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TOWARD A UNIFIED CLASSICAL MODEL OF THE SUN: ON THE SENSITIVITY OF NEUTRINOS AND HELIOSEISMOLOGY TO THE MICROSCOPIC PHYSICS

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ABSTRACT

This paper focuses mainly on the neutrino puzzle and discusses the point of view that neutrinos and helioseismology are two complementary probes of the solar interior. We first analyze the physical differences noticed between already published solar models and their consequences for neutrino predictions. Including improvements achieved in microscopic physics these last 3 years, we propose new results on the solar neutrino predictions and acoustic mode frequencies for $l = 0$–150, in the classical framework of stellar evolution. Doing so, we quantify the influence of precise composition, nuclear reaction rates, screening effect, and opacity calculations on both neutrino and acoustic mode frequency predictions. Our present predictions are 6.4 ± 1.4 SNU for the chlorine experiment, $4.4 \pm 1.1 \times 10^6$ cm^{-2} s^{-1} for the water detector, and 122.5 ± 7 SNU for the gallium detector. Considering that the present experimental situation may support the hypothesis that neutrinos and helioseismology are both representative of the Sun, we then try to derive from the sound speed behavior an estimate of how the neutrino predictions may progress, including phenomena not yet simulated at present; we suggest that a correct description of the region located between the nuclear region and the convective zone justifies an increase of the ^{8}B neutrino prediction by no more than 10%–15%, and that the nuclear reaction rates of the *p-p* chain should be revisited.

Subject headings: elementary particles — nuclear reactions, nucleosynthesis, abundances — Sun: interior

1. INTRODUCTION

The Sun is a unique star because of the quality of its observations (luminosity, mass, radius, and composition). Moreover, its evolutionary stage corresponds to the first stage of hydrogen burning, which is the best known and the longest one. So the astrophysical community is extremely motivated to study this stage in detail in order to put some constraints on the helium initial content and on the age of low-mass stars. In the early 1980s, people working on stellar evolution produced a reference solar model along the framework of the classical stellar evolution (Bahcall et al. 1982; Christensen-Dalsgaard 1982; VandenBerg 1983; Noels, Scuflaire, & Gabriel 1984; Bahcall et al. 1985; Lebreton & Maeder 1986). This activity was encouraged by the detection and interpretation of a large number of acoustic modes. On the neutrino side, the surprise was twofold: the predictions were in disagreement with the first neutrino experiment, the chlorine experiment developed by Davis and collaborators (Davis 1964; Rowley, Cleveland, & Davis 1984), and the predictions differed significantly among various authors, even with rather similar hypotheses. With the development of helioseismology, more stratified information has been offered, and the sensitivity of such models has pushed large and complementary communities to work together.

At the end of the 1980s, more and more astrophysical groups became conscious of the importance of a precise determination of a reference solar model. In fact, the astrophysical community would like to establish some constraints deduced from specific solar observations or solar neutrino detections, on the phenomena which are present in stars but which are still too difficult to include in a stellar calculation: rotation, convection (correctly treated), and mixing (Schatzman et al. 1981; Pinsonneault et al. 1989). A detailed comparison between models of different groups has convinced us (Turck-Chièze et al. 1988a) that the differences were not due to numerical inaccuracy or errors in the computation but came, in fact, from the adopted microscopic description of the Sun. Moreover, the multiplication of the results—Kamiokande neutrinos (Hirata et al. 1989, 1990, 1991), gallium experiments, the detection of about 2700 acoustic modes (Libbrecht, Woodard, & Kaufman 1990)—has stimulated atomic and nuclear communities to improve both the experimental and the theoretical aspects of the solar problem.

The purpose of this paper is to return once more to this first stage of the calculation, that is, in the classical framework of stellar evolution, and extensively study the effect of microscopic physics on the predicted observables: neutrinos and accoustic modes.

We first focus on the interpretation of the differences between the already published solar models (§§ 2 and 3) which reveal the importance of some ingredients and allow us to extend previous similar analysis (Bahcall & Pinsonneault 1992). In order to perform a comparative analysis of neutrinos and accoustic modes, we then introduce the recent improvements noticed in the last 3 years, completing some other recent works (Guenther et al. 1992; Bahcall & Pinsonneault 1992; Berthomieu et al. 1992) by a discussion on screening and photospheric abundances (§ 4). Section 5 presents the effect of these improvements on the structure of the Sun and on the neutrino flux predictions and compares with the recent experimental results. Sections 6 and 7 will be devoted to the helioseismology signature of such improvements. Section 8 is speculative and suggests possible astrophysical consequences of what we have learned from the general experimental situation.

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TABLE 1

COMPARISON OF PRESENT STRUCTURE OBTAINED BY RECENT REFERENCE SOLAR MODELS

Parameter	Bahcall & Ulrich 1988	Turck-Chièze et al. 1988a	Cox et al. 1989	Sackmann et al. 1990
T_C	15.62×10^6 K	15.52×10^6 K	15.68×10^6 K	15.42×10^6 K
ρ_C	148 g cm^{-3}	147.2 g cm^{-3}	162.4 g cm^{-3}	146.6 g cm^{-3}
T_{BCZ}	2.11×10^6 K	2.04×10^6 K	2.29×10^6 K	1.96×10^6 K
d_{CZ}	0.277	0.270	0.286	0.260
Y	0.271	0.275	0.291	0.278
α	...	2.11[a]	1.89	2.07

NOTE.—T_C is the central temperature of the Sun, ρ_C the central density, T_{BCZ} the temperature at the base of the convective zone, d_{CZ} the depth of the convective zone, Y the helium initial mass fraction, and α the mixing-length parameter.

[a] The published value was 1.6, due to an error in the hydrogen equation of state.

2. COMPARISON OF THE DIFFERENT ESTIMATES OF THE SOLAR STRUCTURE

Several solar models were published at the end of the 1980s (Bahcall & Ulrich 1988; Turck-Chièze et al. 1988a; Cox, Guzik, & Kidman 1989; Guenther, Jaffe, & Demarque 1989; Sackmann, Boothroyd, & Fowler 1990). Even if they were generally including updated physics, significant differences or surprising inconsistencies were still present in neutrino predictions, which expressed the limits of the consensus on the input microphysics or emphasized the problems still open. Moreover, the detection of acoustic modes and their interpretation in terms of sound speed revealed that the Sun should really be considered as a unique laboratory of plasma physics. The general agreement on the sound speed behavior has justified complementary questions on these ingredients (see the reviews of Christensen-Dalsgaard & Berthomieu 1992; Christensen-Dalsgaard & Däppen 1992; Turck-Chièze et al. 1993).

To be more quantitative, we would like to discuss first four consistent calculations performed by Bahcall & Ulrich (1988, hereafter BU88), Turck-Chièze et al. (1988a, hereafter TCCCD88), Cox et al. (1989, hereafter CGK89), and Sackmann et al. (1990, hereafter SBF90). They used the reaction rates compiled by Caughlan & Fowler (1988) and the opacity coefficients obtained from the LAOL library (Huebner et al. 1977) and were representative of the solar model calculations published at that time. Tables 1–3 summarize the situation.

2.1. *The Initial Helium Content*

The internal parameters of the Sun appear to be very similar in these four models (Table 1). However, if one considers that the hypotheses of the calculations and the main ingredients are practically the same, one may notice a larger difference in the initial helium content Y_0 than the generally estimated error on this quantity (BU88; Turck-Chièze et al. 1988a). This element, unmeasurable in the photosphere, is a crucial ingredient for stellar modeling and is generally extracted from solar modeling in matching the observed luminosity at the present epoch. The different predictions extend over 7% (see Table 1), and the high value (0.29) of Cox et al. (1989) is also noticed in other recent models (Guenther et al. 1989).

Constraints on this number are not yet available; in fact, the solar helium value should be compared with the values obtained for other stars in our Galaxy, and with its cosmological value. If this primordial helium is now more and more accurately determined from observations of extragalactic H II regions, $Y_P = 0.228 \pm 0.005$, the dependence on metallicity, which results from the enrichment in helium by chemical evolution of the Galaxy, is not sufficiently precise to put severe constraints on the solar value (Pagel 1991 and references therein).

Regarding the solar system, large improvements have been achieved in the determination of helium in the giant planets as a result of the *Voyager* missions. Initially, one had hoped that

TABLE 2

COMPARISON OF EXPERIMENTAL NEUTRINO RESULTS WITH PREVIOUS PREDICTED CALCULATIONS

	THEORETICAL PREDICTIONS		
EXPERIMENTAL RESULTS	BU88	TCCCD88	SBF90
Chlorine experiment:			
2.33 ± 0.25 SNU	7.9 ± 2.5 ($3\ \sigma$)	5.8 ± 1.3 ($1\ \sigma$)	7.7
Ratio experiment/theory (R_H)	$(29.4 \pm 11 \pm 10)$%	$(40 \pm 11 \pm 9)$%	...
Kamiokande II experiment:			
0.28 ± 0.03 events day^{-1} ($E \geq 7.5$ MeV)	0.62 events day^{-1}	0.41 events day^{-1}	0.62 events day^{-1}
2.67 ± 0.29 cm^{-2} s^{-1}	5.8 ± 2.15 ($3\ \sigma$)	3.8 ± 1	5.8
Ratio experiment/theory ($R_{K\,II}$)	$(46 \pm 5 \pm 6 \pm 17)$%	$(70 \pm 8 \pm 9 \pm 19)$%	...
Gallium experiments			
(see Table 8)	132^{+20}_{-16} ($3\ \sigma$)	125 ± 5 ($1\ \sigma$)	125

NOTE.—BU88 = Bahcall & Ulrich 1988; TCCCD88 = Turck-Chièze et al. 1988a; SBF90 = Sackmann, Boothroyd, & Fowler 1990. The ratio experiment/theory includes the experimental errors (statistical or statistical and systematic) and the theoretical ones ($3\ \sigma$ in the BU calculation and $1\ \sigma$ for TCCCD88 according to the authors; see remark on errors in § 5.3). For the Kamiokande experiment, the theoretical estimates in terms of events day^{-1} include a correction for detector efficiency (Kim 1989); the experimental results expressed in terms of flux include the inverse correction and the neutrino energy dependence given by BU88. (1 SNU = 10^{-36} captures atom^{-1} s^{-1}).

TABLE 3

THEORETICAL PREDICTIONS FOR NEUTRINO FLUXES COMING FROM DIFFERENT SOURCES

NEUTRINO SOURCES	NEUTRINO FLUXES (10^{10} cm^{-2} s^{-1}) BU88	TCCCD88	SBF90
pp	$6.0\ (1 \pm 0.02)$	$5.98\ (1 \pm 0.03)$	6.00
pep	$1.4 \times 10^{-2}\ (1 \pm 0.05)$	1.30×10^{-2}	1.29×10^{-2}
^{7}Be	$4.7 \times 10^{-1}\ (1 \pm 0.15)$	4.18×10^{-1}	4.23×10^{-1}
^{8}B	$5.8 \times 10^{-4}\ (1 \pm 0.37)$	$3.83 \times 10^{-4}\ (1 \pm 0.26)$	5.8×10^{-4}
^{13}N	$6.1 \times 10^{-2}\ (1 \pm 0.5)$	6.27×10^{-2}	3.99×10^{-2}
^{15}O	$5.2 \times 10^{-2}\ (1 \pm 0.58)$	5.60×10^{-2}	3.09×10^{-2}
^{17}F	$5.2 \times 10^{-4}\ (1 \pm 0.46)$		4.23×10^{-4}

these measurements would provide the value of the helium abundance of the primitive solar nebula, but the first results have shown that the situation was more complex: $Y_{\rm Saturn}$ was found to be 0.06 ± 0.05 (Conrath et al. 1984), while the Jovian value was 0.18 ± 0.04 (revised by Gautier et al. 1981). These low values have been explained by theories of planetary evolution and high-pressure thermodynamics: at pressures higher than 3 Mbar, hydrogen undergoes a transition to a metallic state (Nellis et al. 1983), helium becomes immiscible in metallic hydrogen at low temperatures, and helium droplets can form and sink toward the center of the planet (Stevenson 1982). Neptune and Uranus seem, according to current interior models, better candidates owing to their thin atmospheres and lower hydrogen pressures in the core. In fact, the recent results confirm this prediction, leading to the following values $Y_{\rm Uranus} = 0.262 \pm 0.048$ (Conrath et al. 1987), $Y_{\rm Neptune} = 0.32 \pm 0.05$ (Conrath et al. 1991). These results, if encouraging, are not yet very precise, partly because of the presence of clouds which may render the extraction of helium content uncertain.

The only observed value of solar helium comes from the study of the prominences, where Heasley & Milkey (1978) deduce a value of $Y = 0.28 \pm 0.05$ which is not very precise and is comparable to those for the outer planets.

