate relationships and pose and test what-if questions.
- By describing how changes affect outcomes, students are making predictions.
- By reading and describing code and algorithms, students are learning to articulate complex mathematical ideas and concepts.
- When students are provided with code and algorithms to solve complex problems, they can alter this code to solve similar problems, thereby gaining a deeper understanding of the mathematical and coding concepts involved.

C4. Mathematical Modelling

apply the process of mathematical modelling to represent, analyse, make predictions, and provide insight into real-life situations

This overall expectation has no specific expectations. Mathematical modelling is an iterative and interconnected process that is applied to various contexts, allowing students to bring in learning from other strands. Students' demonstration of the process of mathematical modelling, as they apply concepts and skills learned in other strands, is assessed and evaluated.

Read more about the mathematical modelling process.

Teacher supports

Key concepts

- The process of mathematical modelling requires: understanding the problem; analysing the situation; creating a mathematical model; and analysing and assessing the model.

Note

- A mathematical modelling task is different from a real-life application due to the cyclic nature of modelling, which involves examining a problem from outside mathematics, modelling it, and then checking the model back against the real-life situation and adjusting as necessary.
- The process of mathematical modelling should not be confused with using a "model" to represent or solve a problem that does not require the whole process.
- Mathematical modelling tasks can be utilized in many ways and can support students with making connections among many mathematical concepts across the math strands and across other curricula.

D. Data

Overall expectations

By the end of Grade 8, students will:

D1. Data Literacy

manage, analyse, and use data to make convincing arguments and informed decisions, in various contexts drawn from real life

Specific expectations

By the end of Grade 8, students will:

D1.1 Data Collection and Organization

identify situations involving one-variable data and situations involving two-variable data, and explain when each type of data is needed

Teacher supports

Key concepts

- A variable is any attribute, number, or quantity that can be measured or counted.
- One-variable data refers to one data set from a sample or a population that can be either qualitative or quantitative. Situations involve representing and analysing the data based on that one variable to answer a question like “What is the average height of all the students in the class?”
- Two-variable data refers to two data sets from the same sample or population that can be either qualitative or quantitative. Situations involve representing and analysing the data based on two variables to answer questions like “Is there a relationship between a person’s height and the length of their arm span?”

D1.2 Data Collection and Organization

collect continuous data to answer questions of interest involving two variables, and organize the data sets as appropriate in a table of values

Teacher supports

Key concepts

- The type and amount of data to be collected is based on the questions of interest. Questions of interest involving two variables require two sets of data to be collected from the same sample or population.
- Depending on the question of interest, the continuous data may need to be collected from a primary or a secondary source.
- Depending on the question of interest, a random sample of the population may need to be taken. Types of sampling methods include simple random sampling, stratified random sampling, and systematic random sampling.
- A table of values for a scatter plot is a list of corresponding values of two variables for each subject in a sample or population.

Note

- A scatter plot does not guarantee that there is a relationship between the two variables. Therefore, it is only when there is a relationship that one variable is dependent on the other variable (i.e., for a relation, the x-value in the set of ordered pairs (x, y) is the independent variable and the y-value is the dependent variable).

- Many science experiments involve the relationship between two variables. The independent variable is what the researcher gets to change, and the dependent variable is what the researcher gets to observe or measure during the experiment.

D1.3 Data Visualization

select from among a variety of graphs, including scatter plots, the type of graph best suited to represent various sets of data; display the data in the graphs with proper sources, titles, and labels, and appropriate scales; and justify their choice of graphs

Teacher supports

Key concepts

- Scatter plots are used to display data points for two continuous variables. The horizontal axis identifies the possible values for one variable and the vertical axis identifies the possible values for the other variable.
- Broken-line graphs are used to show change over time. The value on the horizontal axis is usually time. One or none of the variables are continuous.
- Circle graphs are used to show how categories represent a part of the whole data set for one variable.
- Histograms display data as intervals of numeric data, and their frequencies for one variable that is continuous.
- Pictographs, line plots, bar graphs, multiple-bar graphs, and stacked-bar graphs may be used to display qualitative data and discrete data, and their corresponding frequencies for one variable.