Nowadays, the result of solar modeling on the initial helium content represents the best way to reach such a number, with an estimated accuracy of 4%, so the largest difference noticed in Table 1 was questionable. The origin of such a discrepancy was due to the treatment of the equation of state (Däppen 1991; Christensen-Dalsgaard & Däppen 1992). In the central region of the Sun the equation of state is rather simple, practically a perfect gas with a weakly degenerated electron gas; even if iron is not completely ionized, it plays a very small role in the gas pressure (less than 0.5%), but the Coulomb pressure is more important and represents about 2%. The difference in helium and in density predictions arose from the absence of Coulomb correction effect in the Eggleton, Faulkner, & Flannery (1973) equation of state, well suited for stellar pulsation applications and used by CGK89. The introduction of the Debye-Hückel term modifies the pressure in the solar interior and causes higher ionization fractions particularly sensitive in the ionization zone of hydrogen and helium toward the surface (Christensen-Dalsgaard, Däppen, & Lebreton 1988; Christensen-Dalsgaard & Däppen 1992). This correction was introduced in the simpler treatment of Vardya (1964) used by BU88, TCCCD88, and probably SBF90. Such higher value of the initial helium content obtained by CGK89 corresponded also to a higher central temperature and, consequently, to higher neutrino flux predictions for chlorine and water experiments. In the following, we will only consider the three other calculations for the comparison with neutrino detections.

2.2. *The Convective Zone*

A complete treatment of the convection is not yet available for stellar evolution, and people perform the calculations in the framework of the "mixing-length theory" (MLT; Böhm-Vitense 1958). This approximation conceptually replaces the real situation of "plumes," convective eddies of different sizes, by an average situation where one considers that each convective element travels, on average, a distance Λ before mixing with the surrounding matter. This distance is scaled to the pressure scale height by a parameter α called the mixing-length parameter: $\Lambda = \alpha(d \ln P/dr)^{-1}$. In the solar models, this value α is adjusted to obtain the present solar radius at the present age and largely depends on the opacity coefficients near the surface where both convective and radiative transports are in competition. The present value of α, which is about 2 (see Table 1) correspond to the Los Alamos calculation for temperatures smaller than 10^4 K (Cox 1986) and replaced the typical value of 1.6 corresponding to the Cox & Tabor opacities (Cox & Tabor 1976). The difference came mainly from the introduction of molecular components. More recent results may lead to a different estimate.

The Sun is the only case where one may estimate from helioseismology the depth of the convective zone. From the theoretical point of view, the base of the convective zone depends largely on the opacity coefficients at about 0.7 $R_\odot$. Reference models of Table 1 led to a thinner convective zone than the one deduced from acoustic mode frequencies (see § 6.1) with a small dispersion, which reflects small differences in the opacity coefficients. The better agreement of the CGK89 calculation is due to an ad hoc local increase of the opacity coefficients to reproduce the observed acoustic frequencies. The understanding of this convective depth is crucial for the determination of an eventual presence of undershooting or for the interpretation of the observed solar 99% deficiency of lithium.

2.3. *The Opacity Coefficients*

It is well known that this is a crucial ingredient for the evolution of the Sun. In fact, contrary to the case of massive stars where the radiative transport is dominated by Thomson scattering, the domain of temperature and density of low-mass stars requires precise estimates of the different interactions: Thomson scattering, free-free processes, bound-free and bound-bound processes (Fig. 1). This is important not only for the most abundant elements but also for the heavy elements. It has been established for a long time (Huebner 1986), that the contribution of heavy elements, even with small mass fraction (<2%), represents about 40% in the central part of the Sun and 90% at the base of the convective zone. The reason for such a role is the important contributions of the elements

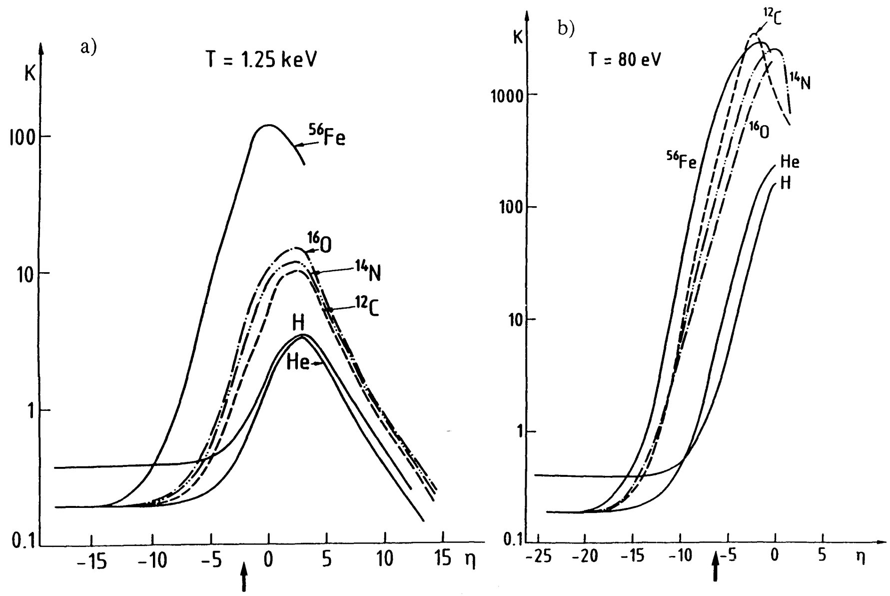

FIG. 1.—Opacity coefficients of an individual element for two thermodynamic situations corresponding to (*a*) the central solar temperature and (*b*) the convective zone temperature as a function of the degeneracy parameter η. The solar case is noted with an arrow. The Thomson scattering, independent of η and of the ion (except for a factor of 2 between hydrogen and the others due to the ratio electrons/nucleons) is clearly visible at low η. In (*a*) only the iron case contains bound-free and bound-bound contributions; the behaviors of C, N, O corresponding to free-free processes are very similar. In (*b*) all the species are partially ionized, except hydrogen and helium.

which are partially ionized. In central solar conditions, all the elements are totally ionized except iron, with four to six bound electrons, and the corresponding contributions of bound-bound and bound-free cross sections largely dominate the photon spectrum in the region where the Rosseland weighting function is maximum (see the contribution of iron in Fig. 1*a*). From the center of the Sun toward the photosphere, iron, silicon, and then neon become partially ionized and add discrete contributions. Near 0.7 $R_{\odot}$, the contribution of partially ionized oxygen, followed by nitrogen and carbon, leads to such a large increase of the opacity coefficients that radiative transport ceases to be sufficiently efficient and gives up to convective instability (see Turck-Chièze et al. 1993, and see Fig. 1*b*, where the contribution of all the heavy elements is important relative to hydrogen or helium).

Turck-Chièze et al. (1988a) have pointed out that the uncertainty in opacity coefficients is an important source of error for neutrino predictions, and Courtaud et al. (1990) have tried to estimate the sources of errors coming from the inaccuracy of the composition (mainly the determination of oxygen, nitrogen, carbon, and iron) and from the inaccuracy of the calculation itself. We will come back to these points quantitatively in § 4.

Let us discuss now the difference in central temperature noticed in Table 1. The 0.7% temperature difference between the three compatible models can be interpreted as due mainly to opacity coefficient variations. One can consider, as an order of magnitude, that 10% difference in opacity coefficients leads to 1% difference in temperature. The origin of such a difference in opacity coefficients has already been studied by Turck-Chièze (1990a, b, 1992). We recall here the conclusions:

1. Bahcall & Ulrich (1988) thought that the correction due to the conversion of carbon and oxygen to nitrogen during hydrogen burning led to a substantial modification of the opacity coefficients in the central part of the Sun. Effectively, 99.5% of carbon and 3% of oxygen are converted to nitrogen, but the opacity coefficients of these three elements are so similar in this region (because they are all three completely ionized; see Fig. 1*a*) that the modification of the Rosseland mean opacity is smaller than 1% and naturally accounted for by an increase of Z by 2% in the central region (Z is the mass fraction of the heavy elements). In the BU88 and the TCCCD88 calculations, this is automatically done by interpolating between tables of different Z to match the Z/X observational ratio at the beginning of the evolution. This point has been confirmed by Iglesias & Rogers (1990). This unjustified correction mainly affected the very center of the Sun, and the opacity coefficients used by BU88 must be reduced by 6% just near the center.

2. Boercker had pointed out the importance of the collective effect of the electrons on Thomson scattering: 30% (Boercker 1987). This correction is taken into account in all calculations. Such a correction was introduced in the Los Alamos library in 1978, and two versions, with and without correction, exist.[2] The Saclay group used the second version, and needed an external correction for this collective electron effect. Turck-Chièze et al. (1988a) have used the correction suggested by Bahcall et al. (1982), which slightly overestimates the effect. Effectively, this correction concerns mainly hydrogen and partly helium (see Fig. 1*a* near $\eta = 1$) but is extremely small for the other elements dominated by free-free processes. We consider, in fact that this effect modifies the intrinsic opacity coefficients by $\kappa' = \kappa - 0.027(1 + X)$ instead of $\kappa' = \kappa - 0.07(1 + X)$, where X is the hydrogen fraction mass content. The reduction was maximum in the central region and has

[2] For those who would like to check the version they use, they may check the hydrogen value at $T = 1.25$ keV, $\eta = 1$: $\kappa(\mathrm{H}) = 2.6729$ cm^2 g^{-1} (without correction) and 2.4847 cm^2 g^{-1} (with correction) (Argo 1989).

introduced some 5% underestimate of the opacity coefficients in TCCCD88 results. This point has been overestimated in the analysis of Bahcall & Pinsonneault (1992).

3. Some other small differences may be due also to different compositions used in the calculation of the opacity coefficients, but the two previous points led to an intermediate value of the central temperature of 15.58×10^6 K. The lower temperature of SBF90's calculation is not yet understood.

2.4. *Other Differences Noticed*

SBF90 and TCCCD88 used the value of the astrophysical factor $S(0)$ recommended by Caughlan & Fowler (1988) for the reaction (^{3}He, ^{3}He): $S(0) = 5.57$ MeV barns; BU88 have adopted a lower astrophysical factor $S(0) = 5.15$ MeV barns to take into account electron screening effects in the laboratory (see discussion in § 3). The consequent difference in central temperature is really small.

Other differences have been noticed in the computations, for which we are unable to estimate the consequences, As one assumes hydrostatic equilibrium, the time step of the evolution is adapted to take into account this hypothesis; it is automatically done by following the most rapidly evolving parameters. Most of the evolutionary codes approach the present age of the Sun in 25–30 steps, but this was not the case for the BU88 calculation, which reaches the present age after 14 steps. Looking at the results, it seems that the consequences might be small, but....

Another problem was pointed out in the BU88 paper regarding the spatial grid of TCCCD88 in the central region. It could be an important point for the neutrino flux of ^{8}B, which is peaked near 0.05 $R_\odot$; thus special attention has been paid to describe precisely the central part. A substantial increase of the number of mesh points has not produced significant modification of the ^{8}B neutrino flux.

3. THE DIFFERENT NEUTRINO PREDICTIONS

Table 2 summarizes the different neutrino predictions of the previously described solar models.[3] The discrepancy between predictions is important for the chlorine and water detectors and justifies a detailed discussion.

As the three different calculations use the same absorption cross sections (see details in Bahcall 1989), the discrepancy between them concerns the astrophysical part of the calculation summarized in Table 3. If the noticed differences of Table 1 seem subtle, it is evident that their consequences for neutrino fluxes should not be ignored, and comparisons with experimental results require understanding such a dispersion.

The agreement in *pp* flux is excellent: this is not surprising, since this flux is independent of solar modeling. For *pep* there is a difference of up to 8% without consequences for neutrino prediction, owing to the absence of screening for this reaction in TCCCD88 calculation (see discussion on screening). The difference in CNO neutrino predictions is due to different reaction rates.

The difference in the ^{7}Be neutrino flux may practically be explained by the difference in temperature previously discussed and understood. As was pointed out by BU88, from 1000 different computations of a solar model with different input parameters, the central temperature dependence of the main neutrino actors is in fact

$$\Phi_{pp} \propto T_C^{-1.2}\,, \quad \Phi_{^7\mathrm{Be}} \propto T_C^{8}\,, \quad \Phi_{^8\mathrm{B}} \propto T_C^{18} \qquad (1)$$

[3] Compared with the experimental results available at the same period.

instead of the dependence deduced directly from the nuclear reaction rates for the central conditions,

$$\Phi_{pp} \propto \rho_C X_C^2 T_C^4\,, \quad \Phi_{^7\mathrm{Be}} \propto \frac{1 - X_C}{1 + X_C}\, T_C^{11.5}\,, \qquad \Phi_{^8\mathrm{B}} \propto \rho_C \frac{1 - X_C}{1 + X_C}\, T_C^{24.5}\,. \qquad (2)$$

This is due to the luminosity constraint (and the resulting composition and density readjustments) and also to the radial extension of the different sources of neutrinos (see Fig. 8 of BU88) producing a different dependence on the temperature. A temperature variation of 0.7% corresponds to a 6% variation of the predicted ^{7}Be neutrino flux. The residual difference of 4% comes from different prescriptions used for the screening effect in the nuclear reactions (see below). In the comparison between BU88 and SBF90 the different ^{7}Be neutrino predictions are totally compatible with the differences in central temperatures, these two groups using the same Salpeter (1954) screening factor for the reaction rates.

The case of the ^{8}B neutrino flux is more puzzling. The absence of a difference between BU88 and SBF90 is surprising, as one would expect a value lower by 25% for SBF90 coming from the difference in central temperature. In fact, the agreement is fortuitous and corresponds to an excessive screening correction on the ^{7}Be electron capture in the SBF90 calculation (Boothroyd 1992) (see the discussion on screening, below). To understand the large difference between BU88 and TCCCD88, one must consider the 0.7% difference in temperature with a consequent 12.5% difference in ^{8}B neutrino flux (see previous section on opacities), a difference in the adopted ^{7}Be$(p, \gamma)^8$B reaction rate with a resulting 16% difference in ^{8}B neutrino flux, and a different screening prescription with 13.5% difference in the same neutrino flux ($3.83 \times 1.42 = 5.5$). Let us comment on the last two effects.

3.1. *The Screening Factor in the Nuclear Reaction Rates*

We have already evoked the role played by the Coulomb effect in the equation of state, which has important consequences for the determination of the initial helium content and consequently of the initial hydrogen content. We have also recalled the collective effects of the electrons on the Thomson scattering in the opacity calculation. We consider here the role of the free electrons and the influence of the electron plasma on the determination of the nuclear reaction rates.

This effect is different in laboratory experiments and in the Sun. In laboratory experiments, one considers, for example, a proton interacting with an atomic target. At a very low energy, typically the range of energy corresponding to stellar thermal energy, the interaction is largely modified by the electron cloud. So if one would like to approach the astrophysical range of energy to avoid theoretical extrapolation of the measured cross section to the range of interest, a proper correction of the data is needed. This is well known since the work of Assembaum, Langanke, & Rolfs (1987). The (^{3}He, ^{3}He) cross section has been measured down to 17 keV, where this effect increases the interaction by about 10%. This correction was not yet introduced in the interpretation of the measurement made by Krauss, Trautvetter, & Rolfs (1987). Consequently, TCCCD88 and SBF90 have not introduced this effect in their solar models: $S(0) = 5.57$ MeV barns, contrary to the case of BU88: $S(0) = 5.15$ MeV barns. Even if this difference is important in absolute value, the consequences for the structure of the

Sun and for the neutrino fluxes are quite small: the chlorine neutrino capture rate is increased by only 0.2 SNU, because the *pp* chain is totally dominated by the slowest reaction: $p + p \rightarrow d + e^+ + \nu_e$.

As has already been noticed by Salpeter (1954), similar effects are found in stars. In the central region of the Sun the atoms are, except for iron, completely stripped of their atomic electrons, and nuclei are immersed in a sea of free electrons. These electrons generally cluster in the area of the nuclei. This shielding effect reduces the Coulomb potential and increases the thermonuclear reaction rates within the star by the factor

$$f = \exp\left(\frac{E_\mathrm{D}}{kT}\right) = \exp\left(\frac{Z_1 Z_2 e^2}{R_\mathrm{D} kT}\right), \tag{3}$$

where R_D is the Debye-Hückel radius, ξ the rms charge average, and θ_e the degeneracy factor:

$$R_\mathrm{D} = \left(\frac{kT}{4\pi e^2 \rho N_\mathrm{A} \xi^2}\right)^{1/2}, \quad \xi^2 = \sum_i \frac{[Z_i(Z_i + \theta_e)]X_i}{A_i}.$$

This is the so-called weak screening approximation. At an intermediate regime of ρ and T, most of the nuclei are bound in a Coulomb lattice structure but the reacting pair of high-energy nuclei is still free. The electrostatic effects are then large, and this regime corresponds to the strong screening interaction, which is also expressed by a multiplicative correction factor (Schatzman 1958; Salpeter & Van Horn 1969). One generally considers that the hypothesis of weak screening is correct if $\Lambda = E_\mathrm{D}/kT \ll 1$. In the central region of the Sun ($T \approx 15 \times 10^6$ K and $\rho \approx 150$ g cm^{-3}), this ratio may reach 0.2–0.4 in an interaction between proton and ^{7}Be or ^{14}N, so the hypothesis of "intermediate screening" is better adapted. It was developed by DeWitt, Graboske, & Cooper (1973), and a tentative generalization to a two-component plasma was made by Graboske et al. (1973, hereafter GDGC). They have considered a statistical-mechanical theory for the screening function *f*, suitable for most astrophysical cases without assumptions on the charge mixture and the charge of the reacting particles. They propose an expression in terms of chemical potentials which may reduce to the case of weak or strong interactions:

$$\log f = k_b \eta_b \Lambda_0^b [(Z_1 + Z_2)^{1+b} - Z_1^{1+b} - Z_2^{1+b}], \tag{4}$$

where Λ_0 is a variable independent of the charge, which characterizes the plasma:

$$\Lambda_0 = 1.88 \times 10^8 (\rho/\mu_I T^3)^{1/2}.$$

The exponent b lies between 1 and $\frac{2}{3}$, corresponding to low- and high-density limits. The parameter η_b takes into account the appropriate charge plasma average ξ, an rms value including degenerate electrons (if necessary). The difficulty for intermediate screening is to find the intermediate average charge.

In the case of weak screening, $b = 1$, $k_b = \frac{1}{2}$, $\eta_b = \xi$, $\zeta_b = 2Z_1 Z_2$, and one retrieves the classical screening factor of equation (3). For intermediate screening, corresponding to $\Lambda > 0.1$,

$$b = 0.860, \quad k_b = 0.380, \quad \eta_b = \frac{\langle z^{3b-1} \rangle}{\xi^{3b-2} z^{2-2b}}. \tag{5}$$

The limitation of such an approach comes from the nonrelativistic treatment and the simplified one or two specific components of the plasma.

This screening correction concerns ion interactions and may not have been used for electron capture rates. In this case, weak interaction is concerned, and because of the kinetic energy of the electrons, the interactions are not at all of the same type: the study of Johnson et al. (1992) shows an order of magnitude of 3%.

Let us study the influence of all these aspects of screening on the neutrino predictions. We have discovered very recently that the prescriptions differ widely between authors: BU88 and SBF90 used the weak screening approximation of Salpeter (1954), and TCCCD88 used the intermediate screening prescription of GDGC. When BU88 and SBF90 only consider weak screening, they overestimate the chlorine prediction by 11% and the Kamiokande prediction by 12%, in comparison with the TCCCD88 calculation. Moreover, SBF90 introduces this Salpeter correction also for the electron capture rate on ^{7}Be; incidentally, they modify the branching ratio between chain II and chain III of the *p-p* chain. A correct treatment would lead to an added reduction of their ^{8}B neutrino flux by 20%, leading to $\Phi_{^8\mathrm{B}} = 4.6$ cm^{-2} s^{-1} and a consequent chlorine reduction (Boothroyd 1992). So their agreement with BU88 calculation was a coincidence. In the TCCCD88 calculation the reaction *pep* was not corrected for screening; this mistake led to a corresponding reduction of the flux by 5% (see Table 3), with a very small error (0.02 SNU) in the chlorine prediction.

3.2. *The* ^{7}Be$(p, \gamma)^8$B *Cross Section*

This reaction is crucial for the determination of the neutrino ^{8}B flux but does not affect the structure of the Sun, because the participation of the chain III in the luminosity of the Sun is very small. The corresponding reaction rate is difficult to determine because the experiment is delicate owing to the radioactivity of the target and must be coupled with precise theoretical extrapolation to the astrophysical range of interest. Thus, everybody was using the mean value of all the experiments performed since 1965, but BU88 and SBF90 used the pioneering Tombrello (1965) calculation for the extrapolation, since TCCCD88 used the Barker (1980) more complete calculation (see also Barker 1983; Barker & Spear 1986). This theoretical difference, mainly due to the treatment of appropriate wave functions, led to a reduction of 15% in ^{8}B flux and 13% in the chlorine prediction (see Turck-Chièze et al. 1993). This point will be discussed again in the next section.

3.3. *Synthesis of the Comparison of Different Theoretical Prescriptions*

We have noticed several differences which affect the neutrino predictions and the helium content. Some of these influence the structure of the Sun: equation of state, opacity coefficients, and nuclear reaction rates. In this case the consequences of a modification are partially limited by the luminosity constraint; accordingly, the gallium prediction is only slightly modified. Some other differences mainly influence the neutrinos produced by heavy nuclei such as ^{8}B, ^{13}N, ^{15}O: the effect of screening and the ^{7}Be$(p, \gamma)^8$B cross section. Due to the absence of constraints from the luminosity, the high-energy neutrinos react strongly to these modifications.

These differences have really pointed out the sensitivity of some neutrino predictions to the microscopic inputs of solar modeling. We consider that taking into account all these remarks with the input physics available at the period of these publications, one of the most correct predictions would have been 6.3–6.5 SNU for the chlorine experiment and a flux of about 4.2×10^6 cm^{-2} s^{-1} for the Kamiokande experiment with, as was previously discussed in TCCCD88, a rather large error bar of at least 15%–20% mainly because of the uncertainties in opacity coefficients and in the ^{7}Be(p, γ)^{8}B reaction rate.

4. IMPROVEMENTS IN THE INPUTS OF THE CALCULATION

Since these last publications, a lot of improvements have appeared in basic atomic and nuclear physics which modified some ingredients of the calculations. Some of them have also been discussed by Guenther et al. (1992), Bahcall & Pinsonneault (1992), or Berthomieu et al. (1992). Let us consider the following topics: reaction rates, screening effect, resolution of structure equations, initial composition and opacity coefficients, and absorption cross sections, in order to analyze their consequent effects on both neutrinos and acoustic modes.

4.1. *The Reaction Rates*

Progress has been made in on the evaluation of the *pp*, the (^{3}He, ^{3}He) and the ^{7}Be(p, γ)^{8}B reaction rates.

The pp reaction.—This is the only reaction which has not yet been measured, because of its extremely small cross section. Two calculations have recently been published to redetermine this electroweak interaction. They include the very new experiment on ultracold neutrons which determines the neutron lifetime, $\tau_\beta = 887.6 \pm 3$ s with an improved accuracy (Mampe et al. 1989). This value is proportional to $(G_A^2 + 3G_V^2)^{-1}$ and allows us, with the determination of G_V from superallowed nuclear Ferni decay, to pin down the axial coupling constant G_A. Gould & Guessoum (1990), in their calculation, improve the determination of the radiative correction: they get a polarization vacuum correction of -1.3% and the radiative correction $+1.5$%, and propose a 2% meson exchange correction without explicit calculation. Carlson et al. (1991) concentrate on the meson exchange currents to conclude that this correction increases the *pp* capture rate by only 1.5%, owing to partial cancellation of effects; they consider the following in their calculation: π and ρ meson exchange seagull or pair currents, π and ρ meson exchange Δ_{33} excitation currents, and $\pi\rho$ exchange current. These two calculations would normally agree, but they lead to different conclusions on the calculation of matrix elements, Gould & Guessoum (1990) introducing accurate parameters for the deuteron wave function from the MIT and Paris groups, and Carlson et al. (1991) using the very recent Argonne nucleon-nucleon interaction. Without details on this part of the calculation in either of the two papers, their conclusions differ by about 5% on the astrophysical factor $S(0)$: $S_{\rm GG}(0) = 4.21 \times 10^{-25}$ MeV barns within 4% accuracy for Gould & Guessoum (1990), and $S_{\rm CRSW}(0) = 4.00 \times 10^{-25}$ MeV barns, $S'(E) = 4.67 \times 10^{-24}$ barns for Carlson et al. (1991).

If one makes a comparison with previous estimates, the GG calculation increases this reaction rate by 3.5%, while the CRSW calculation reduces it by 1.5%. It is amusing to notice that the two main improvements in these last 10 years—better determination of the neutron lifetime (Freedman 1990) and better determination of radiative and meson exchange currents—have modified the prediction by 3% or 4% but that the two effects, of opposite signs, practically cancel each other. As will be detailed later, and may be estimated from the previously published error analysis, the difference between the two calculations has small consequences for the solar structure and for the gallium prediction but leads to about 10% difference on the ^{8}B neutrino flux. This point should be resolved rather quickly.

The astrophysical factor for the interaction ^{3}He, ^{3}He.—This must be corrected for the presence of Coulomb interaction in the laboratory at low energy. We have adopted the value suggested by Assembaum et al. (1987): 5.24 MeV barns. This problem has been reexamined by Parker & Rolfs (1992), who suggest a value of 5. MeV barns. The difficulty in extracting precisely such a Coulomb effect in other experiments is also noticed; at present a 5% error is certainly plausible. In this specific case, the consequent uncertainty in neutrino predictions is very small (smaller than 2%).

The astrophysical factor for the interaction (p, ^{7}Be).—This has already been discussed in § 3. The theoretical calculation of Barker (1980) has been reconsidered by Kajino, Bertsch, & Barker (1989) and Johnson et al. (1992). They confirm the reduction of the S-factor due to the d wave function. Johnson et al. find a reduction of only 8% instead of 15% as in the case of Barker, but the extrapolated value is adjusted to the mean value of two experiments which systematically differ by 20%. Consequently, we introduce a systematic error in addition to the Johnson theoretical error (see the general discussion in Turck-Chièze et al. 1993): $S_{17} = 0.0224 \pm 0.13 \pm 0.003$ keV barns.