Note

- Data that is represented in a table of values displays two pieces of information for each subject in the sample or population. These two pieces of information can be graphed together using a scatter plot. Also, each piece of information can be treated separately and represented using another type of graph such as a histogram or circle graph.

D1.4 Data Visualization

create an infographic about a data set, representing the data in appropriate ways, including in tables and scatter plots, and incorporating any other relevant information that helps to tell a story about the data

Teacher supports

Key concepts

- Infographics are used in real life to share data and information on a topic, in a concise, clear, and appealing way.
- Infographics contain different representations, such as tables, plots, graphs, with limited text such as quotes.
- Information to be included in an infographic needs to be carefully considered and presented so that it is clear and concise. Infographics tell a story about the data with a specific audience in mind. When creating infographics, students need to create a narrative about the data for that audience.

Note

- Creating infographics has applications in other subject areas, such as communicating key findings and messages in STEM projects.

D1.5 Data Analysis

use mathematical language, including the terms “strong”, “weak”, “none”, “positive”, and “negative”, to describe the relationship between two variables for various data sets with and without outliers

Teacher supports

Key concepts

- When data points form close to a line or a curve, this indicates there is a strong relationship between the variables.
- When data points are in a cluster, this indicates there is no relationship.
- The scatter plot of a weak relationship between two variables shows points that are more spread out than those showing a strong relationship.
- The scatter plot of a positive relationship shows points going upwards from the origin and to the right. The scatter plot of a negative relationship shows points going down from the y-axis to the x-axis.
- If a data set has outliers something may have gone wrong in the data collection (or measurement). This requires further investigation. It may represent a valid, unexpected piece of the population needing further clarification. If the investigation uncovers an error, the researcher should fix it. If the data turns out to be from an individual that is not

part of the population, then it should be removed. If none of these are uncovered, then re-sampling may be needed.

Note

- A line of best fit or a curve of best fit can be drawn through the majority of the points and used to make predictions where there is a strong relationship between the two variables.

D1.6 Data Analysis

analyse different sets of data presented in various ways, including in scatter plots and in misleading graphs, by asking and answering questions about the data, challenging preconceived notions, and drawing conclusions, then make convincing arguments and informed decisions

Teacher supports

Key concepts

- Scatterplots are used to determine if a relationship exists between two numerical variables. Analysis of the scatter plot requires identifying how closely the points form a line or curve in order to conclude that there is a relationship.
- The range and the measures of central tendencies may be used to analyse data involving one variable.
- Sometimes graphs misrepresent data or show it inappropriately and this could influence the conclusions that we make about it. Therefore, it is important to always interpret presented data with a critical eye.
- Data presented in tables, plots, and graphs can be used to ask and answer questions, draw conclusions, and make convincing arguments and informed decisions.
- Sometimes presented data challenges current thinking and leads to new and different conclusions and decisions.
- Questions of interest are intended to be answered through the analysis of the representations. Sometimes the analysis raises more questions that require further collection, representation, and analysis of data.

Note

- There are three levels of graph comprehension that students should learn about and practise:

- Level 1: information is read directly from the graph and no interpretation is required.
- Level 2: information is read and used to compare (e.g., greatest, least) or perform operations (e.g., addition, subtraction).
- Level 3: information is read and used to make inferences about the data using background knowledge of the topic.

D2. Probability

describe the likelihood that events will happen, and use that information to make predictions

Specific expectations

By the end of Grade 8, students will:

D2.1 Probability

solve various problems that involve probability, using appropriate tools and strategies, including Venn and tree diagrams

Teacher supports

Key concepts

- Venn diagrams can be used to understand the relationship of probabilities involving multiple events that are given in order to solve a problem. The sum of the components of a Venn diagram is 100% of the total population or sample being referenced.
- Tree diagrams can be used to determine all the possible combinations of outcomes for two or more events that are either independent or dependent.

Note

- Sample space diagrams are a visual way of recording all of the possible outcomes for two events. The diagram below shows the possibilities when two coins are tossed.