Considering the recent work of Mukhamedzhanov & Timofeyuk (1990) and their proposed astrophysical factor of $S_{17} = 0.0165$ keV barns, we warmly encourage the remeasurement of such an experiment over a large range of energy between 50 keV and 1 MeV, and an absolute theoretical estimate of such a cross section.

Other reaction rates.—We have verified the unimportance of the 15% uncertainty on the interaction ($p + d$) due to the very small lifetime of such a reaction (some 10^{-8} yr compared with 10^9 yr for the *pp* reaction). We have reintroduced the expression of the nuclear rates compiled by Caughlan & Fowler (1988) for ^{13}C(p, γ) and ^{14}N(p, γ). This modification has slightly reduced the CNO neutrino predictions in comparison with BU88 and TCCCD88. The recent determination by Kiener et al. (1991) of the ^{13}N(p, γ) reaction rate leads to a modification of the corresponding neutrino flux.

4.2. *The Screening Factors*

Table 4 illustrates the different treatments of screening in the reaction rates. This treatment mainly affects high Z-neutrino fluxes, as the acceleration factor increases with the Z of the reactants (here Z is the number of protons). One sees that the two prescriptions generally used, the weak and the intermediate case (GDGC), differ significantly. If one looks at more

TABLE 4

EFFECT OF ELECTRON SCREENING ON REACTION RATES: WEAK OR INTERMEDIATE REGIME

Reaction	WS	IS (GDGC)	IS (Mitler)	IS (SV)
pp	1.05	1.05	1.05	1.05
pep	1.05	1.05	1.05	1.05
^{7}Be..................	1.21	1.11	1.17	1.18
^{14}N	1.41	1.19	1.29	1.31

NOTE.—Prescriptions of Graboske et al. 1973 (GDGC), Mitler 1977, and Salpeter & Van Horn 1969 (SV). The factor f calculated here corresponds to the solar central conditions.

details, one notices that the GDGC prescription is not sufficiently precise in the case of the Sun, and Dzitko et al. (1992) have suggested in preference to follow Mitler (1977), who has relaxed the simplified hypothesis of spherical symmetry of the electronic cloud. In the specific case of the Sun, such an analysis leads to an intermediate acceleration factor between the classically named weak screening and the GDGC intermediate screening.

4.3. *The Structure Equations in the Central Region*

The structure equations of the classical stellar evolution present a singularity at the center, and the traditional Taylor expansion to the second order used to remove this singularity neglects the contribution of the derivatives of the chemical composition (see, for example, Clayton 1968). If it is correct for massive stars which have a convective core, this approach is not sufficiently accurate for radiative low-mass stars, in particular for the Sun (Christensen-Dalsgaard 1982; Berthomieu 1990). Effectively, a small error in the central structure equation may induce an incorrect central temperature and consequently a greater uncertainty in neutrino fluxes $\Delta\phi/\phi = \eta(\Delta T_C/T_C)$ with $\phi_\nu \sim T_C^\eta$, $\eta(^7\mathrm{Be}) = 8$, and $\eta(^8\mathrm{B}) = 18$. Moreover, the *p*-mode oscillations of low degree ($l = 0, 1, 2, 3$) and the asymptotic frequencies of the gravity modes, dependent on the Brünt-Väissälä frequency, may be affected by this region. So a calculation of a solar model with correct equations in the center constitutes a necessity for a reasonable comparison between predictions and experiment.

At the center, mass and luminosity are both zero, but density, temperature, pressure, and chemical composition have finite values. From the stellar structure equations, one deduces, for $r = 0$, $\partial M_r/\partial r = 0$, $\partial T/\partial r = 0$, and $\partial P/\partial r = 0$; consequently, the expansion of the thermodynamical variables (the density ρ, the temperature T, the pressure P, and the composition $[X]_i$) contains only even terms: $\alpha = \alpha_c + \alpha_2 r^2$, where the central values and the second derivatives α_2 are calculated explicitly or numerically.

In the central region of the Sun, the radial variation of the chemical composition cannot be neglected. In our new version of the code, we introduce the radial second derivative of the hydrogen content in the expansion of ρ, M, and L and in the expansion of the energy production and transport.

At this stage of improvements, the consequences for the structure of the Sun seem sufficiently small to induce only a negligible effect on the neutrino fluxes.

4.4. *The Initial Composition and the Opacity Coefficients*

As was mentioned previously, the precise determination of heavy-element composition is crucial for the determination of the opacity coefficients and the neutrino flux (Courtaud et al. 1990). Some years ago, the photospheric iron composition was found to be 35% greater than the meteoritic value (Anders & Grevesse 1989, hereafter AG89; Pauls, Grevesse, & Huber 1990). We will hereafter refer to this value as $\mathrm{Fe_{high}}$, given as a fractional number: $\mathrm{Fe_{high}} = \mathrm{Fe/H} = (4.68 \pm 0.33) \times 10^{-5}$. Recently two analyses have converge toward a value very near to the meteoritic value, which is the fraction $(3.02 \pm 0.27) \times 10^{-5}$: the analysis of Holweger, Heise, & Kock (1990) based on ionized iron (95% of the total contribution), $\mathrm{Fe_{low}} = (3.24 \pm 0.075) \times 10^{-5}$, and the analysis of Biémont et al. (1991a) on the neutral component, $(3.47 \pm 0.24) \times 10^{-5}$. Specific opacity tables have been recalculated in Saclay with the Los Alamos library for the high and low composition, and solar modeling has been recomputed adjusting the ratio Z/X to the observed values: $Z/X = 0.0273$ for the high iron value and $Z/X = 0.0267$ for the low value. We consider the low value as the most plausible one.

Other improvements concern the CNO abundances. These elements may not be deduced from the analysis of meteorites, and their determination has been largely improved by infrared measurements in space by the *Atmos* SL3 experiment (Farmer & Norton 1989). The accurate oscillator strengths have also been reestimated. The conclusion is the following: an increase of about 10% of carbon in comparison with AG89: $\mathrm{C/H} = (3.98 \pm 0.48) \times 10^{-4}$ (Grevesse et al. 1991) and a reduction of 15% of oxygen abundance, $\mathrm{O/H} = (7.24 \pm 0.72) \times 10^{-4}$ (Biémont et al. 1991b) and 10% of nitrogen, $\mathrm{N/H} = 1 \times 10^{-4}$. Since these elements represent 75% of the heavy elements and oxygen is the most abundant element, such a modification would lead to a reduction of the Z/X value used in solar modeling for the initial composition by about 9%, with consequences for CNO cycle production of energy and for opacity coefficient calculations. Such a reduction has recently been confirmed by Grevesse (1992), and may even be a little larger.

In fact, the reduction of the CNO abundance may be partly compensated for by microscopic diffusion, which concerns all elements heavier than hydrogen. This effect has not yet been introduced in the classical framework of stellar evolution but has been studied by Noerdlinger (1977), Wambsganss (1988), and CGK89 for the solar case. This study has been refined by Proffitt & Michaud (1991), using diffusion coefficients better adapted for stellar interiors and properly considering luminosity and radius constraints. At the solar age, they obtain a central helium increase of 0.013 in mass fraction (1.5% effect), which corresponds to a reduction of 0.033 of the initial helium and an increase of 0.4% in central temperature. The effect on heavier elements is similar: an increase of 3%–5% in the central region and a reduction of 6%–10% at the base of the convective zone for iron. The effect is similar for oxygen, but the small increase in the central region is partly compensated for by the conversion to nitrogen through the CNO cycle. Bahcall & Loeb (1990) have studied the ^{3}He, ^{4}He diffusion with a similar conclusion. But Schatzman (1969, 1984) has shown that this microscopic diffusion must be partly inhibited by turbulent mixing to interpret the absence of large anomalies predicted in stars of type A, for example. Thus, turbulent mixing could reduce the effect of microscopic diffusion by one-third at the solar surface (Proffitt & Michaud 1991). In their recent paper, Bahcall & Pinsonneault (1992) introduce the effect of helium diffusion and confirm the results of Proffitt & Michaud (1991) for this element.

Regarding such processes, we consider our "recent CNO" model as a minimal model for the composition. Effectively, the initial solar composition is certainly intermediate between the AG89 composition and the present photospheric one.

4.5. *The Opacity Coefficients Calculations*

Improvements in this field have been encouraged by the extreme sensitivity of ^{8}B neutrons to the central temperature, the stratified information derived from the detection of solar oscillations, and the difficulty of interpreting the observations of the Cepheid periods (Simon 1982). In these three cases the atomic calculation of opacity coefficients was suspected. New calculations have been performed by a Livermore group using the OPAL code (Iglesias & Rogers 1990). Improvements concern the equation of state (Rogers 1986; Rogers, Wilson, &

Iglesias 1988) and also the treatment of the mixture with coupled equations for the full mixture. The approach is effectively different, considering partition functions and a not quite perfect gas of electrons and ions. Moreover, as bound-bound processes have been shown to be important, the atomic physics has been estimated with care. The consequent opacity increase has eliminated the discrepancy between radial pulsation theory and observations of Cepheids (Andreasen 1988). In the case of the Sun, their results differ from the LAOL estimate by less than 5% in the center up to 20% at the basis of the convective zone. The origin of such differences is not completely evident, but it seems that it is not due to the bound-bound treatment, even if broadening effects have been improved together with plasma corrections, the main actor seems to be the equation of state leading to generally increased partial ionization. One notices a consequent small central temperature increase which exactly compensates for the effect of the improved determination of iron (Tables 5A and 5B). Figure 2 shows the influence of the improvements of compositions and calculations on the opacity coefficients.

4.6. *The Absorption Cross Section for Chlorine*

The expected rate of a solar neutrino detector is given by the convolution of the neutrino flux with the absorption cross section. In the case of the chlorine detector, this absorption cross section concerns the ground state only for low-energy

TABLE 5A

PARAMETERS OF PRESENT SUN AND NEUTRINO PREDICTIONS

Predicted Variables	IS GG*pp* (Fe high) (1)	IS GG*pp* (2)	WS GG*pp* (Fe low) (3)	WS C*pp* (4)
T_C (10^6 K)	15.47	15.33	15.33	15.41
ρ_C (g cm^{-3})	141.8	142.8	142.8	144.3
$Y_{\rm initial}$	0.2736	0.2664	0.2664	0.2661
$r_{\rm BCZ}$	0.729	0.733	0.733	0.733
$T_{\rm BCZ}$ (K)	2×10^6	2.1×10^6	2×10^6	2×10^6
$\Phi_{pp}/10^{10}$	6.05	6.08	6.05	6.02
$\Phi_{pep}/10^8$	1.29	1.30	1.30	1.37
$\Phi_{^7{\rm Be}}/10^9$	4.16	3.91	4.08	4.37
$\Phi_{^8{\rm B}}/10^6$	4.04	3.51	4.28	4.59
$\Phi_{^{13}{\rm N}}/10^8$	3.20	2.98	3.40	3.73
$\Phi_{^{15}{\rm O}}/10^8$	2.62	2.37	2.82	3.17
Chlorine detector (SNU)	5.88	5.22	5.87	6.60
Water detector (10^6 cm^{-2} s^{-1})	4.04	3.51	4.28	4.59
Gallium detector (SNU)	120	116	119	123

NOTE.—The Los Alamos (LAOL) opacity calculations and Anders & Grevesse 1989 composition (except for iron) are used. The effects of two iron compositions are shown, Fe high and Fe low (see text); two prescriptions of screening, intermediate (Graboske et al. 1973; IS) or weak (Salpeter 1954; WS) screening; and two reaction rates for (*p*, *p*), Gould & Guessoum 1990 (GG*pp*) and Carlson et al. 1991 (C*pp*). In the last column we have also considered a low value $S(0) = 5$ MeV barns for (^{3}He, ^{3}He). All the fluxes Φ are expressed in cm^{-2} s^{-1}.

TABLE 5B

EVOLUTION OF DIFFERENT PREDICTIONS

Predicted Variables	IS C*pp* (1)	WS C*pp* (2)	WS C*pp* (recent CNO) (3)	IS C*pp* (recent CNO) (4)
T_C (10^6 K)	15.49	15.50	15.43	15.43
ρ_C (g cm^{-3})	147.3	147.2	146.89	147
$Y_{\rm initial}$	0.2762	0.2760	0.2714	0.2714
$r_{\rm BCZ}$	0.721	0.721	0.725	0.725
$T_{\rm BCZ}$ (K)	2.16×10^6	2.16×10^6	2.11×10^6	2.11×10^6
$\Phi_{pp}/10^{10}$	6.02	5.99	6.01	6.04
$\Phi_{pep}/10^8$	1.38	1.38	1.39	1.39
$\Phi_{^7{\rm Be}}/10^9$	4.42	4.62	4.42	4.25
$\Phi_{^8{\rm B}}/10^6$	4.51	5.15	4.71	4.14
$\Phi_{^{13}{\rm N}}/10^8$	3.49	3.99	4.00	3.65
$\Phi_{^{15}{\rm O}}/10^8$	2.91	3.45	3.39	2.97
Chlorine detector (SNU)	6.51	7.30	6.78	6.07
Water detector (10^6 cm^{-2} s^{-1})	4.59	5.15	4.71	4.14
Gallium detector (SNU)	123	126	124	121

NOTE.—Shown are predictions for the low value of iron, recent Livermore opacity calculations (OPAL) for temperatures greater than 10^6 K and Los Alamos calculations including molecular contributions (Cox 1986) below such limit, Carlson et al. 1991 *pp* reaction rate (C*pp*) and the low value $S(0) = 5$ MeV barns for (^{3}He, ^{3}He). Considered here are the case of intermediate (Graboske et al. 1973; IS) or weak (Salpeter 1954; WS) screening and all the recent abundance determinations including CNO (see text). All the fluxes Φ are expressed in cm^{-2} s^{-1}.

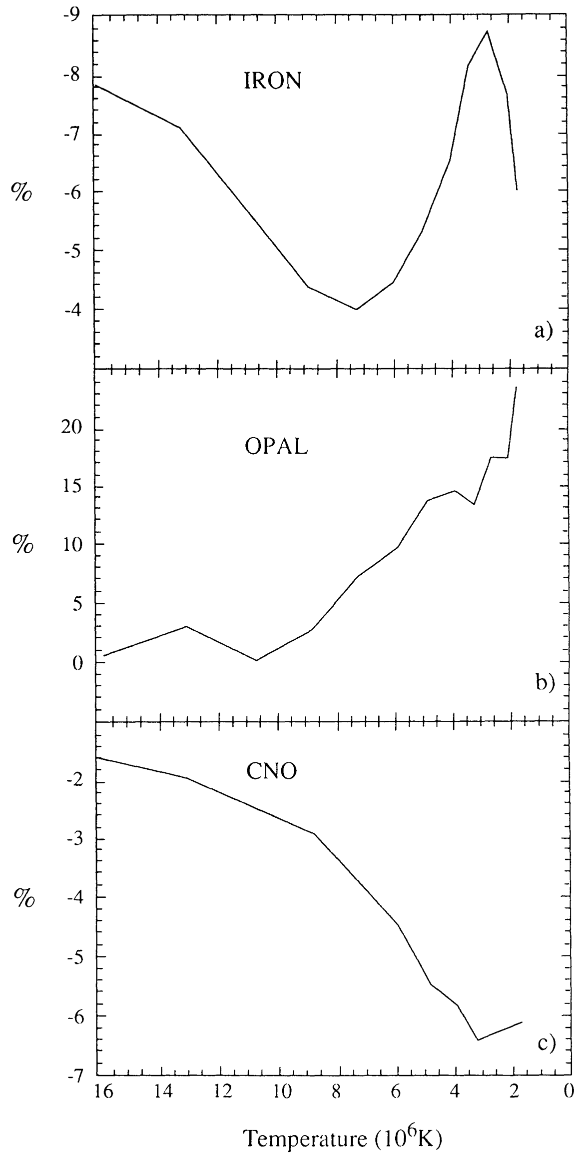

FIG. 2.—Variation of the total opacity coefficients with improvement of the composition and of the opacity calculation. (*a*) Difference in iron composition: $Fe_{high} = Fe/H = 4.68 \times 10^{-5}$ from Anders & Grevesse (1989) and the value, $Fe_{low} = 3.24 \times 10^{-5}$; (*b*) difference between OPAL and LAOL calculations for the same composition; (*c*) difference of CNO abundances between the AG89 composition and the new values: $C/H = 3.98 \times 10^{-4}$, $O/H = 7.24 \times 10^{-4}$, and $N/H = 7.24 \times 10^{-4}$.

neutrinos, but for the ^{8}B neutrinos one needs to know the absorption cross section toward excited states of the daughter nucleus, which means the Gamow-Teller transitions. Recently, the accuracy of the determination of such an absorption cross section has been improved by Garcia et al. (1991), who have measured the β^+ decay of ^{37}Ca. They find a 3% higher value than the one recommended by Bahcall (1989) for the ^{8}B neutrinos. The corresponding error is reduced to 3% instead of 10%.

5. CONSEQUENCES FOR THE REFERENCE MODEL AND THE NEUTRINO FLUXES

The effects of the previously described improvements on the solar structure and on the neutrino fluxes are summarized in Table 5A for Los Alamos opacities and in Table 5B for the OPAL opacities. One notices first that the structure of the star is only slightly modified, and the initial helium varies between 0.266 and 0.276 in mass fraction.

5.1. *Effects of the Improvements on the Neutrino Predictions*

Because the structure of the Sun is not greatly modified by the different improvements, the gallium prediction is quite stable. The predictions concerning the chlorine and the water detectors react more strongly and justify attention. The main improvements in the calculation concern the determination of the opacity coefficients. The improvements due to the OPAL calculations concern the equation of state and do not really affect the central region, but, as the thermodynamical quantities are modified near the basis of the convective zone, the star readjusts itself and the central temperature increases slightly. In our model, the OPAL opacities have been simulated from the tables published in Rogers & Iglesias (1992) and in Bahcall & Pinsonneault 1992 (comparison of col. [4] of Table 5A with col. [2] of Table 5B). The high-energy neutrino prediction is increased by 10%–12%. The improved composition of the photosphere has the opposite consequence for neutrino predictions: globally the Z/X mass fraction is modified from 0.0272 for the composition of Anders & Grevesse (1989) to 0.0267 for the recent determination and 0.0249 for the recent values of carbon, nitrogen, and oxygen. The iron effect is different from the effect of CNO and must be explicitly calculated (Courtaud et al. 1990) (see Table 5A). The effect of CNO modification has been introduced uniquely as an effect of metallicity which is perfectly correct at the base of the convective zone but not totally precise in the central region (see Table 5B, col. [4]). A more detailed calculation must be performed. Nevertheless, it is evident that microscopic diffusion of helium and the heavy elements would partly cancel the reduction of fluxes related to this decrease of the heavy elements. Taking the problem in the opposite way, the increased neutrino flux due to microscopic diffusion (Bahcall & Pinsonneault 1992) would be partly reduced by the improved determination of the carbon, nitrogen, and oxygen photospheric abundances.

In Table 5A we also discuss the impact of the *pp* reaction rate for the two recent calculations of Gould & Guessoum (1990) and Carlson et al. (1991) and the effect of different prescriptions on the screening factor for the reactions (p, ^{7}Be) and those of the CNO cycle. Referring to the discussion of Table 4, in the following we will take the intermediate value between the two prescriptions used (Table 5B, cols. [3] and [4]).

5.2. *Comparison with Updated Calculations*

We have compared our results with the most recent models including some of these improvements (Guenther et al. 1992; Bahcall & Pinsonneault 1992; Berthomieu et al. 1992) and generally find a good agreement when the same inputs have been used. If one compares these with the calculation performed by Bahcall & Pinsonneault (1992), their best model without diffusion can be compared with column (2) of Table 5B. They agree quite well even if small differences persist. Half the difference comes from a slightly different choice for (^{3}He, ^{4}He) (a less than 0.1 SNU effect) and a different choice for the present solar age. Following the remark of Guenther (1989) concerning the pre–main-sequence lifetime, we adjust the present luminosity at $t_\odot = 4.50 \pm 0.05$ Gyr. The temperature is then reduced by $\sim$(0.3–0.4) $\times$ 10^6 K and the chlorine prediction by ~0.3–0.4 SNU. A more precise comparison between authors would necessitate our using exactly the same ingredients for opacity coefficients, equation of state, and nuclear burning: it is not yet justified by the larger discrepancy with the experimental results.

5.3. *Final Results and Estimated Accuracy of the Predictions*

From Table 5B we extract our best predicted values in the classical framework of stellar evolution. We take into account the remark on screening effect introduced in § 4.2 and the low recent CNO value, omit any dynamical effects, and get the following values: chlorine experiment, 6.4 ± 1.4 SNU; water detector, $(4.4 \pm 1.1) \times 10^6$ cm^{-2} s^{-1}; gallium detector, 122.5 ± 7 SNU.

We have mentioned several effects leading to 10%–15% variation of the ^{8}B neutrino flux, but because of their independent origin they are statistically distributed and the final values are, at the present time, not clearly different from the previous ones.

It is not easy to define an uncertainty for such complex calculations, and different points of view have been expressed (BU88; TCCCD88). We consider any theoretical error as a minimal error (absence of error on the assumptions of the calculation) and only indicative of the present status and of the occurring improvements. A statistical distribution is not justified for all the sources of error, so it seems to us dangerous to use the so-called 1 σ or 3 σ error. In Table 6 we use the 1 σ error for all the ingredients which correspond to a measurement, except for some very peculiar cases where problems have been identified. We use a most probable error for the other ones, which are partly or totally dependent on some theoretical estimate. In this second case the determination of error comes from specific analysis and is specific to each author. As all errors are not statistical errors, we believe that it is unjustified to multiply by 3 the error we suggest in order to get a "3 σ" error, as has been sometimes done.

Courtaud et al. (1990) have previously stressed the important error due to the opacity coefficients for the interpretation of the Homestake and Kamiokande results. Considering the impressive progress in this field (in the composition determination and in the calculations), we adopt the value recommended by the different groups: 5% for κ, with a consequent 12% for the high-energy neutrino fluxes. In fact, it is extremely difficult to be more precise on this ingredient. The reaction (^{7}Be, *p*) is always uncertain, since the systematic difference between the experiments is large, so this reaction must be reconsidered, theoretically and experimentally. The uncertainty of Z/X in the central region continues to make a large contribution to the error, even if the photospheric measurements have been considerably improved, and is due to the approximating hypothesis that the initial composition is the photospheric one. The present large uncertainty on the *pp* reaction expresses the present unclear situation on the matrix element and the extremely important role of this major reaction rate (see also § 8). The uncertainty of 5% on the screening factor reflects the difficulty of such a calculation and the difficulty of understanding present experiments measured in the laboratory at very low energy.

TABLE 6

ESTIMATED THEORETICAL UNCERTAINTIES FOR DIFFERENT DETECTORS[a]

Source Uncertainty	Uncertainties (%)		
	Gallium Detector	Chlorine Detector	Water Detector
pp reaction (5%)	2.7	6	8
(^{3}He, ^{3}He) reaction (5%)	<0.1	1.6	2
(^{3}He, ^{4}He) reaction (4%)	1	2.7	3.3
(^{7}Be, *p*) reaction (13%)	1	11	13
Screening effect	1.2	5	5
Luminosity (0.5%)	0.3	3.6	3.6
Age (1%)	<1	1	1.5
Z/X (10%)	1.8	9	11
Opacity (5%)	2	12	13
σ_{abs}	4	3	<1
Total	5.8	21	24

[a] See comments in § 5.3; σ_{abs} is the absorption cross section.

5.4. *Confrontation with the Recent Experimental Results*

The confrontation of experiment with predictions is summarized in Tables 7 and 8, and the error of the experiment/theory ratio includes theoretical estimated error. Table 7 summarizes the different neutrino contributions to the presently available detectors for different calculations discussed in this paper, and they all correspond to a model without inclusion of microscopic diffusion.

Four experiments are now running, and recent results have been published. In order to compare data obtained at about the same time, one considers, for the chlorine experiment, a recent reanalysis concerning the whole experiment (Cleveland 1992), and runs since the beginning of 1988. The mean value is slightly enhanced in comparison with the mean value of the whole experiment: 2.53 ± 0.32 SNU instead of 2.23 ± 0.22 SNU. The discrepancy with the prediction stays greater than a factor of 2, with still a large error. The result of Kamiokande II (Hirata et al. 1989, 1990) seems in better agreement with the prediction, as the experiment/theory ratio is $0.61 \pm 0.06 \pm 0.13$ and even a little more: $0.63 \pm 0.06 \pm 0.09$ if one includes the preliminary Kamiokande III result (Nakamura 1992). In fact, there is no trivial astrophysical explanation for this. Effectively, if one supposes no modification of the neutrino fluxes during the travel toward the detectors, the Kamiokande detector appears as a subset of the chlorine one, since it detects only ^{8}B neutrinos (see Table 7); thus, as the sensitivity with temperature is higher for ^{8}B neutrinos than for any other sources, one would generally think that any modification of the internal structure would mainly influence ^{8}B neutrinos (see the more precise discussion in § 8). Nevertheless, within the large errors the two results are not incompatible, contrary to the case where the predicted values are higher (Lande 1992).

Two experiments, using gallium, have now produced results. The first results of the SAGE experiment (Gavrin et al. 1991) gave a large discrepancy with the theoretical prediction ($R_S <$ 64%) and rejected any astrophysical explanation. But the value corresponding to the year 1991, $85^{+22}_{-32} \pm 20$ SNU (Gavrin 1992), is in excellent agreement with the Gallex result: $83 \pm 19 \pm 8$ SNU (Anselmann et al. 1992a), and they allow any solution to be possible. If one thinks of an astrophysical explanation, these results seem also to favor a large reduction of the ^{7}Be and CNO contributions (see Table 7).