		1st Coin	
		H	T
2nd Coin	H	(H, H)	(H, T)
	T	(T, H)	(T, T)

D2.2 Probability

determine and compare the theoretical and experimental probabilities of multiple independent events happening and of multiple dependent events happening

Teacher supports

Key concepts

- Two events are independent if the probability of one does not affect the probability of the other. For example, the probability for rolling a die the first time does not affect the probability for rolling a die the second time, third time, and so on.
- The more trials completed in an experiment, the closer the experimental probability will be to the theoretical probability.
- The sum of the probability of all possible outcomes is 1 or 100%.
- The probability of an event can be used to predict the likelihood of that event happening again in the future.
- Tree diagrams are helpful to determine all the possible outcomes for multiple independent events and multiple dependent events.

Note

- "Odds in favour" is a comparison of the probability that an event will occur with the probability that the event will not occur (complementary events). For example, the probability that the sum of two dice is 2 is $\frac{1}{36}$ and the probability that the sum of two dice is not 2 is $\frac{35}{36}$. The odds in favour of rolling a sum of 2 is $\frac{1}{36} : \frac{35}{36}$ or 1:35, since the fractions are both relative to the same whole.

E. Spatial Sense

Overall expectations

By the end of Grade 8, students will:

E1. Geometric and Spatial Reasoning

describe and represent shape, location, and movement by applying geometric properties and spatial relationships in order to navigate the world around them

Specific expectations

By the end of Grade 8, students will:

E1.1 Geometric Reasoning

identify geometric properties of tessellating shapes and identify the transformations that occur in the tessellations

Teacher supports

Key concepts

- A tessellation uses tiles to cover an area without gaps or overlaps. The angles where tiles meet always add up to 360°.
- Tessellating tiles are composed of one or more shapes and fit together in a repeating pattern. They are often used to create artistic designs, including wallpaper, quilts, rugs, and mosaics.
- Complex tessellating tiles can be designed by decomposing shapes and rearranging the parts using combinations of translations, reflections, and rotations (see also **SE E1.3**).
- If a shape can be transformed through a series of rotations, reflections and translations (i.e., by being turned, flipped, or slid), and still look the same, the shape is symmetric. Tessellating tiles are symmetric.
- There are different types of symmetries. For example, there is reflective symmetry, rotational symmetry, and translational symmetry. Symmetry is both an adjective (an attribute) and an action (a transformation). Creating and describing tessellating tiles involves symmetry as both an action and an attribute.

E1.2 Geometric Reasoning

make objects and models using appropriate scales, given their top, front, and side views or their perspective views

Teacher supports

Key concepts

- Two-dimensional drawings, if they are accurately constructed and include enough information, can be used to reproduce actual-sized objects or scaled models in three-dimensions (see also **SE E1.3**).

- Two-dimensional drawings can show how things are made, how they can be navigated, or how they can be reproduced, and can be used to represent anything from very small objects to very large spaces.
- Two-dimensional drawings are read and interpreted when navigating a map, following assembly instructions, or building an object from a plan.
- Top (plan) views, and front, and side (elevation) views are "flat drawings" without perspective. They are used in technical drawings to ensure a faithful reproduction in three-dimensions.
- A perspective drawing shows three views (top, front, side) in one illustration. It is preferred for illustrations; however, angles are distorted and backside elements may be hidden. Isometric grids (also called triangular grids) are used to draw different perspectives, including isometric and cabinet projections.

Note

- Cabinet projections are so named because of their early use in the furniture industry. Isometric means "equal measure" and isometric projections use the same scale.
- A scale is a ratio that compares actual dimensions to the dimensions in the drawing. A scale ensures that the intended lengths and proportions can be reproduced. Depending on the type of drawing, angles may or may not be represented accurately.
    - Top, front, and side views of an object or space (plan and elevation drawings) use the actual angle sizes in the drawing and show all lengths in a common scale. For example, a 60° angle in the drawing is 60° in real life, and a scale of 1:100 means that 1 cm on the drawing equals 100 cm in real life (or 1 mm on the drawing equals 100 mm in real life, and so on).
    - Isometric projections, drawn on a triangular grid, show all lengths in the same scale, including the lines that show depth. However, angles are distorted to create the appearance of perspective. Therefore, for example, a 90° angle in real life appears as a 60° in an isometric drawing.
    - Cabinet projections also distort angles but use two scales to create perspective. The "depth" scale is half that of the "base and height" scale. So, for a scale of 1:100, a cabinet projection of a 1 cm cube would have a base and height of 1 cm, but a depth of 0.5 cm.