The present situation encourages progress in any direction. The impression that the neutrino puzzle seems less crucial is artificial, as the solution to such discrepancies is totally unclear. The possible explanation that neutrinos oscillate between different flavors is always justified, and now the range

TABLE 7

Contributions of Different Sources of Neutrinos to Total Capture Rates for Different Theoretical Estimates

Neutrino Sources (Bahcall 1989)	Capture Rates: Bahcall & Ulrich 1988	Turck-Chiéze et al. 1988	Turck-Chièze & Lopes (present study)	Bahcall & Pinsonneault 1992
Chlorine experiment:				
pp	0.0	0.0	0.0	0.0
pep	0.23	0.21	0.22	0.23
^{7}Be	1.12	0.995	1.10	1.17
^{8}B	6.15	4.06	4.63	5.53
^{13}N	0.102	0.104	0.063	0.072
^{15}O	0.34	0.37	0.21	0.24
^{17}F	0.0034	...	0.0028	0.0030
Total (SNU)	7.9	5.75	6.36	7.2
Kamiokande experiment:[a]	5.8	3.8	4.4	5.06
Gallium experiment:				
pp	70.8	70.6	71.1	71.3
pep	3.01	2.795	2.99	3.07
^{7}Be	34.4	30.6	30.9	32.9
^{8}B	14.1	9.31	10.77	12.31
^{13}N	3.77	3.87	2.36	2.68
^{15}O	6.03	6.50	3.66	4.28
^{17}F	0.06	...	...	0.04
Total (SNU)	132	124	122.5	127

Note.—The last calculation of Bahcall & Pinsonneault 1992 is the one without diffusion process. All the calculations use the same absorption cross sections and energy spectra calculated by Bahcall and collaborators (Bahcall 1989).

[a] Uniquely ^{8}B for $E > 7.5$ MeV (in 10^6 cm^{-2} s^{-1}).

of parameters is extremely reduced (Gallex collaboration; Anselmann et al. 1992b). We have shown here that some neutrino fluxes are extremely sensitive to the details of the calculation, and we would like to examine whether a more traditional explanation than the MSW effect may be found in the classical picture of the Sun or in the influence of extra phenomena. The first point will be discussed in § 8. To progress on the second point, we will examine in the next two sections the constraints we may derive from the acoustic mode frequencies.

6. INTRODUCTION TO THE HELIOSEISMOLOGY

The present idea is to explore the helioseismological field with the same solar models previously used for neutrino predictions, in order to distinguish the degree of exigency of the two disciplines and the sensitivity of the two probes (neutrinos and acoustic modes) to the different aspects of the microscopic physics. The second aim is to use helioseismology as a constraint in order to quantify the quality of the present modeling and the room left free for the phenomena not yet introduced in the present classical solar modeling.

TABLE 8

Comparison of Present Experimental Neutrino Results with New Predicted Calculations[a]

Experimental Results	Theoretical Predictions
Chlorine experiment:	
2.53 ± 0.32 SNU	6.4 ± 1.4
Ratio experiment/theory (R_H)	(39.5 ± 10 ± 8)%
Kamiokande experiment:	
0.28 ± 0.03 events day^{-1} ($E \geq 7.5$ MeV)	0.46 ± 0.12 events day^{-1}
2.67 ± 0.28 cm^{-2} s^{-1}	4.4 ± 1.1 cm^{-2} s^{-1}
Ratio experiment/theory ($R_{K\,II}$)	(61 ± 6 ± 13)%
Gallium experiments	123 ± 7 SNU
SAGE I: <79 SNU (95%),	
Ratio experiment/theory ($R_{S\,I}$)	<64.5%
SAGE II: $85^{+22}_{-32} \pm 20$ SNU,	
Ratio experiment/theory ($R_{S\,II}$)	$(69^{+24}_{-30} \pm 5)$%
GALLEX: 83 ± 19 ± 8 SNU,	
Ratio experiment/theory (R_G)	(68 ± 16 ± 4)%

[a] See references in the text.

The observation of the 5 minute oscillations in 1960 by Leighton (Leighton 1961; Leighton, Noyes, & Simon 1962), predicted by Ledoux & Walraven (1958), has opened a new field of research in the study of the solar interior. Although it took 10 years to understand such observations correctly (Ulrich 1970), impressive results have stimulated a lot of work. After the identification of the 2700 observed modes, the development of some asymptotic prescription (Tassoul 1980) has enhanced the influence of the deep interior and correlates specific differences of frequencies to the sound speed. People then began to put some constraints on extra processes not yet introduced in conventional stellar evolution: extra mixing (Provost & Berthomieu 1986; Christensen-Dalsgaard 1986) and the hypothetical existence of weakly interacting particules (Däppen, Gilliland, & Christensen-Dalsgaard 1986; Kaplan et al. 1991; Toutain & Frölich 1992). In parallel, the inversion techniques, previously developed in geophysics, allow us to define the sound speed which determines the propagation of the acoustic modes (Christensen-Dalsgaard et al. 1985; Christensen-Dalsgaard 1988; Shibahashi & Sekii 1988; Vorontsov 1988; Christensen-Dalsgaard, Gough, & Thompson 1991).

6.1. *The Solar Sound Speed*

The square of the sound speed deduced from accoustic modes (Christensen-Dalsgaard 1988) is compared in Figures 3*a* and 3*b* with the sound speed obtained with the present Saclay solar models. It is the first diagnostic which checks the quality of the solar modeling from the helioseismology point of

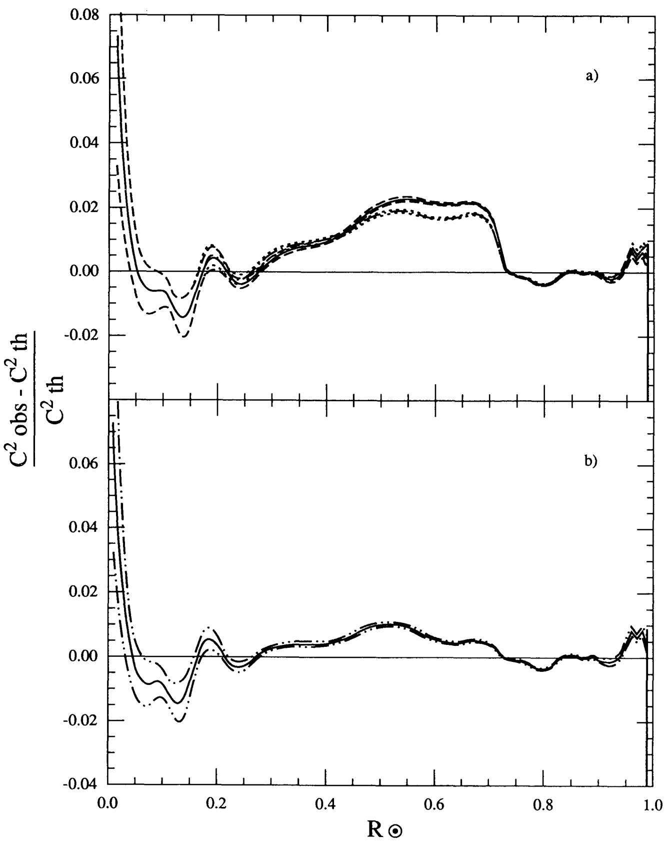

FIG. 3.—Difference, as a function of the position in the Sun, between the square of the sound speed deduced from acoustic modes and that obtained in a theoretical solar model. (*a*) These models correspond to the Los Alamos opacities for the high value of iron (see Table 5A, col. [1]: *dotted line*) and for the most correct low value of iron (see Table 5A, col. [2]: *dashed line*). (*b*) This model corresponds to OPAL opacity calculation and Table 5B, col. (1). The dot-dot-dash line corresponds to the error obtained by inversion of acoustic modes (from Christensen-Dalsgaard 1988).

view, without the introduction of a pulsation code. One notices first that, as is well known, the general behavior is confirmed within 2%, except near the center. Moreover, specific differences allow us to question some well-known peculiar ingredients of the calculation. The two small peaks in the external part correspond precisely to the region of the partial ionization of hydrogen and helium (Christensen-Dalsgaard et al. 1988), showing the limitation of the present equations of state. Such behavior is observed for the prescription of Vardya (1964) (as is the case here) but also for the more complex calculation of Mihalas et al. (1988) (see discussion of Christensen-Dalsgaard & Däppen 1992), leading to the idea that this region could represent an interesting way to independently extract the present photospheric helium (Gough 1984; Däppen & Gough 1984). At the base of the convective zone, the theoretical enhancement of the sound speed could be a priori interpreted as a manifestation of uncertain opacity coefficients coupled with undershooting and microscopic diffusion. The interpretation of the central region is more delicate, due to the interpenetration of nuclear and transport processes. In this case, the sound speed is certainly not a very sensitive variable, as any temperature variation is balanced by a mean molecular weight variation:

$$c^2 = \frac{\Gamma_1 p}{\rho} \approx \frac{\Gamma_1 k_B T}{\mu m_u}, \tag{6}$$

where k_B is the Boltzmann constant, μ the mean molecular weight, and Γ_1 the first adiabatic exponent. Figure 3*a* shows the comparison with solar models using the Los Alamos opacities and the two iron compositions. The major difference appears in the intermediate region, where the most justified low iron value slightly increases the discrepancy with observation. In the central region the modification of opacity noticed in Figure 2 has little impact on the sound speed behavior. Figure 3*b* shows the same comparison, with recent opacity coefficients determined by Iglesias & Rogers (1990): the effect is also concentrated in the intermediate region. In this case one may notice a substantial improvement, reducing the discrepancy between observation and prediction to a 1% effect. The depth of the convective zone is consequently improved. If the depth deduced from the observed frequencies is 0.287 ± 0.003 $R_\odot$ ($r_{BCZ} = 0.713\ R_\odot$) (Christensen-Dalsgaard et al. 1991), the theoretical value using the Los Alamos coefficients and the low iron content is 0.267 $R_\odot$, and the final value using OPAL opacities is 0.279 $R_\odot$ with AG89 composition and low iron, and 0.275 $R_\odot$ with the recent photospheric CNO determination. One may say that half the difference previously noticed is explained, and the other part could be partly explained by microscopic diffusion, even if the present CNO slightly downgrades this comparison, contrary to the effect on neutrinos. The need for a correct treatment of the microscopic diffusion not only for helium but also for heavy elements would certainly improve the sound speed behavior in the whole intermediate region, leading to a reduced effect of undershooting.

It is interesting to notice also that, using the OPAL opacities, the temperature at r_{BCZ} is about 2.3×10^6 K at the beginning of hydrogen burning, which is not far from the temperature of 2.4×10^6 K, where lithium begins to burn.

6.2. *Hypotheses of the Pulsation Calculations and Their Justifications*

If the evolution of the Sun on the main sequence, determined by the nuclear reaction network, the radiation pressure, and gravity, involves periods of a billion years, the dynamical time scale associated with the nonradial oscillations is between several minutes and 1 hour. These extremely different time scales allow the linearization of the hydrodynamic equations around an equilibrium state.

The 5 minute oscillations of the Sun are interpreted as a superposition of eigenmodes. For a star without a magnetic field and without rotation, an eigenmode of oscillation is described in terms of spherical harmonics of degree l. For each l, the modes are characterized by the radial eigenfunction of order n, which can be identified by the radial number of nodes. These oscillations correspond to standing acoustic waves (p-modes), for which the propagation region depends on the sound speed. The acoustic cavity is defined inside the star by the Lamb frequency S_l and outside by the cutoff frequency ω_C, both depending on the sound speed.

Adiabaticity is achieved locally to a high degree of accuracy, from the center to the base of the convective zone. For the more superficial layers and in the atmosphere, the thermal time scale is of the order of the dynamical time scale, and the adiabatic approximation is not entirely correct to determine the theoretical frequencies. The correction required by nonadiabaticity was estimated by CGK89. They found that such an effect reduces the difference between observed frequencies and eigenfrequencies by about 3 μHz for the modes with $3 < l < 10$ and $3000\ \mu\text{Hz} < \nu < 4000\ \mu\text{Hz}$, and is negligible for the modes with $\nu < 3000\ \mu$Hz. In this paper we will only consider the adiabatic approximation, as a first step. Our purpose is not to reach the best comparison with observations, which necessitates very precise numerical adiabatic predictions, but really to quantify the consequences for acoustic frequencies of the previously different physical assumptions which have been revealed as important for the determination of the neutrino predictions.

7. ACOUSTIC MODE PREDICTIONS

The present paper is concerned with the direct problem, i.e., the calculation of acoustic eigenfrequencies corresponding to the equations of oscillations (Unno et al. 1989).

7.1. *From Solar Modeling to the Oscillation Frequency Spectrum*

We have calculated theoretical frequencies in the adiabatic approximation using a relaxation method to solve the system of adiabatic nonradial oscillations with the two conditions of regularity at the center and at the surface. For such a calculation we have used the well-established pulsation code of Christensen-Dalsgaard (1982), and consequently have considered an isothermal atmosphere, which is more realistic than the condition of vanishing surface pressure (Christensen-Dalsgaard 1982; Unno et al. 1989).

Our standard model was computed with approximately 300 spatial mesh points and 26 or 27 time steps between the zero-age main sequence (ZAMS) and the present Sun. To solve the pulsation equations, we need to extract from the present solar structure the partial derivatives of the density ρ and the temperature T versus the pressure P. These derivatives must be carefully determined. Effectively, the errors in the computed frequencies arise not only from the errors in the calculation of the frequencies but also from inaccuracies in the equilibrium model.