E1.3 Geometric Reasoning

use scale drawings to calculate actual lengths and areas, and reproduce scale drawings at different ratios

E1.4 Location and Movement

describe and perform translations, reflections, rotations, and dilations on a Cartesian plane, and predict the results of these transformations

Teacher supports

Key concepts

- When shapes are transformed on a Cartesian plane, the coordinates of the original vertices are transformed to create corresponding coordinates known as image points. Each of the transformations can be defined using a mapping rule in which each point is transformed using that rule.
  - Mapping rule for translations:
    - $(x, y) \longrightarrow (x + a, y + b)$. If a is positive, then the x-value of the image point is to the right 'a' units from the original point. If a is negative, then the x-value of the image point is to the left 'a' units from the original point. If b is positive, then the y-value of the image point is up 'b' units from the original point. If b is negative, then the y-value of the image point is down 'b' units from the original point. For example, $(x, y) \longrightarrow (x - 5, y - 2)$; each of the image points are left 5 units and down 2 units from the original point.
  - Mapping rules for reflections:
    - a shape reflected in the x-axis has a mapping rule $(x, y) \longrightarrow (x, -y)$. For example, the vertex of the shape originally positioned at (2, 3) is now at (2, −3).
    - A shape reflected in the y-axis has a mapping rule $(x, y) \longrightarrow (-x, y)$. For example, the vertex of the shape originally positioned at (2, 3) is now at (−2, 3).
  - Mapping rules for rotations about the origin:
    - a shape rotated 90° counterclockwise has a mapping rule $(x, y) \longrightarrow (-y, x)$.
    - a shape rotated 180° counterclockwise has a mapping rule $(x, y) \longrightarrow (-x, -y)$.
    - a shape rotated 270° counterclockwise has a mapping rule $(x, y) \longrightarrow (y, -x)$.
  - Mapping rules for dilations:

- $(x, y) \longrightarrow (ax, ay)$. For example, $(x, y) \longrightarrow (2x, 2y)$; each of the image points are double the original points. For example, the image point for (−3, 4) would be (−6, 8).

Note

- Transformations on the Cartesian Plane involve points being relocated to another position.
- Translations “slide” a point, segment, or shape by a given distance and direction (vector).
- Reflections “flip” a point, segment, or shape across a reflection line to create its opposite.
- Rotations “turn” a point, segment, or shape around a point centre of rotation by a given angle.
- Dilations (or dilatations) “enlarge” or “shrink” a distance by a given scale factor. Scale factors with an absolute value greater than 1 enlarge the distance, and those with an absolute value of less than 1 reduce the distance. Negative scale factors dilate the shape and rotate it 180°.
- Translations, reflections, and rotations all produce congruent images:
  - Lines map to lines of the same length.
  - Angles map to angles of the same measure.
  - Parallel lines map to parallel lines.
- Dilations (or dilatations) produce scaled images that are *similar:*
  - Lines map to line lengths at a constant scale factor of the same length.
  - Angles map to angles of the same measure.
  - Parallel lines map to parallel lines.

E2. Measurement

compare, estimate, and determine measurements in various contexts

Specific expectations

By the end of Grade 8, students will:

E2.1 The Metric System

represent very large (mega, giga, tera) and very small (micro, nano, pico) metric units using models, base ten relationships, and exponential notation

Teacher supports

Key concepts

- Technology has enabled accurate measurements including very small and very large measures.
- All metric units are based on a system of tens and the metric prefixes describe the relative size of a unit (see **Grade 4, SE E2.2**). Whereas units from kilo- to milli- are scaled by powers of 10, units beyond these are scaled by powers of 1000. Exponents are helpful for representing these relationships.

Metric Prefix	Meaning	Factor
tera-unit	1 trillion units	1 unit × 1 000 000 000 000(10^{12})
giga-unit	1 billion units	1 unit × 1 000 000 000 (10^9)
mega-unit	1 million units	1 unit × 1 000 000 (10^6)
kilo-unit	1 thousand units	1 unit × 1000 (10^3)
unit	**one unit**	**1 unit (10^0)**
milli-unit	one thousandth of a unit	1 unit ÷ 1000 ($\frac{1}{10^3}$)or 10^{-3})
micro-unit	one millionth of a unit	1 unit ÷ 1 000 000 ($\frac{1}{10^6}$) or 10^{-6})
nano-unit	one billionth of a unit	1 unit ÷ 1 000 000 000 ($\frac{1}{10^9}$ or 10^{-9})
pico-unit	one trillionth of a unit	1 unit ÷ 1 000 000 000 000 ($\frac{1}{10^{12}}$ or 10^{-12})

E2.2 Lines and Angles

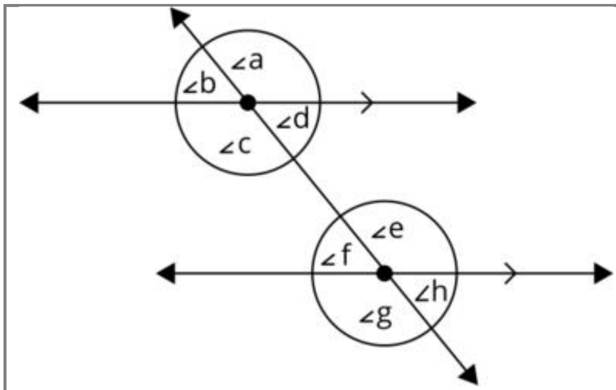
solve problems involving angle properties, including the properties of intersecting and parallel lines and of polygons

Teacher supports

Key concepts

- Angles can be measured indirectly (calculated) by applying angle properties.
- If a larger angle is composed of two smaller angles, only two of the three pieces of information are needed to calculate the third.
- Angle properties can be used to determine unknown angles.

- A straight angle measures 180°; this is used to determine the measure of a supplementary angle.
- A right angle measures 90°; this is used to determine the measure of a complementary angle.
- The interior angles of triangles sum to 180°, the interior angles of quadrilaterals sum to 360°, the interior angles of pentagons sum to 540°, and the interior angles of *n*-sided polygons sum to $(n - 2) \times 180$. The angle properties of a polygon can be used to determine the measure of a missing angle.
- The properties above can be used to determine unknown angles when a line (transversal) intersects two parallel lines:



- Alternate interior angles are equal, so $\angle c = \angle e$ and $\angle d = \angle f$ (Z-pattern).
- Opposite angles are equal, so $\angle b = \angle d$, $\angle a = \angle c$, $\angle f = \angle h$ and $\angle e = \angle g$ (angles formed by two lines intersecting).
- Alternate exterior angles are equal, so $\angle b = \angle h$, and $\angle a = \angle g$ (Z-pattern).
- Corresponding angles are equal, so $\angle b = \angle f$, $\angle c = \angle g$, $\angle a = \angle e$, and $\angle d = \angle h$ (F-pattern).
- Co-interior angles sum to 180°, so $\angle c + \angle f = 180°$ and $\angle d + \angle e = 180°$(C-pattern).

Note

- The aim of this expectation is not to memorize these angle theorems or the terms, but to use spatial reasoning and known angles to determine unknown angles.
- Smaller angles may be added together to determine a larger angle. This is the additivity principle of measurement.
- If two shapes are similar, their corresponding angles are equal (see **SE E1.3**). Recognizing similarity between shapes (e.g., by ensuring that the corresponding side lengths of a shape are proportional) can help to identify their corresponding angles.