We have first checked the accuracy of the equilibrium model, which is verified at a level of 10^{-3} to 10^{-4}. This is certainly representative of most stellar evolution codes developed in the early 1970s. The recent versions of stellar codes developed for the purpose of helioseismology reach a precision of 10^{-4} to 10^{-5} (Christensen-Dalsgaard 1982) and of 10^{-5} to 10^{-6} in the code CESAM developed by Morel, Provost, & Berthomieu (1990). Nevertheless, one may often notice a degradation of this accuracy very near the center and in the low photosphere, where density and pressure change rapidly: in these regions, the accuracy, in our case, is at a level of several percent, even with the structure equations described in § 4.3. The main point is to avoid any discontinuity which may produce an enhancement in the derivatives used in helioseismology. This could be the case at the bottom of the convective zone or if the physics of the code is not consistent—for example, in the treatment of the equation of state or in the treatment of the opacity coefficients. Special care and small modifications have been introduced into our code to avoid such a problem. As far as the physical processes are concerned, uncertainties at a level of several percent were nevertheless noticed in the solar modeling and consequently dominated the errors, except perhaps at the very center. It has been remarked that a simple equation of state corrected with coulomb effects does not give very different results than a more complex one. As was shown by Christensen-Dalgaard (1991), using opacity tables provided by one of us (S. T. C.), the absolute values of the frequencies are extremely dependent on the absolute values of the low photospheric opacity coefficients, and agreement with the data has been improved practically by a factor of 2 at high frequencies with the inclusion of the molecular transitions in the Los Alamos opacities instead of the older Cox & Tabor (1976) opacities and with a rather thin grid of coefficients in the external part in order to reduce any artificial oscillations in the region where noncontinuous bound-bound processes could dominate. This point was confirmed by Kim, Demarque, & Guenther (1991). It is the reason why, even when we introduce OPAL opacities, for the external part we keep the LAOL values, which are more complete than the atomic values of OPAL. If one would like to perform very accurate pulsation calculations, one really needs an even more precise grid and a much improved treatment of the solar atmosphere.

We have also verified that we do not lose precision in the resolution of structure equations, passing from our 300 mesh-point solar model to a grid of 600 or 1200 points, adjusted in accordance with the structure of the p-mode eigenfunctions.

7.2. *The Different Methods*

Adiabatic frequencies were calculated for all the p-modes observed, $l = 0$–700 (see the report of Libbrecht et al. 1990).

Both the eigenfrequencies and the frequencies calculated by the variational method have been estimated. As was indicated by Christensen-Dalsgaard (1982), the error associated with the variational method is proportional to N^{-4} ($N =$ the total number of mesh points); for the eigenfrequencies, the error is of order N^{-2}. If the variational method has clearly converged with 1200 mesh points, one observes even a 2 μHz difference between the two methods, except for $l = 0$. Considering that a mesh of 2400 points (apparently necessary for the determination of the eigenfrequencies) is too far from the mesh of our solar model, we will mainly concentrate here on the results obtained with the variational method (see discussion on the numerical accuracy of Christensen-Dalsgaard & Berthomieu 1992).

For the comparison with the observed acoustic frequencies, all the models were calibrated to have, at the solar age [$(4.50 \pm 0.05) \times 10^9$ yr], the solar radius (assumed as 6.9599×10^{10} cm) and the solar luminosity (3.846×10^{33} ergs s^{-1}), with relative errors in surface radius ($\delta\nu$ varies as $3\delta R$) and luminosity of 10^{-6} and 10^{-4}, respectively.

7.3. *Discussion of the Results*

Figure 4 shows the comparison of the theoretical predictions of acoustic modes with the compilation of Libbrecht et al. (1990) for $l = 0$–150, scaled by $Q_{n,l}$ (see Christensen-Dalsgaard 1988), which removes the part of the variation in $\delta\omega_{n,l}$ due to the dependence of inertia of each mode. Figure 4a corresponds to the model with LAOL opacity and low iron. We notice a difference with observations smaller than $\pm 10\ \mu$Hz (3×10^{-3}) but much greater than the experimental error (0.1–0.5 μHz). This difference translates part of our uncertainty to the external layers of the Sun, part of which is not yet understood in the solar interior. Compared with model 17 of Christensen-Dalsgaard (1991), using the same opacity coefficients, we consider that the agreement is satisfactory and that the general

TABLE 9

ACOUSTIC FREQUENCIES IN μHz FOR THE MOST PENETRATING MODES

l	n	IPHIR	Birmingham	Libbrecht	LAOL + Fe_{ph}	LAOL + Fe_{met}	OPAL + Fe_{met}
0..........	19	2764.42	2764.21	2764.40	2757.28	2756.21	2759.93
	20	2898.99	2899.04	2899.30	2892.15	2890.99	2894.86
	21	3034.10	3033.80	3033.80	3027.27	3026.06	3030.03
	22	3169.13	3168.76	3168.60	3162.38	3161.12	3165.32
	23	3304.06	3303.42	3304.10	3297.93	3296.71	3300.89
	24	3439.24	3438.96	3439.80	3434.14	3432.85	3437.26
1..........	19	2828.47	2828.31	2828.10	2821.42	2820.34	2824.04
	20	2963.59	2963.34	2963.30	2956.52	2955.34	2959.36
	21	3098.44	3098.17	3098.70	3091.73	3090.56	3094.58
	22	3233.66	3233.53	3233.20	3227.21	3225.99	3230.15
	23	3369.01	3368.52	3368.90	3363.07	3361.84	3366.19
	24	3504.90	3504.41	3504.60	3499.61	3498.31	3502.73
2..........	19	2889.82	2889.74	2889.96	2882.40	2881.30	2885.25
	20	3024.92	3024.80	3024.30	3017.85	3016.69	3020.78
	21	3160.17	3159.96	3159.78	3153.26	3152.06	3156.34
	22	3295.18	3295.15	3295.68	3289.11	3287.93	3292.19
	23	3431.20	3430.85	3431.20	3425.54	3424.30	3428.80
	24	3567.04	3567.60	3567.20	3562.25	3560.97	3565.59
3..........	19	...	2947.43	2947.33	2939.45	2938.39	2942.59
	20	...	3082.47	3082.63	3075.22	3074.11	3078.29
	21	...	3218.40	3217.94	3211.11	3209.93	3214.31
	22	...	3354.40	3354.28	3347.38	3346.22	3350.70
	23	...	3490.24	3489.36	3484.37	3483.13	3487.71
	24	...	3627.55	3626.10	3621.45	3620.18	3624.98

NOTE.—We compare three independent sources: the most recent IPHIR results analyzed by Toutain & Fröhlich 1992, the network results of the Birmingham group (Elsworth et al. 1991), and the compilation of Libbrecht et al. 1990 with three of the models discussed in Tables 5A and 5B.

TABLE 10

COMPARISON OF THE SO-CALLED GREAT DIFFERENCE $\nu_{l,n} - \nu_{l,n-1}$

l	n	IPHIR	Birmingham	Libbrecht	LAOL + Fe_{ph}	LAOL + Fe_{met}	OPAL + Fe_{met}
0..........	19	134.54	134.60	134.80	134.67	134.61	134.88
	20	134.57	134.83	134.90	134.87	134.78	134.93
	21	135.11	134.76	134.50	135.12	135.07	135.17
	22	135.03	134.96	134.80	135.11	135.06	135.29
	23	135.93	134.66	135.50	135.55	135.59	135.57
	24	135.10	135.54	135.70	136.21	136.14	136.37
1..........	19	134.96	134.92	134.50	135.19	135.10	135.17
	20	135.12	135.03	135.20	135.10	135.00	135.32
	21	134.85	134.83	135.40	135.21	135.23	135.22
	22	135.22	135.36	134.50	135.47	135.43	135.57
	23	135.35	134.99	135.70	135.84	135.85	136.04
	24	135.89	135.89	135.70	136.53	136.47	136.54
2..........	19	135.76	135.24	135.29	135.24	135.16	135.26
	20	135.10	135.06	134.34	135.45	135.38	135.53
	21	135.25	135.16	135.48	135.40	135.38	135.56
	22	135.01	135.19	135.89	135.84	135.87	135.85
	23	136.02	135.70	135.52	136.42	136.37	136.62
	34	135.84	136.75	136.00	136.70	136.66	136.79
3..........	19	...	135.72	136.04	135.53	135.43	135.72
	20	...	135.04	135.30	135.73	135.72	135.70
	21	...	135.93	135.31	135.88	135.83	136.02
	22	...	136.00	136.34	136.26	136.28	136.39
	23	...	135.84	135.08	136.98	136.91	137.01
	24	...	137.31	136.74	137.06	137.05	137.27

NOTE.—Expressed in μHz for the same calculations and the same experiments as in Table 9.

TABLE 11

COMPARISON OF THE SO-CALLED SMALL DIFFERENCE $\nu_{l,n} - \nu_{l+2,n-1}$

l	n	IPHIR	Birmingham	Libbrecht	LAOL + Fe_{ph}	LAOL + Fe_{met}	OPAL (variational)	OPAL (eigenfrequencies)
0..........	19	10.36	9.71	9.73	10.12	10.06	9.84	9.91
	20	9.17	9.30	9.34	9.75	9.69	9.61	9.64
	21	9.18	9.00	9.50	9.42	9.37	9.25	9.24
	22	8.96	8.80	8.81	9.12	9.06	8.98	8.98
	23	8.88	8.27	8.42	8.82	8.78	8.70	8.70
	24	8.04	8.11	8.60	8.60	8.55	8.46	8.47
1..........	19	...	16.6	16.81	17.45	17.38	17.17	17.22
	20	...	15.91	15.97	17.07	16.95	16.77	16.85
	21	...	15.70	16.07	16.51	16.46	16.29	16.31
	22	...	15.13	15.26	16.10	16.06	15.85	15.91
	23	...	14.12	14.62	15.69	15.63	15.49	15.56
	24	...	14.17	15.24	15.24	15.18	15.02	15.08

NOTE.—Expressed in μHz for n = 19–24 for the same cases. All the theoretical calculations correspond to a variational calculation, except the last one, which corresponds to eigenfrequencies difference.

lation and the determination of the eigenfrequencies. This difference practically disappears in the determination of the differences $\Delta\nu_{0,2}$, $\Delta\nu_{1,3}$ (see Table 11).

7.4. *Sensitivity of the Acoustic Modes to the Microscopic Physics*

Contrary to the case of boron neutrinos, the details of the microscopic physics are not amplified in the acoustic mode frequency determination; nevertheless, a proper description of the ingredients of the solar modeling clearly improves the quality of the calculation.

The improvements in the calculation of the opacity coefficients are clearly visible in the intermediate region, and an effect of 15%–20% on the opacity coefficients leads to 3–4 μHz in the frequencies. The effect of the composition, often connected with a modification of the initial helium abundance, is small and does not seem to improve the agreement with the observations; this may be due to the fact that the order of magnitude of such a modification is similar but of opposite sign to the effect of microscopic diffusion. We have noticed the importance of the grid of κ and of the values of opacity in the outer part. This part of the star, crucial for acoustic modes, has not been discussed here, and progress is still awaited. In fact, external opacities for the detailed precise composition, including molecular contributions and improved atomic physics, are not yet accessible. A calculation of Kurucz (1991) has been used by Guenther et al. (1992) and Berthomieu et al. (1992), but it seems to lead to greater differences from observations than the description of Los Alamos (Cox 1986). The sensitivity of the very internal part to the improved inputs is very small, and modification of opacity in this region (see Fig. 2) is practically invisible on acoustic frequencies. The effect of the reaction rates will be discussed in the next section. Because the sound speed behavior and the difference $\Delta\nu_{l,l+2}$ show reasonable agreement with observations, we may exclude serious problems with the classical framework of stellar evolution.

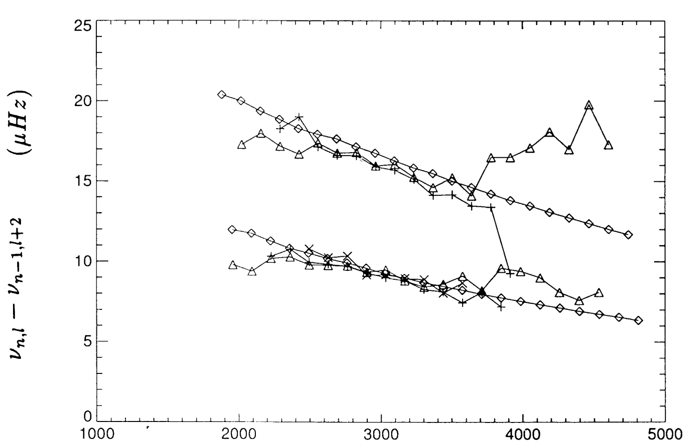

FIG. 6.—Variation of the p-modes frequency splitting for consecutive radial order and pair of degrees $\Delta\nu_{0,2}$ and $\Delta\nu_{1,3}$ as a function of the frequency. The observations are from the Libbrecht compilation (*triangles*), Toutain & Fröhlich (1992) (*crosses*), and the Birmingham group (*plus signs*). The theoretical predictions correspond to the variational calculation (*diamonds*).