E2.3 Length, Area, and Volume

solve problems involving the perimeter, circumference, area, volume, and surface area of composite two-dimensional shapes and three-dimensional objects, using appropriate formulas

Teacher supports

Key concepts

- Two-dimensional shapes and three-dimensional objects can be decomposed into measurable parts.
- The attributes of length (including distance, perimeter, and circumference), area (including surface area), volume, capacity, and mass all have the property of additivity. Measures of parts can be combined to determine the measure of the whole.
- For some attributes and for some shapes, relationships exist that can be expressed as formulas. To apply these formulas to composite shapes and objects, the shapes and objects are decomposed into parts that have known formulas. For example, an L-shaped area could be decomposed into two rectangles, and the smaller areas added together to calculate the whole. (Note: this will not hold true for its perimeter.)
- Applying formulas to real-world contexts requires judgement and thoughtfulness. For example, to apply the formula for the area of rectangle to a garden:
  - determine whether the garden is rectangular;
  - determine whether the garden is close enough to a rectangle that, for the needs of the moment, the formula can be applied;
  - if not rectangular, determine whether the garden can be broken into smaller rectangles (e.g., if it is an L-shaped garden) and the areas combined;
  - if decomposition results in other shapes, apply appropriate area formulas or draw a scale drawing on a grid and approximate the count of squares.
- Known length formulas at this grade level include:
  - *Perimeter* = *side* + *side* + *side* + ...
  - *Diameter* = 2 × *radius* ($2r$)
  - *Circumference* = π × *diameter* (πd)
- Known area formulas at this grade level include:
  - *Area* of a rectangle = *base* × *height*
  - *Area* of a parallelogram = *base* × *height*
  - *Area* of a triangle = $\frac{1}{2}$(*base* × *height*)
  - *Area* of a trapezoid = $\frac{1}{2}$(base 1 + base 2) × *height* (or its equivalent)
  - *Area* of a circle = π × *radius* × *radius* (πr^2)
- Known volume formulas at this grade level include:
  - *Volume* of a prism = (*area of the base*) × *height*

- *Volume* of a cylinder = (*area of the base*) × *height*

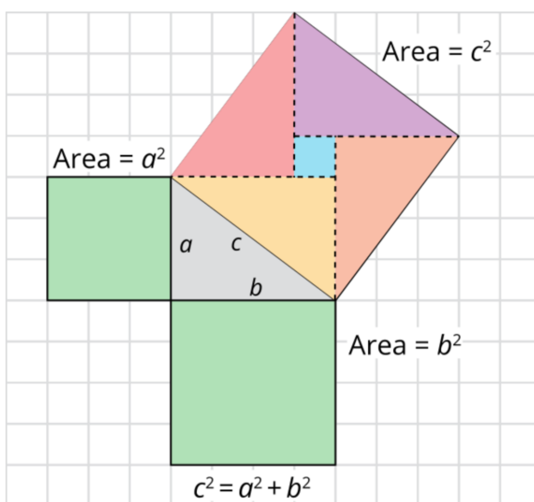
E2.4 Length, Area, and Volume

describe the Pythagorean relationship using various geometric models, and apply the theorem to solve problems involving an unknown side length for a given right triangle

Teacher supports

Key concepts

- The properties of a right triangle can be used to find an unknown side length. The longest side of a right triangle is always opposite the 90°angle and it is called the hypotenuse.
- Given any right triangle, the hypotenuse squared is equal to the sum of the squares of the other two sides. This is known as the Pythagorean relationship. The hypotenuse squared, and each of the sides squared can be represented as three squares formed with the three sides of the triangle, and then visualizing the sum of the areas of the two smaller squares as being equivalent to the area of the larger square.



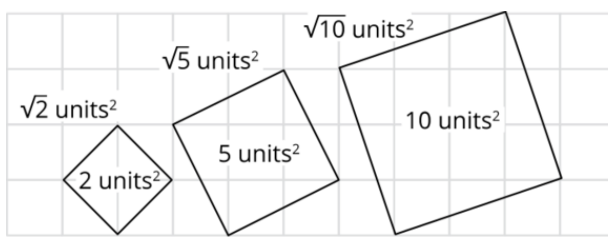
- The Pythagorean theorem expresses this relationship symbolically: $a^2 + b^2 = c^2$, where c is the length of the hypotenuse and a and b are the lengths of the other two sides of the triangle. For example:
  - if side a is 3 units long, then a square constructed on this side has an area of 3^2 or 9 square units;
  - if side b is 4 units long, then a square constructed on this side has an area of 4^2 or 16 square units;

- if the square on side *c* is equal to the combined areas of the squares on sides *a* and *b*, then the square on side *c* must have an area of 25 square units (9 square units + 16 square units);
- if the area of the square on side *c* is 25 square units, then the length of *c* must be $\sqrt{25}$ or 5 units.