8. COMMENTS ON THE COMPARISON WITH EXPERIMENTS

Tables 7 and 8 summarize the present neutrino situation, Tables 9–11 and Figures 3b and 4 the helioseismological one. They are representative of what we have learned from classical stellar evolution, including the best available microscopic physics.

The question now is to know whether they are representative of the real Sun. In fact, if it is evident that such progress reduces the discrepancy with observed acoustic modes, it is difficult to really see the impact of the microscopic physics on the interpretation of the neutrino detections (even if each modification affects the predictions). This section is purely speculative and mainly oriented toward the neutrino puzzle. We follow the simple hypothesis (which may be incorrect if neutrino detections are representative of neutrino properties or if some experiment is incorrect) that the information coming from the two probes, neutrinos and accoustic modes, is representative of the real Sun and must be consistent. The idea is to anticipate the consequences of new modifications on the present observables, neutrinos and acoustic modes, in using what we already understood from the sound speed.

Transport processes, for example, have been extensively studied by several groups, but the quantitative introduction of such processes is still extremely difficult (see chap. 6 of the review of Turck-Chièze et al. 1993 for a summary). The example of microscopic diffusion or gravitational settling may stress the difficulty of such progress: this slow process certainly exists in the internal radiative part of the Sun, but for a complete calculation one needs to consider this effect on all elements relative to hydrogen, and their feedback on opacity calculations and nuclear reaction rates added to an eventual partial inhibition of such process by turbulence, to be consistent with observations of other stars. Today only partial calculations exist, which help to quantify the induced effect on neutrino predictions.

Let us call "primary neutrinos" the neutrinos produced by the main reaction $p(p, D)e^+\nu_e$, and call all the others "secondary neutrinos." If the first are totally constrained by the luminosity of the Sun, and consequently extremely stable, the others may be influenced by a lot of phenomena. They are, of course, determined by the deep internal structure of the Sun, but, as we have seen in Table 5, they are also influenced by modifications of the intermediate region (e.g., the role of opacities) which may induce readjustment of the internal structure. For this reason the sound speed behavior is a good guide to quantify the improvements already made and estimate the consequences of those which are still impossible to introduce. We have seen from Figure 3*b* that the region between 0.2 $R_\odot$ and the surface is not far from being under control, so even if we have not perfectly understood the respective roles of opacity, microscopic diffusion, undershooting, or mixing in this region, we may reasonably predict, in extrapolating the present analysis, the order of magnitude of the resulting effect on neutrinos. In fact, one could predict a new increase of 10%–15% in the chlorine and water predictions (certainly partly due to microscopic diffusion) as far as the intermediate region is concerned. Considering Table 9, the absolute values of the frequencies of low-degree modes could be increased by no more than 2 μHz.

Let us concentrate now on the nuclear region. This internal region is not yet well constrained by helioseismology, owing to the small number of modes already detected and the difficulty of the inversion procedure. Indeed, the number of acoustic modes sensitive to this part of the Sun is about 3 times the number of modes presently detected. So one may hope to improve the present knowledge appreciably with space experiments on *SOHO* (*SOHO* Mission 1988) in the near future. This would be extremely useful to confirm or reject the suggestive present depression noticed in Figure 3 near 0.1 $R_\odot$, which may be due to an incorrect treatment of the energy generation or energy transport, resulting perhaps from the effect of gravity waves or details in the structure equations or some nuclear reaction rates or the presence of a small convective core (see review of Turck-Chièze et al. 1993). This sound speed behavior does not appear a priori in contradiction to the present neutrino results, which could perhaps suggest a smaller reduction of the temperature near the center (where ^{8}B neutrinos are produced) than near 0.1 $R_\odot$ (where ^{7}Be and CNO neutrino sources are produced).

We have shown that large improvements in opacity calculations or even in composition have small impact on the internal sound speed. The reason is that any modification of the luminosity is immediately suppressed by the modification of the initial composition. So only fundamental modification of opacity, such as the introduction of the transition phase in iron, for example, could perhaps influence the internal sound speed (Ruff, Marx, & Dearborn 1988). Since we also know that the central equation of state is reasonably well known, we conclude that the only inputs which may influence the very internal sound speed behavior, in the classical framework, could be the reaction rates, so we will concentrate on this point.

The first detections at typical astrophysical energy have emphasized the importance of screening and the difficulty of understanding this effect in the laboratory, so the present status of screening for reaction rates is not completely reliable even for low-Z nuclei, and an uncertainty of several percent is certainly possible. Partly for this reason and partly because the measurement of such reactions is extremely difficult or even impossible (*pp* reaction), we will explore the possible consequences of a modification of the fundamental reactions involved in the determination of the neutrino fluxes.

If one suspects the ^{7}Be(p, γ) reaction rate, then ^{8}B neutrinos are affected, so we need to determine it precisely for chlorine and water detectors; the effect on gallium is small (see Table 7). For the other reactions, one must distinguish the primary reaction, for which any modification produces a change of the production of energy and consequently of the deep internal structure, from the others which affect only the differential energy between the different chains. In this last case, only an important modification would reduce the neutrino predictions. In Table 12 we consider modifications of the (^{3}He, ^{4}He), (^{3}He, ^{3}He), and (p, p) reaction rates. We have introduced an important modification of these rates: +25% for (p, p), −60% of S_{34} for (^{3}He, ^{4}He), and an increase by a factor of 4 of S_{33} in the hypothesis of a resonance at very low energy, in order to clearly examine the consequences for neutrino fluxes and the sound speed. The ^{7}Be abundance varies as $S_{34}/S_{33}^{1/2}$, and the presence of a resonance at low energy in (^{3}He, ^{3}He) would reduce ^{7}Be neutrinos a little more than ^{8}B neutrinos (Castellani, Degl'Innocenti, & Fiorentini 1992). In the different cases, one may produce a substantial reduction of the neutrino fluxes (Table 12) and clearly follow in Figure 7 the effect on the sound speed profile. For the two reactions (^{3}He, ^{4}He) and (^{3}He, ^{3}He) the sound speed is only slightly changed, following the variation of the mean molecular weight, the temperature being practically unchanged; in the case of (p, p), the sound speed follows the evolution of the temperature. In this case, the

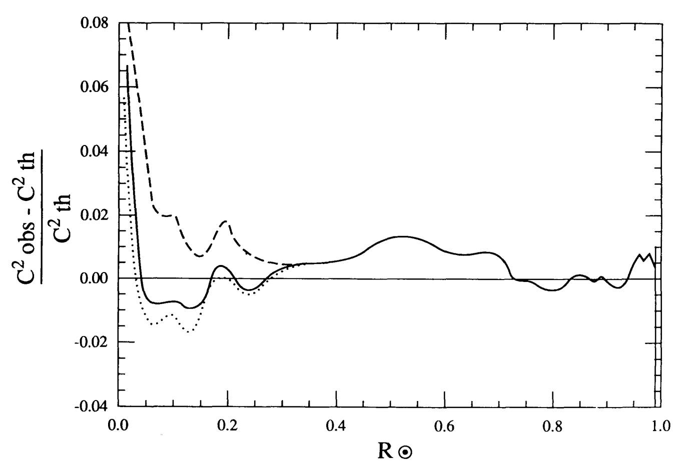

FIG. 7.—Central variation of the square of the sound speed between observation and hypothetical models where some reaction rates have been modified. The continuous line corresponds to the first model of Table 12, the dotted line to modification of (^{3}He, ^{3}He) or (^{3}He, ^{4}He) reaction rates, and the dashed line to a modification by 25% of the *pp* reaction rate.

TABLE 12

EVOLUTION OF NEUTRINO PREDICTIONS WITH HYPOTHETICAL NUCLEAR REACTION RATES

Predicted Variables	Recent CNO	Low (^{3}He, ^{4}He)	High (^{3}He, ^{3}He)	High (p, p)
T_C (10^6 K)	15.43	15.47	15.39	15.00
ρ_C (g cm^{-3})	146.89	148.22	146.85	135.5
$Y_{\rm initial}$	0.2714	0.2750	0.2705	0.2759
$r_{\rm BCZ}$ ($R_\odot$)	0.725	0.724	0.724	0.724
$T_{\rm BCZ}$ (K)	2.11×10^6	2.12×10^6	2.12×10^6	2.12×10^6
$\Phi_{pp}/10^{10}$	6.04	6.25	6.21	6.15
$\Phi_{pep}/10^8$	1.39	1.45	1.45	1.00
$\Phi_{^7{\rm Be}}/10^9$	4.25	1.89	2.23	3.34
$\Phi_{^8{\rm B}}/10^6$	4.14	1.97	2.19	2.23
$\Phi_{^{13}{\rm N}}/10^8$	3.65	3.61	3.72	1.93
$\Phi_{^{15}{\rm O}}/10^8$	2.97	2.99	3.01	1.32
Chlorine detector (SNU)	6.07	3.12	3.44	3.5
Water detector (10^6 cm^{-2} s^{-1})	4.14	1.96	2.19	2.23
Gallium detector (SNU)	121	101	104	107

NOTE.—60% for the (^{3}He, ^{4}He) reaction rate, 4 for the (^{3}He, ^{3}He) reaction rate, +25% on *pp* reaction rate compared with the last result of Table 5B: IS, Carlson et al. 1991 *pp*, Livermore opacities. All the fluxes Φ are expressed in cm^{-2} s^{-1}.

central temperature is reduced by 3% and the behavior of the sound speed in the nuclear region is completely reversed, with a modification of 1.5%.

Of course, such modification of one of these reaction rates is certainly unrealistic, and the corresponding errors proposed in Table 6 suggest a smaller surprise, but it is not totally excluded to discover up to a 10% modification in different reaction rates (including the screening effect) if one looks at the experimental dispersion or at the difficulty of the theoretical predictions. As was mentioned before the (^{7}Be, p) interaction, some predictions may be even 20% smaller than the currently used value. So it would be extremely useful to try to improve the general accuracy of the main reaction rates: (p, p), (^{3}He, ^{3}He), (^{3}He, ^{4}He), (^{7}Be, p). It is interesting to recall that the need accuracy depends on the reaction rates and is less crucial for (D, p), (^{3}He, ^{3}He), and (^{3}He, ^{4}He) than for the two others. Moreover, due to the dependence on the temperature, a modification of the (p, p) reaction rate reduces more ^{8}B neutrinos than ^{7}Be neutrinos. This would not be the case if a small perturbation (as a small convective core) was increasing the central temperature and slightly reducing the temperature corresponding to the region around 0.1 $R_\odot$; such an effect, encouraged both by neutrino results and helioseismology, seems difficult to find in the ingredients of the classical solar model.

As a summary, we have emphasized the importance of the microscopic physics for the quantitative prediction of what we call "secondary" neutrinos (measured in chlorine and water detectors) and for the determination of the acoustic mode frequencies. On the contrary, the primordial *pp* neutrinos are totally determined by the luminosity of the Sun; consequently, the sensitivity of the gallium experiment to the microscopic physics is reduced (see Table 3), and only 30% variation may be supported by the different hypotheses discussed in this paper, leading to a prediction always greater than 100 SNU.

In the classical framework of stellar evolution, one may consider two categories of ingredients: (1) those which influence the structure of the Sun and consequently both neutrinos and seismology: element composition, *pp* reaction, opacity calculations, screening effect on the equation of state and on opacities; (2) those which have no effect on the structure of the Sun and mainly influence energetic neutrinos or CNO neutrinos, without any influence on the acoustic modes: ^{7}Be(p, γ)^{8}B, cross sections of the CNO cycle, intermediate screening effect on reactions involving high-Z nuclei.

Including all the improvements noticed in nuclear and atomic physics, we discuss new predictions for present neutrino detectors and for the measured acoustic frequencies summarized in Tables 5–11. The agreement with pressure modes is really improved but remains at 3 σ or 4 σ from the observations, with a small amount of room for dynamical effects or evolution of some ingredients. The neutrino predictions are not sensibly modified in comparison with previous estimates, in spite of many variations which often mutually cancel. The discrepancy with neutrino observations is always present, and the origin of the discrepancy is not yet understood.

Considering the recent improvements and the present experimental situation, it seems to us that a classical explanation of the neutrino puzzle now excludes problems in opacity calculations, equation of state, or composition and could be found in the nuclear reaction rates including screening correction. But, even in the case of a low-energy resonance in (^{3}He, ^{3}He), as is suggested by the present neutrino experiments, such a solution would not encourage an important greater reduction of ^{7}Be and CNO neutrinos than of ^{8}B neutrinos. If future experiments would confirm the present internal sound speed behavior and a larger reduction of chlorine (also perhaps suggested by the gallium experiment) than in a water detector, an explanation should be sought in a different energy production or energy transport, suggesting perhaps physics not yet included in the classical framework of stellar evolution.

Progress on the central region requires a reexamination of the reaction rates, better accuracy in the neutrino detections, an increase of the detected number of the most penetrating modes, and a very precise description of the real center in the theoretical prediction. For the interpretation of the acoustic modes, one requires also a better determination of the external part.

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