- The inverse relationship between addition and subtraction means that the Pythagorean theorem can be used to find any length on a right triangle (e.g., $c^2 - b^2 = a^2$; $c^2 - a^2 = b^2$).
- The Pythagorean theorem is used to indirectly measure lengths that would be impractical or impossible to measure directly. For example, the theorem is used extensively in construction, architecture, and navigation, and extensions of the theorem are used to measure distances in space.

Note

- Dynamic geometric software can provide students with opportunities to expand their thinking about the Pythagorean relationship related to the areas of shapes other than squares (e.g., semi-circles with diameters that are each of the sides of the triangle.)
- The properties of a square can be used to find its side length or area. The side length of a square is equal to the square root of its area (see also **SE B1.3**). Constructing a square on a line is one way to indirectly measure the line's length.



F. Financial Literacy

Overall expectations

By the end of Grade 8, students will:

F1. Money and Finances

demonstrate the knowledge and skills needed to make informed financial decisions

Specific expectations

By the end of Grade 8, students will:

F1.1 Money Concepts

describe some advantages and disadvantages of various methods of payment that can be used when dealing with multiple currencies and exchange rates

Teacher supports

Key concepts

- There are different ways to make a payment in a different currency, and there are advantages and disadvantages for each.
- Exchange rates should be considered when making a purchase that requires a conversion of funds.

Note

- An understanding of how payments can be made in other currencies extends prior knowledge of money concepts.
- Relevant and meaningful contexts help consolidate their financial and mathematical understanding and skills.

F1.2 Financial Management

create a financial plan to reach a long-term financial goal, accounting for income, expenses, and tax implications

Teacher supports

Key concepts

- Income and expenses have an impact on short-, medium-, and long-term financial goals.
- Income, expenses, and tax implications are important elements to consider when creating a financial plan or budget to achieve a financial goal.

F1.3 Financial Management

identify different ways to maintain a balanced budget, and use appropriate tools to track all income and spending, for several different scenarios

Teacher supports

Key concepts

- Balancing a budget requires tracking income, fixed and variable expenses, spending, and saving.
- Various tools (e.g., spreadsheets, apps) are used to help balance budgets and adjust budgets as needed.

Note

- Social-emotional learning skills and financial management concepts and skills are concurrently developed.

F1.4 Financial Management

determine the growth of simple and compound interest at various rates using digital tools, and explain the impact interest has on long-term financial planning

Teacher supports

Key concepts

- Simple and compound interest grow differently over time in various saving and borrowing scenarios.
- Online tools available to calculate the impact of simple and compound interest over a long period of time.

Note

- Simulated scenarios provide opportunities for students to learn financial literacy concepts in relevant and real-life contexts.

F1.5 Consumer and Civic Awareness

compare various ways for consumers to get more value for their money when spending, including taking advantage of sales and customer loyalty and incentive programs, and determine the best choice for different scenarios

Teacher supports

Key concepts

- Discounts and loyalty programs are offered to consumers as a way for them to get more perceived value for their money.
- It is important to understand the advantages and disadvantages of these programs for both consumers and businesses.

Note

- Simulated scenarios provide opportunities for students to learn financial literacy concepts in relevant and real-life contexts.

F1.6 Consumer and Civic Awareness

compare interest rates, annual fees, and rewards and other incentives offered by various credit card companies and consumer contracts to determine the best value and the best choice for different scenarios

Teacher supports

Key concepts

- When choosing any type of consumer contract, such as for credit cards, or a data plan for example, it is important to identify the interest rates, fees, and incentives offered in order to make an informed decision.

- Before making their decision, informed consumers often compare the incentives offered by various businesses and consider the benefit and costs of these incentives for the consumers versus the benefit for businesses.

Note

- Simulated scenarios provide opportunities for students to learn financial literacy concepts in relevant and real-life contexts.